

Math Ontological Basis of Quasi Fine-Tuning in *Ghc* Cosmologies

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*The subject of fine tuning in physics is a long contentious issue especially now as it has hitched a ride on the Multiverse bandwagon. The mathematics of quadratic forms are predominately featured and relate the physics parameters $G h c$, which in turn are weighted during the Planck Era(s) determined by relative Planck time clocking. This simplifies the search to these three values as being the important apparent fine-tuned parameters (quasi fine tuning) for determining the gravitational build structures restricted to SM-4D type Universes. Two gravitational coupling constants (dimensionless) are prescribed within the *Ghc* complex. Both describe the relative rigidity of gravitational physics in the low energy build of our Universe (General Relativity toward endpoint neutron star, black hole formation). A Master vacuum field symmetry relation (Yang-Mills) is presented using both gravitational coupling constants in their respective degenerate domains (electron to neutron) which shows a relative rigid coherent field of parameters from the Codata set showing the interdependency of these values with each other, particularly G, h, c and particle masses. A consensus example aligns the mass-energy value of the charged pi-meson to 140.050502 MeV. The interdependency demands that the gravitational constant's empirical value to be $6.67354236 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ using Codata 2014 values. The Yang-Mills relation has a perfect symmetry (hidden) due to the inclusion of the very weak gravitational charge (Z_{xx}). This is then the weak gravity unification incorporated into the Standard Model. If the Yang-Mills symmetry relation is true then a double copy pion field permeates the observable Universe.*

"I know a room of musical tunes some rhyme, some ching most of them are clockwork, let's go into the other room and make them work..." from the song 'Bike' by Syd Barrett

As of today there is no other outstanding problem in Modern Physics that looms as large and relevant as that of the Hierarchy problem. It appears to be a huge blank space that juts onto ones face as a great mystery that seems to stand in the way of any further progress in high-energy physics. Looking at the Big Gap between our low-energy World of small particle masses and the extremely large Planck mass/energy there is currently the classically named 'particle desert' where apparently nothing interesting happens until a theory or observational (empirical *sense-data: sensa*) result is manifest near or at the enormous Planck energies. At this point this is approximately where the theory of Quantum Gravity will be realized or manifested. Not much has changed since the early 70's (since the weak force Physics work of the late 1960's early 70's, this also included the required Higgs mechanism for that physics to work, yes even before the Higgs boson was found) until the Standard Model Higgs boson was found by the LHC in 2013. That this is currently a *vanilla* SM Higgs boson compounds the issue of quadratic divergences of the Higgs model from the low energy end of things in our current Universe

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toward the extreme Planck energy. Also, the Higgs mass at 125 GeV has indications that it does not fit within the context of *naturalness* and is perhaps meta-stable, appears hopelessly to be 'fine-tuned', which is not exactly what is hoped for in a rational structure of Nature. However, as it stands the Standard Model of physics is a towering structure of success of how our 'particle zoo' fits together (along with its quantum language as a QFT) to describe Nature. It is the unprecedented success of Science but unfortunately does not include gravitation. The other theory which is also (equally) an unprecedented success of Science is Einstein's General Relativity which is a theory of gravitation. The trick (sic ?) is to find how the two most successful theories of all-time talk to each other or to simply combine them in an overarching theory aptly named *Quantum Gravity*. In the string revolutions of the 1970's and 1980's including Superstring theory there is actually a theory of Quantum Gravity inherent in its mathematics where the much sought after 2 spin graviton resides. It is a true theory of Quantum Gravity as the graviton particle (predicted) is the quantization manifestation of gravity as a quantum particle. As the excitement of Superstring Theory built up into the 1990's some stumbling blocks toward the goal of a final theory appeared. Superstring theory includes the mathematical construct of Super symmetry (SUSY) for which every unique vibration of a string in 10, 11 dimensions is a quantum particle which also has a super-particle (which is generally a little bigger mass) which eliminates quadratic mass divergences of the Higgs boson(s), explain the Hierarchy problem and allow for explanations of why the quantum particles have the masses that they have. In one fell swoop Superstring theory would be manifestly a *Quantum Gravitation Theory* and probably a core part of a *Theory of Everything*. The Standard Model and Gravity would at last be combined into a coherent whole. The stumbling block was Edward Witten's discovery (announcement in 1994) that the five consistent Superstring theories and 11 dimensional Supergravity were all manifestations of the same theory which he newly named *M-Theory*. The problem was which vacua represent our World or which superstring theory would represent the correct version of our World? Actually all are represented at the near Planckian energies (except the supergravity theory) such that really the Planckian energy realm does not represent our current understanding of our low-energy vacua construction of our Universe (things like why do our particle masses have the values that they have. How do the symmetries break in a fashion down the scale from the Planck energy)? That was one problem in that where do the super particles lie (or found) within a range that the various approaches to 'beyond the Standard Model' could be validated. The other stumbling block is the plethora of physics parameters and properties that appear to have no mathematical connections to any Universal mathematical structures which would suggest rationality to the build of our Universe. This is known as the problem of *fine tuning* which just means that parameters (physics) of the Standard Model and some cosmological parameters need to have their numbers adjusted (by hand according the literature) in order to match some empirical observations and math results. Or alternatively that the numbers found through empirical method or theoretical requirement have an appearance of randomization and are not connected to any rational physical explanation by math. The parameters then are like islands of randomness which imply that no complete theory can ever be found or that Science will cease to continue to explain things towards a more fundamental realm (absolute minimum of degrees of freedom). To make matters worse it is not only physics parameters which have this needed (apparent) intervention (anthropic) to make things work it appears that the other Sciences such as Biology may have coincidences which require some anthropic leanings. This is strong language. Examples of fine tunings in physics include the cosmological constant,

dimensionless constants from the Standard Model such as proton electron ratio (or particle masses themselves) nuclear baryogenesis which includes the carbon cycle production in stars so necessary for carbon based life forms and so on and so on. When it comes down to it there are very many parameters and things that appear too good to be true in order for the happenstance of our Universe. There appear way too many things to adjust in order to come to any final sense of how things came to be (which is the same as saying that the current state is so complex that it appears impossible to fix the theory in any rational way). In 2003 the orthodox 'string theorists' (through M-theory/and 'formerly known as String Theory') enumerated a beyond plethora account of the number of false vacua which could create SM like Universes (not all would include intelligent life) and concluded no unique number but on the order of magnitude incomprehensible some numbers were figured (loosely?). A typical number in use is 10^{500} which is a finite number but from a computational viewpoint is kind of like infinity. I once asked the question 'does the Monster Group survive in the other $10^{500} - 1$ Universes?' The answer is yes and so does all of the Mathematics that we have been able to discover and observe in this Universe. The same could be asked of physics as well. But here the answers are different depending on what is meant by the question. If one is assuming that by different numerical results of parameters compared to our Universe then yes maybe that is different physics. But, I would like to argue that for any '4 dimensional SM like Universe' generated the physics (and maths) are generally the same. There exists a dipole of existence of these 4D Universes in a range of cold and empty with no nuclear burning or stars (what I would call arctic like) to Universes consisting of fast burning stars and many black holes (what I would like to call hot desert like) with a middle range of Goldilock type Universes with just the right stuff to include ranges of hostile and hospitable conditions against or for life processes. This middle range (in the thickening and thinning of the forest) might actually be wider than thought and random *fine tuning* is just apparent. In addition as part of any viable 4D Universe the running of the gauge forces to a common GUT point is a necessary requirement of the model and that this point is adjusted up or down the Hierarchal scale (generally not too far from the origin Planck energy, this is not just coincidental) according to the type of 4D Universe generated. Hopefully, this argument is made below.

The plethora of false vacua led to the hypotheses of the 'String Landscape' and its mind boggling potential. The fact that it could explain the random islands of physics parameter values for large ranges of different vacua has led to its powerful injection into the ongoing shifting paradigm of physics.

In the Standard Model there are about 26 parameters which define the Lagrangian of the model minus gravity. Some of these parameters can be put together to create dimensionless parameters or constants which have relevance to fundamental physics. In that a final theory (in principle) could calculate these dimensionless constants would be feature of such a theory. The *NIST* establishes the culturally (stated in SI units) dimension-ful constants as the *fundamental physical constants* in their database named *Codata*. As such there is some arbitrariness (cultural) to them and are not really considered fundamental by the high-energy physics community. However, it is contextual and one can go along with the folklore that the proton is a very messy particle (hadron bag model) and there is nothing fundamental about that or its mass. But, what does one realize about the proton or neutron for that matter anyway? If you reduce it then it becomes a system of three quarks and a very little understood gluon field interacting with the vacuum. The proton's mass and spin are not completely described (its charge is however) by this system.

Sounds even messier? It is suggested that for the matter of the low energy physics vacuum that our present world resides in that Codata's designation is ok and not to be rejected outright in terms of the physical constants usefulness in physics and manufacture of dimensionless ratios. It should be noted that *NIST's* Codata is now under a periodic cycle of 4 years for further improvements to the empirical numerical values of the physical constants [1]. It can be said the establishment of the empirical numerical values is now very good with most values being determined out to nine or more significant figures with relatively low uncertainties. The exception to this rule is the Newton constant G of gravity which is only figured to 5-6 significant figures and has endured large uncertainties in its empirical determination for the last 300 years. Experimenters are certain to close in on this value within the next several years.

The Planck Clock defines the physics parameters

If one looks at the Planck Era in the formation of our Universe you see a set of dimensional numbers defined from that particular moment in the Big Bang. If you are in the mindset of there being only one Universe then these Planckian numbers (units) are not questioned as to the measure of their absoluteness (they are basically not thought about to a degree of reduction). But it is obvious from a physicist's viewpoint if they are questioned to this measure they would have to say that indeed such numbers could not be absolute and they are obviously not fundamental. The three Planckian physics numbers in mind (there are others) are:

$$t_p = \sqrt{\frac{hG}{2\pi c^5}}$$

$$E_p = \sqrt{\frac{hc^5}{2\pi G}}$$

$$m_p = \sqrt{\frac{hc}{2\pi G}}$$

, and are the Planck Time, the Planck Energy and Planck Mass respectively. It is believed that the Planckian units represent conditions that may have existed in the era of the Planck time during which *quantum gravity* effects dominated. The most obvious and common thing about these is the form of the square root value. In addition there is the 5th power of c (speed of light) and these two facts imply the import of quadratic relations in Planckian terms and also in the Hilbert or Hierarchal space from the low energy limit to the Planck energy cutoff. If the Planck Time and the Planck Energy squared are multiplied a dimension-ful Planck's constant squared (reduced) is the result,

$$\frac{hG}{2\pi c^5} \times \frac{hc^5}{2\pi G} = \frac{h^2}{4\pi^2}$$

The domain of Planck's constant is over the entire QFT spectrum of the Hierarchal space. If you take the Planck Energy square from and convert it to its relativistic form then one can obtain a dimensionless Ghc form by multiplying by the inverse of the Planck's constant quadratic form,

$$\frac{hG}{2\pi c^5} \times m^2 c^4 \times \frac{4\pi^2}{h^2} = \frac{2\pi Gm^2}{hc}$$

This is equivalent to,

$$\frac{2\pi Gm^2}{hc} = \frac{m^2}{m_p^2}$$

This is the *gravitational coupling constant* (very weak) where m is a SM particle at the low energy and we keep the ubiquitous quadratic form. Currently, as it stands there is no accepted value in the physics community for the GCC and the NIST has not determined an empirically derived value for this as particle choices for m is ambiguous in the literature. The Planck Clock for a spectrum across a landscape range of SM 4D type Universes can be defined as follows,

$$t_p = \sqrt{\frac{hG}{2\pi c^5}} \times_{\Delta H}$$

Where x sub ΔH is possibly a real number defining a fixed Planck time for a SM-4D Universe where H is the very exponentially large hierarchal or Hilbert space if you like. This would generate a huge number of such Universes which we would at present have a shot at understanding reasonably well. It is like looking at beach sand and deciding that the quartz grains are the only ones to concentrate on and understand while ignoring the other grains (or the other abstruse Universes). A defining feature of most SM-4D Universes generated thus would be that the 3 (SM) gauge forces unify at a GUT level for these SM-4D Universes to exist. Although the gauge forces would have different values for modified Planck times the gravity gauge force (or graviton) would be a truly universal force that leaks or communicates throughout the assemblages of the SM-4D set in the Multiverse. This is demonstrated below. The three parameters which are modified in varietal Planck times are G , h and c . Although these three parameters are not fundamental they are important as a grouping used to define two *Gravitational Coupling Constants* (GCC) which are dimensionless numbers one of which is universal (is pure math and exists in the neutron star/black hole formation upper end point of the low energy realm) for most SM-4D Universes the other being subject to apparent *fine tuning* (in the low energy vacuum) and might be different in other SM-4D Universes. The appearance of two *Gravitational Coupling Constants* is directly indicative of violation of the Einstein Equivalence Principle and is directly related to the isospin degeneracies (essentially degeneracy pressure) from the protonic stable gravitational build structures

(stars) to the extreme gravitational end points of neutron star and black hole formations. This highly suggests that although there is violation of the EP the rigidity of constants is very strong and it would be very difficult to detect any changes in constants any closer than ~ 10 significant figures.

Randomization or Mathematical Distribution of Parameters *Ghc*

It is a difficult question of whether the parameters G , h and c represent some core pure mathematical value or has elements of randomization or is purely random. Hence, it is labeled as having *quasi fine-tuned* elements. It would be thought (best) that it is pure mathematics at core but has random elements due to chaotic conditions brought about during formation in the Planck epoch (or time). For our Universe we have culturally (anthropically) dressed most of the physical constants in units of LMT (Length, Mass and Time) or in SI units, metre, kilograms and seconds (m kg s). The Newton constant G has unit complexity of $m^3 kg^{-1} s^{-2}$ and is definitely not a fundamental representation and is not universal across any number of similar Universes. It should be emphasized that G (although related) is not the same as the GCC which is dimensionless. The GCC because of its degeneracy relation to $SU(2)$ isospin will be seen to change first under neutron star/black hole gravitational fields before the Newton constant G even budges (change in value). If one considers the history of the physical science an overriding feature has always been that absolute concepts (or prejudices) are continually overthrown. Examples of this include Aristotle's theories of motion (prejudices) being overthrown by Galileo's experiments and establishments of inertial frames of reference. Another example is Newton's idea of absolute time and space being replaced with Einstein's space time of GR. If it is the case that time is not absolute and fundamental then the Planck time of $5.39116(13) \times 10^{-44}$ seconds cannot be considered to represent anything definite and final in the World(s) game of physics. When one considers many vacuum solutions to generate many Universes of different types then there has to be many Planck (type) times as well. For this discussion we only consider that Ghc generated cosmologies of SM-4D Universes are the only ones at present we can hope to remotely and locally understand. The relative Planck clock is a simple concept. A random distribution of Real numbers can change the outcome of how a SM-4D Universe is built. When one looks at our Planck time you see the three (not fundamental) physics parameters Ghc . These are empirically derived (in our low energy World) and science has defined these parameters in an anthropic complex manner (SI units of m kg s) to a degree that one could never hope to derive a theory that would predict any one of these values. When one looks at the Planck time you do not see what could be at the core of those three parameters as you see only a clock result as defined by our chosen standard units. In or near the beginning the metre (and hence seconds) was not defined but only later by an established existing sophisticated civilization. Our SI units of measure would not be noted in other SM-4D Universes as any civilizations in those Universes will have culturally different invented standards of measure. So Ghc as defined by us is not Multi-Universal recognizable as anything described by us. Here is an example of this plays out. Using the calculation above let $x_{\text{sub } \Delta H}$ represent a Real number. Let's use the integer 8 for convenience (it could be almost any number such as a fraction or irrational etc.),

$$t_p = \sqrt{\frac{hG}{2\pi c^5}} 8 = \sqrt{\frac{64hG}{2\pi c^5}}$$

Then (keeping it simple) distribute the 8 times the Planck time value as, **8G**, **2h** and **0.757858 c** as $0.757858..^5 = 0.25$. Then the new Universe Planck time could be represented as,

$$t_p = \sqrt{\frac{2h8G}{2\pi 0.25c^5}}$$

This specific Planck time defines the *Ghc* parameters as (in our units of course utilizing Codata 2014) $G = 5.339264 \times 10^{-10} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$, $h = 1.325214008 \times 10^{-33} \text{ Js}$ and $c = 227200113 \text{ ms}^{-1}$. These are very different values than what we observe in our Universe.

As another example to try to drive this point let's say the value is multiplicative inverse of 8,

$$t_p = \sqrt{\frac{hG}{2\pi c^5}} \frac{1}{8} = \sqrt{\frac{hc}{128c^5}}$$

in which case the example could be for the 1/8 Planck time value, **0.5G**, **0.5h** and **1.7411011 c** as $1.7411011...^5 = 16.00$. This newly defined Universe Planck time is represented as,

$$t_p = \sqrt{\frac{0.5h 0.5G}{2\pi 16c^5}}$$

These values would then be represented, $G = 3.33704 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$, $h = 3.31303502 \times 10^{-34} \text{ Js}$ and $c = 521968978 \text{ ms}^{-1}$. Again very different values than what is observed in our Universe.

These are fictitious of course and may not generate but serve as an example of the endless combinations that can be produced in this way. At this point there is no explanation as to how or why the Real numbers would distribute in the tri-way of *Ghc* other than some chaotic or randomization mechanism. In this case the values are like a lottery and it would be impossible to determine what 'pure math' structure or core *Ghc* represents. An end result would be that in order to decode what the actual pure value of what *Ghc* represents is that like the *Enigma Machine* of WWII you have to have access to the analogy of rotor codes in order to crack the message. The rotor code in this case will be to know or having access to the values of these three parameters of at least two or more SM-4D Universes in order to begin to deduce rules for the maths at work on this distribution (if it is not just a randomization it could involve quantisation). Unfortunately, the *Godel Incompleteness Theorem* prevents this specific knowledge from being acquired outside the boundaries of our Universe. The concept of a definite *Ghc* mathematical definition looks to be unobtainable, fuzzy which is why it is termed as having quasi-fine-tuned elements. On the other hand there appears to be uses of *Ghc* in mathematical relations which are pure math constructs. In addition the fact that these physics parameters (precision values from the Codata 2014) can be cobbled to generate accurate well defined physics values points to a Universality of certain mathematical forms.

$$GCC = \frac{2\pi Gm^2}{hc} \quad \alpha = \frac{e^2}{2\epsilon_0 hc}$$

That the classical force equations for gravitation, attractive electric (coulomb) and attractive magnetic fields resemble each other was one motivation for Einstein to seek a Unified Field Theory [2].

$$F = \frac{Gmm'}{d^2} \quad F = \frac{Cq^+q^-}{d^2} \quad F = \frac{Km^N m^S}{d^2}$$

This appears to be no coincidence and no randomization involved for these forms to have similar appearances but is related to something in the deep down of mathematics.

The Heegner Number 163, Imaginary Quadratic Fields and Numerical Relevance

In number theory there is a Heegner number (163) that yields the best 'near integer value'. The relation is,

$$\text{Exp}(\pi\sqrt{163}) = 262537412640768743.99999999999925$$

and having 12 9's in the decimal tail. This apparent coincidence is explained through Field Class Theory through Complex Multiplication (CM) involving *imaginary quadratic fields* and that it definitely is not a coincidence [3]. As expressed it is a Real number and is transcendental. There is a neat trick by which if one divides this number by 24 one gets the relation,

$$\frac{\text{Exp}(\pi\sqrt{163})}{24} = 10939058860032030.99999999999969$$

As it has 13 9's in its decimal tail it is a little bit more toward integral than its famous cousin. This more integral version is unique and is almost the prime 10939058860032031. By having the number 24 embedded in the well-known version could it be related to the 24 dimensional concept of the Leech Lattice or some higher dimensional aspect of sphere packing or lattice points? If one considers that $24^2 = 576$ matrix elements involved in building the Leech Lattice [3] [4] [5] one can make that simple step by insisting the value be squared as $\text{Exp}(2\pi \text{sqrt}163)$ which is a very large 'near integer value' too [6]. One would then have a matrix number form, $24^2, 10939058860032030.99999999999969 \times 10939058860032030.99999999999969$ or it could be generalized to $24^2, 10939058860032031^2$ for a full integer value. The Heegner number 163 (famous for its best representation of a near integer via CM) is possibly related to the Monster group via the 194 mini j-functions found in the character table as one reduces the redundancies of the similar mini functions as one goes down the list (194 171 163) and note that $194-163 = 31$ [7]. Also, note that $31 \times 16 = 496$ SUSY elements and $31 \times 24 = 744$ an apparent magic number that nearly appears at the end of $\text{Exp}(\pi \text{sqrt}163) = 262537412640768743.99999999999925$ and 744 appears in the hauptmodul of the Monster j-function ($j\tau - 744$) although that is generally considered only a convenience. The Leech Lattice is the automorphism group of the Monster group. So

there are elements within quadratic value $\text{Exp}(2\pi \sqrt{163})$ that point to the Leech Lattice and the Monster group.

A Surprising and Curious Identity

Sometimes two disparate things can be placed together to create something which calls for a new explanation (because it looks like it works) or simple refutation (it does not work). If you take the quadratic form of the famous 'near integer value' and multiply it by the number 70^2 you obtain a value that is very close in value to a particular physics form,

$$\text{Exp}(2\pi\sqrt{163}) 70^2 = 337736875876935471466319632506024463200.000000802319\dots$$

or the convenient shorter form of the calculated dimensionless number $3.377368\dots \times 10^{38}$ and is very close to the physics form calculation [8]:

$$\text{Exp}(2\pi\sqrt{163}) 70^2 \sim \frac{hc}{\pi G m_n^2}$$

Where m_n = neutron mass and the value of the physics form is dimensionless (using Codata 2014),

$$\frac{hc}{\pi G m_n^2} = 3.37709(16) \times 10^{38}$$

The gravitational constant G is the limiting parameter as it is only out to 6 significant figures. This physics value retains its quadratic form as,

$$\frac{hc}{\pi G m_n^2} = \frac{m_p^2}{m_n^2}$$

Another way of expressing the pure number form turns out to be quite parsimonious in explanatory power as well as using the 70^2 form.

$$\frac{\text{Exp}(2\pi\sqrt{163})}{24^2} 2^2 840^2 = 3.377368\dots \times 10^{38}$$

Comparing the last three 4 year cycles of Codata:

$$\text{Codata 2014} : \frac{hc}{\pi G m_n^2} = 3.37709(16) \times 10^{38}$$

$$\text{Codata 2010} : \frac{hc}{\pi G m_n^2} = 3.37722(40) \times 10^{38}$$

$$\text{Codata 2006 : } \frac{hc}{\pi G m_n^2} = 3.37700(50) \times 10^{38}$$

It appears that although there is not quite a convergence onto the 'pure math value' the dimensionless number is still within very good range of that value. It is not as good as the 2010 value but is better than the 2006 value. That it is possible due to the uncertainties in the Newtonian gravitational constant that there is this slight scatter in establishing a more precise (eventually out to 9 significant figures or more) value should be considered. One can now consider that the other physical constants are probably near correct out to 10-13 significant figures and hence considered to be precise and accurate. The Newton constant (also known as Big G) is not in the same league of precision and accuracy and has been subject to large empirical fluctuations of its value in the last 300 years. The recent level of 0.05% differences is unacceptable. Currently, there are efforts underway to address this most difficult constant. The Royal Society A convened in February 2014 to create a very large international collaboration whose goal is to standardize the gravitational constant experiments so that they are completely unbiased and repeatable [9] [10]. When the next 4 year cycle ends with publication of Codata 2018 hopefully the Newton constant will be stabilized empirically and a precise value finally established.

The Wild Ride of Numerology and Numerical Relevance of Number Theory and Some Facts

Looking at our near identity we have yet to speak of the appearance of the numbers $70^2 = 4900$ and the number 840. Are they relevant? Interestingly our near identity is a quadratic form and is a near perfect square. The number 70^2 is related to a certain solution of a Diophantine equation (Edouard Lucas' cannonball problem) whereby a perfect square pyramid consisting of 4900 elements (spheres, cannonballs etc.) having a base of 24^2 elements is related to the unique property of summation of the first 24 integers on the number line where each integer is squared $1^2+2^2+3^2+\dots+24^2 = 70^2$. The integers 1 and 24 are the only two integers that have this summation squared property producing a perfect square pyramid. This 4900 pyramid has an fcc packing (face centered cubic) and is the most efficient sphere packing arrangement in 3 dimensions. The other equally efficient packing arrangement in 3 dimensions that may have relevance is the tetrahedral perfect square pyramid which has hcp (hexahedral close packing, also called tetrahedral close packing) arrangement. Earlier it was mentioned that the Heegner number 163 in Class 1 Field theory generates a 'near integer' and that this number is related to the Leech Lattice and the Monster group. An amazing property of $0^2+1^2+2^2+3^2+\dots+24^2 = 70^2$ is that it is directly related to the Leech Lattice as it is involved in the 26 dimensional Lorentzian unimodular lattice $II_{25,1}$ (also called the hyperbolic Leech Lattice) [11]. It should be mentioned here that the normal Leech Lattice has the densest, most efficient and optimized hypersphere packing of spheres in 24 dimensions having a kissing number of 196560 whereby a single sphere touches 196560 spheres without overlap. The lattice $II_{25,1}$ uses the property of the first 24 integers squared generating the perfect square in that its Weyl vector $(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24;70)$ which can be utilized to construct the Leech Lattice [11]. Both the Leech Lattice and $II_{25,1}$ have the Conway Group Co_0 and Co_1 respectively as their automorphism group. The Conway Groups and Mathieu Group M_{24} have very important roles in the moonshine interplays and connections with the Leech Lattice. This is important in that our physical model is structured using the Coxeter-Todd Lattice, a 12 dimensional

object which is a sub-lattice of the 24 dimensional Leech Lattice [12]. Richard Borcherds used the 26D bosonic string to prove the Monstrous moonshine conjectures. This was using a string existing in a compactified torus shape of 22 dimensions plus that of 4D space-time of which the construction is related to the 24 dimensional Leech Lattice. The lattice $II_{25,1}$ is a discretized version of the 26D bosonic string and the moonshine association is mentioned in that there are similar associations of the 'pure math' identity with orbifolding of the Coxeter-Todd Lattice (K_{12}) which exists in 12 dimensions and has a kissing number of 756 [12][13]. Looking at the near identity form there are the numbers 4900 and 840^2 . The number 4900 represents a perfect square pyramidal construction and has an fcc packing arrangement in 3 dimensions (just a note that the Coxeter-Todd lattice in 12 dimensions exhibits something similar to a fcc packing arrangement). The number 4900 may be written as 35×140 where the number 35 is a tetrahedral number and 140 is a pyramidal number. Also, $4 \times 35 = 140$ where 4 is a tetrahedral number. The physical construction could be 140 tetrahedral units (35 elements each) to build 4900 or it could be 35 pyramidal units (140 elements each) to build 4900. One can then think of morphism and what is exactly transforming or how it is possible for a group of pyramids and tetrahedrons to integrate into form to create geometry of vectors and spaces. This might be a property of massless fields and scattering amplitudes representing 3 and 4 gluon vertex form histories [14]. The key to this may be $24 \times 35 = 840$ is the representation of 24 (vectors? or coordinate space) of 35 tetrahedral elements and that $24^2 \times 70^2 = 2^2 \times 840^2$ as a square property coupled to the square of the near integral prime $\text{Exp}(\pi \sqrt{163})/24 = 10939058860032030.999999999999969\dots$. Another interesting way of looking is $4900 = 4 \times 1225$ and these 4 subdivisions can translate into tetrahedrons (when transformed from square pyramid fcc arrangement to tetrahedral pyramid of hcp arrangement) since $1225 = 35^2$ would consist of 35 tetrahedrons with 35 elements each. The number $4900 \times 4 = 19600$ is a tetrahedral number and is a perfect square tetrahedral number $19600 = 140^2$ creating a tetrahedral pyramid consisting of 19600 elements under hcp arrangement. In the Diamond theorem it is interesting to note that the $24 \times 35 = 840$ that 35 structures of $840 \times 4 \times 4$ array of tiles has a geometry related to the Miracle Octad Generator (MOG) used in the construction of the Steiner system $S(5,8,24)$ which in turn is related to the Leech Lattice [15]. The number of permutations in the tile geometry group is $576 \times 560 = 322560$. The Mathieu Group M_{24} is the automorphism group of the $S(5,8,24)$ Steiner system of which the M_{24} group has an octad subgroup having 322560 elements. The number 840 is a highly composite number with 32 divisors and is the smallest number represented by the divisors 1,2,3,4,5,6,7,8. It can be represented by either 24×35 or 12×70 . It is suggestive that this minimalist representation of these 8 divisors is reflective of the gluon octet.

840 as a Color Confinement Number, the Coxeter-Todd Lattice (K_{12}) or Wild Ride of Numerology Part II

Since the development of the theory of the strong force, called quantum chromo-dynamics (QCD) mysteries continue to remain on how hadrons obtain mass from their more fundamental constituents. It appears that the three quarks existing inside of a proton or neutron only contribute about 2% to the total mass [16]. It is not altogether clear how the gluon fields contribute the remaining 98%. The issue of spin is also equally mysterious as the overall quark and gluonic content only contribute about 50% to the total spin [16]. An even more bizarre accounting is how the gluons increase in numbers or saturate (daughter gluon saturation) at some point when the relativistic mass of a hadron such as the proton

increases due to it reaching higher speeds [16]. But besides these questions *QCD* remains a very successful theory. Having a highly composite nature the number 840 has great flexibility in calculation. Besides being the integral part of the Diamond theorem where $24 \times 35 = 840$ here is a numerologic approach to how the color trinity of *QCD* is promoted by this number: $8 \times 35 = 280 = 8$ tetrahedral units with 35 elements (lattice points) as $3 \times 280 = 840$ a trinity group $8 \times 35 + 8 \times 35 + 8 \times 35 = 840$, so there are $3 \times (8 \text{ tetrahedral units with } 35 \text{ elements}) = 840$. The number 840 is color neutral. The number of quarks per hadronic system can be deduced except for the glueball which has none,

Exotic Meson : 280 = Glueball Excitation

$$\text{Meson} : \frac{560}{840} \times 3 = 2 \text{ Quarks}$$

$$\text{Baryon} : \frac{840}{840} \times 3 = 3 \text{ Quarks}$$

$$\text{Exotic Meson} : \frac{1120}{840} \times 3 = 4 \text{ Quarks (Tetraquark)}$$

$$\text{Exotic Baryon} : \frac{1400}{840} \times 3 = 5 \text{ Quarks (Pentaquark)}$$

The number in 840 in our near identity has 84 more elements than the kissing number of the Coxeter-Todd Lattice (K_{12}) in 12 dimensional space which is 756. It could be that the momenta of gluons could be placed on the lattice points of this lattice but due to the uncertainty principle this could never be realized but only as a group space representation. As such, it must be a fuzzier outcome for gluonic fields in such a space and it is proposed the representation of the Coxeter-Todd Lattice as SM structure for physics is true but its natural physics form is for a 840 kissing number in the 12 dimensional space to possibly be represented by the non-lattice 12 dimensional structure P_{12A} [17]. The number 840 can be thought of as a group representation of phase spaces. The number 280 represents the phase space representation (also as a kissing number $240 + 40$ in 8 dimensions) and since a quark cannot be isolated this represents the glueball excitation. The glueball as an 8 dimensional object destabilizes completely and rapidly so as not to reside in 4-dimensional Yang-Mills states [18]. It is interesting to note that $9 \times 84 = 756$ and $10 \times 84 = 840$. The number 840 as a confining value of phase space representation is then color neutral (white). The numbers 560, 1120 and 1400 (and probably 1680) are also color neutral (and by themselves represent phase space numbers) and that 560 is a truncated version of 840 whereas 1120 and 1400 are even more unstable versions of quark makeup. The order of the Coxeter-Todd Lattice is 78382080. If you break it down using the kissing number 756 you obtain $78382080 = 756 \times 576 \times 180$, if you use 840 you obtain $78382080 = 840 \times 576 \times 162$. The number 162 is 1 less than the Heegner number 163 and $162 \times 2 = 324$ which is $196884 - 196560 = 324$ and 196884 is the first coefficient in $j(\tau)$ which in turn must be related to the number of irreducible dimensions 196883 of the Monster group in its character table. The number 24^2 remains in the decompositions of these symmetry numbers and in the transformations to lower and higher order structures. The order of K_{12} $78382080 \div 243 = 322560 = 560 \times$

576 which we see is a connection between the Conway groups and the Leech Lattice involving the Golay Code. Also, $78382080 \times 2048 = 160526499840$ (which is the order of elements of the orbifold Coxeter-Todd Lattice/ z_2) and this numbers decomposition is $160526499840 = 576^3 \times 840$. The number 160526499840 is a very highly composite number. The number 840 is a magic number and its squared form is even more magical.² That it is a squared form in our near identity has something to do with the graviton \sim two copy gluon duality [19],

$$\frac{\text{Exp}(2\pi\sqrt{163})}{24^2} 2^2 840^2 = 3.377368... \times 10^{38}$$

We can decompose the square form $2^2 840^2 = 1680^2 = 2822400$ to an interesting assemblage of highly composite numerical identities [20] [21],

$$\text{Sequence : } 2^8 3^2 5^2 7^2 \text{ or } 256, 9, 25, 49$$

$$576 \times 70^2 = 2822400$$

$$560 \times 7! = 2822400 \quad (7! = 5040)$$

$$840 \times 140 \times 24 = 2822400$$

$$560^2 \times 3^2 = 2822400$$

$$840^2 \times 2^2 = 2822400$$

$$840 \times 560 \times 6 = 2822400$$

If we divide this nice symmetric square form by 2 and invert we obtain the very weak *gravitational coupling constant* utilizing the square gluon fields plus quark fields (using the neutron bag only),

$$\frac{2}{e^{2\pi\sqrt{163}} 70^2} = 5.9217697... \times 10^{-39}$$

This is the *gravitational coupling constant* (dimensionless) that is extant at or near the gravitational endpoints of a neutron star and/or black hole (strong hadronic gravities) in a SM-4D type Universe. The modulus square $|-12994938935542088022.819399144...|^2$ (transcendental) presents histories of a dual simplicial complex and string aspect that represents a unification of QCD with weak gravitation as the ratio Graviton/QCD of particle physics. There is another *gravitational coupling constant* that exists that is expressed in an empirical equation form that exists in weaker hadronic gravities and this will be explained below. But first let's finish out a quick heuristic explanation in the next section. If the near identity is a dual graviton gluon form then Super gravity theories (SUGRA) and super symmetries (SUSY) are realistic descriptions of nature [19].

² Being highly composite enables many interesting combinations and flexibility

The K_{12} (Coxeter-Todd Lattice) Lattice Z2 Orbifold, A Mini-Dual to $T^{24}/Z2$?

The Coxeter-Todd Lattice is a sub-lattice of the Leech Lattice [12][13]. Whereas the Leech Lattice exists in 24 dimensional space the Coxeter-Todd Lattice K_{12} exists in 12 dimensional space. It should be noted for number theory fans that K_{12} can be represented as a 6-dimensional lattice self-dual over the Eisenstein integers. The lattice K_{12} has order 78382080 which can be represented as $840 \times 576 \times 162$ or $756 \times 576 \times 180$ showing continual connections with Golay code and Leech Lattice and possibly *Gellman color matrices*. For a physical construction the lattice points as they have position definiteness to them are not conducive to placing point particles or physical spheres or any type of observation. For a natural construction the gluons (include some tripartite aspect) will be chiral bosonic fields placed on a manifold (in this case an orbifold) which would be the K_{12} lattice Z2 orbifold [22]. This would be done similarly as the Euclidean closed bosonic string is compactified to the 24-dimensional torus T^{24} from quotienting R^{24} with the Leech Lattice [23] except this is analogously done with K_{12} instead. In principle it should be possible and indeed K_{12} is the automorphism group of the Baby Monster Group [24] though there are indications that this arrangement of chiral fields onto the orbifolding will be a complicated construction. The gluons could be placed as a form of tripartite structure of gluon chains (all in a *color superposition* state) instead of the closed bosonic string and the gluon chains are compactified to the 12-dimensional torus T^{12} in the similar manner. Scattering histories would then enable hadronization to various combinations of 2 color flow states of gluons [25]. This structure would be physical and directly related to three sporadic groups M_{24} , the Baby Monster and Monster group [24]. Two copies of the K_{12} lattice Z2 orbifold as such a compactification would have an analogy to gravitation and quark gluon histories. That this construction could naturally have a superconformal symmetry might be probable since the Leech Lattice and the Monster Module both allow for a natural superconformal symmetry [23]. Again this implies that SUSY might be an accurate description of nature. Also, our approach implies that the *GUT* point meets exact.

Weighting of Mass-Energy Values of Physics Particles by the Planck Clock

Earlier we had discussed the concept of the Planck Clock and how it potentially affects the values *Ghc*. Since those three values are not universal the particle masses have to be in lock step with these values since we are generating SM-4D type Universes. It is the range of these values which determine the varietal build of the gravitating macroscopic stellar dynamics that will play out in its evolutionary history. Again if we look at the beach sand analogy this essay is only concentrating on the more common quartz grains (as opposed to feldspathic grains). Although Godel's Incompleteness Theorem prevents us from physics decoding of the other Universes there is at least a commonality that exists with the SM-4D Universes. That commonality is required to be gravity. If the graviton being a closed string is not affected or attached to the 3D-brane then it has to be considered that gravitons leak into the other Universes. This means at least for most SM-4D type Universes that the graviton is a Multi-Universal type of interaction. It is proposed that the near physics math identity we have established is a Universal (and invariant to a degree) constant that synergistically interacts with *Ghc* to establish the workings of a classical 4D Universe. The particle mass of the neutron can be determined using the identity as a constant and setting the known *Ghc* parameters with the mass squared as the variable. We will use the prior examples;

$$t_p = \sqrt{\frac{hG}{2\pi c^5}} 8 = \sqrt{\frac{64hG}{2\pi c^5}}$$

$$t_p = \sqrt{\frac{2h8G}{2\pi 0.25c^5}}$$

Setting this as 8 times the Planck time value the distribution we used was **8G**, **2h** and **0.757858 c**. and,

$$\text{Exp}(2\pi\sqrt{163}) 70^2 \sim \frac{hc}{\pi G m_n^2}$$

$$\frac{hc}{\pi G m_n^2} = \frac{m_p^2}{m_n^2}$$

We can isolate m_{nx} (our new neutron mass value),

$$m_{nx}^2 = \frac{\pi G m_n^2}{hc} \frac{2h 0.757858c}{2\pi 8G}$$

Our new neutron mass m_{nx} is calculated to be 2.297 times lighter than our neutron mass m_n . For our new calculated Universe X we have the following values (we of course still have to use our best determined empirical values Codata 2014 for the left side of the equation);

$$G = 5.3392 \times 10^{-10} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$h = 1.3252 \times 10^{-33} \text{ Js}$$

$$c = 227200198 \text{ ms}^{-1}$$

$$m_{nx} = 7.29055 \times 10^{-28} \text{ kg}$$

For our other 1/8 times the Planck time value example the distribution was **0.5G**, **0.5h** and **1.74110c** we obtain the following new Universe values;

$$G = 3.3704 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$h = 3.3130 \times 10^{-34} \text{ Js}$$

$$c = 521968986 \text{ ms}^{-1}$$

$$m_{nx} = 2.1991 \times 10^{-27} \text{ kg}$$

In this case the new neutron mass m_{nx} is 1.3129 times heavier than our neutron mass m_n . Surprisingly we obtain information about particle mass as well but this still leaves the parameters Ghc with a quasi-fine tuning nature. These two toy models might not exist or even generate as SM-4D Universes but the parameters generated show that they are all interrelated due to some underlying mathematical basis. One might object and say that the speed of light couldn't be values such as these (and they might be right) but in the context of each individual Universe it might mesh with the physics in the Lorentzian manner too. It actually calculates that the gravitational lengths of these two examples are the same as their respective Planck lengths suggesting that the Lorentz symmetries are maintained. The reader may check this out using these two relations minus the factors of 2.

$$r_s = \frac{GM_x}{c^2} \qquad l_p = \sqrt{\frac{hG}{\pi c^3}}$$

M_x is the new Planck mass and r_s is the gravitational length or Schwarzschild radius.

Or it could be that there are tolerances or limits involved disallowing such a working Universe and that the parameters for SM-4D Universes have to be just right. One scenario would be that there are quantization rules involved in the randomization and a discrete spectrum of Ghc SM-4D Universes may exist (in which case we call it a subset of the entire *Godel Spectrum*³ and all other abstruse Universes are either discrete as well or fill in the gaps for a continuous spectrum). The subject of colliding Universes suggests that the spectra are brutally discrete. Who knows? But what is really fascinating about this approach is that it highly suggests that there is a direct correlation between gravity as we know it and the mass-energy of particles within a SM like framework. If we assume that because of the rigid maths involved that the *GUT* points take on a similar or near similar ratio value to the new Planck energy cutoff then Universes with heavier baryons than ours might only produce fast burning first generation stars which will not produce heavy metals and carbon. The heavier baryons scenario could be offset by a weaker (new) Newton constant G and do something totally different (combinations and permutations). It should be noted that the Newton constant G which can vary greatly from Universe to Universe is not the same as the very weak *gravitational coupling constant* (utilising the neutron mass) which does not vary from Universe to Universe. Or conditions for life could be closer to one edge of the Goldilocks zone or the other and so on. It is mind boggling. A real problem for this approach to generating these various Universes is whether or not the *GUT* energy ratio to the cutoff energy is variable in a tight tolerance or whether it is rigid in respect to the underlying mathematics. At this point we consider that it is fixed in the hierarchical space due to mathematical constraints that we have been building with. It is possibly related to the neutron/proton mass particle ratio = 1.00137841898 and the relation of $SU(2)$ Lie algebra. If the square root of (neutron based) *gravitational coupling constant* is multiplied by whatever neutron mass your Universe has it produces the Planck energy cutoff value.

$$\sqrt{\frac{e^{2\pi\sqrt{163}} 70^2}{2}} = 12994938935542088022.81939914...$$

³ The unknowable Godel Spectrum is the same as the Metaphysical Landscape.

For example if you use the very well determined Codata 2014 value of the neutron mass (10 significant figures) you obtain,

$$m_n \times \sqrt{\frac{e^{2\pi\sqrt{163}} 70^2}{2}} = 2.17655802 \times 10^{-8} \text{ kg}$$

Compare to the Codata 2014 value of the Planck mass at $2.176470(51) \times 10^{-8}$ kg. This is highly suggestive of color flow histories in the hierarchal space between the low and high energy domains. We did say that this number can be represented as a modulus square value. The proton and neutron can appear to be near degenerate objects in that the gluon fields and not the quarks are the main contributors of mass in hadrons. They almost have the same mass with the neutron being slightly more mass. If mathematical connections are there then there might be the analogy of the neutron/proton mass ratio to that of the Planck energy to the *GUT* energy. Isospin or the *SU(2)* group Lie algebra of translating between the proton and neutron does not feel the strong force. Since at the *GUT* energy the three gauge forces unify some modification of group transformations (more reliance on string physics) must occur and are probably connected. A numerologic suggestion would be to $1.00137841898 - 1 = 0.00137841898$ and obtain the inverse $725.4688\dots$ and step down the Planck mass energy equivalent $1.220910(29) \times 10^{19}$ GeV $\div 725.4688 = 1.682925 \times 10^{16}$ GeV to obtain the *GUT* energy value. The square root of 725.4688 is 26.93 and that inverse could be close to the string coupling constant. That might be one way to determine *GUT* energies in the other toy Universes that is if the number 1.00137841898 is a pure math number (maybe not). But it does look like it just might be a determined pure math number.

A Violation of the Einstein EP, Degeneracy Pressure and Isospin

We have been discussing a ‘pure math’ square form using different concepts from *number theory* and *group theory*. Its inverse form divided by 2 is the *gravitational coupling constant* for extreme gravitational fields at or near neutron star and black hole structure as such it is related to neutron degeneracy. Its counterpart in the lesser gravitational extremes for up through ‘electron degeneracies’ the gravitationally weaker dwarf stars is an *empirical relation* and is a not a pure math relation because it involves the messier relations of the proton (charge) and the beautiful *QED*.

$$\sqrt[4]{\frac{e}{2}} 2\alpha^4 e^{\frac{\pi}{4\alpha}} = 3.382023703\dots \times 10^{38} \text{ (dimensionless)}$$

This is a modified relation taken from Damour where A and B are of natural order unity and’t Hooft suggested that $B = \pi/4$ [26],

$$\text{gravitational coupling constant} \simeq A e^{\frac{-B}{\alpha}}$$

Our form is close to the physics form calculation [27],

$$\text{Codata 2014 : } \frac{hc}{\pi G m_p m_n} = 3.38175(16) \times 10^{38}$$

(m_n = neutron mass and m_p = proton mass and α is the fine structure constant). Not quite as good as the pure math identity involving the neutron fields. Dividing our new dimensionless number form which utilizes the fine structure constant (not the particle physics form) by factor 2 and inverting we obtain the *gravitational coupling constant* that operates in the domain of protonic stellar physics in the degeneracies below that of neutron stars,

$$\frac{2}{\sqrt[4]{\frac{e}{2}} 2\alpha^4 e^{\frac{\pi}{4\alpha}}} = 5.9136191... \times 10^{-39}$$

Comparing this to the neutronic form this is a slightly weaker coupling constant. In the next section it will be demonstrated that the empirical form using the fine structure constant (out to 13 precise significant figures) may be converging to the Codata set in the future (really it would be the Newton constant G that may converge for correction). So there are two *gravitational coupling constants*, one for on top of neutron star structure and the other for up to the *Chandrasekhar Limit* and past this for continuing degeneracies close to neutron degeneracies. This suggests a slight dynamism (but still relatively rigid) between these two numbers in the gravitational fields in the observable Universe. Recently, observations have been made with white dwarf stars concerning any changes in the dimensionless constants, *fine structure constant*, *proton /electron mass ratio* and have concluded no changes of these numbers in the strong gravitational fields of these stars to a certain degree of error (from 10,000 to 30,000 earth's gravitational fields) [28]. This is good news for General Relativity's Equivalence Principle (EP) which holds that certain fundamental constants (historical constants) are rigid in order for GR to remain consistent across the Universe [29]. This rigidity is probably necessary for our current Universe structures build such as galaxies, stable solar systems. If the rigidity is absolute this sets up an asymmetry between GR's space-time with absolute constants and QM's with no absolutes (in general). History has shown that if anything there has always been the elimination of absolutes from Science. For example, Newton's absolute space and time was replaced with Einstein's dynamical space-time. If there are two low-energy build *gravitational coupling constants* this would be a direct violation of EP. A consequence is that the equivalence principle (EP) is not going to be a firm absolute principle of physics. Observable violations are going to be very difficult. To date there is no proof that any physics constant or dimensionless constant has changed or been measured to have changed since the initial fires 14 billion years ago. It is likely that most physics values were made relatively rigid (not absolutely) at about 10^{-28} seconds after the Big Bang after the drop of the reheating time. In that we exist in the low-energy end of the hierarchal space we cannot perceive and detect the small amount of dynamism that does exist. If one looked across the interstellar gulf of time and the constants varied as large as some of the literature suggest what a wreck it would be for the isotropy of space. It's like the aether argument all over again. The proton and neutron have almost the same mass with the neutron being slightly heavier with no charge compared to the proton which has a positive charge. There is the neutron to proton mass ratio (1.00137841898). The strong force does not differentiate between the two

(and is thus democratic). Heisenberg came up with isospin to explain this democracy of the strong force on these hadrons. The isospin is the $SU(2)$ Lie algebra group of translating between the proton and the neutron. In theory one can perform the isospin (internal translations, rotations) on a proton to convert it to a neutron. This has the effect of changing the quark makeup from uud to ddu with some change of vacuum contributions thus making the neutron slightly more mass. If this is what is occurring as we move from the weaker gravitational fields of protonic stars and through the electron degeneracies of dwarf stars toward the stronger gravitational fields of neutron degeneracies it is apparent that degenerate pressures from these crushing gravities drives the isospin from protonic to neutronic degeneracy. If it is true these two identities are *gravitational coupling constants* for their respective domains then the EP is not an absolute principle.

Congruences of Big G

Currently, as it stands what are considered the fundamental mass values, Planck constant and fine structure constant as reported by the NIST through Codata 2014 are accurate and precise (up to 10-13 significant figures) and hence considered very good numerical (dimension-ful) values. This is not the case with the gravitational constant G . Ever since Cavendish determined a value ~ 300 years ago that is good to three significant figures the constant has varied to a current spread of 0.05% and experimentalist have not been able to obtain any accuracy or precision past three to four significant figures [30]. Compared to the other constants in Codata 2014 this is a very poor showing and is not acceptable. However, this is changing as the Royal Society A and NIST have created a consortium to address this issue, especially by standardizing the G measuring approach and removing biasing [9] [31]]. The current value of G in Codata 2014 $6.67408(31) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ (SI units) is more than likely a compromise of current experimental values. Hopefully, by the next Codata cycle (2018) it will have been determined to be more accurate and precise. If one looks at the last three (4 year) cycles of Codata; Codata 2006, 2010 and 2014 the 'pure math' near identity we have established is hovering around the Codata value calculations determined using the constants. If G is getting better as far as becoming more accurate and precise one would hope that all the Codata values would converge in the calculation onto the 'pure math' version if it is true. At this point this has not happened as Codata 2014 yields,

$$\text{Codata 2014 : } \frac{hc}{\pi G m_n^2} = 3.37709(16) \times 10^{38}$$

Not as good as Codata 2010 but better than the Codata 2006 value which is still very good. Looking at both identities if one considers that the other constants are very well determined (accurate and precise to many significant figures) the gravitational constant G can be isolated and set up against the determined values along with the constants to align G to the determined value. As a consequence of doing this straightforward calculation a congruence of a new G constant value occurs with the identities which seem to suggest that the two identities are directly related.

$$G = \frac{hc}{\pi m_n^2} \frac{1}{e^{2\pi\sqrt{163}} 70^2}$$

The calculated G value is,

$$G = 6.67354141 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

Isolating G using the empirical relation,

$$G = \frac{hc}{\pi m_p m_n} \frac{1}{\sqrt[4]{\frac{e}{2}} 2\alpha^4 e^{\frac{\pi}{4\alpha}}}$$

The calculated G value is,

$$G = 6.67354236 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

This congruency points out the how well the accuracy and precision of the Codata sets of fundamental constants have been established. This congruency is better than the Codata 2010 congruency which was $6.67354189 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ and $6.67354317 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ respectively. Here is something that should finally convince that this is not just coincidence. If we use the Codata 2014 and the isolated G from the (weaker *gravitational coupling constant*) protonic form there is congruence to the empirical equation,

$$\frac{hc}{\pi G_{pn} m_p m_n} = 3.38202370 \times 10^{38}$$

(where $G_{np} = 6.67354236 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)

Compare this to the empirical equation which uses the very accurate and precise (to 13 significant figures) *fine structure constant*.

$$\sqrt[4]{\frac{e}{2}} 2\alpha^4 e^{\frac{\pi}{4\alpha}} = 3.382023703... \times 10^{38}$$

Since we exist in the weaker fields of the weaker protonic *gravitational coupling constant* our empirical value may possibly be close to this value,

$$G_{pn} = 6.67354236 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

Comparing this to the Newton Constant determined at the stronger neutronic *gravitational coupling constant* regime,

$$G_{nn} = 6.67354141 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

It is noticed that this numerical value is slightly less than the G_{np} value. This is puzzling but it could be due to factors unknown or an underlying flaw in the dimension-ful and anthropic approach using a complication of LMT type units. It could be that the Codata values will improve even more and at some point these values might cross over or it is a manifestation of *QED* at the lower energy. Looking at the

past Codata values there are two Codata sets that actually have the arrangement whereby the electron degenerative value (protonic) is smaller than the neutronic degenerative value:

$$\text{Codata 1998 : } G_{pn} = 6.6735404\text{\underline{1}} \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\text{Codata 1998 : } G_{nn} = 6.6735426\text{\underline{0}} \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

and set,

$$\text{Codata 2006 : } G_{pn} = 6.6735406\text{\underline{2}} \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\text{Codata 2006 : } G_{nn} = 6.6735423\text{\underline{9}} \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

So it is possible that Codata value improvement will eventually maintain this weaker to stronger relationship. What is interesting about the 1998 and 2006 Codata set is that they present really nice numerical agreements and an excellent alignment with a very large symmetry as done below. This does not mean that the *Ghc* and Codata values are better values than the other Codata set values but that the proper ratio of the values are possibly near correct. If *QED* and the proton is a messier proposition than *QCD* and the neutron then it might be that the symmetry of the gravity gluon duality preserves the ‘pure math’ aspect of the slightly stronger neutronic *gravitational coupling constant* and that looking for ‘pure math’ constructs of the lower energy *QED* related dimensionless constants such as the *fine structure constant* is pointless or too difficult. This just might be the inconsistency in the presented maths that suggests *Godel’s Incompleteness Theorem* particularly in the larger ‘core’ structure which will be mentioned below. The random like mixing of *Ghc* in the Planck Time produces a quasi-fine tuning aspect of the *fine structure constant* which appears to be a major complication to a first principle calculation. Instead there are empirical equation forms (quadratic appearances) which have form structure that are probably related to math in the deep down more than likely having something to do with *modular forms*. Here are two physics forms (dimensionless) that have the close quadratic structure that is probably not a coincidence.

$$GCC = \frac{2\pi Gm^2}{hc} \qquad \alpha = \frac{e^2}{2\epsilon_0 hc}$$

In a final note on the Newton constant in 2012 Vadim Milyukov and Shu-hua Fan adjusted the Codata 2010 *G* value to $G = 6.67349(81)\dots \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ which is close to the above values [32].

The *Ghc* Master Vacuum Fields are Players of a Very Large Symmetry (Yang-Mills)

The Monster group has a very large number of elements to its symmetry.

80801742479451287588645990496171075700575436800000000

This is an exact integer consisting of approximately 8×10^{53} elements. There is a curious 'number theoretic' physics calculation form which aligns very close to this number and like the two gravitational coupling constants (GCC) above there is an electron and neutron degenerate form.

$$\frac{hc}{2\pi Gm_e^2} = \frac{m_p^2}{m_e^2} \text{ where } m_e = \text{electron mass, } m_p = \text{Planck mass}$$

$$\text{Neutron Degenerate Form : } \frac{4 m_p^2}{\alpha^4 m_e^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{e^{2\pi\sqrt{163}} 70^2 \frac{\alpha^4}{2\pi} - 1.00}}} \quad (1)$$

$$\text{Electron Degenerate Form : } \frac{4 m_p^2}{\alpha^4 m_e^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\sqrt[4]{8e} \alpha^4 e^{\frac{\pi}{4\alpha}} \frac{\alpha^4}{2\pi} - 1.00}}} \quad (2)$$

Both relations if true should end up calculating the Monster group number of elements to 7-8 significant figures. This means that these two relations should be equivalent when the *Ghc* Codata and their respective fixed *G* become coherent. Currently we can only obtain the Monster elements out to 8 significant figures since historically the Planck constant and the electron mass is less precise at about 9 significant figures with high uncertainties. The small term on the right of each form has a slight difference due to the slight differences of the Newton constant *G* in the gravitational fields from microgravity to electron degeneracies (10,000 to 30,000 g's) up through neutron star formations at about 200,000,000,000 g's and final black hole end point. We define the small terms respectively (neutron and electron degenerative forms) as follows.

$$Z_{nn} = \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{e^{2\pi\sqrt{163}} 70^2 \frac{\alpha^4}{2\pi} - 1.00}}}$$

$$Z_{pn} = \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\sqrt[4]{8e} \alpha^4 e^{\frac{\pi}{4\alpha}} \frac{\alpha^4}{2\pi} - 1.00}}}$$

These can be compactly displayed as;

$$Z_{nn} = \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\frac{1}{GCC_{nn}} \frac{\alpha^4}{2\pi}} - 1.00}}$$

$$Z_{pn} = \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\frac{1}{GCC_{pn}} \frac{\alpha^4}{2\pi}} - 1.00}}$$

The GCC_{nn} and GCC_{pn} are the gravitational coupling constants in their respective degenerative domains. We can compare Codata sets from 1998 to 2014 using both degenerate formulas (1) and (2) in determining the Monster group number of elements out to 8 significant figures.

Codata 1998	Codata 2002	Codata 2006	Codata 2010	Codata 2014
$M = 8.08017244 \times 10^{53}$	$M = 8.08017216 \times 10^{53}$	$M = 8.08017243 \times 10^{53}$	$M = 8.08017229 \times 10^{53}$	$M = 8.08017231 \times 10^{53}$
$G_{nn} = 6.673542595 \times 10^{-11}$	$G_{nn} = 6.673542183 \times 10^{-11}$	$G_{nn} = 6.673542390 \times 10^{-11}$	$G_{nn} = 6.673541889 \times 10^{-11}$	$G_{nn} = 6.673541406 \times 10^{-11}$
$Z_{nn} = 1.00336611230842$	$Z_{nn} = 1.00336611230828$	$Z_{nn} = 1.00336611230840$	$Z_{nn} = 1.00336611230828$	$Z_{nn} = 1.00336611230829$

Table 1: Number of elements M determined using neutron degenerate formula (1). G_{nn} has dimensions $m^3 kg^{-1} s^{-2}$

Codata 1998	Codata 2002	Codata 2006	Codata 2010	Codata 2014
$M = 8.08017500 \times 10^{53}$	$M = 8.08017026 \times 10^{53}$	$M = 8.08017449 \times 10^{53}$	$M = 8.08017066 \times 10^{53}$	$M = 8.08017107 \times 10^{53}$
$G_{pn} = 6.673540413 \times 10^{-11}$	$G_{pn} = 6.673543277 \times 10^{-11}$	$G_{pn} = 6.673540620 \times 10^{-11}$	$G_{pn} = 6.673543169 \times 10^{-11}$	$G_{pn} = 6.67354236 \times 10^{-11}$
$Z_{pn} = 1.00336610225792$	$Z_{pn} = 1.00336610226141$	$Z_{pn} = 1.00336610225838$	$Z_{pn} = 1.00336610226159$	$Z_{pn} = 1.00336610226125$

Table 2: Number of elements M determined using electron degenerate formula (2). G_{pn} has dimensions $m^3 kg^{-1} s^{-2}$

Two values are closer to the Monster integer value than the others and they are the Codata 1998 value and the Codata 2006 value from the proton electron degenerate form in table 2. Interestingly, the fixed G_{pn} gravitation constants (for Codata 1998 and Codata 2006) are very similar to each other and the G_{nn} constants are similar to each other as well. These are the only cases where the Newton constant is a lesser strength constant in the proton-electron form than the neutron-neutron form as should be expected. If you do not use the fixed Newton constant values but use the respective Codata values instead you obtain a shotgun set of M values. The alignment of the 1998 and 2006 Codata sets with M

elements does not mean that those respective Codata sets are correct but could indicate that the Gravitational constant G_{pn} (or an empirical determined G) could be close to the value $6.6735405 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$. It remains for future Codata cycles to improve upon the values especially the electron mass and Planck constant to see if there is a coherent consistency which suggests such a Master vacuum field. We do not suggest that this is a prediction or a theory explaining the constant G but is an aligning of values within their specific Codata set whereby Ghc and mass energy have mathematical dependence on the set.

Coherency and Consistency of the Master Set and a General Analysis of the Ghc Codata Field

For lack of a better way to state this there is an equivalent (and better) physics form than the Master vacuum field forms previously addressed. Again there are two degenerate forms,

$$\text{Neutron Degenerate Form : } \frac{m_p^2}{m_{e+e-}^4} \frac{(4\pi^{+-})^2}{\alpha^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{e^{2\pi\sqrt{163}} 70^2 \frac{\alpha^4}{2\pi} - 1.00}}}$$

$$\text{Electron Degenerate Form : } \frac{m_p^2}{m_{e+e-}^4} \frac{(4\pi^{+-})^2}{\alpha^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\sqrt[4]{8e} \alpha^4 e^{\frac{\pi}{4\alpha}} \frac{\alpha^4}{2\pi} - 1.00}}}$$

$m_{e+e-} = 2m_e$ (2 x electron mass = combined masses of electron positron pair)

π^{+-} = charged pion mass, α = fine structure constant

Since the latest consensus is the Codata 2014 we can solve for the pi-meson mass and obtain a value that is in line for the 2014 Codata set. The pi-meson and other particles not included in the Codata set are included in the latest Particle Data Group (PDG 2014) publication [33].

$$\frac{m_p^2}{m_{e+e-}^4} \frac{(4\pi^{+-})^2}{\alpha^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{e^{2\pi\sqrt{163}} 70^2 \frac{\alpha^4}{2\pi} - 1.00}}} = 8.08017231 \times 10^{53}$$

SI Units : $\pi^{+-} = 2.49662697 \times 10^{-28} \text{ kg}$

Mass Energy Equivalent in MeV : $\pi^{+-} = 140.050503 \text{ MeV}$

$$\frac{m_p^2}{m_{e+e-}^4} \frac{(4\pi^{+-})^2}{\alpha^2} \frac{1}{\sqrt[2^{11}]{\sqrt[2^{16}]{\sqrt[4]{8e} \alpha^4 e^{\frac{\pi}{4\alpha}} \frac{\alpha^4}{2\pi}} - 1.00}}} = 8.08017107 \times 10^{53}$$

SI Units : $\pi^{+-} = 2.49662695 \times 10^{-28} \text{ kg}$

Mass Energy Equivalent in MeV : $\pi^{+-} = 140.050502 \text{ MeV}$

The 2014 PDG Value : $\pi^{+-} = 139.57018(35) \text{ MeV}$

That both degenerate forms calculate very close numerical values suggests that both forms should calculate the Monster number of elements and are equivalent forms at the level that the *Ghc* Codata set becomes correct and coherent. Again for this calculation we used the fixed G_{pn} and G_{nm} for the 2014 set and in order to align with the consensus utilized the Monster number of elements on tables 1 and 2 for 2014. The result on the pi-meson suggests that (with the current 2014 Codata) its value should move up⁴. One wonders whether the calculation should be done using the 1998 and/or 2006 Codata values since better matches were made to the symmetry and that the Newton values seemed a proper order of weaker to stronger but those Codata cycles are no longer consensus and the parameter mixing could be such that it appears to work and is thus an illusion. Looking at the above new forms one can see that *QED* is playing an active role in the relation as we have the electron positron vacuum energies in conjunction with the charged pion masses (the charge on the charged pion is due to photon exchange via *QED* of the pions interaction with the electromagnetic vacuum and not *QCD* [34][35]). The charge state of the pi-meson is also the same as the *isospin* state and is involved in the *SU(2)* transformation of the proton to the neutron and the reverse [34][35]). The fine structure constant is a coupling via the photon exchanges in a *QED* interaction. You can observe that a power square of the fine structure constant has melded into the new form because of an equivalency. If this equation has any certainty then the following relation are true and the equations can be made into nice and symmetric forms.

Neutron Degenerate Form : $\frac{m_p^2}{m_e^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} Z_{nn} = \text{Monster Group Order of Elements}$

Electron Degenerate Form : $\frac{m_p^2}{m_e^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} Z_{pn} = \text{Monster Group Order of Elements}$

⁴ The current PDG value for the mass of the charged pi-meson has not been changed since the late 1990's. See Ref. [36]

Both forms are equivalent.

$$\frac{m_p^2}{m_e^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} Z_{pn} = \frac{m_p^2}{m_e^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} \frac{2(\pi^{+-})^2}{m_{e+e-}^2} Z_{nn}$$

Throughout the isospin symmetry $SU(2)$ transformation it is evident that the empirical determined Codata values will change to reflect the new energy and gravitational conditions at or near the neutron star formation. The Z_{pn} is smaller than the Z_{nn} term out at 9 decimal places so that some parameter values will change near this placement as well. Assume that the Planck mass stays invariant but since the Newton constant G changes this would indicate that the Planck constant changes in concordance for the ratio to stay the same with the speed of light c unchanged (Lorentz invariance is maintained at least at the low-energy vacuum values used in the equations). The Planck mass squared suppresses quantum gravitational effects in the low-energy vacuum fields and allows weak gravity to act like a charge value for the pseudo-Nambu-Goldstone Bosonic (pNGB) fields. One value which we do know that changes slightly which affects the equation is the *fine structure constant* which is the gauge force of QED . There is an equivalency of the charged pion mass in a simple mass ratio to the numerical value of the charged pion obtained by using the Master vacuum relation (uses the fixed value of G_{pn}). This ratio can be used to determine the new pi-meson value using the 2014 Codata:

$$m_{e+e-} = 2m_e \quad \frac{m_{e+e-}}{\pi^{+-}} = \alpha$$

The new pi-meson value, $\rightarrow \pi^{+-} = 140.050502 \text{ MeV}$

This is in excellent agreement with the Master vacuum field relation in the lower energy electron degeneracy realm. At this point it is hard to say if the electron masses changes or the pi-meson mass changes or if both change but that in some manner the positron-electron combined mass ratio to the charged pion mass does change in a slight running (eighth decimal place) toward the gravitational endpoint of a neutron star. Since we assume the Planck mass is fixed (and is the high-energy cutoff) in the cosmological Planck era this reduces some complexity to the nice symmetric final Master vacuum field forms. It will be interesting to see if the next 2018 Codata will demonstrate better results. The final degenerate forms have an interesting quadratic grouping of the charged pi-meson along with the electron positron masses which is indicative of a very active low-energy vacuum the symmetry of which is that of the Monster group. If this is the effective low-energy expectation value field of the SM then it suggests that there is a double copy pion field (throughout our observable Universe) which permeates the low-energy aspect of the SM observable Universe. It is then highly suggestive that the pi-meson is the bottom of the hill expectation value of the lightest particle of the QCD group $SU(3)$ and that this corroborates this particle as being the *pseudo Nambu-Goldstone* boson representative of the famous *mass gap*. The *glueball* will then be the last particle to finish out the good ole SM. In that these calculations include the particle antiparticle values in the QED vacuum along with the fine structure constant and the pi-meson particle masses of QCD hints very strongly this being a core conception of the

Standard Model including (possibly graded or spectrally quantized) gravitational degeneration of stable hadronic matter creating strong gravitational build structures in SM-4D Universes.

Conclusion

Hopefully, the reader can walk away from this and understand at least the simple rational dynamism of having two *gravitational coupling constants* operating in the familiar SM-4D type Universe(s). In that if true then the EP is violated through hadronic isospin under great degenerate pressures. At the same time the EP violation is not great and still leaves the constants having enough rigidity (up to parts per million or more) to allow a stable build to these class of Universes. If this generation of these types of Universes occurs this tones down the metaphysical and quasi-religious leanings of the rhetoric around the Multiverse controversy (at least of 4 dimensional type Universes). We have nothing to say about the other classes or types and it is felt that Godel's Incompleteness Theorem prevents much to say about this. We also point out the gravity gluon dual feature of the 'pure math' neutronic *gravitational coupling constant*. In addition it possibly includes SUSY in that its group theory and manifestation as a 12 dimensional structure T^{12} being on the cusp of instability in the *QCD* vacuum allows for a *super conformal field theory* as the mini-dual (baby-dual ?) to the 24 dimensional Leech Lattice embedded on the torus T^{24} and its *super conformal field theory*. The *QCD* vacuum instability related to the confinement of quarks and gluons is due to the magic number 840 (which is color neutral and has a slight instable interaction with the vacuum) is also a kissing number in the 12 dimensional non-lattice P_{12A} . The kissing number 756 of the 12 dimensional Coxeter-Todd Lattice is the degenerate form of 840 and can be farther complexed to its orbifold $T^{12}/Z2$ its dual action related to the gravity gluon duality. The number 840 can peel off simplicial histories (tetrahedrons and pyramids which quickly hadronise or meld back into the vacuum) of daughter gluons and quark-antiquark pairs in the increasing momentum of hadrons or scattering histories as evidenced by 3 and 4 gluon vertex color flows. A simple approach to determination of physics values rely on the concept of the Planck clock involving the 3 parameters *Ghc*. That these values depend on a relative time period determine the nature of the SM-4D Universe that is created and the graded concept of these Universes from utterly hostile to hospitable for life processes. A Universe might exist (as a fun concept) which is very hospitable where there exists many earth-like planets with the propensity for intelligent life to form. It might be common for such Worlds to be in contact with one another at some point in their evolution. Another example (more serious) might be that a Universe only has fast burning stars and hence no complex chemistries for varied life processes and little or no complex planet manufacture. The list goes on. The values *Ghc* generated also are integrated by the central concept that gravity or the *gravitational coupling constant* is Meta Universal and is the only common invariant in the Multiverse of SM-4D type Universes. This is represented by the pure math *gravitational coupling constant* which exists at the gravitational endpoint of most SM-4D type Universe and balances the books for the *Ghc* coding. That the three parameters *Ghc* are apparently *fine-tuned* (*quasi-fine-tuned*) suggests that at the heart of these parameters lies a mystery which may have explanation by way of the *theory of modular forms*. This mixing of *Ghc* parameters suggests that initial conditions are the arbiter of how the particular Universe evolves and that clone Universes of this type are very rare or do not exist. That there maybe two *gravitational coupling constants* present in our current observable Universe would mean that the Equivalence Principle of Einstein's General Relativity

is violated and this opens up new possibilities for a more accurate description of gravitational dynamics in the Galactic/Stellar build in our Universe. The physical constants of Nature are relatively constant and rigid for there to be a consistent isotropy of space in all directions. Observable violations of these constants are going to be difficult in that changes in the constants will be to 9 or more decimal positions. It should be an open problem as to the 'near identities' presented are true or false. Finally, a Master vacuum field relation to the Monster group symmetry is presented which integrates the *Ghc* Codata as an interdependent field involving particle masses of the SM and the *very weak* dimensionless gravitational gauge force. We have also presented a (hidden) highly symmetric version of this field which shows that the pion is potentially a major part of the effective low-energy physics of the SM. If true then a pion condensate permeates the vacuum fields [37] [38] of our observable Universe and the pion will be the lightest *QCD* particle after the zero expectation value of the *mass gap*. Let us make clear that the theory presented in this paper (the low-energy SM) is not *quantum gravity* but is a form of unification of *Gravity/QCD*. If the pion field is there then the *glueball* should be the last important particle found to finish the low energy end of the vanilla SM. By themselves the two dimensionless *gravitational coupling constants* exhibits this relation of gravity to the strong force. It is possible that we have touched upon the mathematical 'core' of the Standard Model having elements of the *Gravity/QCD* dynamics in a Yang-Mills expression. Perhaps this may lead to higher energy/dimensional realizations of quantum gravity and if the Monster group and graviton gluon duality is involved implies that some form of *SUGRA* is a likely candidate. We leave it to the master (whose intuition made for a good road map time and again) to say something which may be true,

"In a reasonable theory there are no (dimensionless) numbers whose values are only empirically determinable" A. Einstein, Letter to Ilse Rosenthal-Schneider, Princeton, October 13, 1945

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