

On Problems and Solutions of General Relativity ~ (Commemoration of the 100th Anniversary of General Relativity)

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In commemoration of the 100th anniversary of general relativity, many others would look back on its history and achievements, so I will write mine focusing on the problems of general relativity and their solutions.

Pieces of writing hereunder are the rearrangement of my past idea from thesis [1] and postings on my personal blog [2] and science community by focusing on the theme of problems and solutions of general relativity.

I. Several problems of general relativity

Since its publication [3], general theory of relativity has been numerously challenged but time-tested in the process through numerous experiments(Gravitational time dilation and frequency shift, Precession of apsides, Gravitational lensing...). [4] [5]

However, the possibility and demand for revised general relativity is also coming to the fore. This arises from its own problem of general relativity or is required from newly found observations. In my eyes, currently, there are three big problems with respect to general relativity.

1) Problems of integration with quantum mechanics and the case of a very big curvature in space-time [6]

2) The theory includes the existence of singularity, which denies the application of general relativity itself, as a solution for field equation. [4]

3) Rising of the problems of dark energy and dark matter [7] [8]

I would investigate these problems in reverse order and propose my ideas to work them out or solutions.

II. Problems of dark energy and dark matter

1. Rising of the problems of dark energy and dark matter

To explain the phenomena currently observed from the universe, we assume the need for something unknown called dark matter and dark energy. [7] [8] However, since its quantity is so great(95% level of total mass in the universe) [9] [10] but not detected from our surroundings, [11–13] opinions are coming forth that general relativity should be revised into other forms as to the outside of the solar system. [14]

While the standard model makes up a whole system with its entirety tested through experiments, dark matter is expected not to belong to this standard model. [15] One of the ways to solve this problem is a revision of general relativity.

Accelerated expansion of the universe suggests the existence of new force similar to anti-gravity, [5] which is a result different from the initial expectation of general relativity. [7] [8]

Although general relativity may be able to explain the universe's accelerated expansion within its own system of theory by adding cosmological constant, [7] [8] [16] this can be essentially nothing but a makeshift for general relativity.

Besides, even though the cosmological constant has met the purpose as a makeshift to lengthen the life of general relativity, we still have not succeeded in explaining the origin of the cosmological constant. [5] [16]

It is obvious that dark matter and dark energy are an observation that must be explained by general relativity, so to explain it, many scholars in the mainstream system of physics are seeking to revise the general relativity. We call them MOND [14] or F(R) theory backers. [17]

2. Some ways to solve the problem of dark energy and dark matter [1]

2-1. Understanding of the problem

Since dark matter and dark energy comes from Friedman equation that governs cosmic dynamics, so we need to look into this equation again.

Friedman equation can be induced from 00 component of field equations.

$$R_{00} - \frac{1}{2}g_{00}R = 8\pi GT_{00} \tag{1}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \tag{2}$$

But we can also induce this from conservation of energy in classical mechanics, which helps capture the situation definitely.

$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} + \frac{1}{6}\Lambda mc^2 r^2 = \text{const.}$$

$$v = \frac{dr}{dt} = Hr = HR\omega$$

$$\frac{1}{2}m\omega^2 \left[\left(\frac{1}{R} \frac{dR}{dt}\right)^2 - \frac{8\pi G\rho}{3} + \frac{1}{3}\Lambda c^2 \right] R^2 = \frac{1}{2}m\omega^2 [-kc^2]$$

Figure 1: The Friedmann equation can derive from field equation or mechanical energy conservation equation. We judge the components of the universe by gravitational effect (or gravitational potential energy) rather than mass energy.

The present cosmological constant can be obtained by adding potential energy $U_\Lambda = -\frac{1}{6}\Lambda mc^2 r^2$ to mechanical energy conservation equation. [5]

If we insert 'new potential energy term' into mechanical energy conservation equation, we will get a similar term such as $-\frac{1}{3}\Lambda c^2$ term.

From the relations above, we become aware that existence of gravitational potential energy brings the corresponding energy density into being, assuming what ratio of gravitational potential energy means what ratio of energy density.

Such relationship suggests that with two more terms of gravitational potential energy corresponding to the current dark energy and dark matter, it could explain the problem of these two satisfactorily.

Now, where could we find the source of these two new gravitational potential energies?

2-2. Solution of the problem [1]

2-2-1. Necessity to introduce negative energy

Solution of a certain problem may be found within the system of theory but at times outside of the theory. Let's consider the source of our cosmic energy regardless of dark matter and dark energy.

If the conservation of energy that “energy will be always conserved” is applied to the initial state of the universe, this gives rise to the question of “where the energy of our universe did come from?”. The most natural answer to this question is the assumption that the energy itself was not something created and that zero-energy state went through phase transition keeping conservation of energy and generating negative and positive energy. Therefore, in order to offset the known positive energy of matter, negative energy is needed.

$$E_T = 0 = (+E) + (-E) = (\sum m_+ c^2) + (\sum -m_- c^2) + (\sum U) = 0 \quad (3)$$

The theory of ‘universe birth model from nothing’ by Hawking, Guth and Vilenkin etc. can be explained by the fact that the gravitational potential energy offsets the mass energy. [18]

And calculating this actually,

For the sphere with uniform density, gravitational self-energy or gravitational binding energy ($-U_{gs}$) is given as follows:

$$U_{gs} = -\frac{3}{5} \frac{GM^2}{R} \quad (4)$$

From Mass energy equivalence principle,

$$-\frac{3}{5} \frac{GM^2}{R_{gs}} = -Mc^2 \quad (5)$$

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} \quad (6)$$

The calculation above suggests that if matter or energy is distributed within the radius R_{gs} , gravitational potential energy with a negative value will completely offset mass energy to make the state of zero-energy.

However, Schwarzschild black hole's radius is given by $R_B = \frac{2GM}{c^2}$

$$R_{gs} = 0.3R_B < R_B \quad (7)$$

If gravitational potential energy exactly balances the energy in matter, our universe has to be black hole. But, our universe is not black hole.

At present, the gravitational potential energy does not completely offset mass energy. And for the birth of the universe from “nothing or zero energy” and energy conservation at the birth of the universe, “negative mass”, which corresponds to “negative energy”, is of utmost necessity.

2-2-2. The error of denying negative energy [19] [20]

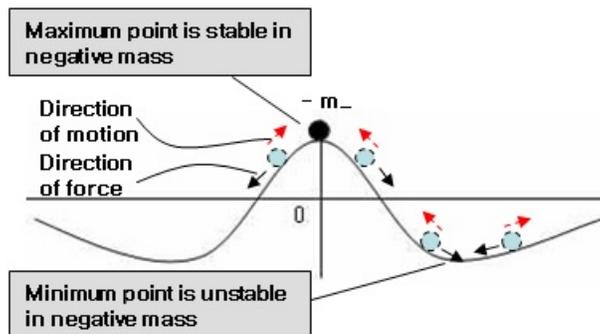


Figure 2: When there is negative mass in potential which has a point of maximum value and a point of minimum value.

If negative mass exists, is it stable at a lower energy state?

$$\vec{F} = -m_- \vec{a} \quad (8)$$

$$(m_- > 0)$$

$$\vec{a} = -\frac{\vec{F}}{m_-} \quad (9)$$

The acceleration of negative mass is opposite to the direction of force. Therefore, the negative mass has harmonic oscillation at the maximum point and it is also stable at the maximum point. [19] [20]

If there is no force to direct negative mass toward the minimal point, how negative mass can go to the minimal point?

Negative mass is stable at the state of high energy. This is quite contrary to the case of positive mass where, stability occurs at the minimum point, i.e., at the lowest energy state. [19] [20]

As we have examined above, “the problem of transition to minus infinite energy level” does not occur, and thus positive mass and negative mass can exist in the same space-time. This is a very important result because it means that negative mass and negative energy can exist stably in our universe. [19] [20]

If negative mass and positive mass coexist, gravitational potential energy consists of the following three items.

$$U_T = \sum_{i < j} \left(-\frac{Gm_+ m_+}{r_{++ij}} \right) + \sum_{i < j} \left(-\frac{G(-m_-)(-m_-)}{r_{--ij}} \right) + \sum_{i,j} \left(-\frac{G(-m_-)m_+}{r_{-+ij}} \right) \quad (10)$$

$$U_T = \sum_{i < j} \left(-\frac{Gm_+ m_+}{r_{++ij}} \right) + \sum_{i < j} \left(-\frac{Gm_- m_-}{r_{--ij}} \right) + \sum_{i,j} \left(+\frac{Gm_- m_+}{r_{-+ij}} \right) \quad (11)$$

$$U_T = U_{++} + U_{--} + U_{-+} \quad (12)$$

The present cosmological constant can be obtained by adding potential $U_\Lambda = -\frac{1}{6}\Lambda mc^2 r^2$ to mechanical energy conservation equation. [5] Refer to figure 1. If we insert ‘new potential energy term’ into mechanical energy conservation equation, we will get a similar term such as $-\frac{1}{3}\Lambda c^2$ term.

At this time, let’s insert the new gravitational potential energy term(eq.(11)) into it.

If U_{++} , U_{--} , U_{-+} has a ratio(4.9% : 26.8% : 68.3%) between each other, maybe, we will estimate that ratio of energy density such as 4.9% : 26.8% : 68.3% exist. [21]

2-2-3. Theoretical demonstration of constituent ratio of the universe by Pair Creation Model

2-2-3-1. Total gravitational potential energy in the universe that consists of negative mass and positive mass, and dark Energy

Let’s find values at the easiest state when matters are evenly distributed and then expand them into the state that shows the current universe.

Total gravitational potential energy U_T is as shown in equation (10). As we maintain that each gravitational potential energy corresponds to matter, dark matter, and dark energy, the following formula can be drawn.

$$U_T = \sum_{i < j} \left(-\frac{Gm_+ m_+}{r_{++ij}} \right) + \sum_{i < j} \left(-\frac{Gm_- m_-}{r_{--ij}} \right) + \sum_{i,j} \left(+\frac{Gm_- m_+}{r_{-+ij}} \right) \quad (13)$$

$$= U_m + U_d + U_\Lambda \quad (14)$$

$$(m_+ \geq 0, m_- \geq 0)$$

As positive masses are evenly distributed in radius R, $U_m = -\frac{3}{5} \frac{GM_+^2}{R}$

As negative masses are evenly distributed in radius R, $U_d = -\frac{3}{5} \frac{GM_-^2}{R}$

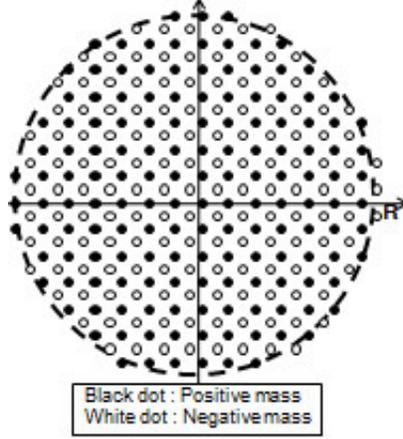


Figure 3: Universe where negative mass(energy) and positive mass(energy) are evenly distributed.

The concept of gravitational self-energy which consists of two types of source masses (positive and negative) corresponds to the state that all gravitational potential energy terms have negative values.

$$U_S = \sum_{i<j} \left(-\frac{Gm_+i m_+j}{r_{++ij}}\right) + \sum_{i<j} \left(-\frac{Gm_-i m_-j}{r_{--ij}}\right) + \sum_{i,j} \left(-\frac{Gm_-i m_+j}{r_{-+ij}}\right) \quad (15)$$

$$= U_m + U_d - U_\Lambda \quad (16)$$

$$= -\frac{3}{5} \frac{GM^2}{R} \quad (17)$$

Let's consider that any gravitational potential energy terms that constitute the system do not disappear while evaluating U_T or U_S .

$$U_m + U_d - U_\Lambda = -\frac{3}{5} \frac{GM_+^2}{R} - \frac{3}{5} \frac{GM_-^2}{R} - U_\Lambda = -\frac{3}{5} \frac{GM^2}{R} \quad (18)$$

$$(M_+ = M_- \geq 0, M = M_+ + M_-)$$

Therefore, it can be inferred that the dark energy itself will have the following forms.

$$U_\Lambda = +k_h(t) \frac{GM^2}{R} \quad (19)$$

In the above, a simple case that positive mass and negative mass are evenly distributed was assumed, but in our real universe, the distribution of positive mass and negative mass are asymmetrical. For this reason, coefficient $k_h(t)$ is introduced.

We can find the current $k_h(t)$ values in observing the universe.

2-2-3-2. Current universe : If positive mass constitutes galaxy or cluster of galaxies and negative mass are completely distributed evenly [1]

$U_{++} = -\frac{3}{5} \frac{GM_+^2}{R}$ needs to be corrected because it's not that positive mass is completely distributed evenly.

Let's get a hint from the movement between the sun and the earth!

Total gravitational potential energy between the sun and the earth is

$U_T = U_s = \text{Sun's gravitational self-energy} + \text{Earth's gravitational self-energy} + \text{gravitational potential energy between the sun and the earth}$

$$U_T = U_S = U_{self-Sun} + U_{self-Earth} + U_{Sun-Earth} \quad (20)$$

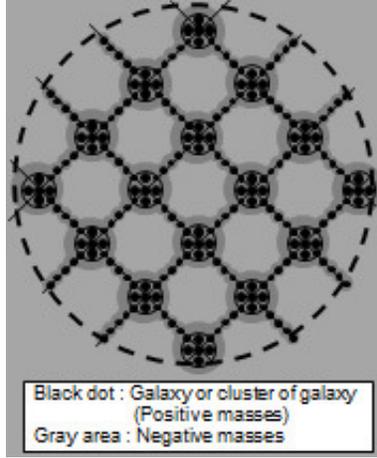


Figure 4: The current universe presumed in this model: Although the positive mass(black dot) constitute galaxy or cluster of galaxies and negative mass(gray) are almost evenly distributed in the entire universe, it is presumed that the density of negative mass(dark gray) near the galaxy would be higher than that of negative mass in the void area(gray), because negative mass receives attractive gravitational effect from large positive mass.

$$U_{Sun-Earth} = U_S - (U_{self-Sun} + U_{self-Earth}) \quad (21)$$

However, the particles that constitute the sun always move together because they are gravitationally strongly bonded. Likewise, the particles that constitute the earth always move together because they also share strong gravitational bond. Therefore, what determines the movement between the sun and the earth is only the ‘gravitational potential energy between the sun and the earth’.

In other words, the gravitational potential energy of objects that have strong gravitational bonds between them, does not change. It means that the gravitational potential energy of objects that are gravitationally strongly bonded does not contribute to the movement of other objects.

We can see that galaxies or cluster of galaxies are strongly bonded gravitationally. Thus, we should subtract these gravitational potential energy terms.

$$U_{++} = U_m = -\frac{3}{5} \frac{GM_+^2}{R} - N \left(-\frac{3}{5} \frac{Gm_+^2}{r_0} \right) = -\frac{3}{5} \frac{GM_+^2}{R} \left[1 - N \left(\frac{m_+}{M_+} \right)^2 \frac{R}{r_0} \right] \quad (22)$$

* M_+ : Total mass of matters within Hubble’s radius

* R : Hubble’s radius

* m_+ : Average mass of objects strongly bond gravitationally (Probably, mass of galaxy or cluster of galaxies)

* r_0 : Average radius of objects strongly bond gravitationally

* N : Number of objects strongly bond gravitationally

* $M_+ = M_-$, $M = M_+ + M_-$

On the other hand, **negative mass has gravitation effect which is repulsive to each other. Accordingly, if we assume that the entire universe is almost evenly distributed,**

$$U_{--} = U_d = -\frac{3}{5} \frac{GM_-^2}{R} = -\frac{3}{5} \frac{GM_+^2}{R} \quad (23)$$

$$\frac{U_{--}}{U_{++}} = \frac{U_d}{U_m} = \frac{-\frac{3}{5} \frac{GM_-^2}{R}}{-\frac{3}{5} \frac{GM_+^2}{R} \left[1 - N \left(\frac{m_+}{M_+} \right)^2 \frac{R}{r_0} \right]} = \frac{1}{1 - N \left(\frac{m_+}{M_+} \right)^2 \frac{R}{r_0}} = c \quad (24)$$

$$N \left(\frac{m_+}{M_+} \right)^2 \frac{R}{r_0} = 1 - \frac{1}{c} \quad (25)$$

$$N\left(\frac{\rho_{m+}^2 r_0^5}{\rho_{M+}^2 R^5}\right) = 1 - \frac{1}{c} \quad (26)$$

From observation: $c_{WMAP} = \frac{23.3}{4.6} \simeq 5.065$ [9], $c_{Planck} = \frac{26.8}{4.9} \simeq 5.469$ [10]

Now, let's calculate dark energy:

$$\text{If } U_S \approx -\frac{3}{5} \frac{GM^2}{R},$$

From the equation (16)(17), we get,

$$-\frac{3}{5} \frac{GM_+^2}{R} \left[1 - N\left(\frac{m_+}{M_+}\right)^2 \frac{R}{r_0}\right] - \frac{3}{5} \frac{GM_-^2}{R} - U_\Lambda \approx -\frac{3}{5} \frac{GM^2}{R} \quad (27)$$

$$U_\Lambda \approx +\frac{3}{5} \frac{GM^2}{R} - \frac{3}{5} \frac{GM_+^2}{R} \left[1 - N\left(\frac{m_+}{M_+}\right)^2 \frac{R}{r_0}\right] - \frac{3}{5} \frac{GM_-^2}{R} \quad (28)$$

1) **WMAP:**

$$\begin{aligned} U_\Lambda &= +\frac{3}{5} \frac{G(2M_-)^2}{R} - \frac{3}{5} \frac{GM_-^2}{R} \left(\frac{1}{5.065}\right) - \frac{3}{5} \frac{GM_-^2}{R} \\ &= \left(4 - \frac{1}{5.065} - 1\right) \frac{3}{5} \frac{GM_-^2}{R} \\ &= (-2.803)U_d \end{aligned} \quad (29)$$

Therefore,

$$U_{++} : U_{--} : U_{-+} = U_m : U_d : U_\Lambda = -0.197 : -1 : +2.803 = 4.93\% : 25.00\% : 70.07\% \quad (30)$$

It's very close to observed WMAP value.

2) **Planck:**

$$\begin{aligned} U_\Lambda &= +\frac{3}{5} \frac{G(2M_-)^2}{R} - \frac{3}{5} \frac{GM_-^2}{R} \left(\frac{1}{5.469}\right) - \frac{3}{5} \frac{GM_-^2}{R} \\ &= (-2.817) \left(-\frac{3}{5} \frac{GM_-^2}{R}\right) \\ &= (-2.817)U_d \end{aligned} \quad (31)$$

Therefore,

$$U_{++} : U_{--} : U_{-+} = U_m : U_d : U_\Lambda = -0.183 : -1 : +2.817 = 4.58\% : 25.00\% : 70.43\% \quad (32)$$

Also, it's similar to the estimation data of Planck satellite. Therefore, this model has potential.

Average of WMAP and Planck - Matter : Dark Matter : Dark Energy = 4.75% : 25.05% : 70.20%

Average of Pair Creation Model - Matter : Dark Matter : Dark Energy = 4.75% : 25.00% : 70.25%

We found the ratio above with Pair Creation Model for negative and positive masses, and the value being similar to the current cosmic observation suggests the need to review this model seriously.

2-2-3-3. Pair Creation Model with negative and positive masses [1] [21]

In the above process of finding the ratios of matter, dark matter and dark energy, I used very strong constraints.

1) The universe consists of negative mass(energy) and positive mass(energy), with the same size of them.

2) Matter, dark matter and dark energy correspond to the gravitational potential energy made by positive masses, negative masses, and between positive and negative masses, respectively.

3) We judge the components of the universe by gravitational effect rather than mass energy. If U_{++} , U_{--} , U_{-+} has a ratio(4.9% : 26.8% : 68.3%) between each other, maybe, we will estimate that ratio of energy density such as 4.9% : 26.8% : 68.3% exist.

This does not mean that 68.3% of dark energy exists independently, but it means that the explanation of gravitational potential energy (U-+) occurring from negative energy, which is the same as positive energy, is possible. [21] This discovery implies that our belief that size of gravitational effect and size of components of the universe would always 1:1 correspond was wrong. [21]

Moreover, this negative energy is the energy which is inevitably required from zero energy, which is the most natural total energy value in the universe.

Trimming up the model, I can explain the current observations accurately. For details, see this thesis. [1]

2-2-4. Strengths of Pair Creation Model with negative and positive masses

2-2-4-1. Demands from an fundamental law

Pair Creation Model is the one essentially demanded from conservation of energy.

As seen in chapter 2-2-1, since only gravitational potential energy cannot offset the positive mass energy, there is need for negative energy at the time of cosmic birth, too, in order to make conservation of energy valid.

$$E_T = 0 = (+E) + (-E) = \left(\sum m_+ c^2\right) + \left(\sum -m_- c^2\right) + \left(\sum U\right) = 0 \quad (33)$$

Think that the universe started with a specific energy E_1 instead of zero energy. Logically, such universe would permit all real number times energy state of this specific energy value E_1 and eventually allow the possibility of the existence and logics of the universe that has an infinite number of different energy values.

However, “0” is a very special value, and the universe was born from absolutely nothing, in other words, zero energy state and the birth process implies that it is a phase transition of a zero energy state. This suggests that the law of conservation of energy applies for the birth of the universe and **the zero energy value is the demand and outcome of the law of conservation of energy.**

Therefore, if it is verified that this model is correct, then it can be seen as the verification of that multiverse and anthropic principle are incorrect. [22] [23]

2-2-4-2. Negative mass is a valid solution for the universe’ accelerated expansion.

Λ CDM, has been gracefully accepted as our current standard model of cosmology. But neither Λ nor CDM has been successfully proven till now. [5] [24] [25] At this point, what we can trust is the information, that a certain repulsive gravitational (accelerating expansion) effect and an attractive gravitational (centripetal force) effect exists in the universe. At least the above fact is consistent with our observation.

At the present, it is understood that dark matter and dark energy are completely different in nature. Dark matter corresponds to the attractive effect, whereas dark energy corresponds to the repulsive effect. Therefore, dark matter and dark energy have a completely different significance.

In 1998, observations by both the HSS team and SCP team resulted in the determination of negative energy density from inspected field equations devoid of cosmological constant.

The findings were as follows:

HSS(The High-z Supernova Search) team : If $\Lambda = 0$, $\Omega_M = -0.38(\pm 0.22)$ [7]

SCP(Supernova Cosmology Project) team : If $\Lambda = 0$, $\Omega_M = -0.4(\pm 0.1)$ [8]

Therefore, negative mass is a valid solution for the universe' accelerated expansion.

However, “the problem of transition to minus infinity energy level” took the better of them and the two teams concluded that negative mass and negative energy level could not exist in our universe. They instead revised the field equation by inserting the cosmological constant. [7] [8]

Moreover, we considered vacuum energy as the source of cosmological constant Λ , but the current result of calculation shows difference of 10^{120} times between the two, which is unprecedented even in the history of Physics. [5]

However, if “the problem of transition to minus infinity energy level” does not occur and negative and positive mass can coexist, what would happen?

2-2-4-3. Explanation of the failing observation of dark matter on earth or in the solar system

Currently, we are failing to detect dark matter from the solar system or the earth. [11–13] Dark matter consisting of negative mass exists outside the galaxy structure with only gravitational effect within the structure. Therefore, this enables us to explain the current situation in which gravitational effect does exist but the substance is not detected.

2-2-4-4. Pair Creation Model coincides with the assumption that particles considered the origin of dark matter should not exist in the Standard Model. [15]

Negative mass complies with conservation of energy or conservation of momentum, and exists outside of the standard model.

2-2-4-5. CCC(Cosmological Constant Coincidence) Problem

We can answer the CCC problem, “Why does dark energy share the similar scale with matter?” It is because it has the same gravitational effect as them.

2-2-4-6. Explanation of Fine Tuning Problem

Pair creation model secures that cosmic curvature is close to 0.

Positive energy and negative energy are counterbalanced in a zero energy universe, thus explaining the fact that the universe is almost flat.

2-2-5. Method to test Pair Creation Model

We must not throw away the other models without reviews even if the Λ CDM model is right overall. It's because if Λ CDM model is right, the model has to account for Λ and CDM as completely as possible, but they are not successful till now. [25]

The reason why the entire explanation of Λ CDM at this point in time seems to be right is that the repulsive gravitation effect corresponding to Λ and the attractive gravitational effect corresponding to cold dark matter are required. However, this can also be properly explained by this model (Pair creation of positive energy and negative energy). The negative mass(energy) is the object that satisfies energy conservation and should be an indispensable concept.

Λ CDM model expects that the ratio of matter and dark matter will be constant, [5] but this model suggests that as the universe expands, the gravitational effect of matter vs dark matter differs.

Finally, in the current mainstream physical description, we will describe that the amount of dark matter gradually increases. Even though the gravitational effects of dark matter increase, the gravitational effects corresponding to dark energy also increase. So the universe is estimated to continue to accelerate its expansion. However, gravitational potential energy is a conserved quantity and decreases as the distance increases, so it appears to be lower than the Λ CDM model in acceleration.

Therefore, the past and the future predicted by two models (Λ CDM and pair creation) are completely different.

$$\frac{\Omega_d}{\Omega_m} = 1 \Rightarrow \frac{\Omega_d}{\Omega_m} = 5.47$$

Thus, it is necessary to investigate the change of the ratio $\frac{\Omega_d}{\Omega_m}(t)$

We need to make new Friedmann equations and new field equation on the assumption that negative energy(mass) and positive energy(mass) coexist and compare two models.

2-2-6. Expansion of general relativity [1] [21]

Field equation of the existing general relativity is a special case of that for Pair Creation Model, that is, a version when there exists only positive mass.

Einstein's field equation is given by :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} \quad (34)$$

We need to frame a new Friedmann equation and field equation, on the assumption that negative energy(mass) and positive energy(mass) coexist.

If negative energy and positive energy coexist, gravitational potential energy consists of the below three terms.

$$U_T = U_{++} + U_{--} + U_{-+}$$

$$\text{Matter (Positive mass)} : \sum_{i>j} -\frac{Gm_+i m_+j}{r_{++ij}} \rightarrow 8\pi G(^{++}T_{\mu\nu})$$

$$\text{Dark Matter (Negative mass)} : \sum_{i>j} -\frac{Gm_-i m_-j}{r_{--ij}} \rightarrow 8\pi G(^{--}T_{\mu\nu})$$

$$\text{Dark Energy(gravitational potential energy between negative mass and positive mass)} : \sum_{i,j} +\frac{Gm_-i m_+j}{r_{-+ij}} \rightarrow 8\pi G(^{-+}T_{\mu\nu})$$

Therefore, new field equation is

$$G_{\mu\nu} = 8\pi GT_{\mu\nu} \quad (35)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(^{++}T_{\mu\nu} + ^{--}T_{\mu\nu} + ^{-+}T_{\mu\nu}) \quad (36)$$

At this time, we should consider that negative mass surrounds the galaxy or galaxy clusters composed of positive mass.

Only the positive mass world, the Earth and the Solar system

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(^{++}T_{\mu\nu}) \quad (37)$$

Thus, we get an Einstein's field equation.

But negative energy(mass) exists outside of this galaxy structure, So, we observe the dark matter term and dark energy term in the universe.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(^{++}T_{\mu\nu} + ^{--}T_{\mu\nu} + ^{-+}T_{\mu\nu}) \quad (38)$$

We need solve this new field equation.

Friedmann equation utilized the concept of mechanical energy conservation applied to positive mass. This gave rise to the equation of motion in the universe that is composed of '**one kind**' of gravitational characteristic (positive mass source).

However, **if there exist two kinds of gravitational sources**, gravitational potential energy is composed of three terms. U_{11} (gravitational potential energy between type 1s)+ U_{22} (gravitational potential energy between type 2s)+ U_{12} (gravitational potential energy between type 1 and type 2)

We can get a very similar result with the Λ CDM model. Also, it has a better condition. gravitational potential energy U_{-+} , which belongs to this dark energy, has a positive value, and generates repulsive force. We can explain the basis for this term. It is positive gravitational potential energy.

We cannot conclude that this model is wrong although this model is different from the values of the existing Λ CDM model. All results from the Λ CDM model are consistent within the system. The universe that consists of two gravitational sources differs in their movement compared to conventional models of single gravitational sources. Thus, we need to set up new Friedmann equation and field equation to solve problems.

Nevertheless, this model provides similar results to Λ CDM model, because ‘ U_{-+} (repulsive gravitational potential energy)’ and ‘ U_{--} (attractive gravitational potential energy)’ that plays a role of Λ and CDM respectively, are added one by one separately.

III. The existence of singularity, which denies the application of general relativity itself, as a solution for field equations. [2]

1. Understanding of the problem : Generation of singularity [26]

Generally, stars are under gravitational contraction by their own gravity and it is known that if this is contracted within a certain radius (like Schwarzschild radius), it makes a black hole which even lights cannot escape from.

Since gravity is generally an attractive force, gravitational contraction continues to exist in the black hole, too. Thus, in the central part of black hole exists an area with infinite density of energy, which point we call singularity. [4] [26]

Such singularity denies application of the existing laws of physics and it is unnatural for a certain substantial object to have infinite density of energy. Besides, such singularity has never been observed as substance but is just a mathematical result of general relativity, which is considered a defect or limit of the theory.

We assume that the solution for this singularity consists in quantum mechanics. Though exact explanation is not available because quantum gravity theory in integration of quantum mechanics and gravity has not been completed yet, [6] we are looking for ways to avoid the problem of singularity by supposing the minimum size in the superstring theory. [27]

This writing will prove that an object of positive energy has the minimum size for its existence and that since this size is in proportion to that of energy, there is no singularity with infinite density.

2. Solution of the problem [2]

2-1. Gravitational potential energy with negative values

Many teachers in the school are teaching that gravitational potential energy is a hypothetical energy by which to set randomly reference point, explaining that “The infinite point” has just been set for convenience.

Consequently, the majority of people simply consider that gravitational potential energy is a hypothetical or potential energy to be eliminated by setting randomly reference point, not an object that has a fixed value or substance like mass energy.

2-1-1. Gravitational potential energy is an object that has a certain value at all spatial positions.

Let’s consider the case below.

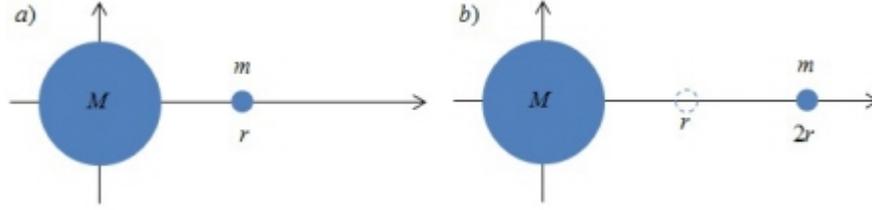


Figure 5: The mass (m) exists in a stationary state at the points r and $2r$ away from M .

When m is situated at r and $2r$, we can see that these two have difference in gravitational potential energy and total energy.

While a) has rest mass energy and gravitational potential energy at position r ,

In case of b), the m initially located at $2r$ becomes accelerated from a halting state to pass position r . Because b) has kinetic energy as well as rest mass energy and gravitational potential energy at position r . It seems obvious that this kinetic energy comes from gravitational potential energy, so gravitational potential energy is an object that has different values when r , a distance from gravitational source for all spatial positions, varies.

2-1-2. All energies are a gravitational source. [4]

We know that kinetic energy and thermal energy increase the mass of an object, and that these kinetic energy and thermal energy are also a kind of energy that plays the role as a gravitational source. [28]

In general relativity, curvature in space-time is determined by the energy of its system while energy is a gravitational source. [4]

If we set gravitational potential energy as such energy allowing randomly reference point, total energy of the system has an infinite number of values according to the reference point chosen by each of people who exists in the same inertial system at a single situation, while curvature of space-time must be changing numerously in real time by the threshold set optionally by people. This seems unnatural.

It seems logical that the curvature of space-time must have a single value at its same position and that the value of gravitational potential energy must also be in the form of a single value.

2-1-3. $U(r) = -\frac{GMm}{r}$ (except $r=0$) is considered to provide consequently the right explanation for all points

Teachers and professors have explained that it is alright to set the randomly reference point for gravitational potential energy because, since the variation of gravitational potential energy has caused kinetic change in the problem under review, there was no problem in dealing with only the variation of gravitational potential energy.

From the equation $K + U = const.$ we obtain such equation as $\Delta K = -\Delta U$, which can explain motion with variation, but this neither means that all observers in the same inertial system may set randomly reference point at random nor confirms that U is an object with an optional value.

Let's consider the following case that the value of gravitational potential energy has been fixed for the distance of 0 to infinity from gravitational source.

$$U(r) = -\frac{GMm}{r} \quad (39)$$

$$\Delta U = U_f - U_i = \left(-\frac{GMm}{r+h}\right) - \left(-\frac{GMm}{r}\right) = \frac{GMm}{r(r+h)}h \approx \frac{GMm}{r^2}h = mgh \quad (40)$$

Even though, as above, gravitational potential energy has the value of energy defined for $r=0$ to $r=\infty$ from gravitational source, we can obtain the right result in a problem in which its variation matters.

2-1-4. Effect of mass defect in atomic scale caused by binding energy

Equation of Coulomb potential energy ($U = -\frac{kq_1q_2}{r}$), which is similar to gravitational potential energy, explains mass defect and energy level exactly against all positions of r , while we observe that the effect of mass defect occurs in as large the size as binding energy. [29]

Gravitational potential energy is another binding energy, which we should handle as substantial energy.

2-2. Gravitational self-energy or Gravitational binding energy [30]

The concept of gravitational self-energy is the total of gravitational potential energy possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dM , it involves the existence of gravitational potential energy among these dM s and is the value of adding up these.

$$M = \sum dM$$

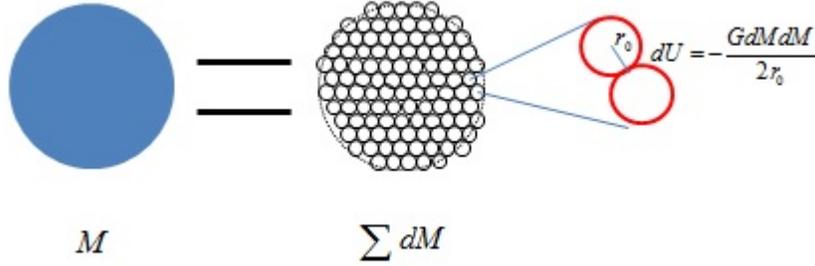


Figure 6: Since all mass M is a set of infinitesimal mass dM s and each dM is gravitational source, too, there exists gravitational potential energy among each of dM s. Generally, gravitational potential energy by infinitesimal mass that consists of an object itself is reflected on the mass of the object itself. Mass of an object measured from its outside corresponds to the value of dividing the total of all energy into c^2 .

Gravitational self-energy or Gravitational binding energy ($-U_{gs}$) in case of uniform density is given by:

$$U_{gs} = -\frac{3}{5} \frac{GM^2}{R} \quad (U_{gs} : \text{gravitational self-energy})$$

2-3. For black hole or singularity, never fail to consider gravitational self-energy

In the generality of cases, the value of gravitational self-energy is small enough to be negligible, compared to mass energy mc^2 .

- 1) The earth's gravitational self-energy is roughly 4.6×10^{-10} times as large as the earth's rest mass energy.
- 2) The moon's gravitational self-energy is roughly 0.2×10^{-10} times as large as the moon's rest mass energy.

Therefore, in usual cases, $|U_{gs}| \ll Mc^2$, so generally, there was no need to consider gravitational self-energy.

Meanwhile, looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = \left| -\frac{3}{5} \frac{GM^2}{R_{gs}} \right| = Mc^2 \quad (41)$$

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} \quad (42)$$

This equation means that if infinitesimal mass is uniformly distributed within the radius R_{gs} , gravitational self-energy for such an object equals mass energy in size. So, in case of such an object, mass energy and gravitational self-energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing R_{gs} with R_B , the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} < R_B = \frac{2GM}{c^2} \quad (43)$$

$$R_{gs} = 0.3R_B \quad (44)$$

This means that there exists the point where gravitational self-energy becomes equal to mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists at the point $0.3R_B$, a 30% level of the black hole radius. Even with kinetic energy and virial theorem applied only the radius diminishes as negative energy counterbalances positive energy, but no effects at all on this point: “there is a zone which cannot be compressed anymore due to the negative gravitational potential energy”

Since this value is on a level not negligible against the size of black hole, we should never fail to consider “gravitational self-energy” for case of black hole.

2-4. Black hole doesn't have singularity.

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point $R=0$. From the point R_{gs} , gravity is 0, and when it enters into the area of R_{gs} , total energy within R_{gs} region corresponds to negative values enabling antigravity to exist.

This $0.3R_B$ region comes to exert repulsive effects of gravity on the particles outside of it, therefore it interrupting the formation of singularity at the near the area $R=0$.

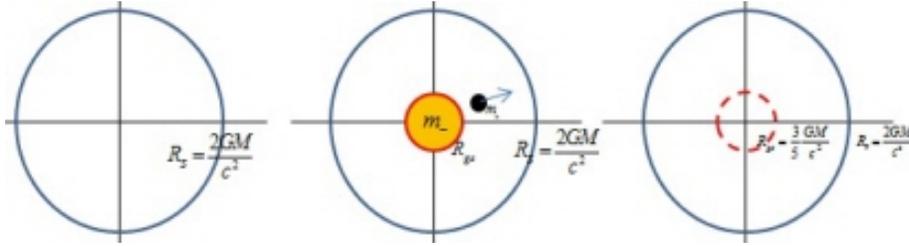


Figure 7: Considering gravitational potential energy for black hole, the area of within R_{gs} has gravitational self-energy of negative value, which is larger than mass energy of positive value. This area (within R_{gs}) exercises antigravity on all particles entering this area anew, and accordingly prevents all masses from gathering to $r=0$.

However, it still can perform the function as black hole because R_{gs} is only 30% of R_B with a large difference in volume and, comparing total mass, it still can correspond to a very large quantity of mass. Therefore, it still can perform the function as black hole on the objects outside of R_B .

There are a few interesting cases here:

- 1) For case of forming black hole and then bursting

In the process of a star's gravitational contraction (or collapse), matters outside the radius of black hole flows inside black hole, where even if these particles enter inside the event horizon of black hole, velocity vector still has $-\hat{r}$ direction.

Thus, it can occur that these particles reach the area of R_{gs} and then pass over it.

On the above, situation of mass in uniform distribution was considered but for the mass not uniform in distribution, or in case of central density being much higher, these would expand themselves again. In some cases, they may pop out of the black hole. Maybe, it can be a solution to the problem of gamma-ray arising from the merger of the black holes. [31]

If a situation where R_{gs} becomes bigger than the minimum radius of the Kerr black hole occurs due to the particles absorbed from the outside, then it is possible for energy (and particles) to escape from the inside of the black hole.

2) For case of particles becoming stable while vibrating inside black hole

By locking horns between gravitational self-energy and mass energy, particles inside black hole or distribution of energy can be stabilized.

3) The analysis above is also applicable to birth of the universe.

Birth of high-density energy in such an area of Planck scale simply means that negative gravitational self-energy is larger than the mass energy of matter. So it generates repulsive effect and should result in cosmic expansion.

It can explain the reasons for expansion in high-density state in which gets over the density of black hole in the beginning of the universe, and expansion immediately after cosmic birth.

2-5. The minimal size of existence

Differential and integral concepts we have acquired and used in mathematics and physics comprise the idea that an object can be considered “a set of some infinitesimals.”

[**Existence = the sum of infinitesimal existences composing an existence**]

A single mass M for some object means that it can be expressed as $M = \sum dM$ and, for energy, $E = \sum dE$. The same goes for elementary particles, which can be considered a set of dM s, the infinitesimal mass.

The equation ($R_{gs} = \frac{3}{5} \frac{GM}{c^2}$) above means that if infinitesimal masses are uniformly distributed within the radius R_{gs} , the size of negative-value binding energy becomes equal to that of mass energy. This can be the same that the rest mass, which used to be free for the mass defect effect caused by binding energy, has all disappeared.

This means the total energy value representing “some existence” coming to 0 and “extinction of the existence (with positive energy)”. **Therefore, R_{gs} is considered to act as “the minimal radius” or “a bottom line” of existence with some positive energy.**

For case of proton,

$$R_{gs} = \frac{3}{5} \frac{GM}{c^2} = 7.457 \times 10^{-55} [m] \quad (45)$$

Currently, the size of proton is about $10^{-15}m$, which isn't against the assumption of this model

2-6. Expansion of general relativity

2-6-1. We can solve the problem of singularity by separating the term ($-M_{gs} = \frac{U_{gs}}{c^2}$) of gravitational self-energy from mass and including it in the solutions of field equation.

In the Schwarzschild solution,

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (46)$$

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5} \frac{GM^2}{Rc^2} \quad (47)$$

$$ds^2 = -\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right) c^2 dt^2 + \frac{1}{\left(1 - \frac{2G(M - M_{gs})}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (48)$$

In general, $M \gg M_{gs}$. But we should never fail to consider “gravitational self-energy” for case of black hole.

- 1) If $M \gg | -M_{gs} |$, in other words if $r \geq R_S$, we get the Schwarzschild solution.
- 2) If $M \ll M_{gs}$,

$$ds^2 \simeq -\left(1 + \frac{2GM_{gs}}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{2GM_{gs}}{c^2 r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (49)$$

i) $r \geq R$, By eq.(47)

$$ds^2 \simeq -\left(1 + \frac{6G^2 M^2}{5c^4 Rr}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{6G^2 M^2}{5c^4 Rr}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (50)$$

ii) $0 \leq r < R$

The area of within R_{gs} has gravitational self-energy of negative value, which is larger than mass energy of positive value. This area(within R_{gs}) exercises antigravity on all particles entering this area anew, and accordingly prevents all masses from gathering to $r=0$. Besides, negative mass has gravitation effect which is repulsive to each other. Therefore, we can assume that $-M_{gs}$ is almost evenly distributed. Therefore ρ_{gs} is constant. And we must consider the Shell Theorem.

$$-M_{gs} = -\frac{4\pi r^3}{3} \rho_{gs} \quad (51)$$

$$\left(1 + \frac{2GM_{gs}}{c^2 r}\right) = 1 + \frac{2G\left(\frac{4\pi}{3} r^3 \rho_{gs}\right)}{c^2 r} = 1 + \frac{8\pi G \rho_{gs} r^2}{3c^2} \quad (52)$$

$$ds^2 \simeq -\left(1 + \frac{8\pi G \rho_{gs} r^2}{3c^2}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{8\pi G \rho_{gs} r^2}{3c^2}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (53)$$

If $r \rightarrow 0$,

$$ds^2 \simeq -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (54)$$

There is no singularity.

In practice, gravitational collapse must be stopped at the point where $M_{shell} = M_{shell-gs}$.

For Schwarzschild black hole, the Kretschmann scalar is,

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{48G^2 M^2}{c^4 r^6} \quad (55)$$

In the preexisting model, if $r \rightarrow 0$, it does diverse.

However, this model is

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{48G^2 (M - M_{gs})^2}{c^4 r^6} \approx \frac{48G^2 (-M_{gs})^2}{c^4 r^6} = \frac{(16\pi G \rho_{gs})^2}{3c^4} \quad (56)$$

If $r \rightarrow 0$, it does not diverge.

2-6-2. Gravitational self-energy can provide the concept of minimal size, one of the reasons for introducing string theory.

In quantum mechanics we could think of De Broglie’s matter wave theory about length of some objects. However this formula does restrict the upper limit of velocity to c , but does not have the upper limit for mass. In other words this implies there is no lower limit of wavelength and it can go to 0 in quantum mechanics. It’s the same with Heisenberg’s Uncertainty Principle.

Furthermore, 0.02 milligram of Planck mass ($m_p = \sqrt{\frac{c\hbar}{G}}$) they introduced is not even the size for the role of upper or lower limit among “ $0 \sim \infty$ ”. Since Planck mass cannot perform a role as the upper limit, Planck length, inversely related to the Planck mass, cannot do a role of lower limit either.

Planck mass and length means that “within such a size we should consider the quantum mechanical effects.” but does not indicate “no more or no less can exist.”

To remove singularity, considering gravitational self-energy is only enough without need to assume some minimal unit like a string. [27] Thus, the existing relations need to be transformed so that they may include the minimal length by dint of gravitational self-energy.

$$\Delta x \sim \frac{\hbar}{\Delta p} \tag{57}$$

$$\Delta x \sim \frac{\hbar}{\Delta p} + R_{gs} \geq \frac{3}{5} \frac{GM}{c^2} = \frac{3}{5} \frac{GE}{c^4} \tag{58}$$

Since the increase of momentum is put into the increase of energy and mass, Δx cannot go to 0. Therefore, we can introduce the minimal length naturally and work out the problems of singularity or infinity.

IV. Integration with quantum mechanics and a very large curvature in space-time

To solve this problem, we must reject the error originating with Bohr and start from the fact that the XXX theory is not beautiful. Specifically, $\sim \sim$

Ouch, it’s time to go fry chicken.
Next time for the remainder

References

- [1] Hyoyoung Choi, Is the State of Low Energy Stable? Negative Energy, Dark Energy and Dark Matter. (2013). [<http://www.researchgate.net/publication/263468413>]. [<http://vixra.org/abs/1309.0035>]. Hyoyoung Choi & Prabir Rudra, Pair Creation Model of the Universe From Positive and Negative Energy (2014) [<http://www.researchgate.net/publication/275056453>] [<http://vixra.org/abs/1403.0180>].
- [2] Hyoyoung Choi, <http://blog.naver.com/hbar108>.(2009 - 2015).
- [3] Einstein, Albert, “Die Grundlage der allgemeinen Relativittstheorie”, *Annalen der Physik* 49: 769822, (1916), Bibcode:1916AnP...354..769E, doi:10.1002/andp.19163540702, archived from the original (PDF) on 2006-08-29, retrieved 2006-09-03. [<http://web.archive.org/web/20060829045130/http://www.alberteinstein.info/gallery/gtext3.html>].
- [4] Wikipedia, *General relativity*. [http://en.wikipedia.org/wiki/General_relativity].
- [5] Bradley W. Carroll, Dale A. Ostlie. *Introduction to Modern Astrophysics*. 2nd Edition. Pearson Education, Inc.. (2007)
- [6] Wikipedia, *Quantum gravity*. [http://en.wikipedia.org/wiki/Quantum_gravity].
- [7] Riess, A. G. et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* 116, 10091038 (1998).
- [8] Perlmutter, S. et al. Measurements of omega and lambda from 42 high-redshift supernovae. *Astrophys. J.* 517, 565586 (1999).
- [9] E. Komatsu et al. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *Astrophys.J.Suppl.*192:18,(2011).
- [10] Planck Collaboration. Planck 2013 results. XVI. Cosmological parameters. (2013). [[arXiv:1303.5076](https://arxiv.org/abs/1303.5076)].

- [11] LUX Collaboration. First results from the LUX dark matter experiment at the Sanford Underground Research Facility. (2013). [arXiv:1310.8214].
- [12] E. Aprile et al. [The XENON100 Collaboration]. Dark Matter Results from 100 Live Days of XENON100 Data. Phys. rev. Lett. 107, 131302(2011).
- [13] C. Moni Bidin et al., NO EVIDENCE FOR A DARK MATTER DISK WITHIN 4 kpc FROM THE GALACTIC PLANE. (2010). [http://arxiv.org/PS_cache/arxiv/pdf/1011/1011.1289v1.pdf].
- [14] Wikipedia, *Modified Newtonian dynamics*. [[http://en.wikipedia.org/wiki/Modified Newtonian dynamics](http://en.wikipedia.org/wiki/Modified_Newtonian_dynamics)].
- [15] Wikipedia, *Standard Model*. [[http://en.wikipedia.org/wiki/Standard Model](http://en.wikipedia.org/wiki/Standard_Model)].
- [16] Wikipedia, *Cosmological constant*. [[http://en.wikipedia.org/wiki/Cosmological constant](http://en.wikipedia.org/wiki/Cosmological_constant)].
- [17] Wikipedia, *f(R) gravity*. [[https://en.wikipedia.org/wiki/F\(R\) gravity](https://en.wikipedia.org/wiki/F(R)_gravity)].
- [18] Stephen Hawking. *A Brief History of Time*(Bantam Dell Publishing Group, New York, 1992).
- [19] Hyoyoung Choi, Study of Interaction between Negative mass and Positive mass, (Konkuk University, Seoul).(1997).
- [20] Hyoyoung Choi, Hypothesis of Dark Matter and Dark Energy with Negative Mass. (2009). [<http://www.researchgate.net/publication/289251017>][<http://vixra.org/abs/0907.0015>].
- [21] Hyoyoung Choi. The Change of Gravitational Potential Energy And Dark Energy in the Zero Energy Universe, (2011). [<http://www.researchgate.net/publication/274732484>][<http://vixra.org/abs/1110.0019>].
- [22] Wikipedia, *Multiverse*. [<http://en.wikipedia.org/wiki/Multiverse>].
- [23] Wikipedia, *Anthropic principle*. [[http://en.wikipedia.org/wiki/Anthropic principle](http://en.wikipedia.org/wiki/Anthropic_principle)].
- [24] P. J. E. Peebles and B. Ratra, The Cosmological Constant and Dark Energy. (2002). [<http://arxiv.org/abs/astro-ph/0207347>].
- [25] Wikipedia, *Lambda CDM model*. [[http://en.wikipedia.org/wiki/Lambda CDM model](http://en.wikipedia.org/wiki/Lambda_CDM_model)].
- [26] Wikipedia, *Gravitational singularity*. [[http://en.wikipedia.org/wiki/Gravitational singularity](http://en.wikipedia.org/wiki/Gravitational_singularity)].
- [27] Wikipedia, *Superstring theory*. [[http://en.wikipedia.org/wiki/Superstring theory](http://en.wikipedia.org/wiki/Superstring_theory)].
- [28] Wikipedia, *Equivalence principle*. [[http://en.wikipedia.org/wiki/Equivalence principle](http://en.wikipedia.org/wiki/Equivalence_principle)].
- [29] Wikipedia, *Nuclear binding energy*. [[http://en.wikipedia.org/wiki/Nuclear binding energy](http://en.wikipedia.org/wiki/Nuclear_binding_energy) Mass defect].
- [30] Wikipedia, *Gravitational binding energy*. [[http://en.wikipedia.org/wiki/Gravitational binding energy](http://en.wikipedia.org/wiki/Gravitational_binding_energy)].
- [31] NASA, *NASA's Fermi Preps to Narrow Down Gravitational Wave Sources*. (2016). [<http://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=12216>].