Wave function collapse in Linguistic interpretation of quantum mechanics

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Abstract

Recently I proposed the linguistic interpretation of quantum mechanics, which is characterized as the linguistic turn of the Copenhagen interpretation of quantum mechanics. This turn from physics to language does not only extend quantum theory to classical theory but also yield the quantum mechanical world view. Although the wave function collapse is prohibited in the linguistic interpretation, in this paper I show that the phenomenon like wave function collapse can be realized. Hence, I propose the justification of the projection postulate in the linguistic interpretation.

Key phrases: Copenhagen interpretation, Wave function collapse, von Neumann-Lüders projection postulate

1 Preparations

Recently in [3]-[6], I proposed measurement theory (i.e., quantum language, or the linguistic interpretation of quantum mechanics), which is characterized as the linguistic turn of the Copenhagen interpretation of quantum mechanics. This turn from physics to language does not only extend quantum theory to classical theory but also yield the quantum mechanical world view. The linguistic interpretation says that

(A) *Only one measurement is permitted.* And thus, the state after a measurement is meaningless since it can not be measured any longer. Thus, the collapse of the wavefunction is prohibited. We are not concerned with anything after measurement. That is, any statement including the phrase "**after the measurement**" is wrong. Also, the causality should be assumed only in the side of system, however, a state never moves. Thus, the Heisenberg picture should be adopted, and thus, the Schrödinger picture should be prohibited. (For details, see [4, 6].)

Therefore, the wave function collapse is meaningless in the linguistic interpretation. In this sense, the linguistic interpretation and the Copenhagen interpretation are different.

Although my idea proposed in this paper was discovered in the investigation of quantum language, it may be understood without the knowledge of quantum language. Hence, the readers are not required to have the usual knowledge of quantum language, but that of quantum mechanics.

1.1 Hilbert space

According to ref.[10], we briefly introduce the mathematical formulation of quantum mechanics as follows.

Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|F\|_{B(H)} = \sup_{\|u\|_H=1} \|Fu\|_H$), in which quantum mechanics is formulated. Define $Tr(H)$, the trace class, by $Tr(H) = B(H)_*$ (i.e., pre-dual space). For any $u, v \in H$, define $|u\rangle\langle v| \in B(H)$ such that

$$(|u\rangle\langle v|)w = \langle v, w\rangle u \quad (\forall w \in H). \tag{1}$$

The trace map $\mathrm{Tr}_H : Tr(H) \to \mathbb{C}$(= the complex field) is defined by

(B) $\quad \mathrm{Tr}_H(T) = \sum_{k=1}^{\infty} \langle e_k, Te_k\rangle \quad (\forall T \in Tr(H))$

where it does not depend on the choice of the complete orthonormal system $\{e_k\}_{k=1}^{\infty}$. The mixed state space $Tr_{+1}(H)$ is defined by $\{\rho \in Tr(H) \mid \rho \geq 0, \quad \mathrm{Tr}_H(\rho) = 1\}$.

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1.2 Observables, state, Markov operator

We define the observable $\mathsf{O} = (X, \mathcal{F}, F)$ in $B(H)$ (or, POVM, *cf* [1]) such that

(C$_1$) X is set, $\mathcal{F}$ ($\subseteq 2^X$: the power set of X) is a σ-field.

(C$_2$) $F : \mathcal{F} \to B(H)$ is a map such that $0 = F(\emptyset) \le F(\Xi) \le F(X) = I$ (= the identity) ($\forall \Xi \in \mathcal{F}$),

(C$_3$) for any countable decomposition $\{\Xi_1, \Xi_2, \ldots, \Xi_n, \ldots\}$ of Ξ $\Big($ i.e., $\Xi = \bigcup_{n=1}^{\infty} \Xi_n$, $\Xi_n \in \mathcal{F}, (n = 1, 2, \ldots)$, $\Xi_m \cap \Xi_n = \emptyset \ \ (m \ne n)$ $\Big)$, it holds that

$$\langle u, F(\Xi)u \rangle = \lim_{n\to\infty} \sum_{k=1}^{n} \langle u, F(\Xi_k)u \rangle \qquad (\forall u \in H) \tag{2}$$

Also, a pure state is represented by $\rho = |u\rangle\langle u|$ (where $u \in H$, $\|u\| = 1$).

Let H_1 and H_2 be Hilbert spaces. A continuous linear operator $\Phi : B(H_2) \to B(H_1)$ is said to be a Markov operator, if the pre-dual operator $\Phi_* : Tr(H_1) \to Tr(H_2)$ satisfies that $\Phi_*(Tr_{+1}(H_1)) \subseteq Tr_{+1}(H_2)$.

1.3 Axioms

A measurement of an observable $\mathsf{O} = (X, \mathcal{F}, F)$ for a state $\rho(= |\rho\rangle\langle u|)$ is denoted by $\mathsf{M}_{B(H)}(\mathsf{O} :=(X, \mathcal{F}, F)$, $S_{[\rho]})$.

Now we introduce two axioms as follows.

Axiom 1 [Measurement]. *The probability that a measured value x ($\in X$) obtained by the measurement $\mathsf{M}_{B(H)}(\mathsf{O} :=(X, \mathcal{F}, F)$, $S_{[\rho]})$ belongs to a set $\Xi(\in \mathcal{F})$ is given by*

$$\rho(F(\Xi)) \Big(= \mathrm{Tr}_H(\rho F(\Xi)) = \langle u, F(\Xi)u \rangle \Big)$$

Axiom 2 is presented as follows:

Axiom 2 [Causality]. *Let $t_1 \le t_2$. The causality is represented by a Markov operator $\Phi_{t_1,t_2} : B(H_{t_2}) \to B(H_{t_1})$.*

2 The wave function collapse (i.e., the projection postulate)

2.1 Problem: The von Neumann-Lüders projection postulate

Let λ be a real-valued one-to-one function on $\mathbb{N}(= \{1, 2, ...\})$. Consider the self-adjoint operator $L = \sum_{k=1}^{\infty} \lambda(k) P_{\lambda(k)}$, where $\mathbb{P} = [P_{\lambda(k)}]_{k=1}^{\infty}$ is a spectral decomposition in $B(H)$, that is, $P_{\lambda(k)}(\in B(H))$ is a projection ($\forall k = 1, 2, ...$) such that

$$\sum_{k=1}^{\infty} P_{\lambda(k)} = I$$

$\Big($ As mentioned in [7], it may suffice to discuss the simple case that $\lambda(k) = k$ ($k = 1, 2, ...$) since discussions are the same. However, in this paper. we discuss the general $L = \sum_{k=1}^{\infty} \lambda(k) P_{\lambda(k)}$. $\Big)$ Put $\Lambda = \{\lambda(k) \mid k = 1, 2, ...\}$. The self-adjoint operator L is identified with the observable $\mathsf{O}_L = (\Lambda, 2^{\lambda}, P)$ in $B(H)$ such that

$$P(\{\lambda(k)\}) = P_{\lambda(k)} \qquad (\forall k = 1, 2, ...) \tag{3}$$

Axiom 1 says:

(D$_1$) The probability that a measured value $\lambda(n)$ $(\in \Lambda)$ is obtained by the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_L := (\Lambda, 2^\lambda, P)$, $S_{[\rho]})$ is given by

$$\mathrm{Tr}_H(\rho P_{\lambda(n)}) (= \langle u, P_{\lambda(n)} u \rangle), \quad (\text{ where } \rho = |u\rangle\langle u|) \tag{4}$$

Also, the von Neumann-Lüders projection postulate (in the Copenhagen interpretation, *cf.* [9, 2]) says:

(D$_2$) When a measured value $\lambda(n)$ $(\in \Lambda)$ is obtained by the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_L := (\Lambda, 2^\lambda, P)$, $S_{[\rho]})$, the post-measurement state ρ_a is given by

$$\rho_a = \frac{P_{\lambda(n)} |u\rangle\langle u| P_{\lambda(n)}}{\|P_{\lambda(n)} u\|^2}$$

And furthermore, when a measurement $\mathsf{M}_{B(H)}(\mathsf{O}_F := (X, \mathcal{F}, F)$, $S_{[\rho_a]})$ is taken, the probability that a measured value belongs to $\Xi (\in \mathcal{F})$ is given by

$$\mathrm{Tr}_H(\rho_a F(\Xi)) \Big(= \langle \frac{P_{\lambda(n)} u}{\|P_{\lambda(n)} u\|}, F(\Xi) \frac{P_{\lambda(n)} u}{\|P_{\lambda(n)} u\|} \rangle \Big) \tag{5}$$

Problem 1. In the linguistic interpretation, the phrase: "post-measurement state" in the (D$_2$) is meaningless. Also, the above (=(D$_1$)+(D$_2$)) is equivalent to the simultaneous measurement $\mathsf{M}_{B(H)}(\mathsf{O}_L \times \mathsf{O}_F, S_{[\rho]})$, which does not exist in the case that O_L and O_F do not commute. Hence the (D$_2$) is meaningless in general. Therefore, we have the following problem:

(E) How should the projection postulate (=(D$_1$)+(D$_2$)) be modified and improved? Or, how should it be understood?

In the following section, I answer this problem within the framework of the linguistic interpretation.

2.2 The justification of von Neumann-Lüders projection postulate in the linguistic interpretation

Let $\mathbb{P} = [P_{\lambda(k)}]_{k=1}^\infty$ be as in Section 2.1, and let $\{e_{\lambda(k)}\}_{k=1}^\infty$ be a complete orthonormal system in a Hilbert space K. Define the predual Markov operator $\Psi_* : Tr(H) \to Tr(K \otimes H)$ by, for any $u \in H$,

$$\Psi_*(|u\rangle\langle u|) = |\sum_{k=1}^\infty (e_{\lambda(k)} \otimes P_{\lambda(k)} u)\rangle\langle \sum_{k=1}^\infty (e_{\lambda(k)} \otimes P_{\lambda(k)} u)| \tag{6}$$

or

$$\Psi_*(|u\rangle\langle u|) = \sum_{k=1}^\infty |e_{\lambda(k)} \otimes P_{\lambda(k)} u\rangle\langle e_{\lambda(k)} \otimes P_{\lambda(k)} u| \tag{7}$$

Thus the Markov operator $\Psi : B(K \otimes H) \to B(H)$ is defined by $\Psi = (\Psi_*)^*$.
Define the observable $\mathsf{O}_G = (\Lambda, 2^\Lambda, G)$ in $B(K)$ such that

$$G(\{\lambda(k)\}) = |e_{\lambda(k)}\rangle\langle e_{\lambda(k)}| \qquad (k \in \mathbb{N} = \{1, 2, ...\})$$

Let $\mathsf{O}_F = (X, \mathcal{F}, F)$ be arbitrary observable in $B(H)$. Thus, we have the tensor observable $\mathsf{O}_G \otimes \mathsf{O}_F = (\Lambda \times X, 2^\Lambda \boxtimes \mathcal{F}, G \otimes F)$ in $B(K \otimes H)$.

Fix a pure state $\rho = |u\rangle\langle u|$ $(u \in H, \|u\|_H = 1)$. Consider the measurement $\mathsf{M}_{B(H)}(\Psi(\mathsf{O}_G \otimes \mathsf{O}_F), S_{[\rho]})$. Then, we see that

(F) the probability that a measured value $(\lambda(k), x)$ obtained by the measurement $\mathsf{M}_{B(H)}(\Psi(\mathsf{O}_G \otimes \mathsf{O}_F)$, $S_{[\rho]})$ belongs to $\{\lambda(n)\} \times \Xi$ is given by

$$\begin{aligned}
&\mathrm{Tr}_{H}[(|u\rangle\langle u|)\Psi(G(\{\lambda(n)\})\otimes F(\Xi))] = \mathrm{Tr}_{K\otimes H}[(\Psi_*(|u\rangle\langle u|))(G(\{\lambda(n)\})\otimes F(\Xi))] \\
=&\mathrm{Tr}_{K\otimes H}[(|\sum_{k=1}^{\infty}(e_{\lambda(k)}\otimes P_{\lambda(k)}u)\rangle\langle\sum_{k=1}^{\infty}(e_{\lambda(k)}\otimes P_{\lambda(k)}u)|)(|e_{\lambda(n)}\rangle\langle e_{\lambda(n)}|\otimes F(\Xi))] \\
=&\langle P_{\lambda(n)}u, F(\Xi)P_{\lambda(n)}u\rangle \quad (\forall\Xi\in\mathcal{F})
\end{aligned}$$

(In a similar way, the same result is easily obtained in the case of (7)).
Thus, we see:

(G_1) if $\Xi = X$, then

$$\mathrm{Tr}_{H}[(|u\rangle\langle u|)\Psi(G(\{\lambda(n)\})\otimes F(X))] = \langle u, P_{\lambda(n)}u\rangle \tag{8}$$

(G_2) when a measured value $(\lambda(k), x)$ belongs to $\{\lambda(n)\}\times X$, the conditional probability such that $x\in\Xi$ is given by

$$\frac{\langle P_{\lambda(n)}u, F(\Xi)P_{\lambda(n)}u\rangle}{\langle u, P_{\lambda(n)}u\rangle} = \langle\frac{P_{\lambda(n)}u}{\|P_{\lambda(n)}u\|}, F(\Xi)\frac{P_{\lambda(n)}u}{\|P_{\lambda(n)}u\|}\rangle \quad (\forall\Xi\in\mathcal{F}) \tag{9}$$

where it should be recalled that O_F is arbitrary. Also note that the above is a consequence of Axioms 1 and 2.

Considering the correspondence: (D) $\Leftrightarrow$ (G), that is,

$$\mathsf{M}_{B(H)}(\mathsf{O}_L, S_{[\rho]}) \Big(\text{or, meaningless } \mathsf{M}_{B(H)}(\mathsf{O}_L\times\mathsf{O}_F, S_{[\rho]})\Big) \Leftrightarrow \mathsf{M}_{B(H)}(\Psi(\mathsf{O}_G\otimes\mathsf{O}_F), S_{[\rho]}),$$

namely,

$$(4)\Leftrightarrow(8), \quad (5)\Leftrightarrow(9)$$

there is a reason to assume that the true meaning of the (D) is just the (G). Also, note the taboo phrase "**post-measurement state**" is not used in (G_2) but in (D_2). Hence, we have the answer of Problem 1.

3 Conclusions

As mentioned in Section 1 (A), the wave function collapse (or more generally, the post-measurement state) is prohibited in the linguistic interpretation. Hence, some asked me "How is the projection postulate?" In this paper I answer this question as follows:

(H) The von Neumann-Lüders projection postulate (D_2) concerning the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_L, S_{[\rho]})$ does not hold (i.e., (D_2) is wrong). However, in the linguistic interpretation (i.e., without the phrase: "post-measurement state"), the similar result (G_2) concerning $\mathsf{M}_{B(H)}(\Psi(\mathsf{O}_G\otimes\mathsf{O}_F), S_{[\rho]})$ holds

And therefore, the "projection postulate" is a theorem in the linguistic interpretation.
I hope that my assertion will be examined from various points of view.

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