The Many Meanings of the Planck Mass

In this article I explain the concepts behind the multifaceted Planck mass through each of its facets, highlighting the relation of the Planck mass with the origins of the universe, with the physics of white dwarfs and black holes, with the Heisenberg uncertainty principle, and, finally, with the microscopic world of elementary particles. The paper also suggests a new definition for this "multicoloured" mass. In certain way the Planck mass is a unit that unifies all of physics. Finally, one of the meanings proves, beyond reasonable doubt, the existence of the Pre-universe and suggests, also, the existence of parallel universes.

by R. A. Frino

November, 2015

Keywords: *Planck units, Planck's constant, reduced Planck's constant, Newton's Gravitational constant, speed of light in vacuum, Planck mass, Planck time, Planck length , Planck mass density, Planck spherical volume, Planck energy, Planck wavelength, planck frequency, Planck momentum, Planck force, Planck acceleration, Planck temperature, Planck charge, Planck current, Planck Schwarzchild radius, the Heisenberg uncertainty principle, the energy-time uncertainty relation, the momentum-position uncertainty relation, black hole entropy, normal time.*

Index

1. Introduction

In 1900 the German physicist Max Planck proposed a series of units known as the Planck units [1, 2, 3, 4, 5, 6]. These units are based on only five physical constants - the Newton's gravitational constant, *G*, the speed of light in vacuum, *c*, and the Planck's constant, *h*, the permittivity of vacuum, ϵ_0 , and the Boltzmann constant, k_B (plus one geometrical constant - the number *π*). It is also customary to write the Planck units in terms of the reduced Planck constant or "h-bar", denoted by $\hbar$ (The reduced Planck constant is defined as $\hbar = h/2\pi$). With respect to these units, Planck [7] points out that the units *''necessarily retain their significance for all times and for all cultures, even alien and non-human ones''.* So based on the above quote we draw the conclusion that Planck believed his units were universal, and he was right.

Some of the most well known Planck units are the Planck length, the Planck time and the Planck mass. These units depend on three fundamental physical constants (*G, c* and *h*) only**.** The value of the Planck mass, which is about $2.17651(13)\times 10^{-8}\,Kg$ [8], seems to be enormous in comparison to the masses of the elementary particles. Despite this great disparity, I must emphasize that the Planck mass is not a meaningless unit of mass as one might think. On the contrary, the Planck mass has a number of well defined physical meanings associated not only to astrophysics [9] and cosmology [10] but also to particle physics. In the remainder of this paper I explain these meanings. The formulas for some of the most common Planck units are shown in table 1.

Eq.	Planck Unit Name	Formula (with *h*)	Formula (with *h-bar*)
(1)	Planck length	$L_P=\sqrt{\frac{hG}{2\pi c^3}}$	$L_P=\sqrt{\frac{\hbar G}{c^3}}$
(2)	Planck time	$T_P=\sqrt{\frac{hG}{2\pi c^5}}$	$T_P=\sqrt{\frac{\hbar G}{c^5}}$
(3)	Planck mass	$M_P=\sqrt{\frac{hc}{2\pi G}}$	$M_P=\sqrt{\frac{\hbar c}{G}}$
(4)	Planck momentum/ Planck Impulse	$P_P=\sqrt{\frac{hc^3}{2\pi G}}$	$P_P=\sqrt{\frac{\hbar c^3}{G}}$
(5)	Planck energy/ Planck Work	$E_P=\sqrt{\frac{hc^5}{2\pi G}}$	$E_P=\sqrt{\frac{\hbar c^5}{G}}$
(6)	Planck frequency	$\nu_P=\sqrt{\frac{2\pi c^5}{hG}}$	$\nu_P=\sqrt{\frac{c^5}{\hbar G}}$

(7)	Planck wavelength	$\lambda_P = \sqrt{\frac{2\pi h G}{c^3}}$	$\lambda_P = 2\pi\sqrt{\frac{\hbar G}{c^3}} = 2\pi L_P$
(8)	Planck acceleration	$a_P = \sqrt{\frac{2\pi c^7}{h G}}$	$a_P = \sqrt{\frac{c^7}{\hbar G}}$
(9)	Planck mass density	$\delta_P = \frac{2\pi c^5}{h G^2}$	$\delta_P = \frac{c^5}{\hbar G^2}$
(10)	Planck force	$F_P = \frac{c^4}{G}$	—
(11)	Planck Spherical Area	$A_{SP} = 4\pi L_P^2$	—
(12)	Planck Spherical Volume	$V_{SP} = \frac{4}{3}\pi L_P^3$	—
(13)	Planck electric charge	$Q_P = \sqrt{2\epsilon_0 h c}$	—
(14)	Planck current	$I_P = c^3\sqrt{\frac{4\pi\epsilon_0}{G}}$	—
(15)	Planck Schwarzchild radius	$R_{SP} = 2\sqrt{\frac{h G}{\pi c^3}}$	—
(16)	Planck entropy (black holes)	$S_{BHP} = \pi k_B$	—
(17)	Planck speed	$c_P = c$	—

TABLE 1: Some Planck units.

The nomenclature used throughout this paper is included in Appendix 1.

2. The Meanings of the Planck Mass

In this section I shall discuss the definitions of the gravitational coupling constants, the role of Planck mass in relativistic kinematics, the relation of the Planck mass with the Heisenberg uncertainty relations, the relation with photons, the mass of the smallest possible black hole, the mass of the universe at the very beginning of normal time, the Chandrasekhar mass limit for white dwarfs, and finally the relation of the Planck mass with the mass and the radius of the proton. There are some meanings relating to astrophysics that I left out intentionally, but not without including a reference for those readers interested in the topic.

2.1. Meaning 1
Gravitational Coupling Constant for the Proton

An excellent paragraph from a Wikipedia web page [11] reads

" α_G can be defined in terms of any pair of charged elementary particles that are stable and well-understood. A pair of electrons, of protons, or one electron and one proton all satisfy this criterion."

I shall define two gravitational coupling constants, one for the proton and one for the electron (see subsection 2.2). The Planck mass is used in both definitions. The gravitational coupling constant for the proton is defined as follows

$$\alpha_{Gp} = \alpha_G = \left(\frac{m_p}{M_P} \right)^2 \tag{2.1.1}$$

It is worthwhile to remark that this formula is the definition of the gravitational coupling constant for the proton and not the definition of the Planck mass. We can get to this formula very easily. We shall start from the definition of the Planck mass given by equation (3) of table 1

$$M_P = \sqrt{\frac{h\,c}{2\,\pi\,G}} \tag{2.1.2}$$

This formula may be written as

$$G\,M_P^2 = \frac{h\,c}{2\,\pi} \tag{2.1.3}$$

On the other hand, we shall also consider the gravitational force, F, between two protons at rest (the proton mass is denoted by m_p) which are separated by a distance denoted by r

$$F = \frac{G\,m_p^2}{r^2} \tag{2.1.4}$$

This equation may be written as

$$G\,m_p^2 = F\,r^2 \tag{2.1.5}$$

Now we define the gravitational coupling constant, α_{Gp} , for the proton as the ratio between equations (2.1.5) and (2.1.3). This gives

$$\alpha_G = \frac{G\,m_p^2}{G\,M_P^2} \tag{2.1.6}$$

Finally, after simplification we get

$$\alpha_G = \frac{m_p^2}{M_P^2} \tag{2.1.7}$$

Which is equation (2.1.1). The the gravitational coupling constant for the proton (and for any other particle) is a dimensionless number and its value is

$$\alpha_G \approx 5.9057 \times 10^{-39} \qquad \text{(value of } \alpha_G \text{)}$$

It is interesting to compare this coupling constant with the electromagnetic coupling constant, α , by evaluating the following ratio

$$\frac{\alpha}{\alpha_G} = \frac{e^2}{2\epsilon_0 h c}\left(\frac{M_P}{m_p}\right)^2 \approx 1.236 \times 10^{36} \tag{2.1.8}$$

Thus, the electromagnetic coupling constant is about 1.236×10^{36} times bigger than the gravitational coupling constant for the proton. This result shows the weakness of the gravitational force in comparison to the electromagnetic force. By the way, it is worthwhile to point out that gravitation is the weakest of all known forces of nature.

2.2. Meaning 2
Gravitational Coupling Constant for the Electron

The Planck mass is used to define the gravitational coupling constant for the electron

$$\alpha_{Ge} = \left(\frac{m_e}{M_P}\right)^2 \tag{2.2.1}$$

It is worthwhile to remark that this formula is the definition of the gravitational coupling constant for the electron and not the definition of the Planck mass. This formula may be derived in a similar way as we did for the proton.

We may compare this coupling constant with the electromagnetic coupling constant by computing the following ratio

$$\frac{\alpha}{\alpha_{Ge}} = \frac{e^2}{2\epsilon_0 h c}\left(\frac{M_P}{m_e}\right)^2 \approx 4.166 \times 10^{42} \tag{2.2.2}$$

Thus, the electromagnetic coupling constant is about 4.166×10^{42} times bigger than the gravitational coupling constant for the electron. Once again, this result shows the weakness of the gravitational force in comparison to the electromagnetic force.

2.3. Meaning 3
The Planck Mass and the Velocity of a Proton

From a point of view of relativistic kinematics, the Planck mass is the mass of a proton that is travelling at a velocity

$$v_p = c\sqrt{1-\alpha_{Gp}} \tag{2.3.1}$$

where α_{Gp} is the gravitational coupling constant for the proton which is given by equation (2.1.1). Let us derive the above formula for any particle (proton, electron, etc.). I shall start the derivation with Einstein's relativistic mass law

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2.3.2}$$

I shall also use the definition of Planck mass given by equation (3) of table 1

$$M_P = \sqrt{\frac{h\,c}{2\,\pi\,G}} \tag{2.3.3}$$

Now we ask the question: what is the velocity a particle (proton, electron, etc.) must travel to, so that its relativistic mass to be equal to the Planck mass? The velocity we are looking for has to satisfy the following equation

$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{h\,c}{2\,\pi\,G}} \tag{2.3.4}$$

Thus, we solve this equation for the velocity of the particle, *v*. This is very easy and gives

$$v = c\sqrt{1-\frac{2\,\pi\,m_0^2\,G}{hc}} \tag{2.3.5}$$

This equation may be written as

$$v = c\sqrt{1-\frac{m_0^2}{\frac{hc}{2\,\pi\,G}}} \tag{2.3.6}$$

But the denominator inside the square root is the square of the Planck mass, so that we may write

$$v = c\sqrt{1-\frac{m_0^2}{M_P^2}} \tag{2.3.7}$$

But the ratio inside the square root is the definition of gravitational coupling constant for any massive particle, $\alpha_{any\ particle}$ (whose rest mass is m_0). Therefore we may write

$$v_{any\ particle} = c\ \sqrt{1-\alpha_{G\ any\ particle}} \tag{2.3.8}$$

If the particle is a proton, equation (2.3.8), turns out to be equal to equation (2.3.1)

$$v_{proton} = c\sqrt{1-\alpha_{G\ proton}} \tag{2.3.9}$$

Knowing the value of α_G and the speed of light, we are able to calculate the speed of the proton so that its relativistic mass to be equal to the Planck mass. This gives

$$v_{proton} = c\sqrt{1-5.9057\times 10^{-39}} < c \tag{2.3.10}$$

As we can see, the velocity of the proton must be extremely close to the speed of light (but as shown by the last equation it is less than the speed of light) in order to have a relativistic mass equal to the Planck mass.

2.4. Meaning 4
The Planck Mass and the Velocity of an Electron

The Planck mass is the mass of an electron that is travelling at a velocity given by

$$v_e = c\sqrt{1-\alpha_{Ge}} \tag{2.4.1}$$

where α_{Ge} is the gravitational coupling constant for the electron which is given by equation (2.2.1). This formula is derived from equation (2.3.7) (which applies to all particles) by replacing the rest mass m_0 by m_e .

2.5. Meaning 5
The Planck Length as a function of the Planck Mass

The Planck mass may be used to define the Planck length as follows

$$L_P = \frac{h}{2\pi M_P c} \tag{2.5.1}$$

This equation may be written as

$$M_P c L_P = \frac{h}{2\pi} \tag{2.5.2}$$

or

$$\frac{M_P}{2} c L_P = \frac{\hbar}{2} \tag{2.5.3}$$

This is an extremely fundamental equation. In the following subsections (2.7) I shall explain the reason why this equation is so fundamental.

2.6 Meaning 6
The Planck Mass Defines the Planck Orbital Momentum

In classical physics the orbital momentum, L, of a body of mass m that turns along a circle of radius r at a velocity v is defined by the following relation

$$L = m\, v\, r \tag{2.6.1}$$

Similarly, The Planck mass, the Planck Length and the Planck speed (the speed of light) may be used to define the Planck orbital momentum, C_P, as follows

$$C_P = M_P\, c\, L_P \tag{2.6.2}$$

Using equations (1) and (3) from table 1 we write the above equation as

$$C_P = \sqrt{\frac{h\, c}{2\,\pi\, G}} \sqrt{\frac{h\, G}{2\,\pi\, c^3}}\, c = c \sqrt{\frac{h^2}{(2\,\pi)^2 c^2}} \tag{2.6.3}$$

Hence we get expression for the Planck momentum

$$C_P = \frac{h}{2\pi} = \hbar \tag{2.6.4}$$

Thus, the Planck momentum is equal to the reduced Planck's constant, $\hbar$. This means that equation (2.6.2) may be written as

$$M_P\, c\, L_P = \frac{h}{2\pi} \tag{2.6.5}$$

Figure 1 shows the Planck orbital momentum graphically.

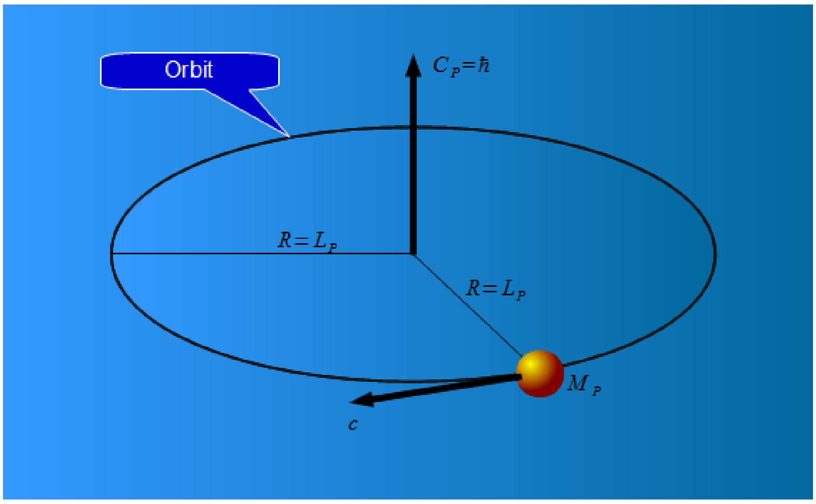

FIGURE 1: The Planck orbital momentum C_P

In order to express the above equation in the form of the Heisenberg uncertainty principle we divide both sides by 2 (I shall explain this equation in full in Meaning 7). We also replace $h/2\pi$ by $\hbar$. This yields

$$\frac{M_P}{2} c\, L_P = \frac{\hbar}{2} \tag{2.6.6}$$

It is worthwhile to observe that this meaning is equivalent to the previous meaning. This is so because the Planck momentum, C_P , turned out to be $\hbar$. This equivalence emphasizes the fact that the reduced Planck's constant is the value of the Planck momentum.

2.7 Meaning 7
The Planck Mass as the Solution to the Equation $p\,r = \hbar$

The Planck mass is the mass, *m*, that satisfies the following equation

$$p(m)\, r = \hbar \tag{2.7.1}$$

Where $p = p(m)$ is the momentum of a photon whose total (or kinetic) energy is equal to the absolute value of the gravitational potential energy, *U*, between two equal masses (each mass is denoted by *m*) separated by a distance *r*. This momentum is given by

$$p = p(m) = \frac{G\, m^2}{r} \frac{1}{c} \tag{2.7.2}$$

And the corresponding energy of the photon is

$$U = \frac{G\, m^2}{r} \tag{2.7.3}$$

In other words, the Planck mass is the mass that satisfies the equation

$$\left(\frac{G\, m^2}{c r}\right) r = \hbar \tag{2.7.4}$$

As may be easily verified, the solution to this equation is the Planck mass

$$m = \sqrt{\frac{\hbar\, c}{G}} = \sqrt{\frac{h\, c}{2\, \pi\, G}} = M_P \tag{2.7.5}$$

One more thing before we finish this point. Because of Einstein's equation of equivalence between mass and energy

$$E = m\, c^2 = U \tag{2.7.6}$$

Equation (2.7.3) may be written as

$$m\, c^2 = \frac{G\, m^2}{r} \tag{2.7.7}$$

This, in turns, means that

$$r = \frac{G\, m}{c^2} \tag{2.7.8}$$

In virtue of equation (2.7.5) we substitute the mass *m* with the Planck mass, M_P . This gives

$$r = \frac{G}{c^2}\sqrt{\frac{\hbar\, c}{G}} = \sqrt{\frac{\hbar\, G}{c^3}} = L_P \tag{2.7.9}$$

Thus we have proved that when $m = M_P$ (the solution to equation (2.7.1)), the distance between the masses is the Planck length, L_P .

Now we focus on the two results we got:

equation (2.7.5) : $m = M_P$, and
equation (2.7.9): $r = L_P$.
Thus substituting, in equation (2.7.1), *p(m)* $M_P\, c$ and *r* by L_P , we get the following equation

$$M_P\, c\, L_P = \hbar \tag{2.7.10}$$

Or, dividing by 2 both sides

$$\frac{M_P}{2} c\, L_P = \frac{\hbar}{2} \tag{2.7.11}$$

I said earlier that this equation was an extremely fundamental equation. The reader will surely recognise this relationship as the Heisenberg uncertainty relation

$$\Delta p_x\, \Delta x \geq \frac{\hbar}{2} \tag{2.7.12}$$

with three "modifications":

(a) the uncertainty in the momentum, Δp, has been replaced by the Planck momentum over 2, $\frac{P_P}{2} = \frac{M_P c}{2}$,
(b) the uncertainty in the position (e.g. Δx) along one of the coordinate axis (e.g. the x axis), has been replaced by the Planck length, L_P, and
(c) the inequation sign (≥) has been replaced by an equation sign (=).

Thus we draw the conclusion that this meaning is equivalent to meanings 5 and 6 analysed above.

Equation (2.5.3), (2.6.6) and (2.7.11) is so fundamental because it represents the Heisenberg uncertainty equation at the beginning of time (It is worthwhile to remark that this is an equation and not an inequation). I shall explain more about this equation at the end of this paper (see conclusions).

2.8 Meaning 8
The Planck Mass as the Equivalent Mass of a Photon whose Frequency is the Inverse of the Planck Time

The Planck mass is the equivalent mass of a photon whose frequency is the inverse of the Planck time (T_P) divided by 2π.

$$M_P = \frac{m_\gamma\left(f = 1/T_P\right)}{2\pi} \tag{2.8.1}$$

The derivation of this formula is very simple. Let's start with the Einstein formulas for the total (or kinetic) energy of a photon

$$E = h\, f \tag{2.8.2}$$

Because the frequency, *f*, of the photon is the inverse of the period, *T,* we can write

$$E = \frac{h}{T} \tag{2.8.3}$$

Now we multiply ad divide by the square of the speed of light, c

$$E = \frac{h}{T c^2} c^2 \tag{2.8.4}$$

According to Einstein's law of equivalence between mass and energy

$$E = m c^2 \tag{2.8.5}$$

In this case the mass *m* is the equivalent mass of a photon (this is different from the rest mass of a photon which is zero). If we compare equation (2.8.4) with (2.8.5) we find that the equivalent mass is

$$m = \frac{h}{T c^2} \tag{2.8.6}$$

Now if we assume that the period *T* is equal to the Planck time, T_P, then the equivalent mass of the photon will be

$$m(T_P) = \frac{h}{T_P c^2} \tag{2.8.7}$$

If we replace, T_P, by its definition given in table 1, we get

$$m(T_P) = \frac{h}{c^2} \sqrt{\frac{2 \pi c^5}{h G}} \tag{2.8.8}$$

Or

$$m(T_P) = 2 \pi \sqrt{\frac{h c}{2 \pi G}} \tag{2.8.9}$$

Because the square root is the definition of the Planck mass, we may write

$$m(T_P) = 2 \pi M_P \tag{2.8.10}$$

Hence

$$M_P = \frac{m(T_P)}{2 \pi} \tag{2.8.11}$$

Which is equation (2.8.1)

2.9. Meaning 9
The Mass of the Smallest Black Hole

The mass of the black hole of minimum radius is equal to the Planck mass over 2.

$$M_{BH\,min} = \frac{M_P}{2} \tag{2.9.1}$$

In order to derive this equation I shall borrow the formula for the Schwarzchild radius (or the radius of a black hole) from Einstein's theory of General Relativity. Thus, the Schwarzchild radius is

$$R_S = \frac{2\,G\,M}{c^2} \tag{2.9.2}$$

Solve this equation for the mass *M* we get

$$M = \frac{R_S\,c^2}{2\,G} \tag{2.9.3}$$

The mass, *M*, of the black hole will be minimum when the value of its radius, R_S ,to be minimum. But, according to Postulate 2 (space quantization postulate) [12], the minimum distance in the Universe is the Planck length, L_P . Thus, the minimum radius of any sphere in the Universe must be the Planck length (this is because the centre of the sphere must be accessible). Consequently, when R_S is equal to L_P , the mass of the black hole will be minimum. I shall denote this mass with $M_{BH\,min}$. This fact can be expressed mathematically as follows

$$M_{BH\,min} = \frac{L_P\,c^2}{2\,G} \tag{2.9.4}$$

Considering the expression for the Planck length given by the corresponding equation of table 1

$$L_P = \sqrt{\frac{h\,G}{2\,\pi\,c^3}} \tag{2.9.5}$$

we may write equation (2.9.4) as follows

$$M_{BH\,min} = \sqrt{\frac{h\,G}{2\,\pi\,c^3}}\,\frac{c^2}{2\,G} = \sqrt{\frac{h\,G\,c^4}{2\,\pi\,c^3\,4\,G^2}} = \sqrt{\frac{h\,c}{2\,\pi\,4\,G}} = \frac{1}{2}\sqrt{\frac{h\,c}{2\,\pi\,G}} = \frac{M_P}{2} \tag{2.9.6}$$

For clarity reasons I shall rewrite the first and the last side of equation (2.9.6). This yields

$$M_{BH\,min} = \frac{M_P}{2} \tag{2.9.7}$$

Figure 2 shows the smallest possible black hole as a sphere of radius equal to the Planck length.

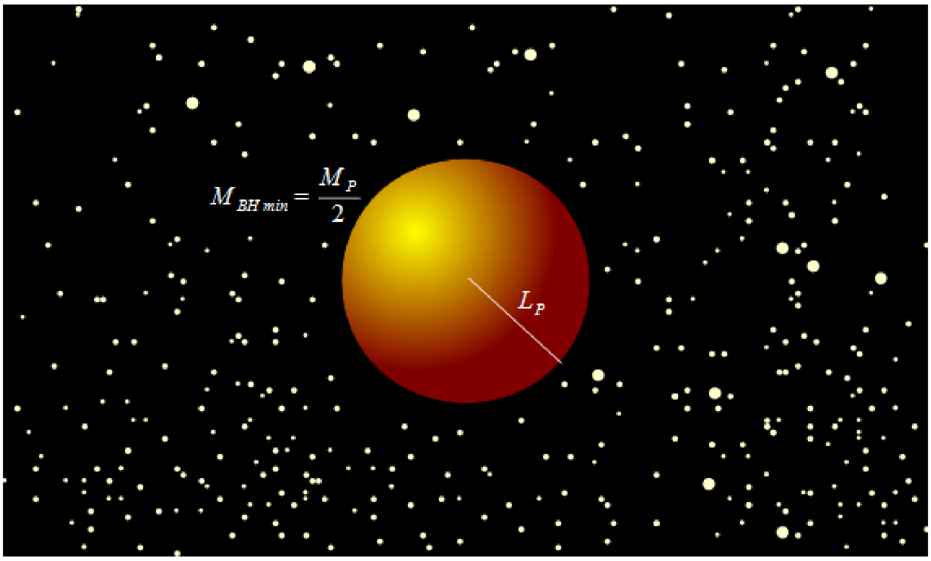

FIGURE 2: A black hole of minimum size. Although this black hole is depicted here in yellow-brownish colour, all black holes are actually invisible. However we could "see" a black hole due to three reasons: (a) the absence of distant stars behind it, (b) the gravitational lensing effects on distant galaxies behind it, and (c) the Hawking radiation which is, normally, extremely weak.

Thus, the minimum mass of a black hole is one half of the Planck mass (It is worthwhile to remark that some people erroneously suggest that the minimum mass of a black hole is the Planck mass).

2.10. Meaning 10
The Mass of the Universe at the Beginning of Time

The mass of the entire Universe, $M_{U0} = M_U(T = T_P)$, at the beginning of time, was equal to the Planck mass over 2. Mathematically

$$M_U(T = T_P) = \frac{M_P}{2} \tag{2.10.1}$$

As we can see from equations (2.8.1) and (2.9.1) the mass of the Universe at the beginning of time was equal to the mass of the smallest possible black hole [10].

Let us derive the above equation. In order to do that I shall assume that, in the beginning, the following equation, based on the Heisenberg uncertainty principle, is valid

$$\Delta E \Delta T = \frac{\hbar}{2} \tag{2.10.2}$$

It is worthwhile to observe that I have used the energy-time Heisenberg uncertainty relation (or temporal Heisenberg uncertainty relation) under the form of an equation. This is the primordial form of the Uncertainty principle. In this equation the energy uncertainty, ΔE, and the time uncertainty, ΔT, are given by

$$\Delta E = \frac{E_P}{2} = \frac{1}{2}\sqrt{\frac{h c^5}{2\pi G}} = \frac{1}{2} c^2 \sqrt{\frac{h c}{2\pi G}} = \frac{M_P}{2} c^2 \tag{2.10.3}$$

and

$$\Delta T = T_P \tag{2.10.4}$$

respectively. Where E_P is the Planck energy, M_P is the Planck mass and T_P is the Planck time. Substituting ΔE and ΔT in equation (2.10.2) with equations (2.10.3) and (2.10.4) we get

$$\frac{M_P}{2} c^2 T_P = \frac{\hbar}{2} \tag{2.10.5}$$

It is worthwhile to remark that at the beginning of normal time, the temporal Heisenberg uncertainty relation was an equation and not an inequation. Equation (2.10.5) suggests that, in the very beginning, the initial mass of the Universe, M_{U0}, was the Planck mass divided by 2. In other words:

$$M_{U0} = \frac{M_P}{2} \tag{2.10.6}$$

Figure 3 shows the Universe at the beginning of time.

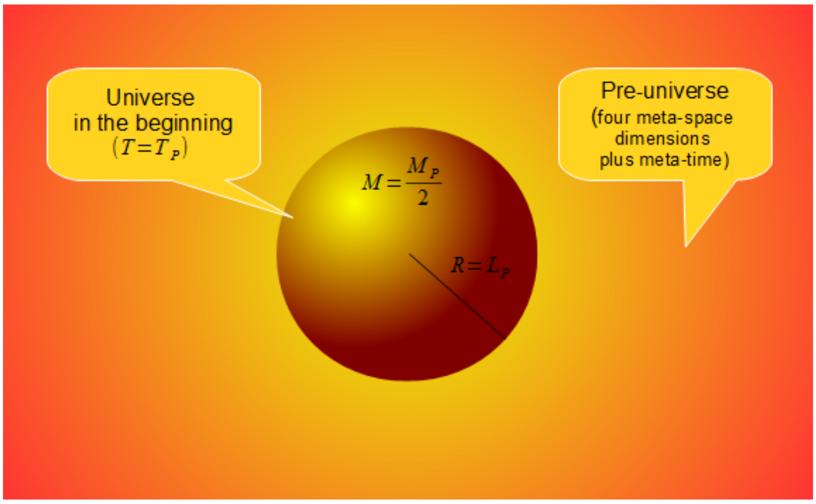

FIGURE 3: The Universe at the beginning of time $T = T_P$

In virtue of equation equation (2.10.6), equation (2.10.5) may be rewritten as

$$M_{U0} c^2 T_P = \frac{\hbar}{2} \tag{2.10.7}$$

Finally, considering that

$$E_{U0} = M_{U0} c^2 \tag{2.10.8}$$

We may rewrite equation (2.10.7) as

The "initial" energy-time Heisenberg uncertainty principle

$$E_{U0} T_P = \frac{\hbar}{2} \tag{2.10.9}$$

This equation may be considered the energy-time Heisenberg uncertainty principle that held at the beginning of time, or to put it into other words, the "initial" energy-time Heisenberg uncertainty principle (note that this relation is an equation and not an inequation).

2.11. Meaning 11
The Chendrasekhar Mass Limit for White Dwarfs

The Planck mass also plays a role in astrophysics. The formula for the Chendrasekhar mass limit for white dwarfs is an example. This formula may be found in reference [13, 14] and is given by the following relation

$$M_{ch} = \frac{M_P^3}{m_p^2} \tag{2.11.1}$$

This mass limit is called the Chendrasekhar limit in honor of the astrophysicist who discovered it. A star which has consumed his usable nuclear fuel and whose mass exceeds this limit will collapse to form a neutron star. The reason of this collapse is that when the Chendrasekhar limit is exceeded, electrons will "combine" (interact) with protons to form neutrons and therefore electrons won't be able to provide the necessary outward pressure (technically called: *electron degeneracy pressure* which is the outward pull caused by the electrons of the star) to prevent gravitational compression (the inward pull caused by gravity). If the star's mass is sufficiently large, the inward gravitational pull will overcome the electronic pressure, causing the star to collapse into an even denser object known as: black hole.

2.12. Meaning 12
The Ratio between the Planck Force and the Planck Acceleration

Before getting to the Planck mass I shall derive Newton's law of universal gravitation from the scale principle [15]. In 1687 Isaac Newton published his Principia where he introduced his law of universal gravitation. This law is described by the following equation

$$F_G = \frac{G\, m_1 m_2}{r^2} \qquad (2.12.1)$$

Where

F_G = Gravitational force between two any bodies of masses m_1 and m_2 (this force is also known as force of universal gravitation, gravity, gravity force, force of gravitational attraction, force of gravity, Newtonian force of gravity, force of universal mutual gravitation, etc.)
G = Gravitational constant (also known as constant of gravitation, constant of gravity, gravitational force constant, universal constant of gravity, universal gravitational\ constant, Newtonian gravitational constant, etc.)
m_1 = mass of body 1
m_2 = mass of body 2
r = distance between the centers of body 1 and body 2

We draw the following scale table

Work	Work	Energy	Energy
W_G	W	E_1	E_2

TABLE 2: This scale table (or scaling table) is used to derive Newton's law of universal gravitation.

For table 2 to work, the quantities shown on this table must be defined as follows

$$W_G = F_G\, r \qquad (2.12.2)$$

$$W = F_P\, r = M_P\, a_P\, r \qquad (2.12.3)$$

$$E_1 = m_1\, c^2 \qquad (2.12.4)$$

$$E_2 = m_2\, c^2 \qquad (2.12.5)$$

From the table we establish the following relationship

$$W_G W = S E_1 E_2 \tag{2.12.6}$$

Replacing the variables W_G , W , E_1 *and* E_2 by equations (2.12.2), (2.12.3), (2.12.4) and (2.12.5), respectively, we get

$$F_G r F_P r = S m_1 c^2 m_2 c^2 \tag{2.12.7}$$

$$F_G = S \frac{G}{c^4} \frac{1}{r^2} m_1 c^2 m_2 c^2 \tag{2.12.8}$$

$$F_G = S G \frac{m_1 m_2}{r^2} \tag{2.12.9}$$

If $S = 1$ we obtain the Newton's law of universal gravitation (see equation (2.12.1)). But, where does the Planck mass fit in all this? The Planck mass may be derive from the Planck force, F_P and the Planck accelerationm. The derivation is as follows. Let us consider the Newton's second law of motion

$$F = m a \tag{2.12.10}$$

Where we have assumed that the mass of the body or particle, *m*, does not vary with time. The Planck force is defined as

$$F_P \equiv M_P a_P \tag{2.12.11}$$

Where a_P is the Planck acceleration. Therefore equation (2.12.7) may be rewritten as

$$F_G r M_P a_P r = S m_1 c^2 m_2 c^2 \tag{2.12.12}$$

Hence, solving for the Planck mass

$$M_P = S \frac{c^4}{F_G a_P} \frac{m_1 m_2}{r^2} \tag{2.12.13}$$

In virtue of equation (2.12.1) and making the scaling factor, *S,* equal to 1, we get

$$M_P = \frac{c^4}{G a_P} \tag{2.12.14}$$

Therefore the ratio c^4/G must be a force, more exactly it must be the Planck force (there are other ways of getting this result)

$$F_P = \frac{c^4}{G} \tag{2.12.15}$$

Thus, we have derived the formula for the Planck force without using the Planck mass. Because the Planck mass is the Planck force divided by the Planck acceleration all we need to do now is to calculate the Planck acceleration. To calculate the Planck acceleration (which is constant) we use the concept of acceleration which is defined as the change in the speed of a given body divided by the time taken to complete the change (*a = dv/dt* for the expert). Then we define the Planck acceleration as

$$a_P = \frac{c-0}{T_P} = \frac{c}{T_P} \tag{2.12.16}$$

Thus, the Planck acceleration is the acceleration of a body or particle when its velocity changes from zero (e.g. from rest) to the speed of light (the fastest speed) in a time interval equal to the Planck time (the shortest time interval). Thus, the Planck acceleration, according to the Einsteinian philosophy, is the fastest possible acceleration. However, according to Einstein, massive bodies cannot travel at the speed of light or faster than this speed. Then, we draw the conclusion that the Planck acceleration cannot be achieved by any massive body or particle. However, it seems there are particles that could achieve this acceleration when they come into existence. These particles are photons (or gravitons if they really exist). If photons are really massless particles, then a photon can achieve the Planck acceleration when is generated (when “is born”) provided that it is created in a time equal to the Planck time. But, how do we get the formula for the Planck acceleration? To get the formula for the Planck acceleration we use equation (2.12.16) where we replace the Planck time by the corresponding formula given in table 1. This yields

$$a_P = \frac{c}{\sqrt{\frac{hG}{2\pi c^5}}} \tag{2.12.17}$$

Finally, we get the equation for the Planck acceleration

$$a_P = \sqrt{\frac{2\pi c^7}{hG}} \tag{2.12.18}$$

Because we have derived the expressions for both the Planck force and the Planck acceleration independently from the Planck mass (without using the formula for the Planck mass), then we might define the Planck mass from equation (2.12.11) by solving it for the Planck mass. This result can then be taken as a possible definition of the Planck mass. This definition would be as follows

$$M_P \equiv \frac{F_P}{a_P} \tag{2.12.19}$$

2.13. Meaning 13
The Mass of the Proton

In a previous article [16] I proposed a formula for the mass of the proton based on the Planck mass and the electron rest mass. The formula is

$$m_p = \frac{m_e^2}{\alpha^{12} M_P \left(1 - \alpha^{12} \frac{M_P}{m_e}\right)} \tag{2.13.1}$$

Taking into account the definition of the gravitational coupling constant for the electron, equation (2.2.1), the above formula may be written in terms of this constant

$$m_p = \frac{m_e}{\frac{\alpha^{12}}{\sqrt{\alpha_{\text{Ge}}}} \left(1 - \frac{\alpha^{12}}{\sqrt{\alpha_{\text{Ge}}}}\right)} \tag{2.13.2}$$

Please refer to the above mentioned article for the details relating to this equation.
One more thing before I finish this point. If we define the proton-electron mass ratio as

$$R_{pe} \equiv \frac{m_p}{m_e} \approx 1836.152\ 672 \tag{2.13.3}$$

We may write equation (2.13.2) in a more compact form

$$\frac{m_p}{m_e} = \frac{1}{\frac{\alpha^{12}}{\alpha_{\text{Ge}}^{0.5}} \left(1 - \frac{\alpha^{12}}{\alpha_{\text{Ge}}^{0.5}}\right)} \tag{2.13.4}$$

Thus, this equation relates three physical constants: the proton to electron mass ratio, R_{pe}, the electromagnetic coupling constant, α, and the gravitational coupling constant for the electron, α_{Ge}. This is one of the laws the Standard Model is unable to formulate.

2.14. Meaning 14
The Radius of the Proton

Now I shall show another law that the Standard Model is unable to formulate. The Planck mass in conjunction with the Planck length may be used to find the formula for the radius of the proton. In a previous article [17] I showed how to use the scale law to derive two approximate formulas for the radius of the proton. The formulas are derived from the following equation

$$\frac{r_p}{L_p} = S\,\frac{M_P}{m_p} \tag{2.14.1}$$

where S is the scale factor (or scaling factor). Because the scale factor is an unknown parameter we need to estimate it. Then, we compare the results with the experiment. The first estimate I made uses the following scale factor

$$S = \sqrt{\frac{m_\tau}{m_\mu}} = \sqrt{16.8167} = 4.100\,816\,992 \tag{2.14.2}$$

Thus, the first formula for the radius of the proton, which is, by the way, the least accurate of the two I shall present, is obtained by substituting, into equation (2.14.1), the scale factor with equation (2.14.2), the Planck length with equation (1) and the Planck mass with equation (3). Then, after some simple mathematical work we get

$$r_p = \left(\sqrt{\frac{m_\tau}{m_\mu}}\right)\frac{h}{2\pi\, m_p c} \tag{2.14.3}$$

According to this formula the value of the proton radius is

$$r_p = 8.624\,383\,532\times 10^{-16} m = 0.86244\, fm \qquad \text{(estimate 1)}$$

The second estimate uses the following scale factor

$$S = 4 \tag{2.14.4}$$

As professor J. Barrow explains in his book [3], an interesting point to observe, which was also noticed by A. Einstein, is that, in general, the scale factors (he calls them constants) in physics are relatively small numbers, such as $S = -1, -1/2,\ 1,\ 2, 4, \pi$, 2π , 4π , 8π , 16π , $8\pi/3$, etc. This is another point in favour of the scale law (equation 2.14.1) since this equation works for small scaling factors such as 4. (of course the equation would "work" for huge scaling factors as well, but we would discard these solutions).

By making similar substitutions as we did before, except that we use the value of 4 for the scale factor, we get a more accurate formula (the second formula) for the proton radius

$$r_p = \frac{2h}{\pi\, m_p c} \tag{2.14.5}$$

According to this formula the value of the proton radius is

$$r_p = 8.412\,356\,415\times 10^{-16} m = 0.84124\, fm \qquad \text{(estimate 2)}$$

The proton radius was measured in 2013 by A. Antognini et al. [18] through an experimental method based on the Lamb shift in muonic hydrogen. The observed value for the proton radius was

$$r_{p\ exp} = 0.84087(39)\ fm \qquad \text{(measured)}$$

The two digits surrounded by parenthesis indicate the overall experimental error in the last two digits of the measured value. Therefore the true value of the proton radius must fall between the following minimum and maximum values:

$$r_{p\ exp\ min} = 0.84048\ fm \ \text{ and } \ r_{p\_exp\_max} = 0.84126\ fm$$

,respectively. By comparing the second estimate with this error band we find that the estimate falls into the interval

$$r_{p\_exp\_min} < 0.84124\ fm < r_{p\_exp\_max} \qquad \text{(comparison)}$$

This indicates that the second estimate is very close to the true value of the radius of the proton. This examples shows how the scale law may be used to estimate quantum mechanical properties that are, otherwise, extremely difficult to evaluate.

3. Summary

The following table summarizes the meanings of the Planck mass discussed in the previous section.

MEANING (number)	EQUATION	PHYSICAL MEANING
1	$\alpha_{Gp} = \alpha_G \equiv \left(\frac{m_p}{M_P}\right)^2$	Gravitational coupling constant for the proton is based on the Planck mass
2	$\alpha_{Ge} \equiv \left(\frac{m_e}{M_P}\right)^2$	Gravitational coupling constant for the electron is based on the Planck mass
3	$v_p = c\sqrt{1-\alpha_{Gp}}$	Relativistic kinematics The mass of a proton that is travelling at a velocity v_p is equal to the Planck mass
4	$v_e = c\sqrt{1-\alpha_{Ge}}$	Relativistic kinematics The mass of an electron that is travelling at a velocity v_e is equal to the Planck mass
5	$L_P = \frac{h}{2\pi M_P c}$ implies $M_P\, c\, L_P = \hbar$	The Planck length as a function of the Planck mass

MEANING (number)	EQUATION	PHYSICAL MEANING
6	$C_P = M_P\, c\, L_P$ implies $M_P\, c\, L_P = \hbar$	The Planck momentum depends on the Planck mass
7	$p(m)\, r = \hbar$ implies $\left(\frac{G m^2}{c r}\right) r = \hbar$ implies $M_P\, c\, L_P = \hbar$	Particle physics The Planck mass is the solution to the equation: $p\, r = \hbar$
8	$M_P = \frac{m_\gamma\left(f = 1/T_P\right)}{2\pi}$	Particle physics The Planck mass is the equivalent mass of a photon whose frequency is the inverse of the Planck time
9	$M_{BH\,min} = \frac{M_P}{2}$	Astrophysics The mass of the smallest black hole is the Planck mass divided by 2
10	$M_{U0} = \frac{M_P}{2}$	Cosmology The Mass of the universe at the beginning of time is the Planck mass divided by 2
11	$M_{ch} = \frac{M_P^3}{m_p^2}$	Astrophysics The Chendrasekhar mass limit for white dwarfs is the cube of the Planck mass over the square of the proton mass
12	$M_P \equiv \frac{F_P}{a_P}$	The Planck mass may be defined as the Planck force divided by the Planck acceleration
13	$\frac{m_p}{m_e} = \frac{1}{\frac{\alpha^{12}}{\alpha_{\mathrm{Ge}}^{0.5}}\left(1 - \frac{\alpha^{12}}{\alpha_{\mathrm{Ge}}^{0.5}}\right)}$	Particle physics The mass of the proton is a function of the Planck mass (The formula relates three fundamental ratios)
14	$\frac{r_p}{L_P} = S\,\frac{M_P}{m_p}$ where $S \approx 4$	Particle physics The ratio of the proton radius to the Planck length is proportional to the ratio of the Planck mass to the proton mass. The proportionality constant is the scale factor S

TABLE 3: The equations that give rise to the different meanings of the Planck mass and the corresponding physical meanings.

3. Conclusions

Without doubt the Planck mass is a multicoloured unit - No other Planck unit presents so many facets as those presented by the Planck mass. We have seen that, at the time of the big bang, this is at time equal to the Planck time, the amount of matter was equal to the Planck mass over 2 and the radius of the universe was equal to the Planck length. This means that from the very beginning the Heisenberg uncertainty relations were established in its primordial form as shown by the following two equations:

$$E_{U0}\, T_P = \frac{\hbar}{2} \qquad (3.1 = 2.10.9)$$

$$M_{U0}\, c\, L_P = \frac{\hbar}{2} \qquad (3.2)$$

But taking a closer look we realize that these two equations are identical. In fact, if we consider Einstein's most famous formula

$$E_{U0} = M_{U0}\, c^2 \qquad (3.3)$$

and the fact that the Planck time is the Planck length divided by the speed of light

$$T_P = \frac{L_P}{c} \qquad (3.4)$$

We may write equation (3.1) as

$$M_{U0}\, c^2 \frac{L_P}{c} = \frac{\hbar}{2} \qquad (3.5)$$

And this result is identical to equation (3.2). Now, in virtue of equation (2.10.6), which is

$$M_{U0} = \frac{M_P}{2} \qquad (3.6 = 2.10.6)$$

we can rewrite equation (3.2) in the form of the Heisenberg uncertainty principle:

Fundamental law

$$\frac{M_P}{2}\, c\, L_P = \frac{\hbar}{2} \qquad (3.7)$$

This coincides with equations (2.5.3), (2.6.6) and (2.7.11). This is an amazing result that emphasizes how fundamental the Heisenberg relations are. We are now in the position of understanding the reason why the Planck mass has the relatively enormous value it has. This is so because the initial mass and size (radius) of the universe had to comply with the Heisenberg uncertainty equation given by the above fundamental law. Because the shape of the universe at the beginning was a sphere of radius equal to the Planck length, the mass of the universe at that time had to be equal to the Planck mass divided by 2 in order to satisfy the fundamental law (3.7). In summary, the reason why the Planck mass is so unusually big is because the Planck length is extremely small, or from a temporal point of view, because the Planck time is extremely small. But there is still something even more amazing. The fact that in the beginning there was a momentum, P_{U0} , given by

$$P_{U0} = \frac{M_P}{2} c \tag{3.8}$$

indicates that our universe, at the beginning of time, was in motion with respect to something else, otherwise this momentum wouldn't be meaningful. Then, it is natural to ask the question: with respect to what was our universe in relative motion? The answer to this question is that the universe was in relative motion with respect to the Meta-universe or Pre-universe whose existence I postulated in 2014 [19]. It is worthwhile to observe that equation (3.8) indicates the motion was at the speed of light. This clearly violates the Einstein's principle that states that a massive object cannot reach or exceed the speed of light. However, there are, at least, two possibilities. (a) The first possibility is that the universe was created already at the speed of light with respect to the pre-universe. Therefore, the universe did not undergo any acceleration, since, at $T=0$, there was no universe. The universe started its existence at a time equal to the Planck time before that time there was no universe (only pre-universe). I have called this process *luminar creation* [20]. (b) The second possibility is that the relativistic mass law, given by equation (2.3.2), does not apply to the initial mass of the universe, $M_P/2$, when the motion is relative to the pre-universe. This simply means that special relativity (SR) does not hold between the pre-universe and the universe. SR only holds within our universe (This seems to be true up to certain extent since the quantum phenomenon known as entanglement seems to violate the maximum speed limit postulated by Einsten's SR, at least, with respect to the speed of the information between entangled particles. Einstein referred to entanglement as a *spooky action at a distance*).

It is also possible that our universe was not only in motion with respect to the pre-universe, but also in relative motion with respect to other universes. Theses universes are the so called "parallel universes". Thus, we draw the main conclusion of this paper: the above fundamental law proves the existence of a pre-universe and, it also suggests, the existence of parallel universes.

Appendix 1
NOMENCLATURE

The following are the symbols used in this paper

$c=$ speed of light in vacuum
$h=$ Planck's constant
$\hbar=$ reduced Planck's constant
$G=$ Newton's gravitational constant
$\epsilon_0=$ permittivity of vacuum
$k_B=$ Boltzmann's constant
$e=$ elementary electrical charge

$r =$ distance between two particles or bodies
$f =$ frequency of a photon (frequency of the electromagnetic radiation)
$v =$ speed of a body or particle of mass m
$m =$ in 2.3: mass of any particle. In 2.7: equivalent mass of a photon.
$m_0 =$ rest mass of any particle or body
$m_1 =$ mass of a particle 1 or body 1
$m_2 =$ mass of a particle 2 or body 2
$m_\gamma =$ equivalent mass of a photon
$m_p =$ proton rest mass
$m_e =$ electron rest mass
$v_p =$ velocity of a proton
$v_{proton} =$ velocity of a proton
$v =$ velocity of any particle or body
$v_{any\ particle} =$ velocity of any particle
$v_e =$ velocity of an electron
$r_p =$ radius of the proton
$r_{p\ exp} =$ measured value of the radius of the proton (mean value)
$r_{p\ exp\ max} =$ measured value of the radius of the proton plus the overall error
$r_{p\ exp\ min} =$ measured value of the radius of the proton minus the overall error
$M_{BH\ min} =$ minimum mass of a black hole
$M_U(T_P) =$ initial mass of the entire Universe
$M_{U0} =$ initial mass of the entire Universe
$P_U(T_P) =$ initial momentum of the entire Universe
$P_{U0} =$ initial momentum of the entire Universe
$E_U(T_P) =$ initial energy of the entire Universe
$E_{U0} =$ initial energy of the entire Universe
$M_{ch} =$ The Chandrasekhar mass limit
$p =$ momentum of a photon (in general is the momentum of a body or particle)
$p(m) =$ momentum of a photon (in general is the momentum of a body or particle)
$E =$ total relativistic energy of a body or particle
$E_1 =$ total relativistic energy of a body or particle
$E_2 =$ total relativistic energy of a body or particle
$E_{U0} =$ initial energy of the entire Universe
$U =$ absolute value of the gravitational potential energy between two equal masses separated by a distance r.
$W_G =$ work done by the gravitational force over certain distance r
$W =$ work done by the Planck force over certain distance r
$S =$ scale factor (or scaling factor)
$\Delta p =$ uncertainty in the momentum of a particle
$\Delta p_x =$ uncertainty in the momentum of a particle along the x axis
$\Delta x =$ uncertainty in the position of a particle
$\Delta E =$ uncertainty in the energy of a particle (or in the energy of the Universe at the beginning of normal time)
$\Delta T =$ uncertainty in the lifetime of a particle (or in the lifetime of the Universe at the beginning of normal time)
$\alpha_{Gp} =$ gravitational coupling constant for the proton
$\alpha_G =$ gravitational coupling constant for the proton

$\alpha_{Ge}=$ gravitational coupling constant for the electron
$\alpha_{G\,any\,particle}=$ gravitational coupling constant for any particle
$\alpha=$ electromagnetic coupling constant, fine-structure constant or atomic structure constant
$L_P=$ Planck length
$T_P=$ Planck time
$M_P=$ Planck mass
$P_P=$ Planck momentum
$C_P=$ Planck orbital momentum
$E_P=$ Planck energy
$a_P=$ Planck acceleration
$\delta_P=$ Planck mass density
$F_P=$ Planck force
$A_{SP}=$ Planck spherical area
$V_P=$ Planck spherical volume
$Q_P=$ Planck electrical charge
$I_P=$ Planck current
$\lambda_P=$ Planck wavelength
$\nu_P=$ Planck frequency
$R_{SP}=$ Planck Schwarzchild radius
$S_{BHP}=$ Planck black hole entropy
$c_P=$ Planck speed in vacuum (which is the same as the speed of light in vacuum, c)

REFERENCES

[1] NIST, *Fundamental Physical Constants—Extensive Listing*, pdf file retrieved from http:// physics.nist.gov/constants, (2010).
[2] P. J. Mohr, B. N. Taylor, D. B. Newell, *CODATA Recommended Values of the Fundamental Physical Constants: 2010*, pdf file retrieved from the internet, (2010).
[3] J. D. Barrow, *The Constants of Nature*, Chap 2, (2002).
[4] Wikipedia, *Planck Units*, retrieved from: http: //en.wikipedia.org/wiki/Planck_units, (2007).
[5] F. Wilczek, *Scaling Mount Planck I, A View from the Bottom*, Physics Today, June 2001, p12-13, (2001).
[6] F. Wilczek, *Scaling Mount Planck III, Is That All There Is?*, Physics Today, August 2002, p10-11, (2002).
[7] M. Planck, *Uber Irreversible Strahlungsvorgange*, S.B. Preuss Akad Wis, 5, pp. 440-480. Translation into English M. Masius, The Theory of Heat Radiation, Dover, New York, 1959. (1899).
[8] NIST, *Fundamental Physical Constants: Planck mass*, retrieved from: http: //physics.nist.gov/cgi-bin/cuu/Planck mass, (2010).
[9] C. Sivaram, *WHAT IS SPECIAL ABOUT THE PLANCK MASS?*, Indian Institute of Astrophysics, (2007).
[10] R. A. Frino, *The Quantum Gravitational Cosmological Model without Singularity*, vixra.org: vixra 1508.0203, (2015).
[11] Wikipedia, *Gravitational coupling constant*, retrieved from: http://

en.wikipedia.org /wiki/Gravitational_coupling_constant.html, (2012).
[12] R. A. Frino, *Quantum Gravitational Relativity – Part I*, vixra.org: vixra 1505.0160, (2015).
[13] C. Sivaram, *WHAT IS SPECIAL ABOUT THE PLANCK MASS?*, Indian Institute of Astrophysics, pdf file: 0707.0058.pdf, p 6, (2007).
[14] S. Chandrasekhar, *The Role of General Relativity in Astronomy: Retrospect and Prospect,* pdf file, J. Astrophys. Astr. 1 (1), (1980).
[15] R. A. Frino, *Newton's Law of Universal Gravitation and the Scale Principle*, vixra.org: vixra 1407.0023, (2014).
[16] R. A. Frino, *The Mass of the Proton*, vixra.org: vixra 1408.0090, (2014).
[17] R. A. Frino, *Scale Factors and the Scale Principle (The Scale Law)*, vixra.org: vixra 1405.0270, (2014).
[18] A. Antognini et al, *Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen*, Science 339 (6118): 417–420, (2013).
[19] R. A. Frino, *Where Do the Laws of Physics Come From?*, vixra.org: vixra 1407.0103, (2014).
[20] R. A. Frino, *Elementary Differences between Energy and Mass,* vixra.org: vixra 1409.0128, (2014).