A note on the Wilson's theorem

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Abstract:

We obtained two interesting congruence relations related to Wilson's theorem.

Introduction:

In[1] David M.Burton , Elementary Number Theory,[93] 2007

John Wilson has proved following theorem

i. (p-1)!+1 is divisible by p, where p is any prime number
$(p-1)! \equiv -1 \pmod{p}$

ii.(p-2)!-1 is divisible by p, where p is any prime number
$(p-2)! \equiv 1 \pmod{p}$

Wilson appears to have guessed this on the basis of numerical computations; at any rate neither he nor Waring knew how to prove it. Confessing his inability to supply a demonstration, Waring added " Theorems of this kind will be very hard to prove, because of the absence of a notation to express prime numbers.''

As an extension of the above theorem we obtained the following results in the empirical process.

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Main Results

Proposition 1:

(p-3)! - $\frac{(p-1)}{2}$ is divisible by p, where p is any prime number greater than 2

(p-3)!≡$\frac{(p-1)}{2}$ (mod p)

Table i.

R= Remainder

p=Prime number

Prime Number (p)	Remainder(R) in (p-3)!
5	2
7	3
31	15
47	23
97	48
641	320
997	498
1277	638
1933	966
2647	1323
6257	3128
30529	15264
1213253	606626
1215649	607824
15,485,863	7742931
32,452,843	16226421
49,979,687	24989843
295,075,147	147537573
472,882,027	236441013
533,000,389	266500194
920,419,813	460209906
982,451,653	491225826
961,748,941	480874470
4,294,967,296	2147483648

Proposition 2 :
As we know any prime number greater than 3 can be expressed as (6k-1) or (6k+1)

Case I:

Let p is primer greater than 3 and of the form (6k-1) then
(p-4)! - k is divisible by p
(p-4)! ≡k(mod p)

Table ii.
R= Remainder
p= Prime number

Prime number p=(6k-1)	Remainder (R) in (p-4)! =k
5	1
11	2
29	5
47	8
89	15
197	33
641	107
1229	205
4259	710
9011	1502
560297	93383
49,979,687	8329948
334,214,459	55702410
817,504,253	136250709

Case II:

Let p is prime number greater than 3 and of the form 6k+1 then
(p-4)!-5k is divisible by p
(p-4)! ≡ 5k(mod p)

Table iii.

Prime Number p=(6k+1)	Remainder(R) in (p-4)!=5k
7	6
31	26
97	81
571	476
2719	2266
12073	10061
30661	25551
538333	448611
1213129	1010941
15,485,863	12904886
32,452,843	27044036
920,419,813	767,016,511

Conclusion and Remarks:

I followed the empirical process and we got the above results. We tried our best and checked up to some high extent using prime numbers. We believe that we can prove those propositions using some advanced Number theory techniques.

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2. http://mathworld.wolfram.com/WilsonsTheorem.html

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