Expanding MOND with baryon intrinsic Dark Matter, Helmholtz work, an entropic force and a new dimension parameter

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Abstract

In this paper I present a baryon intrinsic Dark Matter halo model. The model gives a correct first order galactic rotation curve, leads to the baryonic Tully-Fisher relation and to the MOND force for the weak acceleration regime. Then I show that the MOND force can be derived from the combination of my model's potential and the first law of thermodynamics in the Helmholtz energy $A = U - TS$ formulation. In my model the MOND work is identical to the Helmholtz work. The entropy connected to the intrinsic Dark Matter halo allows the derivation of the Dark Matter force, the deviation from Newton, as an entropic force. The definition of the entropy leads to a new parameter, of dimensional degrees of freedom, added to MOND. This new parameter solves the galaxy cluster mass discrepancy problem of MOND and produces and exact relationship between the MOND acceleration and the Hubble acceleration, with cosmological implications. In my model the cosmic structure formation degree of freedom value $N = \sqrt{cH_0/a_0} = 2.1$, is also the minimum mass discrepancy in the MOND cluster analysis. The realization that MOND is a theory based on Helmholtz work shifts the question regarding its relativistic formulation towards the larger problem of a relativistic formulation of thermodynamics, a highly discussed and accepted problem in physics.

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I. THE PROBLEMS FOR WHICH THE DARK MATTER HALO OF GALAXIES IS A SOLUTION

In 1933 Dark Matter was mentioned as "dunkle Materie" in a paper by Zwicky. Fritz Zwicky was studying the Coma Cluster of galaxies and found that his calculations for orbital acceleration and stellar mass within it was off by a large factor. He concluded that there should be a much greater density of dark matter within the cluster than there was luminous matter. Zwicky concluded that this constituted an unsolved problem [1]. In 1937 Zwicky regarded his study on the Coma Cluster a test of Newton's law of gravity on the largest cosmological scale possible, by applying the virial theorem on a cluster of galaxies. He also mentioned in his 1937 paper the possibility to test the virial theorem by applying it to the rotational velocities of the individual stars in the separate galaxies. But he concluded that this was technologically out of reach [2].

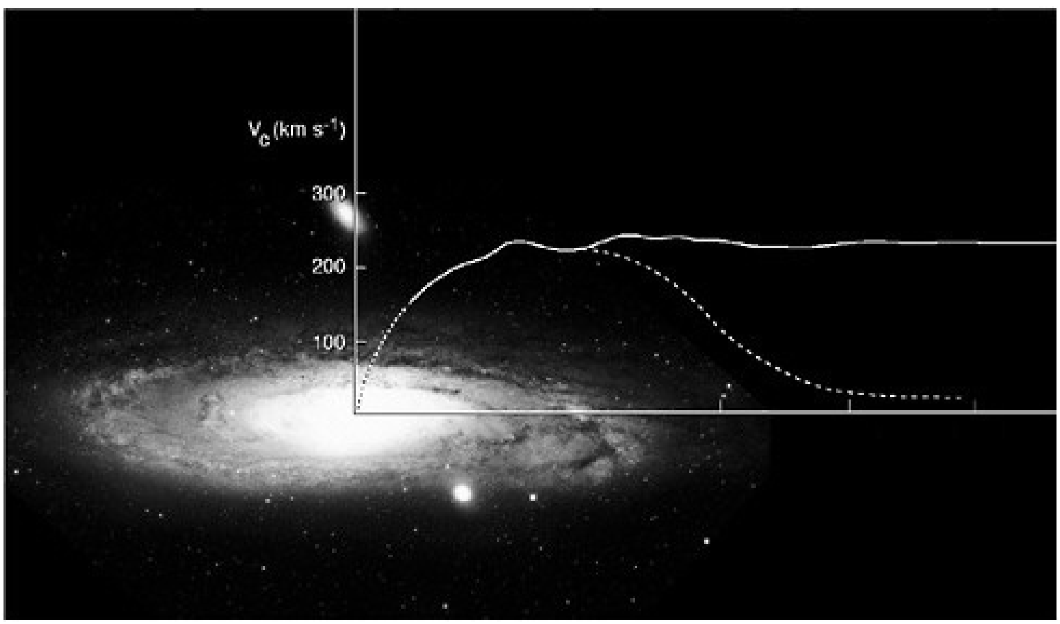

FIG. 1. A typical galactic rotation curve. The dotted line is the Newtonian expectation based on visible mass from the stars. The straight line is the observed rotation velocity. The outer velocities are mainly based upon radio astronomy measurements of neutral hydrogen gas clouds.

The breakthrough research of Rubin and Ford around 1970-1975 established beyond doubt the outer rotational velocity curves of individual galaxies, which turned out to be flat [3]. This was in conflict with velocity curves that resulted from the application of the virial theorem to the luminous mass of these galaxies. Rubin and Ford cited colleagues who suggested the existence of a large galactic halo of dark matter. In a 1980 paper presenting further research they concluded that the form of the rotation curves implied that significant non-luminous mass should be located at large distances beyond the optical galaxy. The total mass of a galaxy should, for large distances, increase at least as fast as the distance from the center [4].

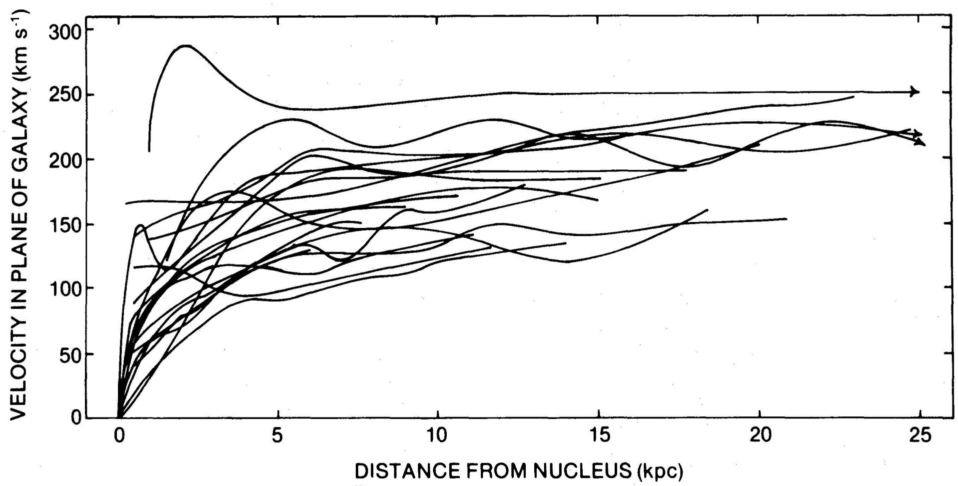

FIG. 2. Plotting velocities against distance for several galaxies. Reprint from Rubin 1980 [4].

The third major evidence for Dark Matter is the gravitational lensing effect of clusters of galaxies. The mass of stars and hot gas in clusters who collectively act as a gravitational lens is too small to bend the light from the background galaxies so much. A large density of dark matter in the center of these cluster is needed to explain the strength of the observed lensing effects.

For our paper, the galaxy rotation curve results are most relevant. All three mentioned astrophysical observations use the hypothesis of a galactic Dark Matter halo that extends far beyond the luminous part of the galaxies in order to explain the results of measurements. But where the first two effects relate to Newtonian gravity, gravitational lensing directly involves Einstein's General Relativity. Our model will focus on the Newtonian virial theorem and deviations from it. In [5], Aguirre et.al. summarize several observational constraints that any modification of Newtonian gravity replacing the Cold Dark Matter model must satisfy. In our paper we will show that the proposed model meets the requirements set by these constraints.

II. A BARYON INTRINSIC DARK MATTER HALO MODEL

It is common knowledge that a Dark Matter source mass function linear in r can explain the flatness of galaxy rotation curves at large r [6]. This is the empirical starting point of our Dark Matter model. Given a rest mass M_0 at $r = 0$, it will have an additional spherical Dark Matter halo containing an extra mass, with Dark Matter properties only, in the sphere with radius r as

$$M_{\rm DM} = \frac{r}{r_{\rm DM}} M_0 \tag{1}$$

in which the Dark Matter radius $r_{\rm DM}$ should have a value somewhere around 10 kpc, so approximately once or twice the radius of an average luminous galaxy. This radius turns out be galaxy specific, determined by the galaxies baryonic mass and its constant of final rotational velocity in the galaxies outer region.

As everything indicates that Dark Matter only interacts as being a source mass of gravity, this extra mass $M_{\rm DM}$ only comes into play when the rest mass M_0 acts as a source of gravity. So $M_{\rm DM}$ doesn't contribute to the inertial mass of M_0, nor does it contribute to its gravitational charge when acted upon by a force of gravity. Our choice of the Dark Matter halo as being only the source of a field of gravity and not at the same time the charge in a field of gravity of another particle, is based on what astrophysicist do not see. As for example in a science journalists impression of a galaxy cluster collision research:

> Surprisingly, the study discovered that dark matters in galaxy cluster collisions simply pass through each other. This implies that dark matter particles do not interact with themselves, which would have caused dark matter to slow down. Instead, it appears that while dark matter could interact "non-gravitationally" with visible matter, this is not the case when it interacts with itself. More importantly, the study challenges the view that dark matter consists of proton-like particles - or perhaps any particles whatsoever. "We have now pushed the probability of two 'dark matter particles' interacting below the probability of two actual protons interacting, which means that dark matter is unlikely to consist of just 'dark-protons'," says David Harvey. "If it did, we would expect to see them 'bounce' off each other". [7]

In this Dark Matter model there aren't additional baryonic particles in the Universe like

Cold Dark Matter or WIMP's. The Dark Matter halo of the proposed model is an additional property of every already known elementary particle with a rest mass m_0 in the Universe.

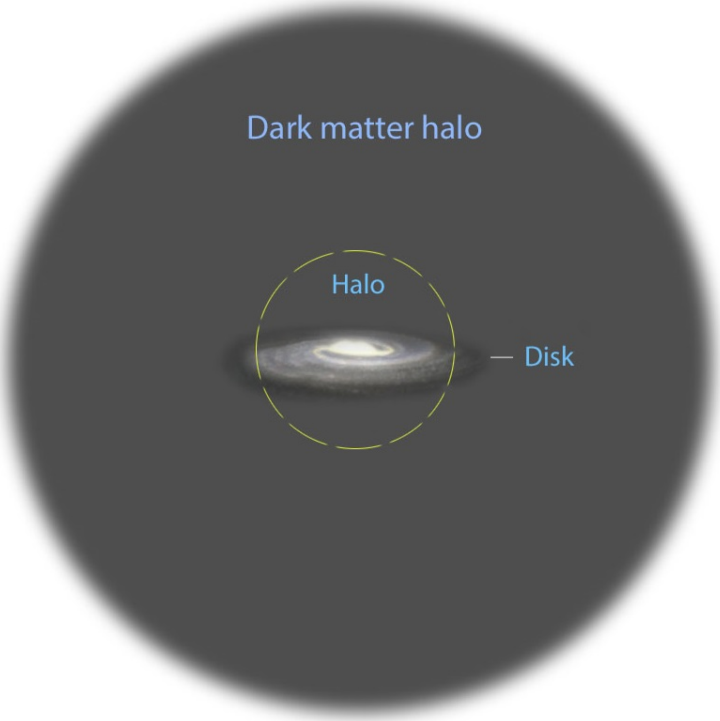

FIG. 3. Disk shaped galaxy with baryonic halo environment and Dark Matter halo.

The total gravitational source mass M_g of an elementary particle M_0 contained within a sphere with radius r will then be given by

$$M_g = M_0 + M_{\text{DM}} = M_0 + \frac{r}{r_{\text{DM}}} M_0 = m_0 \left(1 + \frac{r}{r_{\text{DM}}}\right). \tag{2}$$

The total mass of an elementary particle at rest inside a galactic sphere of radius r_{DM} wil be twice the original rest mass at $r = 0$.

From a recent paper by Koopmans et.al. we quote:

> In both spiral and elliptical galaxies with prominent baryonic components, there appears to be a conspiracy between dark-matter and baryons, leading to a nearly universal total mass distribution out to the largest measured radii that is very close to isothermal (i.e. $\rho \sim r^{-2}$), with only a small intrinsic scatter between systems. [6]

This is a key motivation for our proposed axiom. The observation in the quote indicates towards some kind of a source like connection between baryons and their Dark Matter halo.

The elementary particle Dark Matter halo mass content has been derived from a mass density that is inversely proportional to $4\pi r^2$. So the mass density of the halo drops of or dilutes at the surface of an ever larger sphere in the same way that all classical central sources do. We therefore define a Dark Matter halo mass density as

$$\rho_{\rm DM} = \frac{M_0}{4\pi r^2 r_{\rm DM}} \tag{3}$$

and then the spherically symmetric gravitational source mass m_g inside a sphere of radius r is given by

$$M_g = \int_V \rho_{\rm DM} dV = \int_r \rho_{\rm DM} 4\pi r^2 dr = \int_r \frac{M_0}{r_{\rm DM}} dr = \frac{M_0}{r_{\rm DM}} r + M_0 \tag{4}$$

with the last factor as the obvious constant of integration, given the starting point of our model that we have $M_g = M_0$ at $r = 0$. The density and mass functions in our proposal of the elementary particle's halo are chosen to match the values necessary to arrive at the constant velocity rotation curve of galaxies. It is the astrophysical experimental input, especially the $\rho \sim r^{-2}$ Dark Matter density distribution observation, see [6], that is turned into axiomatic definitions regarding the properties of elementary rest masses.

III. THE POTENTIAL, THE VIRIAL THEOREM AND GALAXY ROTATION CURVES

Given the definition of the gravitational potential as

$$\phi = -\frac{GM}{r} \tag{5}$$

with gravitational source mass M as

$$M = M_0 + \frac{rM_0}{r_{\rm DM}} \tag{6}$$

we get a gravitational potential at r as

$$\phi = -\frac{GM_0}{r} - \frac{GM_0}{r_{\rm DM}} = \phi_0 + \phi_{\rm DM} \tag{7}$$

The extra term in the potential is effecting the gravitational energy of a satellite mass m in the field of a source mass M. This gravitational energy is given by

$$U_g = m\phi = m\phi_0 + m\phi_{\rm DM} = -\frac{GM_0 m}{r} - \frac{GM_0 m}{r_{\rm DM}}. \tag{8}$$

Now we assume that the virial theorem is still valid, giving $2U_k = -U_g$, so $v^2 = -\phi$ for orbiting satellites and

$$v^2 = -\phi = \frac{GM_0}{r} + \frac{GM_0}{r_{\rm DM}}. \tag{9}$$

If we let $r \to \infty$ then

$$v_f^2 = \frac{GM_0}{r_{\rm DM}}, \tag{10}$$

which is a constant, the galaxy rotational velocity curves' final constant value.

This result allows us to give an estimate of $r_{\rm DM}$ by applying this to the Milky Way galaxy. We get

$$r_{\rm DM} = \frac{GM_0}{v^2} \approx \frac{6,67.10^{-11} \cdot 1,99 \cdot 10^{30} \cdot 1,4.10^{11}}{(230.10^3)^2} = 3,5.10^{20} m = 11,4 kpc. \tag{11}$$

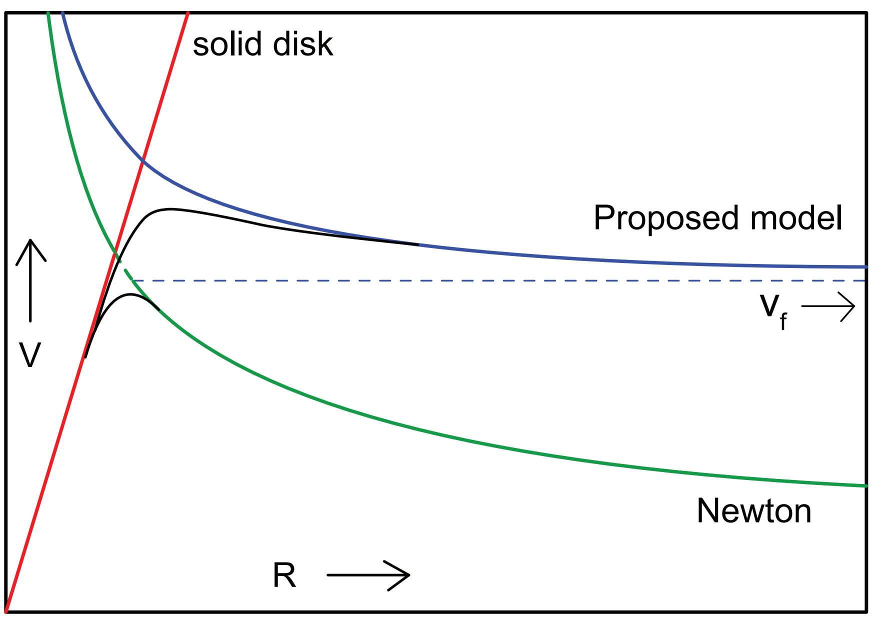

FIG. 4. Plotting velocity against distance for the Newtonian versus our model's expectation.

Actual galaxy velocity rotation curves vary considerably from our model with its point like mass distribution. Real galaxies have disk like or spherical like mass distributions which cause deviations from our single particle model. But if we combine the solid disk model with our model we get the result of the upper curve in Fig.(4), with the lower curve as the Newtonian expectation. Relative to the simplicity of our model, this result matches the overall galactic rotation curves quite nicely.

IV. THE FORCES IN OUR MODEL AND THE DIRECT CONNECTION TO MOND AND THE BARYONIC TULLY-FISHER RELATION

For the resulting force of gravity on a classical mass m we get the Newtonian result

$$\boldsymbol{F}_N = -m\nabla\phi = -m\nabla\phi_0 + -m\nabla\phi_{\text{DM}} = -m\nabla\phi_0 = -\frac{GM_0 m}{r^2}\hat{r}. \tag{12}$$

This is due to the fact that the new mass factor varies linear over r and thus results in a additional potential term that is constant. Our Dark Matter halo acts as a gauge term in the source that produces a constant term ϕ_{DM} in the potential and thus has zero effect on the classical Newtonian force as the divergence of the potential. This of course presents a problem for our model because MOND phenomenology clearly indicates that a deviation of Newtonian forces on the galactic scale matches the experimental findings, see [8] and [9]. Somehow this gap between model and phenomena has to be closed. Put in a simple equation, we have

$$F_{MOND} - F_{Newton} = F_{Emergent}, \tag{13}$$

at least as far as our model is used.

In the previous section we arrived at a acceptable first approximation plot for galactic rotation curves using the virial theorem and the gravitational energy from our model. The virial theorem starts with setting the gravitational force equal to the centripetal force, as in $F_g = F_c$ and then the connected energy relation is derived. I we apply the same procedure to our gravitational potential energy, assuming the orbits to be quasi circular, we can insert Eqn.(9) in the formula for F_c to get

$$F_c = \frac{m_0 v^2}{r} = \frac{GM_0 m_0}{r^2} + \frac{GM_0 m_0}{r\ r_{\text{DM}}} = F_N + F_{\text{DM}} \tag{14}$$

This means that an additional Dark Matter force F_{DM} is needed to provide the necessary centripetal force. This Dark Matter force needed in our model must be equal to

$$\boldsymbol{F}_{\text{DM}} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}}\hat{r} \tag{15}$$

and it has to be equal to $F_{MOND} - F_{Newton} = F_{Emergent}$ in order for our model to match the basic MOND phenomenology.

MOND works with a universal acceleration a_0, the value of which is calculated with the baryonic Tully-Fisher relation, see [10]. In our model on the other hand, we introduced a Dark Matter radius r_{DM}. Using the virial acceleration with the final rotation velocity of galaxies v_f^2 and our Dark Matter radius r_{DM}, we get the special acceleration of our model as

$$a_{\text{DM}} \equiv \frac{v_f^2}{r_{\text{DM}}} \tag{16}$$

so r_{DM} can be given by

$$r_{\text{DM}} = \frac{v_f^2}{a_{\text{DM}}} \tag{17}$$

which, inserted into Eqn.(10) gives

$$v_f^2 = \frac{G a_{\text{DM}} M_0}{v_f^2} \tag{18}$$

and this gives

$$v_f^4 = G a_{\text{DM}} M_0, \tag{19}$$

a relation that we recognize as Milgrom's form of the Baryonic Tully-Fisher relation. This allows us to identify our a_{DM} with Milgrom's a_0 [9]. If we then insert r_{DM} from Eqn.(17) in Eqn.(15), we get

$$F_{\text{DM}} = \frac{G a_{\text{DM}} M_0 m_0}{r\ v_f^2} \tag{20}$$

and combined with

$$v_f^2 = \sqrt{G a_{\text{DM}} M_0} \tag{21}$$

this gives the MOND force for the $r \gg r_{\text{DM}}$ regime

$$F_{\text{DM}} = \frac{m_0}{r} \sqrt{G a_0 M_0}. \tag{22}$$

This shows that our gravitational energy combined with the virial theorem reproduces the basic 1983 MOND premisses, the MOND force and the baryonic Tully-Fisher relation, see [8]. The original Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity [11]. The physical basis of the Tully-Fisher relation is the relation between a galaxy's total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation [10]. In 2005 McGaugh determined the baryonic version of the LT relation as $M_d = 50 v_f^4$, see [10] and Fig(5). In this form, M_d is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}\ kg$ units and the final velocity of the galactic rotation velocity curve v_f is

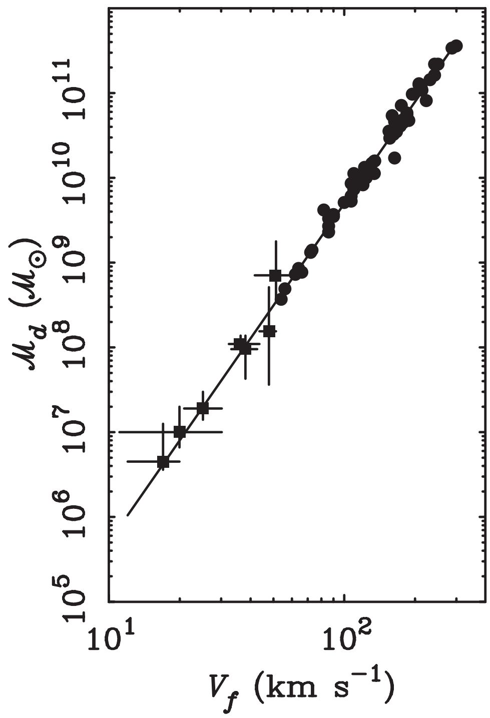

FIG. 5. The Baryonic Tully-Fisher relation. Reprint from McGaug 2005 [10].

expressed in km/s. If we express the galactic mass in kg and the velocity in m/s we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20} v_f^4$.

In 1983, Milgrom interpreted the BTF relation as an indication of a deviation from Newtonian gravity, making a modification of Newtonian dynamics or MOND necessary [8]. Using McGaug's 2005 values in SI units, Milgrom presented the BTF relation in the form $v_f^4 = 1,0 \cdot 10^{-20} M_b = G a_0 M_b$, resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}$ m/s^2 in McGaug's values. Milgrom hypothesized that this relation should hold exactly, thus interpreting it as an inductive law of nature instead of looking at it as just an empirical relation [9]. The resulting acceleration can be written as $5 \cdot a_0 \approx cH_0$, with the velocity of light c and the Hubble constant H_0. According to Milgrom, the deeper significance of this relation between this special galactic acceleration and the Hubble acceleration should be revealed by future cosmological insights [8].

The conclusions from this empirical input is that our $r_{\rm DM}$ is galaxy specific, just as v_f^2 is. The acceleration on the other hand is determined, as Milgrom foresaw already in 1983, by the intersect of the BTF-relation. Since then, this law has been confirmed for over more than 5 powers of ten in the baryonic mass.

V. MOND AND THE ENTROPIC FORCE FROM THE FIRST LAW OF THERMODYNAMICS,

Our Dark Matter force emerged from virial requirements in combination with our potential function $\phi_g = \phi_N + \phi_{\text{DM}}$ but we couldn't derive the emergent Dark Matter force, identical to MOND's force, from the usual way as the divergence of the potential. But according to the general approach of Verlinde, we should be able to connect the DM force to emergent gravity, using Boltzmann's entropy. We have for the entropic force of gravity the general requirement

$$F\Delta r = T\Delta S, \tag{23}$$

see Eqn. 3.7 in [12], in which T stands for the absolute temperature connected to m_0 and S for the entropy of m_0 in the DM field of M_0. This restricted formulation of the first law of thermodynamics can be enlarged to the Helmholtz free energy equation $A = U - TS$. For the simplicity of our model's sake let's assume reversibility of all processes involved and assume isothermal processes only. Then with the Helmholtz free work done as $dA = -dW = -Fdr$ its infinitesimal formulation results in $Fdr = -dU + TdS$. From this we can get the derivative equation for the Helmholtz free force

$$F = -\left(\frac{dU}{dr}\right)_S + T\left(\frac{dS}{dr}\right)_U \tag{24}$$

and if gravity in the extreme weak gravity regime can be derived from this formulation of the first law of thermodynamics then this should give as a result

$$F_{Helmholtz} = F_{MOND} = -\frac{GM_0m_0}{r^2} - \frac{GM_0m_0}{r\ r_{\text{DM}}} = F_N + F_{\text{DM}} \tag{25}$$

so we should get

$$F_N = -\left(\frac{dU}{dr}\right)_S = -\frac{GM_0m_0}{r^2} \tag{26}$$

and

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_U = -\frac{GM_0m_0}{r\ r_{\text{DM}}}. \tag{27}$$

For the energy U we have the gravitational potential energy $m_0\phi_g = m_0\phi_N + m_0\phi_{\text{DM}}$ and that results in the Newtonian force of gravity. The entropy can be derived from the integral form

$$\int_{r_i}^{r_f} Fdr = \int_{S_i}^{S_f} TdS. \tag{28}$$

The situation in the Dark Matter regime is isothermal and we get

$$\int_{S_i}^{S_f} dS = \int_{r_i}^{r_f} -\frac{GM_0 m_0}{r\ r_{\text{DM}} T} dr = -\frac{GM_0 m_0}{r_{\text{DM}} T} \int_{r_i}^{r_f} \frac{1}{r} dr \tag{29}$$

and so

$$S_f - S_i = -\frac{GM_0 m_0}{r_{\text{DM}} T} \ln \frac{r_f}{r_i}, \tag{30}$$

which can also be written as

$$S_f - S_i = \frac{U_{\text{DM}}}{T} \ln \frac{r_f}{r_i} = k_B \frac{U_{\text{DM}}}{k_B T} \ln \frac{r_f}{r_i}. \tag{31}$$

We have the Boltzmann definition of entropy S as

$$S = k_B \ln W \tag{32}$$

with for W the number of microstates. We assume that

$$\ln W \propto \ln \frac{r}{r_m} \tag{33}$$

with r_m as a microstate radius of the particle with mass m_0 that is at distance r in the field of M_0. We further assume that the particles on the galactic disk are all moving with v_f with one single degree of freedom perpendicular to r, so the number of microstates of m_0 on this circumference is given by $W = r/r_m$. This can be combined with the above to

$$S_f - S_i = k_B \frac{U_{\text{DM}}}{k_B T} \ln \frac{r_f}{r_m} - k_B \frac{U_{\text{DM}}}{k_B T} \ln \frac{r_i}{r_m} \tag{34}$$

resulting in a Dark Matter entropy on the outer galactic disk as

$$S = k_B \frac{U_{\text{DM}}}{k_B T} \ln \left(\frac{r}{r_m} \right) \tag{35}$$

with T related to the particle in the Dark Matter field in question. Define a ξ as

$$\xi = -\frac{U_{\text{DM}}}{k_B T} \tag{36}$$

then in the flat rotation curve domain, the Dark Matter entropy is given by

$$S = -k_B \xi \ln \left(\frac{r}{r_m} \right) \tag{37}$$

producing a decreasing entropy as we go further away from the baryonic matter M_0. A decrease of entropy outwards produces an entropic force inwards.

So we sort of followed the thermodynamic guidance of Verlinde, but we didn't derive Newton's force of gravity as an entropic force. The use of the first law of thermodynamics with an energy part related to Newtons force and an entropic part related to Dark Matter was enough to reach our specific goal to derive the force needed to match the centripetal force that emerged from our potential energy. The entropy was needed only to cover the Dark Matter part of the story, to fill in the missing part, the deviation from Newtonian gravity.

Now let's finish this section by inversing the derivation, starting with a gravitational charge m_0 in the Dark Matter field of the gravitational source M_g. This gives us the DM potential

$$\phi_{\text{DM}} = -\frac{GM_0}{r_{\text{DM}}}, \tag{38}$$

the DM energy as the constant

$$U_{\text{DM}} = -\frac{GM_0 m_0}{r_{\text{DM}}}, \tag{39}$$

and the Dark Matter entropy

$$S = k_B \left(\frac{U_{\text{DM}}}{k_B T}\right) \ln\left(\frac{r}{r_m}\right) \tag{40}$$

From the entropy we can derive the entropic DM force at constant U using

$$F_{\text{DM}} = T\left(\frac{dS}{dr}\right)_U = T\frac{d}{dr}k_B\left(\frac{U_{\text{DM}}}{k_B T}\right)\ln\left(\frac{r}{r_m}\right) = U_{\text{DM}}\frac{d}{dr}\ln\left(\frac{r}{r_m}\right) = \frac{U_{\text{DM}}}{r} = -\frac{GM_0 m_0}{r\ r_{\text{DM}}}. \tag{41}$$

This inverse derivation demonstrates that we can arrive at the Dark Matter force needed in MOND's weak acceleration regime by starting with our model's Dark Matter halo mass M_{DM} for every elementary particle. Assuming for the moment that all processes are reversible, the Helmholtz free force of gravity F_H is then given by $F_H = F_{MOND} = F_N + F_{\text{DM}}$. Of course, the thermodynamic reversibility question in relation to cosmology will turn out to be little bit more complex than we assumed in our model. That is why it still is just a model.

VI. THE PARAMETERS OF THE ENTROPIC MODEL

The number of microstates W of an ideal gas particle m with a microstate radius r_m in a volume with macrostate radius r is traditionally given by the times one can fit this small or microscopic volume in the large or macroscopic volume, so by the thermodynamic gas molecule in a bottle number of microstates

$$\ln W = \ln \frac{V}{V_m} = 3 \ln \frac{r}{r_m}. \tag{42}$$

But if we have such a particle moving with only one degree of freedom on a circular trajectory with radius r and this particle can be considered a quantum particle with a de Broglie wavelength $\lambda = h/p$, then the number of microstates can be defined as the number of microscopic wavelengths that fit onto the macroscopic circumference. This gives the number of microstates as

$$\ln W = \ln \frac{2\pi r}{\lambda} = \ln \frac{rp}{\hbar} = \ln \frac{S}{\hbar} \tag{43}$$

with S as the phase space of the particle. In the case of galactic neutral hydrogen gas particles, this phase space is huge. This approach also gives us a way to look at a relativistic generalization of the model. And it gives hint regarding a possible connection of the Dark Matter halo of elementary particles to a de Broglie subquantum thermodynamics. Such an interpretation could open a new Quantum Gravity research program.

If we focus on the $\ln W$ part of the entropic force on a particle with N space-like degrees of freedom, we have

$$\frac{d}{dr} \ln W = \frac{d}{dr} \ln \left(\frac{r}{r_m}\right)^N = N \frac{d}{dr} \ln r - N \frac{d}{dr} \ln r_m = N \frac{d}{dr} \ln r = \frac{N}{r} \tag{44}$$

then it is clear that r_m is a free parameter of our theory from the perspective of the derived F_{DM}. This means that both a thermodynamic and a quantum interpretation of r_m are possible, in principle. Practical considerations should determine the choice of model for r_m. The quantum interpretation has the advantage to look like a move towards a quantum theory of gravity, but as long as it remains a free parameter, this has no specific use. On the other hand, a star orbiting a galaxy at a large distance has an incredible small de Broglie wavelength but clearly a much smaller number of microstates relative to the length of its orbit, as compared to a neutral hydrogen atom. So for large objects the volume of classical mass radius interpretation seems to make the most sense.

The other free parameter of our theory is the temperature T, because its interpretation doesn't effect the resulting F_{DM} either, as can be seen in

$$F_{\text{DM}} = T \left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = T \frac{d}{dr} k_B \ln \left(\frac{r}{r_m}\right)^{\frac{U_{\text{DM}}}{k_B T}} = T \frac{U_{\text{DM}}}{T} \frac{d}{dr} \ln \left(\frac{r}{r_m}\right) = \frac{U_{\text{DM}}}{r}. \tag{45}$$

The temperature as a free parameter of our theory only works as far the temperature of the orbiting objects is independent from the radius at which they orbit.

The key non-free parameters of this entropic force derivation are the Dark Matter energy U_{DM}, the radius r and the spacial degree of freedom N. The first two of these variables are already part of MOND. The third variable, the geometrical degree of freedom, is not at present part of the MOND theory. According to my model, it should be added to MOND.

VII. ADDING THE DIMENSION PARAMETER TO MOND: FROM GALAXIES TO THE COSMOS

In my model, objects on a disk have one degree of freedom, objects moving freely on a sphere have two degree's of freedom and objects that behave as in a mono-atomic gas have three degrees of freedom. The natural logarithm of the number of microstates wil then be

$$\ln W = \ln \left(\frac{r}{r_m}\right)^N = N \ln \frac{r}{r_m}. \tag{46}$$

For the entropic Dark Matter force, adding this degree of freedom parameter N with value between 1 and 3 results in

$$F_{\text{DM}} = T \left(\frac{dS}{dr}\right)_{U_{\text{DM}}} = U_{\text{DM}} \frac{d}{dr} \ln \left(\frac{r}{r_m}\right)^N = \frac{N U_{\text{DM}}}{r} = -\frac{G(NM_0)m_0}{r\ r_{\text{DM}}}. \tag{47}$$

Without this number of microstates degree of freedom related factor N, in certain situations the needed baryonic mass might be overestimated by a factor between 2 and 3. The parameter N will never be exactly three because such systems behave as a free gas and do not display gravitational attraction phenomena. The spacial degree of freedom N is a new parameter for MOND that follows logically from the entropy considerations.

In the case of galaxy clusters, this degree of freedom cannot be 2 or smaller because then the clusters should have been shaped like a disk or a recognizable sphere. Neither can it be 3 because then the clusters would disperse like a free gas. So its degree of freedom should be somewhere in between 2 and 3, giving it an apparent baryonic mass $2M_b < M_a < 3M_b$. This is confirmed by the observations.

> What one may call, generically, "the dark matter problem" first became evident with radial velocity studies of rich clusters of galaxies (Zwicky 1933 [1]). The discrepancy between the visible and Newtonian dynamical mass, quanti

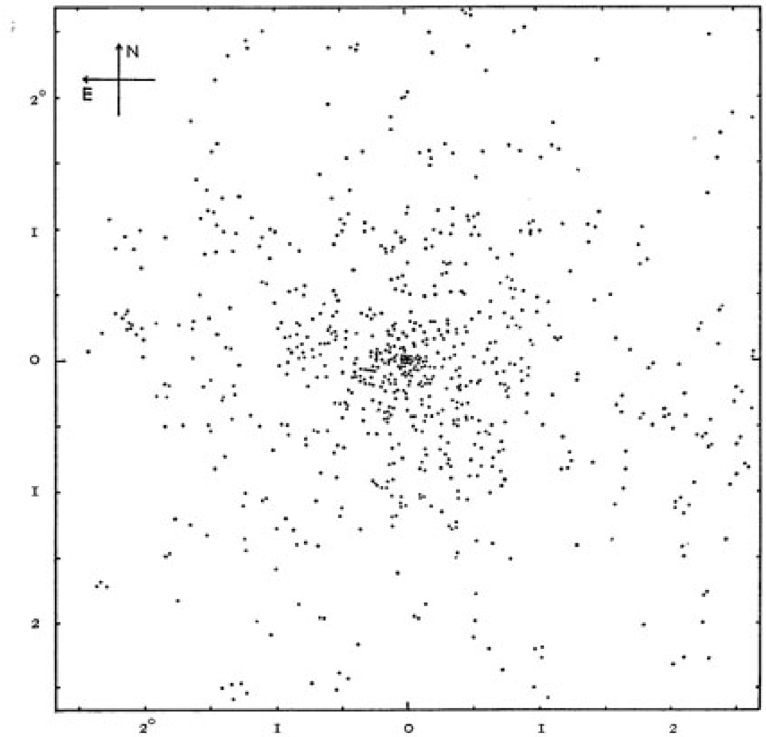

FIG. 6. The Coma cluster, in between a 2D sphere and a free gas. Reprint from Zwicky 1937 [2].

> ed then interms of mass-to-light ratio, was more than a factor of 100. With the advent of X-ray observatories and the detection of hot gas in clusters, this discrepancy between dynamical and detectable mass was reduced to a factor of 10. Modified Newtonian dynamics reduces the discrepancy further, to a factor of 2 to 3, but it is clear that a discrepancy remains which cannot be explained by detected gaseous or luminous mass [13].

The fact that MOND has this problem, indicates a distinct difference between MOND and my model. In my model this would not be a problem but a chance to measure the degree of freedom parameter of galaxy clusters. This parameter should contain information regarding the process of formation of such clusters. The more the mass of a cluster is overestimated towards a factor 3, the more the cluster should look like and behave as a free gas of galaxies. According to McGaugh, *there is roughly a factor of two of residual missing mass in these objects* ([14], p. 81).

The same reasoning can be applied to the cosmos as a whole, so because cluster formation takes place in the universe, its entropic degree of freedom should be somewhere in between

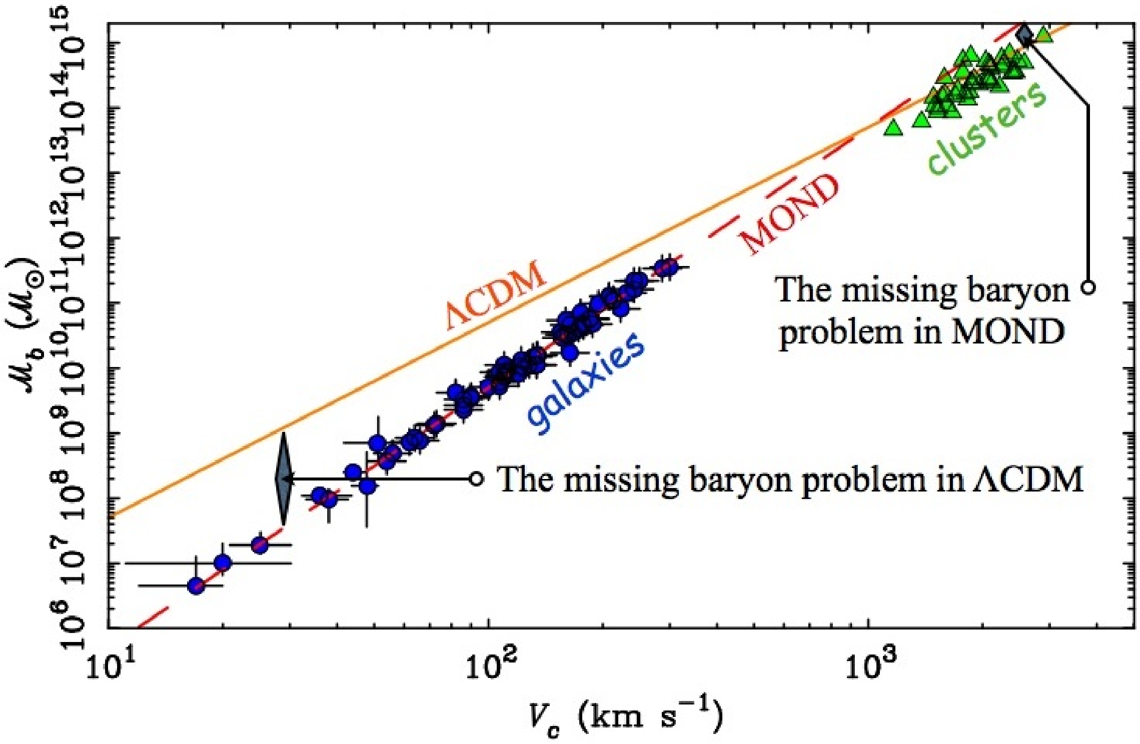

FIG. 7. The Baryonic Tully-Fisher relation with missing baryon mass problem in clusters with MOND and in galaxies with ΛCDM. For MOND this can be solved with the new parameter N.

the values $N = 2$ and $N = 3$. If we add the degree of freedom to Milgrom's MOND form of the entropic force and reformulate the result in the familiar MOND variables we get

$$F_{\text{DM}} = -\frac{G(NM_0)m_0}{r\ r_{\text{DM}}} = F_{\text{DM}} = -\frac{m_0}{r}\sqrt{GN^2 a_0 M_0}. \tag{48}$$

If we adopt Milgrom's interpretation but now include the entropic degree of freedom parameter N we get

$$N^2 a_0 = cH_0 \tag{49}$$

so

$$N = \sqrt{\frac{cH_0}{a_0}} = 2,1. \tag{50}$$

In our model, the cosmic entropic degree of freedom is given by this value $N = 2,1$. In the calculation of N we used the value of $Ga_0 = 1,0 \cdot 10^{-20}$ from McGaugh [10].

This parameter should be important in understanding cosmic structure formation. It should give the degree of freedom in large scale structure formation systems as the ratio of the galaxy cluster special acceleration a_0 relative to the Hubble acceleration cH_0. So we

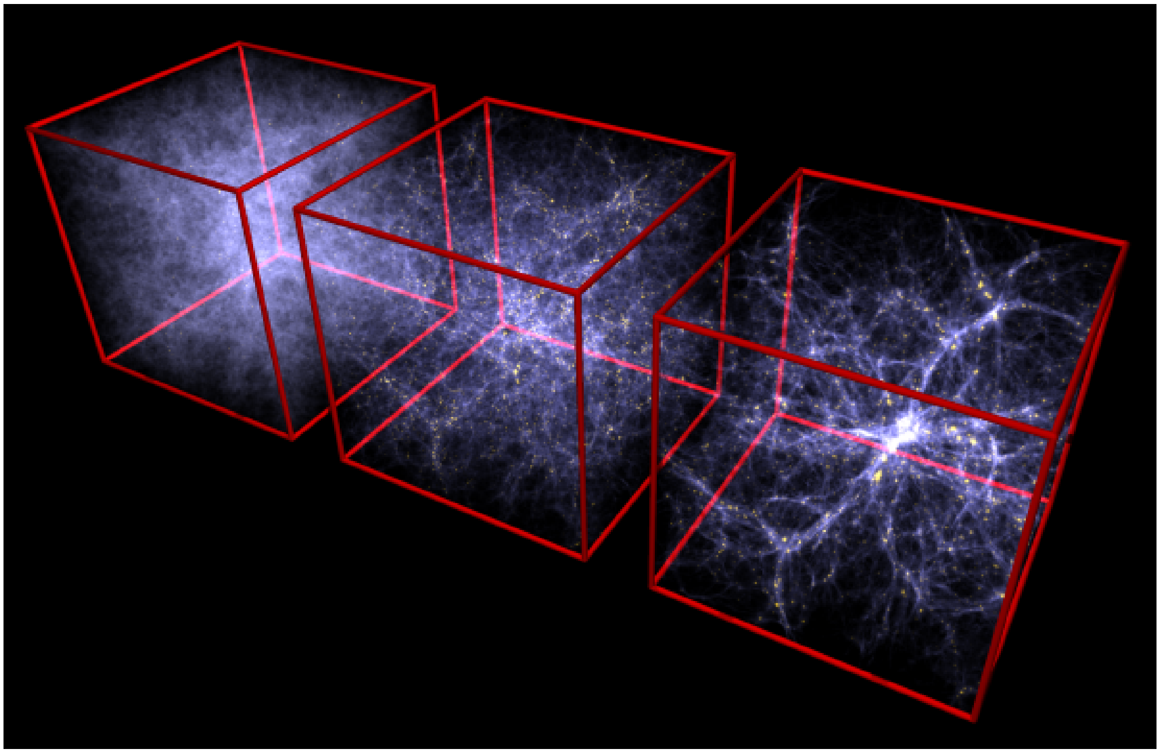

FIG. 8. Structure formation of galactic superclusters during the lifetime of the universe, starting as a 3D gas and from then onward as a thermodynamic irreversible process with a $N = 2.1$ limit.

can predict that the MOND missing mass calculations with regards to the largest structure formation systems should result in a factor $N = 2.1$. With a cosmic factor $N = 2$ as a lowest limit we would all end up living on the surface of a sphere, as on the event horizon of a black hole, so the fact that the cosmic $N = 2.1$ prevents this cosmic black hole collapse from happening. With a cosmic $N = 3$ as a lowest limit all matter would have remained in the state of a free gas, as it was not long after the Big Bang, something that clearly didn't happen.

VIII. THE RADIAL VELOCITY OF LIGHT AND GALACTIC EINSTEIN LENSING

In GR the gravitational index of refraction is then given by

$$n_\phi = \frac{c_0}{c_\phi} \approx 1 + \frac{2GM}{rc^2} > 1 \tag{51}$$

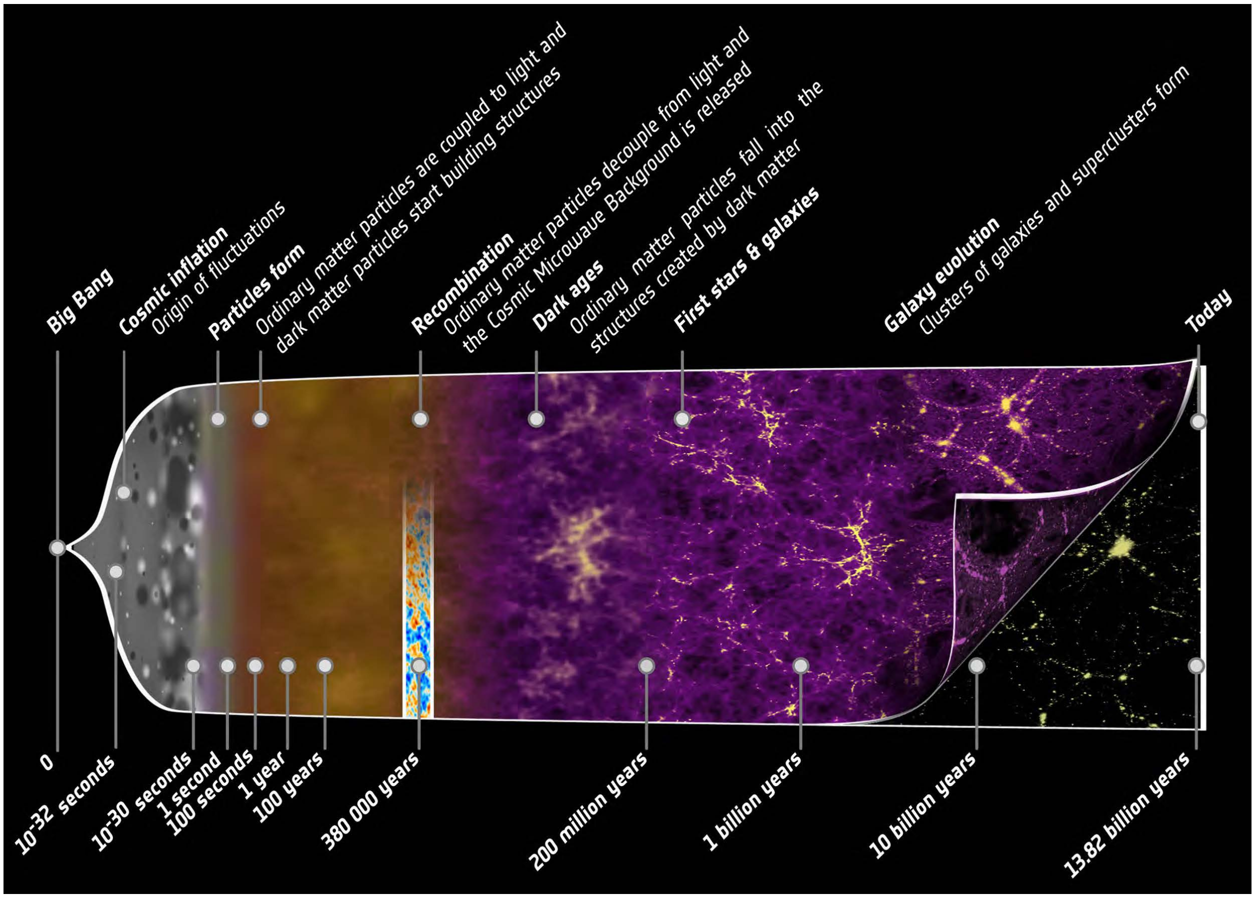

FIG. 9. Timeline of the Universe, as an irreversible process with a cosmic $N = \sqrt{\frac{cH_0}{a_0}} = 2.1$ limit that prevents total black hole collapse everywhere on the baryonic Tully-Fisher mass scale.

explaining the bending of light rays that pass by close to the sun. The use of Eqn.(51) in the context of our model is independent of the theory of relativity from which it is obtained. Our Dark Matter model just has to deliver a suitable ϕ to insert in Eqn.(51). Because our model is based on a change in the source mass of gravity and hence also in the potential energy and the potential itself, the problem of lensing by the Dark Matter halo is just a matter of getting the right potential ϕ in Eqn.(51).

If we use the potential from which we obtained a correct galaxy rotation curve, we get an extra term ϕ_{DM} in the radial velocity of light in the Dark Matter halo as

$$c_\phi = \left(1 + \frac{2\Phi}{c^2}\right) c_0 = \left(1 - \frac{2GM}{rc^2} - \frac{2GM}{r_{\text{DM}}c^2}\right) c_0 \tag{52}$$

so now even far away from the gravitating baryonic mass, will there still be Einstein lensing as

$$c_\phi = \left(1 - \frac{2GM}{r_{\text{DM}} c^2}\right) c_0 \tag{53}$$

with a Dark Matter outer halo constant index of refraction as

$$n_{\text{DM}} = 1 + \frac{2GM}{r_{\text{DM}} c^2} = 1 + \frac{2v_f^2}{c^2} \tag{54}$$

where we used the final velocity of the rotation curve v_f. For a galaxy as the Milky Way, this has the value $n_{\text{DM}} = 1,0000012$. This is a small value, but because the distances are so huge, the effect can still be pronounced.

In this approach we didn't include the Helmholtz free energy in the derivation of the gravitational bending. Because no work is done on photons in the phenomenon of Einstein lensing, this seems an acceptable approach. All we did was to assume that the Dark Matter halo, through its energy, has the capability to bend space in a GR context in the same way ordinary matter does, because in GR having a gravitational effect as a source and bending space are one and the same thing. But this first order approach needs refinement.

As for Einstein Lensing by galaxy clusters, the approach of this section has the limitation of being dependent on the theoretical r_{DM} or the empirical v_f^2, with the last to be restricted to galaxies only. Further research as to its enlargement towards other systems, such as galaxy clusters, is needed in order to arrive at a way to determine the gravitational potential ϕ_g of such clusters to be used in Einstein lensing calculations. We quote from Mannheim [15]:

> As with our discussion of ΛCDM generated galactic halos, what is needed to make cluster dark matter into a falsifiable theory is a prediction of the amount of lensing given only a knowledge of the luminous content of the cluster. Currently the amount of dark matter in clusters is inferred only after the lensing measurements have been made. While making lensing predictions based only on the known luminous content of a cluster thus serves as a goal for dark matter, at the present time it also remains an objective for the alternate gravitational theories we shall discuss below, including those for which galactic rotation curve predictions can be made from luminous information alone. Progress on cluster gravitational lensing should thus be definitive for both dark matter and its potential alternatives.

It should be clear that the model proposed in this paper is a research program in itself.

IX. DE BROGLIE'S SUBQUANTUM THERMOSTAT: A QUANTUM GRAVITY RESEARCH PROGRAM?

Modern post-orbital or post-"Bohr-Sommerfeld" wave quantum mechanics began with de Broglie's hypothesis of the existence of matter waves connected to particles with inertial mass. De Broglie started with the assumption that every quantum of energy U should be connected to a frequency ν according to $U = h\nu$ with h as Planck's constant [16],[17]. Because he assumed every quantum of energy to have an inertial mass m_o and an inertial energy $U_0 = m_0c^2$ in its rest-system, he postulated $h\nu_0 = m_0c^2$. De Broglie didn't restrict himself to one particular particle but considered a material moving object in general [16]. This object could be an electron, an atom or any other quantum of inertial energy. If this particle moved, the inertial energy and the associated frequency increased as $\nu_i = \gamma\nu_0$. De Broglie assumed the inertial energy of the moving particle to behave as a wave-like phenomenon, so the inertial wave associated with a moving particle not only had a frequency ν_i but also a wave-length λ_i, analogous to the fact that any inertial energy U_i of a moving particle had a momentum p_i associated to it according to the relation $p = h/\lambda$.

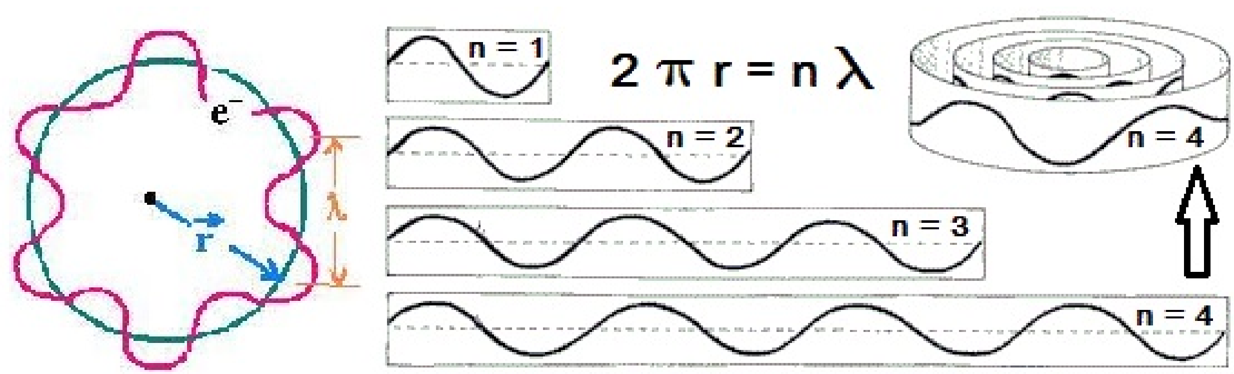

FIG. 10. De Broglie matter waves of an electron orbiting a proton in a neutral H1 atom

In the subsequent Kopenhagen Interpretation the square of the amplitude of the wave became a measurement of the probability (density) of finding the particle in the wave, $P = |\Psi|^2$, with the matter wave function as a function of the phase of the wave n:

$$\Psi = Ae^{i\frac{2\pi r}{\lambda}} = Ae^{in} = Ae^{i\frac{rp}{\hbar}} = Ae^{i\frac{S_\alpha}{\hbar}}. \tag{55}$$

.

De Broglie never could specify what exactly oscillated in relation to m_0 with frequency ν and wavelength λ, but using these relations physicists could make major progress in theory and experiments. De Broglie's ideas regarding matter waves surrounding moving masses proved useful in the further development of quantum mechanics and the success of such chimeras is what finally counts in science, regardless of the ontological status of postulated ideas. Discussions regarding the experimental reality of the frequency of rest masses is ongoing in quantum mechanics, especially discussions related to our most accurate atomic clocks in the earth's field of gravity, see [18], [19] and [20] .

From the beginning de Broglie tried to give a realistic or ontological interpretation to the matter waves of his postulates as for example in his pilot wave theory. But the Kopenhagen or Born probability interpretation of the matter waves proved the most succesful and for many years de Broglie accepted this approach. But in his later years, de Broglie came back to his original ideas and started to work again on the pilot wave theory. In this theory he assumed the existence of a substratum, a subquantum thermostat with stochastic or thermodynamic properties embodying the de Broglie matter waves [21].

In our model we need to give the Dark Matter halo similar properties as the de Broglie subquantum matter wave medium. This analogy is necessary in order to assures our proposal to be in full accordance with Special Relativity. With these matter wave like properties, disturbances in the elementary Dark Matter halo due to changes in position and momentum of m_0 at $r = 0$ will travel with matter wave velocity through the halo, with

$$v_{wave} = \frac{c^2}{v_{particle}} \tag{56}$$

If the elementary particle has velocity zero, the disturbances at the source can travel instantaneously through the entire halo, producing a Newtonian instantaneous force field of gravity. The DM halo functions as a medium similar to the stochastic or subquantum thermodynamic de Broglie matter waves. This implies that the halo has some kind of local Born-interpretation probability-density properties. The Dark Matter halo is a storage place of stochastic (sub)quantum gravity information regarding M_0. Our elementary particele intrinsic dark matter halo is a storage place for source information regarding gravity, information that is accessed by particles that act as charges in this field of gravity. Due to the fact that the source particle itself is in a stochastic nondeterministic motion in its own matter wave, the information stored in its halo must be of stochastic nondeterministic nature as well.

In our view we connect the de Broglie's later thoughts about the matter wave field as a subquantum thermostat to Verlinde's latest ideas of gravity as emergent from quantum information [21],[12]. The idea of gravity as an entropic force caused by changes in the information associated with the positions of material bodies of Verlinde, and de Broglie's ideas of the matter wave field as an subquantum thermostat connect with the observations regarding Dark Matter.

In special relativity, causal information cannot travel faster than light. But in quantum mechanics, subquantum information can travel faster than light without conflicting with Special Relativity. The velocity of waves in matter waves is always faster than light but because causal information can only travel at group velocity SR's axiom is upheld. In our model, Dark Matter disturbances travel with matter wave velocity through the Dark Matter halo of every elementary particle. Particles that will be kicked out of an original $r \approx 0, p \approx 0$ position and momentum and acquire a velocity approaching c will have a matter wave velocity approaching c from above and there will be a considerable delay regarding the Dark Matter halo information adjustment to the new situation of the source particle. Considerable has to be interpreted as on galactic travel and time scales with $r_{\text{DM}} \approx 50.000 ly$. In such circumstances, large retardation effects should be expected, diluting the conspiracy between dark-matter and baryons on cosmic scales.

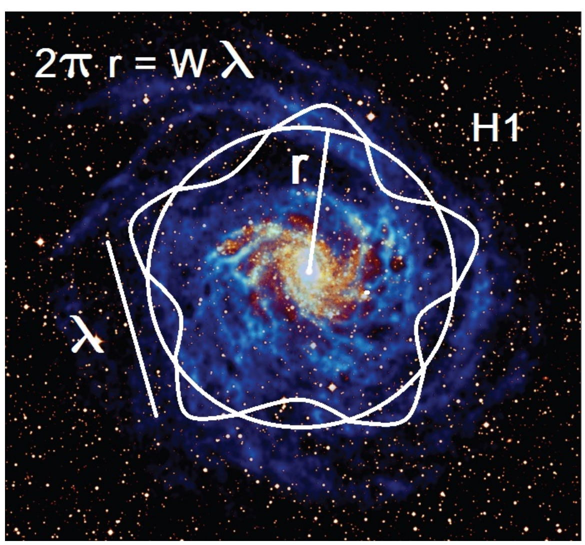

FIG. 11. De Broglie matter waves of a hydrogen atom orbiting the visible stars in a galaxy. The wavelength of the atom $\lambda = 1,7 \cdot 10^{-12} m$ and the circumference is $2\pi r \approx 10^{19} m$ so $n = W \approx 10^{30}$.

Because we have connected an entropy and a number of microstates to an external particle in our Dark Matter halo's mass and potential, so to our Dark Matter field, we can go a bit further in the analogy between our model's halo and the de Broglie's subquantum thermodynamic medium.

For an elementary particle as the neutral hydrogen gas molecule in galactic gas clouds we can connect its number of microstates in its orbit with radius r to the probability of finding this particle in a certain location on this trajectory. This particle can be considered a quantum particle with a de Broglie wavelength $\lambda = h/p$, and the number of microstates can be defined as the number of microscopic wavelengths that fit onto the macroscopic circumference, so its phase. This gives the entropy S_ϵ and the number of microstates or the phase $n = W$ roughly as

$$S_\epsilon = A \ln W = A \ln \frac{2\pi r}{\lambda} = A \ln \frac{rp}{\hbar} = A \ln \frac{S_\alpha}{\hbar} \tag{57}$$

with S_α as the galactic action of the particle as a charge in the field and the "amplitude" containing the information regarding the Dark Matter halo as the source of the field. For the "amplitude" we have

$$A = k_B \cdot \left(\frac{U_{\text{DM}}}{k_B T} \right) . \tag{58}$$

This "amplitude" is an entropy. If we compare this to Quantum Mechanics, with the probability as a matter wave density $P = |\Psi|^2$, with the matter wave function as

$$\Psi = A e^{iW} = A e^{i \frac{S_\alpha}{\hbar}} . \tag{59}$$

Regarding the relation between entropy and action in his pilot wave theory, de Broglie wrote: *In agreement with an idea of Eddington's some fifty years back, it is tempting to try and establish a relation between the two major "invariants" of physics, Action and Entropy [21].* For our context, not de Broglie's result but his attempts are interesting. We want to use a similar medium as the subquantum hidden thermostat for our Dark Matter halo. The analogy/connection between them in the appearance of the action in both approaches. In Quantum Mechanics it appears as

$$\Psi \propto e^{i \frac{S_\alpha}{\hbar}} . \tag{60}$$

and in our Dark Matter entropic model it shows up in

$$\frac{S_\epsilon}{k_B} \propto \ln \frac{S_\alpha}{\hbar} \tag{61}$$

There are important differences. Instead of having the problem of finding the particle as a wave packet with a position and momentum on its galactic trajectory, in Quantum Mechanics the problem is to find the particle's trajectory with a position and a momentum in its wave-packet. The actions in both cases have different relations of trajectory $2\pi r$ relative to the matter wavelength λ. In Quantum Mechanics free particles usually have $\lambda \gg 2\pi r$ and in our Dark Matter model $\lambda \ll 2\pi r$, with for QM bounded particles the intermediate $n\lambda = 2\pi r$. Somehow, the action in QM and the action in our model seem vaguely related through some kind of a Legendre Transformation. The action in QM is in the exponential and the action in the DM halo is in the natural logarithm and the Legendre Transformation of the exponential function contains the natural logarithm.

The point I want to make is that a search for a hypothetical connection between Quantum Mechanics' matter waves and our halo model for Dark Matter gravity will probably go through thermodynamics and information regarding probabilities, microstates, hidden thermostats and phase-spaces. So the more the Helmholtz energy part of our model will be researched, the more likely it will be that at a certain moment a real connection with Quantum Mechanics will be found. Such a connection will then be a theory of Quantum Gravity. Our model contains a research program.

X. WHERE NEXT? RESEARCH QUESTIONS.

In [5], Aguirre et.al. summarize several observational constraints that any modification of Newtonian gravity as a theory explaining Dark Matter phenomena must satisfy. Our model reproduces MOND in most area's relevant to Aguirre's constraints and improves MOND in the area of galaxy clusters and cosmic structure formation.

In the area of Einstein lensing, further research is needed. Especially regarding the relation between Dark Matter halo energy, Newtonian field energy and Helmholtz free energy on the one hand and the curvature of the metric or the appearance of a reduced radial velocity of light on the other hand. But also the way to arrive at a Dark Matter halo potential around galaxy clusters is an important subject for further research.

In our model we simplified reality by focusing either on the Newtonian domain where $a \gg a_0$ or on the MOND domain with $a \ll a_0$. In the first domain we assumed a constant entropy S and in the second domain a constant energy U. We skipped the intermediate domain and further research is needed in this area where S and U are simultaneously variables of r.

Regarding galaxy cluster, the thermodynamics of such clusters and the question to what extend the use of the Helmholtz free energy equation can explain the clusters thermodynamics are important for further research.

As for the intrinsic Dark Matter halo and test particles in orbit in these halo's, the suggestions related to de Broglie are an interesting subject. In relation to this, the inverse Legendre Transformation of the Helmholtz-MOND force equation is an interesting issue. This might be an area of Quantum Gravity.

We were inspired by Verlinde's approach to gravity as an entropic force but we didn't use this relative to Newton's force of gravity. So the connection between Verlinde's theory, where also Newton's force of gravity is reduced to thermodynamics, and our Dark Matter halo's entropic force is in interesting question or topic for further research.

Thermodynamics and Relativity, Special or General, do not easily fuse. To this day different approaches exist as to the adequate formulation of a relativistic thermodynamics. One conclusion from our paper is significant, if our approach is correct then the relativistic formulation of MOND as an independent topic is not relevant any more because then MOND can be derived from the first law of thermodynamics in its Helmholtz free energy formulation. So the question regarding relativistic MOND transforms into the question of the relativistic version of the first law of thermodynamics. On this topic an extensive literature is available. Further study in this field seems appropriate. Of course, Verlinde's approach is to reduce gravity entirely to thermodynamics. A complicated field altogether, which makes it necessary to be selective.

As for cosmology, what is the role or status of our model's of intrinsic Dark Matter in the early universe? More questions than can be answered, that is for sure.

I conclude that the proposed thermodynamic or Helmholtz force version of MOND is not so much a theory as a research program.

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