The Nucleon-Nucleon Effective Range Parameters and Scattering Lengths at Low Energies

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Abstract: We know that the Quantum Chromodynamics (QCD) fails at low energies. For example, within QCD, we cannot calculate precise mass and spin of nucleons. It does not concern the Scale-Symmetric Theory (SST). Moreover, experimental data show that at low energies the effective range parameters for nn, np, and pp scatterings are similar, but different. Why? Here, applying SST, we calculated the effective range parameters and scattering lengths for nn (respectively 2.732 fm and -18.806 fm), for np (2.778 fm and -23.867 fm), and pp (2.824 fm and -16.995 fm). These theoretical results are consistent with experimental data. At low energies, the effective range parameters follow from exchanges of virtual pions (which are responsible for the nuclear strong interactions) and production of the virtual condensates (which are responsible for the nuclear weak interactions at low energies). The condensates insignificantly increase the effective ranges. Just at low energies, the effective range parameters for nucleons follow from their strong-weak interactions described within SST. On the other hand, the scattering lengths we interpret as the circumferences of the produced gluon/photon loops with maximum radius. The obtained here and within SST results show that QCD is the incomplete and partially incorrect theory - the same concerns the electroweak theory at low energies.

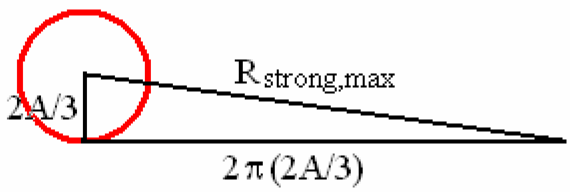
1. Introduction

The extended General Relativity leads to the inflation field composed of the non-gravitating tachyons (the modified Higgs field) [1]. The succeeding phase transitions of such field lead to different scales described within the Scale-Symmetric Theory (SST) [1]. Within the theory of one of such scales, we proved that baryons have the atom-like structure [1].

In baryons, there is the core composed of the torus/charge responsible for the nuclear strong interactions (it produces the pions) and of the ball/condensate in its centre responsible for the nuclear weak interactions. Both the charge and condensate consist of the entangled or confined Einstein-spacetime components i.e. of the neutrino-antineutrino pairs. Equatorial radius of the torus/charge is $A = 0.6974425$ fm. Radius of the condensate is $r_{p(proton)} = 0.8710945 \cdot 10^{-17}$ m $= 0.008710945$ fm ([1]: see formula (49)). Outside the core are the four S states of pions. Radius of the last shell is $R = A + 4B$, where $B = 0.5018395$ fm ([1]: see explanation below formula (31)). The four S states are placed on the plane perpendicular to the half-integral spin of the core. Calculated here the effective range parameters concern such plane so the spins of nucleons must be aligned.

On the surface of the torus/charge are produced virtual bosons and range of the remainder with a mass of $M_{Remainder}$ = 187.537 MeV is 4B = 2.007358 fm ([1]: see the explanation between formulae (31) and (33)). Knowing this range, we can calculate ranges of virtual pions produced by the core of baryons. Initially, centers of the emitted virtual pions overlap with centre of the core. There appear the virtual spheres which radii are defined by the ranges of the virtual pions.

The virtual condensates appear as well on surfaces of the virtual spheres. Condensates that surfaces are tangent to surfaces of the virtual spheres produced by pions increase the range of the strong-weak interactions of nucleons. The condensates produce the standing waves in such a way that the distance between the standing-wave nodes is equal to the size of the condensates i.e. $\lambda_p = 4\ r_{p(proton)}$. Such method we applied with very good effect in following papers [4], [5]. The increased range of the strong-weak interactions, $R_{range,strong-wak}$, is $R_{range,strong-weak} = R_{range,strong} + \lambda_p$.



Maximum range of nuclear strong interactions:

$R_{strong,max}$ = 2.9582094 fm

Notice as well that the virtual pions in the strong interactions behave analogically as the photons in the electromagnetic interactions i.e. the virtual pions are emitted when the core of nucleons is charged i.e. are emitted by the charged.core-charged.pion state of the neutrons and by the charged.core-neutral.pion state of the protons ([1]: see formulae (36)-(39)), i.e. at low energies, the neutrons emit virtual charged pions whereas protons emit virtual neutral pions.

Here, our interpretation of the effective range parameter, r_o, is as follows. The effective range parameters are equal to the ranges of emitted by nucleons virtual pions insignificantly increased by produced virtual condensates.

In quantum mechanics, the effective range parameters and scattering lengths describe low-energy scattering. At low energies the s-wave phase shift, δ (i.e. the phase difference between incoming and outgoing wave), can be written as follows

$$k \cot \delta = -1 / a + r_o k^2 / 2, \qquad (1)$$

where a is the scattering length, r_o is the effective range parameter, and $k = 1/\lambda$ is the wave number (i.e. the number of wavelengths per unit distance).

When a slow nucleon (its de Broglie wavelength is very long) scatters off a short ranged scatterer (i.e. other nucleon) it cannot resolve the structure of the nucleon. The idea is that it is not important how the potential V(r) looks at short length scales but only how it looks at long length scales. It means that to lowest order we have only a spherical symmetric outgoing wave, the so-called s-wave scattering (angular momentum $l = 0$). At higher energies we must take into account p-, d-wave ($l = 1, 2$) scattering, and so on.

The solution to the radial Schrödinger equation ($l = 0$) outside of the well is

$$u(r) = A \sin(kr + \delta), \tag{2}$$

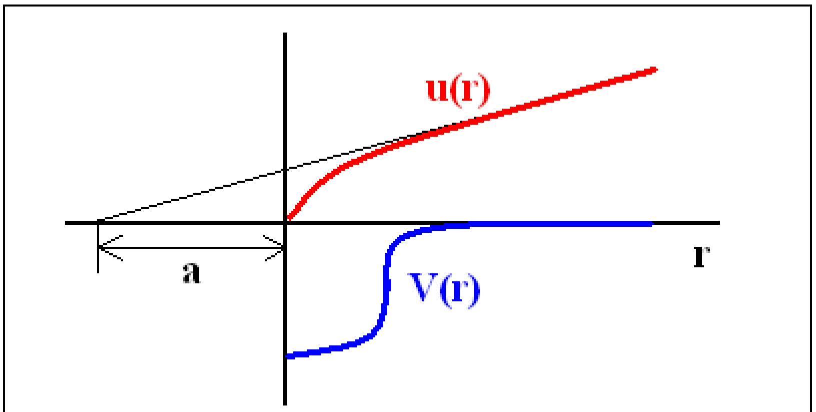
where u(r) is the wave function, $k = (2mE)^{1/2}/\hbar$, and $\delta = - k\, a$. The scattering length, a, is the radius of sphere that is equal to circumference of produced gluon/photon loop when the loop is open.

Here, our interpretation of the scattering length, a, is as follows. Due to the scattering processes, there are produced photon circles/loops which can be open. The circumference of the photon circle/loop with the maximum radius is equal to the scattering length, a. Notice as well that the sign "–" in the definition of scattering length is conventional.

To relate the scattering length to physical observables that can be measured in a scattering experiment, we need to compute the cross section, σ,

$$\sigma = 4\, \pi\, a^2. \tag{3}$$

At zero-energy the asymptotic NN wave behaves as $1 - r / a$.



a < 0 so attractive potential and scattering state

2. The effective-range-parameter calculations

Range, R_{range}, of a virtual or real particle is inversely proportional to its mass M

$$R_{range} = 1 / M. \tag{4}$$

The remarks in Introduction and formula (4) lead to following formula for the effective range parameter r_o

$$r_o = 4\ B\ M_{Remainder} / M + 4\ r_{p(proton)}\ , \qquad (5)$$

where B = 0.5018395 fm, $M_{Remainder}$ = 187.537 MeV, $r_{p(proton)} = 0.8710945 \cdot 10^{-17}$ m = 0.008710945 fm.

Table 1. *Effective range parameters r_o*

Aligned-spin NN scattering	**Exchanged pions**	**Effective range parameter (fm)** Experimental data [2]	**Effective range parameter (fm)** Scale-Symmetric Theory: formula (5)
nn	$\pi^{-,+}$	2.75 ± 0.11	2.732
np	$\pi^{-,+}$ or π^{o}	2.77 ± 0.05	2.778
pp	π^{o}	2.85 ± 0.04	2.824

Applying formula (5), for charged pion, $M_{charged\text{-}pion}$ = 139.57018(35) MeV [3], we obtain

$$r_{o,nn} = 2.73208\ \text{fm} \approx 2.732\ \text{fm}\ . \qquad (6)$$

For neutral pion, $M_{neutral\text{-}pion}$ = 134.9766(6) MeV [3], we obtain

$$r_{o,pp} = 2.82387\ \text{fm} \approx 2.824\ \text{fm}\ . \qquad (7)$$

For the np scattering is the mean value

$$r_{o,np} = (r_{o,nn} + r_{o,pp}) / 2 \approx 2.778\ \text{fm}\ . \qquad (8)$$

3. The scattering-length calculations

The maximum range for the strong-weak interactions, $R_{range,strong\text{-}weak}$, is

$$R_{range,strong\text{-}weak} = R_{range,strong} + 4\ r_{p(proton)} = 2.99305\ \text{fm}, \qquad (9a)$$

and it is the maximum radius of produced photon loop. It leads to conclusion that scattering length for nn scattering is

$$a_{nn} = -2\ \pi\ R_{range,strong\text{-}weak} \approx -18.806\ \text{fm}. \qquad (9b)$$

Contrary to the neutrons, the protons are charged so there dominate the shells. The last shell has following radius [1]

$$R_{d=4} = A + 4\ B = 2.7048005\ \text{fm}, \qquad (10a)$$

and it is the maximum radius of produced photon loop. It leads to conclusion that scattering length for pp scattering is

$$a_{pp} = -2\ \pi\ R_{d=4} \approx -16.995\ \text{fm}. \qquad (10b)$$

For np scattering, the maximum radius of photon loop should be the sum

$$R_{sum} = R_{range,strong\text{-}weak} + R_{d=4} \approx 5.69785 \text{ fm}. \quad (11)$$

But according to the Scale-Symmetric Theory, radius of photon loops produced by a nucleon cannot be greater than $A + 8\,B = 4.71216$ fm (in the $d = 4$ state, the pions decay to photons so they reach the $d = 8$ state) [1]. According to SST, the gluon/photon loops can transform into torus with mean radius equal to 2/3 of the equatorial radius of the torus which is equal to the radius of the initial loop – such tori are most stable [1]. Such tori produce the gluon/photon loops that radii are $f = 2/3$ of the initial radius of the loops. It leads to conclusion that the scattering length for np scattering is

$$a_{np} = -2\,\pi\, f\, R_{sum} \approx -23.867 \text{ fm}. \quad (12)$$

Table 2. *Scattering lengths* a_{NN}

Aligned-spin NN scattering	**Scattering lengths (fm)** Experimental data [2]	**Scattering lengths (fm); formula (…)** Scale-Symmetric Theory
nn	–18.9 ± 0.4	–18.806 (9b)
np	–23.740 ± 0.020	–23.867 (12)
pp	–17.3 ± 0.4	–16.995 (10b)

4. Summary

Contrary to the Scale-Symmetric Theory (SST), the Quantum Chromodynamics (QCD) fails at low energies. For example, within QCD, we cannot calculate precise mass and spin of nucleons and effective range parameters for aligned-spin NN scatterings at low energies.

Here, applying the Scale-Symmetric Theory, we calculated the effective range parameters and scattering lengths for nn (respectively 2.732 fm and –18.806 fm), for np (2.778 fm and –23.867 fm), and pp (2.824 fm and –16.995 fm). These theoretical results are consistent with experimental data.

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The obtained here and within SST results show that QCD is the incomplete and partially incorrect theory. The same concerns the electroweak theory at low energies. For example, the neutrinos at low energies can exchange the entanglons only which are responsible for the quantum entanglement [1]. At low energies, the neutrinos cannot emit the virtual W or Z bosons. If at low energy the neutrinos could emit the virtual W and Z bosons then matter should not be practically transparent for them.

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