Proof of Fermat's last theorem (Part I of III)

\[ a^n + b^n = c^n \quad (n > 1 \text{ and odd}) \]

Objet: Proof of Fermat's last theorem with conventional means.

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1) Introduction:
- I am not a professional of mathematics.
- My English is poor. I use google translate to write these pages.

2) I prove that \( c < (a + b) \)

\[(a + b)^n = a^n + b^n + \sum_{k=1}^{n-1} \binom{n-1}{k} a^k b^{n-k} \]

Then \((a + b)^n > (a^n + b^n)\)
Then \((a + b)^n > c^n\) because \(c^n = a^n + b^n\)
Then \((a + b) > c\)

d is a natural number. It is the complement of c to \((a + b)\).
\[ c + d = a + b \]
\[ c = a + b - d \]
\[ c - b = a - d \]
\[ c - a = b - d \]

3) I prove a parity of d
Whatever the parity of a, b and c, we can easily verify that d is always even.

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\( 2^n \text{ divide } d^n. \)
4) **Conditions (supposition) to prove Fermat's last theorem:**
- a, b, c, d and n are non-zero positive integers
- a, b and c are pairwise coprime
- n > 1 and odd
- \(a^n + b^n = c^n\)
- a + b = c + d
- a < b

5) **I prove that c is not coprime with d**
\[d^n = d^n\]
\[c^n - a^n - b^n = 0\]
\[d^n = d^n + c^n - a^n - b^n\]
\[d^n = (d^n + c^n) - (a^n + b^n)\]
\((c + d)\) divide \((d^n + c^n)\)
\((a + b)\) divide \((a^n + b^n)\)
\((c + d) = (a + b)\)
\(\boxed{(c + d) \text{ divide } d^n}\)

Any integer which divide \((c + d)\) divide \(d^n\)
Any prime number which divide \((c + d)\) divide \(d^n\)
Any prime number which divide \((c + d)\) divide \(d\)
Any prime number which divide \([(c + d) \text{ and } d]\) divide \(c\)

**c is not coprime with d**
\[(c + d) = (a + b)\]
\(\boxed{(a + b) \text{ divide } d^n}\)

Any integer which divide \((a + b)\) divide \(d^n\)

6) **I prove that a is not coprime with d**
\[d^n = d^n\]
\[c^n - a^n - b^n = 0\]
\[d^n = d^n + c^n - a^n - b^n\]
\[d^n = (c^n - b^n) - (a^n - d^n)\]
\((c - b)\) divide \((c^n - b^n)\)
\((a - d)\) divide \((a^n - d^n)\)
\((c - b) = (a - d)\)
\(\boxed{(a - d) \text{ divide } d^n}\)
\(\boxed{(c - b) \text{ divide } d^n}\)
Any integer which divide \((a - d)\) divide \(d^n\)
Any prime number divide \((a - d)\) divide \(d^n\)
Any prime number divide \((a - d)\) divide \(d\)
Any prime number divide \([a - d) and d]\) divide \(a\)

\(a\) is not coprime with \(d\) (Except if \((a - d) = 1\)).

\((c - b) = (a - d)\)
\((c - b) \) divide \(d^n\)

Any prime number which divide \((c - b)\) divide \(d\).

7) \(I\) prove that \(b\) is not coprime with \(d\)
\(d^n = d^n\)
\(c^n - a^n - b^n = 0\)
\(d^n = d^n + c^n - a^n - b^n\)
\(d^n = (c^n - a^n) - (b^n - d^n)\)
\((c - a) \) divide \((c^n - a^n)\)
\((b - d) \) divide \((b^n - d^n)\)
\((c - a) = (b - d)\)
\((b - d) \) divide \(d^n\)
\((c - a) \) divide \(d^n\)

Any integer which divide \((b - d)\) divide \(d^n\)
Any prime number divide \((b - d)\) divide \(d^n\)
Any prime number divide \((b - d)\) divide \(d\)
Any prime number divide \([b - d) and d]\) divide \(b\)

\(b\) is not coprime with \(d\)
\((c - a) = (b - d)\)
\((c - a) \) divide \(d^n\)

Any prime number which divide \((c - a)\) divide \(d\)
8) Contraductions

I proved that:

1. Any prime number which divide \((a + b)\) divide \(d\).
2. Any prime number which divide \((c + d)\) divide \(d\).
3. Any prime number which divide \((a - d)\) divide \(d\).
4. Any prime number which divide \((c - b)\) divide \(d\).
5. Any prime number which divide \((b - d)\) divide \(d\).
6. Any prime number which divide \((c - a)\) divide \(d\).

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