

Gedankenexperiment for refining the Unruth metric tensor uncertainty principle via Schwartzshield Geometry and Planckian Space-time . To answer a question the author had with Mukhanov in the Marcel Grossman 14 meeting about casual barriers.

Andrew Walcott Beckwith

Physics Department, Chongqing University, College of Physics, Chongqing
University Huxi Campus, No. 44 Daxuechen Nanlu, Shapinba District, Chongqing
401331, People's Republic of China

Rwill9955b@gmail.com; abeckwith@uh.edu

Abstract. First of all, we restate a proof of a highly localized special case of a metric tensor uncertainty principle first written up by Unruth. Unruth did not use the Roberson-Walker geometry which we do, and it so happens that the dominant metric tensor we will be examining, is variation in the δg_{tt} . The metric tensor variations given by δg_{rr} , $\delta g_{\theta\theta}$, and $\delta g_{\phi\phi}$ are almost nonexistent, as compared to the δg_{tt} contribution. Afterwards, what is referred to by Barbour as the increment of emergent duration of time δt is extracted from the HUP applied to δg_{tt} in such a way as to give, in the Planckian space-time regime a nonzero minimum non zero lower ground to a massive graviton, $m_{graviton}$. The lower bound to the massive graviton, as configured in addition is influenced by δg_{tt} and kinetic energy which is rendered in the Planckian emergent duration of time δt as $(E - V)$. Finally, we find that although the $\Delta t \cdot \Delta E$ value of the HUP can be reconstructed from δg_{tt} version of the HUP, that the quantum value of the $\Delta t \cdot \Delta E$ HUP is likely not ever recoverable due to $\delta g_{tt} \neq O(1) \sim g_{tt} \equiv 1$. i.e. $\delta g_{tt} \neq O(1)$ is consistent with non curved space, so $\Delta t \cdot \Delta E \geq \hbar$ no longer strictly holds. This even if $T_{ii} = \text{diag}(\rho, -p, -p, -p)$ fluid approximation is used. Our treatment of the mass of the inflaton turns the supposition of inflation on its head, via a cite of W J Handley et al, in that we are considering the lower mass limits of the graviton as due to when the presumed inflaton is many times larger than a presumed Potential energy, with in addition, the KE initially proportional to $\rho_w \propto a^{-3(1-w)} \sim g^* T^4$, with g^* initial degrees of freedom, and T the initial temperature just before the onset of inflation.

i. Introduction

The first matter of business will be to introduce a framework of the speed of gravitons in “heavy gravity”, and this is important since eventually, as illustrated by Will [1,2] it could possibly be observed. Secondly, it also will involve an upper bound to the rest mass of a graviton. The second aspect of the inquiry of our manuscript will be to come up with a variant of the HUP, involving a metric tensor, as well as the Stress energy tensor, which will in time allow us to establish a lower bound to the mass of a graviton, preferably at the start of cosmological evolution. The article concludes in its last section as to why a statement by Mukhanov in Marcel Grossman 14, 2015, Rome, that a multiverse contribution to a new universe would have a causal barrier averaging of time contributions even if there were contributions from a multiverse, so there was only one space-time contribution is possibly indefensible.

We reference what was done by Will in his living reviews of relativity article as to the ‘Confrontation between GR and experiment’. Specifically we make use of his experimentally based formula of [1,2] , with $v_{graviton}$ the speed of a graviton, and $m_{graviton}$ the rest mass of a graviton, and $E_{graviton}$ in the inertial rest frame given as:

$$\left(\frac{v_{graviton}}{c} \right)^2 = 1 - \frac{m_{graviton}^2 c^4}{E_{graviton}^2} \quad (1)$$

Furthermore, using [2], if the rest mass of a graviton is very small we can make a clear statement of

$$\begin{aligned} \frac{v_{graviton}}{c} &= 1 - 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D} \right) \cdot \left(\frac{\Delta t}{1sec} \right) \\ &\doteq 1 - 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D} \right) \cdot \left(\frac{\Delta t = \Delta t_a - (1+z) \cdot \Delta t_b}{1sec} \right) \\ &\Leftrightarrow \frac{2m_{graviton} c^2}{E_{graviton}} \approx 5 \times 10^{-17} \cdot \left(\frac{200Mpc}{D} \right) \cdot \left(\frac{\Delta t_a - (1+z) \cdot \Delta t_b}{1sec} \right) \end{aligned} \quad (2)$$

Here, Δt_a is the difference in arrival time, and Δt_e is the difference in emission time/in the case of the early Universe, i.e. near the big bang, then if in the beginning of time, one has, if we assume that there is an average $E_{graviton} \approx \hbar \cdot \omega_{graviton}$, and

$$\begin{aligned} \Delta t_a &\sim 4.3 \times 10^{17} \text{ sec} \\ \Delta t_e &\sim 10^{-33} \text{ sec} \\ z &\sim 10^{50} \end{aligned} \quad (3)$$

Then, $\left(\frac{\Delta t_a - (1+z) \cdot \Delta t_b}{1sec} \right) \sim 1$, and if $D \sim 4.6 \times 10^{26} \text{ meters} = \text{radii(universe)}$, so one can set

$$\left(\frac{200Mpc}{D} \right) \sim 10^{-2} \quad (4)$$

And if one sets the mass of a graviton [3] into Eq. (1) , then we have in the present era, that if we look at primordial time generated gravitons, that if one uses the

$$\begin{aligned} \Delta t_a &\sim 4.3 \times 10^{17} \text{ sec} \\ \Delta t_e &\sim 10^{-33} \text{ sec} \\ z &\sim 10^{55} \end{aligned} \quad (5)$$

Note that the above frequency, for the graviton is for the present era, but that it starts assuming genesis from an initial inflationary starting point which is not a space – time singularity.

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.

We will next discuss the implications of this point in the next section, of a non zero smallest scale factor . Secondly the fact we are working with a massive graviton , as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of modification of the values[3]

$$\left\langle (\delta g_{uv})^2 (\hat{T}_{uv})^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2}$$

$$\xrightarrow{uv \rightarrow tt} \left\langle (\delta g_{tt})^2 (\hat{T}_{tt})^2 \right\rangle \geq \frac{\hbar^2}{V_{Volume}^2} \quad (6)$$

$$\& \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$$

The reasons for saying this set of values for the variation of the non g_{tt} metric will be in the 3rd section and it is due to the smallness of the square of the scale factor in the vicinity of Planck time interval.

2.Non zero scale factor, initially and what this is telling us physically. Starting with a configuration from Unruth.

Begin with the starting point of[4,5]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \quad (7)$$

We will be using the approximation given by Unruth [4,5], of a generalization we will write as

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \quad (8)$$

$$(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A$$

If we use the following, from the Roberson-Walker metric[6].

$$g_{tt} = 1$$

$$g_{rr} = \frac{-a^2(t)}{1 - k \cdot r^2} \quad (9)$$

$$g_{\theta\theta} = -a^2(t) \cdot r^2$$

$$g_{\phi\phi} = -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2$$

Following Unruth [4,5] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \quad (10)$$

Then, the surviving version of Eq. (7) and Eq. (8) is, then, if $\Delta T_{tt} \sim \Delta \rho$

$$\begin{aligned} V^{(4)} &= \delta t \cdot \Delta A \cdot r \\ \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\ \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}} \end{aligned} \quad (11)$$

This Eq. (11) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time[6]

$$T_{ii} = \text{diag}(\rho, -p, -p, -p) \quad (12)$$

Then

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}} \quad (13)$$

Then, Eq.(11) and Eq. (12) and Eq. (13) together yield

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (14)$$

$$\text{Unless } \delta g_{tt} \sim O(1)$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \quad (15)$$

In fact, we have that from Giovannini [6], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \quad (16)$$

Then, there is no way that Eq. (14) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$. Hence, the Mukhanov suggestion as will be discussed toward the end of this article, is not feasible. Finally, we will discuss a lower bound to the mass of the graviton.

3 . How we can justifying writing very small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ values.

To begin this process, we will break it down into the following co ordinates

In the rr , $\theta\theta$, and $\phi\phi$ co ordinates, we will use the Fluid approximation, $T_{ii} = \text{diag}(\rho, -p, -p, -p)$ [7] with

$$\begin{aligned}\delta g_{rr} T_{rr} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0 \\ \delta g_{\theta\theta} T_{\theta\theta} &\geq - \left| \frac{\hbar \cdot a^2(t)}{V^{(4)}(1 - k \cdot r^2)} \right| \xrightarrow{a \rightarrow 0} 0 \\ \delta g_{\phi\phi} T_{\phi\phi} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot \sin^2 \theta \cdot d\phi^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0\end{aligned}\tag{17}$$

If as an example, we have negative pressure, with T_{rr} , $T_{\theta\theta}$, and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, **since there is no way that $p = -\rho$ is zero valued.**

Having said this, the value of δg_{tt} being non zero, will be part of how we will be looking at a lower bound to the graviton mass which is not zero.

4. Lower bound to the graviton mass using Barbour's emergent time

In order to start this approximation, we will be using Barbour's value of emergent time [8,9] restricted to the Plank spatial interval and massive gravitons, with a massive graviton [10]

$$(\delta t)_{\text{emergent}}^2 = \frac{\sum_i m_i l_i \cdot l_i}{2 \cdot (E - V)} \rightarrow \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)}\tag{18}$$

Initially, as postulated by Babour [8,9], this set of masses, given in the emergent time structure could be for say the planetary masses of each contribution of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton.

If $(\delta t)_{\text{emergent}}^2 = \delta t^2$ in Eq.(11), using Eq.(11) and Eq. (18) we can arrive at the identification of

$$m_{\text{graviton}} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E - V)}{\Delta T_{tt}^2}\tag{19}$$

Key to Eq. (19) will be identification of the kinetic energy which is written as $E - V$. This identification will be the key point raised in this manuscript. Note that [11] raises the distinct possibility of an initial state, just before the 'big bang' of a kinetic energy dominated 'pre inflationary' universe. I.e. in terms of an inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [7]. The key finding which is in [11] is, that, if the kinetic energy is dominated by the 'inflaton' that

$$K.E. \sim (E - V) \sim \dot{\phi}^2 \propto a^{-6} \quad (20)$$

This is done with the proviso that $w < -1$, in effect, what we are saying is that during the period of the 'Planckian regime' we can seriously consider an initial density proportional to Kinetic energy, and call this K.E. as proportional to [7]

$$\rho_w \propto a^{-3(1-w)} \quad (21)$$

If we are where we are in a very small Planckian regime of space-time, we could, then say write Eq.

(21) as proportional to $g^* T^4$ [7], with g^* initial degrees of freedom, and T the initial temperature as low

Just before the onset of inflation. The question to ask, then is, what is the value of the initial degrees of freedom, and what is the temperature, T, at the start of expansion? For what it is worth, the starting supposition, is that there would then be a likelihood for an initial low temperature regime

5. Multiverse, and answering the Mukhanov hypothesis. Influence of the Einstein spaces

Here, the initial $a_0 \sim a_{initial} \sim 10^{-55}$, or so and so the density in Eq. (21) at Planck time would, be proportional to the Planck Frequency[7]

$$\omega_p = \frac{1}{t_p} = \sqrt{\frac{c^5}{\hbar G}} = 1.85487 \times 10^{43} s^{-1} \sim 1.85 \times 10^{43} Hz \quad (22)$$

This is at the instant of Planck time. We can then ask what would be an initial time contribution before the onset of Planck time. i.e. does Eq. (22) represent the initial value of graviton frequency?

This value of the frequency of a graviton, which would be red shifted enormously would be in tandem with an initial time step of as given by[12]

$$t_{initial} \approx \frac{1}{\sqrt{6\pi\rho_{initial}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \quad (23)$$

This value for the initial time step would be probably lead to Pre Planckian time, i.e. smaller than 10^{-43} seconds, which then leads us to consider, what would happen if a multi verse contributed to initial

space-time conditions as seen in Eq. (11) above. If the cosmic fluid approximation as given by Eq. (12) were legitimate, and one could also look at Eq. (13), then

$$m_{graviton} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \Rightarrow (\delta g_{tt})^2 \geq \frac{2\hbar^2}{m_{graviton} \cdot l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2} \quad (24)$$

But, then if one is looking at a multiverse, we first will start at the Penrose hypothesis for a cyclic conformal universe, starting with[13]

$$\hat{g}^{uv} = \Omega_{uv} g^{uv}$$

$$\Omega_{uv} (new-universe) = (\Omega_{uv}^{-1} old-universe) \quad (25)$$

i.e.

$$\Omega_{uv} \rightarrow \Omega_{uv}^{-1} (inversion)$$

However, in the multiverse contribution to Eq.(12) above, we would have, that

$$\Omega_{uv}^{-1} old-universe \rightarrow \frac{1}{N} \sum_{j=1}^N [\Omega_{uv}^{-1} (inversion)]_j \quad (26)$$

So, does something like this hold ? In a general sense ?

$$\begin{aligned} & (\delta g_{uv})^2 \Big|_{initial} \hat{g}^{uv} = \Omega_{uv} g^{uv} \\ & \rightarrow \frac{1}{N} \sum_{j=1}^N [\Omega_{uv}^{-1} (inversion)]_j g^{uv} \\ & \sim ?? \sim 1 / [\beta M_{Planck}^2] + \varepsilon^+ \end{aligned} \quad (27)$$

If the fluid approximation as given in Eq. (12) and Eq. (13) hold, then Eq. (27) conceivably could be identifiable as linkable to.

$$\begin{aligned} \delta t \Delta E & \geq \frac{\hbar}{(\delta g_{tt})} \equiv \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_{tt}^{-1}]_j \right) \cdot \delta g_{tt} \right)} \\ \Leftrightarrow \delta t & \geq \frac{1}{\Delta E} \cdot \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_{tt}^{-1}]_j \right) \cdot \delta g_{tt} \right)} \approx \frac{m_{graviton} l_P \cdot l_P}{2 \cdot (E-V)} \end{aligned} \quad (28)$$

If we could write, say

$$\frac{1}{\Delta E} \cdot \frac{N \cdot \hbar}{\left(\left(\sum_j [\Omega_u^{-1}]_j \right) \cdot \delta g_u \right)} = \frac{N \cdot \hbar / \Delta E}{\left(\sum_j [\Omega_u^{-1} \cdot (\delta g_u)]_j \right)} \sim \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)}$$

$$\Leftrightarrow \Delta E \sim 2 \cdot (E - V)$$

(29)

&

$$m_{\text{graviton}} l_P \cdot l_P \sim \frac{N \cdot \hbar}{\left(\sum_j [\Omega_u^{-1} \cdot (\delta g_u)]_j \right)}$$

Then, if each j is the jth contribution of N “multiverse” contributions to a new single universe being nucleated, one could say that there was, indeed, likely an “averaging” and that the causal barrier which Mukhanov spoke of, as to each δt , and actually to each graviton entering into the present universe, one could mathematically average out the results of a sum up of each of the contributions from each prior to a present universe, according to

$$\frac{N \cdot \hbar}{\left(\sum_j [\Omega_u^{-1} \cdot (\delta g_u)]_j \right)} \equiv \frac{\hbar}{\frac{1}{N} \cdot \sum_j [\Omega_u^{-1} \cdot (\delta g_u)]_j}$$

(30)

If Eq. (30) held, then we could then write

$$\delta t \geq \frac{1}{\Delta E} \cdot \frac{\hbar}{\left(\frac{1}{N} \sum_j [\Omega_u^{-1} \cdot \delta g_u]_j \right)} \approx \frac{m_{\text{graviton}} l_P \cdot l_P}{2 \cdot (E - V)}$$

(31)

Instead, we have, Eq. (28), and that it is safe to say that for each collapsing universe which might contribute to a re cycled universe that the following inequality is significant.

$$\frac{1}{N} \cdot \sum_j [\Omega_u^{-1} \cdot (\delta g_u)]_j \neq \frac{1}{N} \cdot \sum_j [\Omega_u^{-1}]_j \cdot \delta g_u$$

(32)

Hence, the absence of an averaging procedure, due to a multiverse, would then rule against a causal barrier, as was maintained by Mukhanov, in his discussion with the author, in Marcel Grossman 14, in Italy. Then the possible approximation say of

$$(T_u)^2 \sim \omega_{\text{graviton}}^2 \propto \beta M_{\text{Planck}}^2 \sim \left(t_{\text{initial}} \approx \frac{1}{\sqrt{6\pi\rho_{\text{initial}}}} \sim \frac{a_0^2}{\sqrt{6\pi}} \right)^{-2}$$

(33)

Would not hold, and that in itself may lead to a break down of the Causal barrier hypothesis of Mukhanov, which the author emphatically disagreed with.

6. Conclusion. Considering Eq. (6) and Eq. (11) in lieu of Einstein space, and further research questions

A way of solidifying the approach given here, in terms of early universe GR theory is to refer to Einstein spaces, via [14] as well as to make certain of the Stress energy tensor [15] as we can write it as a modified Einstein field equation. With, then \aleph as a constant.

$$R_{ij} = \aleph g_{ij} \quad (34)$$

Here, the term in the Left hand side of the metric tensor is a constant, so then if we write, with R also a constant [15]

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ij}} = -\frac{1}{8\pi} \cdot [\aleph - R + \Lambda] \cdot g_{ij} \quad (35)$$

The terms, if we use the fluid approximation given by Eq. (12) as well as the metric given in Eq. (9) will then tend to a constant energy term on the RHS of Eq. (35) as well as restricting i, and j, to t and t

So as to recover ,via the Einstein spaces, the seemingly heuristic argument given above. Furthermore when we refer to the Kinetic energy space as an inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [7], we can also then utilize the following operator equation for the generation of an ‘inflaton field’ given by the following set of equations

$$\begin{aligned} \phi(t, \cdot) &= \cos(t\sqrt{K})f + \frac{\sin(t\sqrt{K})}{\sqrt{K}}g \\ f(x) &= \phi(0, x) \\ g(x) &= \frac{\partial \phi(0, x)}{\partial t} \\ -\frac{\partial^2 \phi}{\partial t^2} &= K\phi \end{aligned} \quad (36)$$

In the case of the general elliptic operator K if we are using the Fulling reference, [16] in the case of the above Robertson-Walker metric, with the results that the elliptic operator, in this case become,

$$\begin{aligned} K &= -\nabla^2 + (m^2 + \xi R) \\ &= -\sum_{i,j} \frac{\partial_i \left(g^{ij} \sqrt{|\det g|} \partial_j \right)}{\sqrt{|\det g|}} + (m^2 + \xi R) \\ &\xrightarrow{i,j \rightarrow t,t} -\frac{\partial^2}{\partial t^2} + (m^2 + \xi R) \end{aligned} \quad (37)$$

Then, according to [16] , if R above, in Eq. (37) is initially a constant, we will see then, if m is the inflation mass, that

$$\begin{aligned}
\phi(t, \cdot) &= \cos(t\sqrt{K})f \\
-\frac{\partial^2}{\partial t^2} &\rightarrow \omega^2 \\
\Leftrightarrow \phi(t, \cdot) &= \cos(t\sqrt{\omega^2 + (m^2 + \xi R)})
\end{aligned} \tag{38}$$

Then c_1 as an unspecified, for now constant will lead to a first approximation of a Kinetic energy dominated initial configuration, with details to be gleaned from [16,17,18] to give more details to the following equation, R here is linked to curvature of spacetime, and m is an inflaton mass, connected with the field $\phi(t, \cdot) = \cos(t\sqrt{K})f$ with the result that

$$\dot{\phi}^2(t, \cdot) \approx [\omega^2 + (m^2 + \xi R)] \cdot c_1 \gg V(\phi) \tag{39}$$

If the frequency, of say, Gravitons is of the order of Planck frequency as in Eq.(22) , then this term, would likely dominate Eq.(39). More of the details of this will be worked out, and also candidates for the $V(\phi)$ will be ascertained, most likely, we will be looking the Rindler Vacuum as specified in [19] as well as also details of what is relevant to maintain local covariance in the initial space-time fields as given in [20]

Why is a refinement of Eq. (39) necessary?

The details of the elliptic operator K will be gleaned from [16,17,18] whereas the details of inflaton $\dot{\phi}^2 \gg (P.E \sim V)$ [7] are important to get a refinement on the lower mass of the graviton as given by the left hand side of Eq. (24) . We hope to do this in the coming year. The mass, m , in Eq.(37) for the inflaton, not the Graviton, so as to have links to the beginning of the expansion of the universe. We look to what Corda did, in [21] for guidance as to picking values of m relevant to early universe conditions.

Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

References

- [1] C. Will, “ Was Einstein Right? A Centenary Assessment”, pp49-96, of ‘General Relativity and Gravitation, A Centennial Perspective’ Edited by A. Ashtekar, B. Berger, J. Isenberg, M. MacCallum, Cambridge University Press, Cambridge, UK, 2015
- [2] C. Will, “The Confrontation between General Relativity and Experiment”, <http://relativity.livingreviews.org/Articles/lrr-2014-4/download/lrr-2014-4Color.pdf>
- [3] T. G. Downes, G. J. Milburn, “Optimal Quantum Estimation for Gravitation “, gr-qc arXiv:1108.5220
- [4] W. G. Unruh; “Why study quantum theory?”, **Canadian Journal of Physics**, 1986, Vol. 64, No. 2 : pp. 128-130; (doi: 10.1139/p86-019)
- [5] W. G. Unruh; “Erratum: Why study quantum gravity?”, *Can. J. Phys.* 64, 128 (1986)

- [6] M. Giovannini, ***A Primer on the Physics of the Cosmic Microwave Background*** World Press Scientific, Hackensack, New Jersey, USA, 2008
- [7] E. W. Kolb, M. S. Turner, ***The Early Universe***, Addison-Wesley Publishing company, The Advanced Book Program, Redwood city, California, USA, 1990
- [8] J. Barbour, "The Nature of Time", <http://arxiv.org/pdf/0903.3489.pdf>
- [9] J. Barbour, "Shape Dynamics: An Introduction", pp. 257-297, in ***Quantum Field Theory and Gravity, Conceptual and Mathematical Advances in the Search for a Unified Framework*** edited by F. Finster, O. Muller, M. Nardmann, J. Tolksdorf, E. Zeidler, by Birkhauser, Springer- Verlag, London, UK, 2010
- [10] A. Goldhaber, M. Nieto, "Photon and Graviton Mass Limits",
Rev.Mod.Phys.82:939-979,2010, <http://arxiv.org/abs/0809.1003>
- [11] W J Handley, S D Brechet, A N Lasenby, and M P Hobson," Kinetic initial conditions for inflation",
<http://arxiv.org/pdf/1401.2253v2.pdf>
- [12] J. D. Walecka, ***Introduction to Modern Physics, Theoretical Foundations***, World press scientific Co, Pte. Ltd. 5 Tok Link, Singapore, Republic of Singapore, 596224
- [13] R. Penrose, ***Cycles of Time: An Extraordinary New View of the Universe*** ,The Bodley Head,London, UK, 2010
- [14] A.Z. Petrov, ***Einstein Spaces***, Pergamon Press, Oxford London, UK, 1969
- [15] . D. Gorbunov, V. Rubakov, ***Introduction to the Theory of the Early Universe, Cosmological Perturbations and Inflationary Theory***, World Scientific Publishing Pte. Ltd, Singapore, Republic of Singapore, 2011
- [16] S. A. Fulling ***Aspects of Quantum Field Theory in Curved Space- Time*** , London Mathematical Society Student texts 17, Cambridge University Press, Cambridge, UK, 1991
- [17] H. Gutfreund, J. Renn, ***The Road To Relativity, The history and Meaning of Einstein's " The Foundation of General Relativity", (featuring the Original Manuscript of Einstein's Masterpiece)*** , Princeton University Press, Princeton and Oxford, Princeton, New Jersey, USA, 2015
- [18] J. Griffiths , J. Podolsky, ***Exact Space-Times in Einstein's General Relativity***, Cambridge Monographs in Mathematical Physics, Cambridge, UK, 2009
- [19] R. M. Wald. ***Quantum Field theory in Curved Spacetime and Black Hole Thermodynamics*** , University of Chicago Press, Chicago, Illinois, USA, 1994.
- [20] K. Fredenhagen, and K. Rejzner, Local Covariance and Background Independence, pp 15-23, in ***Quantum Field Theory and Gravity, Conceptual and Mathematical Advances in the Search for a Unified Framework*** edited by F. Finster, O. Muller, M. Nardmann, J. Tolksdorf, E. Zeidler, by Birkhauser, Springer- Verlag, London, UK, 2010
- [21] C. Corda, "Primordial gravity's breath", <http://arxiv.org/abs/1110.1772>