Smarandache–Boolean–Near–Rings and Algorithms

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Abstract. In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure $A_0$ on N such that there exists a proper subset $M$ of N, which is embedded with a stronger algebraic structure $A_1$, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its algorithms through Boolean-ring with left-ideals, direct summand, Boolean-$l$-algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities.

Keywords: Boolean-ring, Boolean-near-ring, Smarandache-Boolean-near-ring, left-ideal, direct summand, Boolean-$l$-algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities

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1. Introduction

In order that New notions are introduced in algebra to better study the congruence in number theory by Smarandache [4]. By <proper subset> of a set A we consider a set P included in A, and different from A, different form the empty set, and from the unit element in A – if they rank the algebraic structures using an order relationship: They say that the algebraic structures $S_1$<< $S_2$ if: both are defined on the same set; all $S_1$ laws are also $S_2$ laws; all axioms of an $S_1$ law are accomplished by the corresponding $S_2$ law; $S_2$ law accomplish strictly more axioms that $S_1$ laws, or $S_2$ has more laws than $S_1$.

For example: Semi group <<Monoid<< group << ring<< field, or Semi group<< commutative semi group, ring<< unitary , ring etc. They define a General special structure to be a structure SM on a set A, different form a structure SN, such that a proper subset of A is an structure, where SM<< SN <<

2. Preliminaries

Definition 2.1. A left near-ring A is a system with two binary operations, addition and multiplication, such that

(i) the elements of A form a group $(A, +)$ under addition,
(ii) the elements of \( A \) form a multiplicative semi-group,

(iii) \( x(y + z) = xy + xz \), for all \( x, y, z \in A \)

In particular, if \( A \) contains a multiplicative semi-group \( S \) whose elements generate \( (A,+) \) and satisfy

(iv) \((x+y)s = xs + ys\), for all \( x, y \in A \) and \( s \in S \), then we say that \( A \) is a distributively generated near-ring.

**Definition 2.2.** A near-ring \((B,+,\cdot)\) is Boolean-Near-Ring if there exists a Boolean-ring \((A,+,\Lambda,1)\) with identity such that \( \cdot \) is defined in terms of \( + \), \( \Lambda \) and 1, and for any \( b \in B \), \( b.b = b \).

**Definition 2.3.** A near-ring \((B,+,\cdot)\) is said to be idempotent if \( x^2 = x \), for all \( x \in B \). If \((B,+,\cdot)\) is an idempotent ring, then for all \( a, b \in B \), \( a + a = 0 \) and \( a.b = b.a \).

**Definition 2.4.** A Boolean-near-ring \((B,+,\cdot)\) is said to be Smarandache-Boolean-near-ring whose proper subset \( A \) is a Boolean-ring with respect to same induced operation of \( B \).

**Definition 2.5.** (Alternative definition for S-Boolean-near-ring) If there exists a non-empty set \( A \) which is a Boolean-ring such that it superset \( B \) of \( A \) is a Boolean-near-ring with respect to the same induced operation, then \( B \) is called Smarandache-Boolean-near-ring. It can also written as S-Boolean-near-ring.

3. Algorithms

**Left – Ideal:** Clay and Lawver [2] have introduced the left-ideals of \((B,+,\cdot)\) in \( P(x) \) are the subgroups of the groups \((P(x),+)\), where \( P(x) = \{b \in B / b \land x = b\} = B_x \) is a maximal sub-\( z \)-ring. It also contained in an ideal. Let \( A = I_0 \). Now to construct a set \( B \) as follows.

\( B \) contains a unique minimal ideal \( I_0 \) contained in all other non-zero ideals. According to Pilz [4, Theorem (1.60 (d))], \( B \) is Boolean-near-ring. Now by definition, \( B \) is a Smarandache-Boolean-near-ring.

**Algorithm 3.1.**

Step 1: Consider a Boolean-ring \( A \)
Step 2: Let \( A = I_0 \), be an ideal
Step 3: Let I_i, i = 0,1,2,3,…… be supersets of \( I_0 \)
Step 4: Let \( B = \bigcup I_0 \)
Step 5: Choose the sets \( I_j \) from \( I_i \)’s subject to \( a, b \) and \( c \in B \) such that

\[(a + b).c + a.c + b.c = x \land c \in I_j \text{ and } x \in B \text{ we have } P(x) \subseteq I \]

Step 6: Verify that \( \bigcap I_j = I_0 \neq \{0\} \)
Step 7: If step (6) is true, then we write \( B \) is a Smarandache-boolean-near-ring.

**Direct Summand**

Clay and Lawver [2] has introduced the concept of direct summand. Let \( A \) be an ideal of \( B \), then \( A \) is a direct summand if and only if \( A = P(x) \). Now to construct a set \( B \) as
follows. B contains a unique minimal direct summand $M_0$ contained in all other non-zero direct summands. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.2.
Step 1: Consider a Boolean-ring A
Step 2: Let $A = M_0$, be a direct summand.
Step 3: Let $M_i, i = 0,1,2,3,\ldots$ be supersets of $M_0$.
Step 4: Let $B = \bigcup M_i$.
Step 5: Choose the sets $M_j$ from $M_i$’s subject to for all $x \in B$ such that $M_0$ is a direct summand we have $M_0 = P(x)$ and $B = P(x) + P(x^1)$, where $P(x)$ and $P(x^1)$ are ideals of $B$ and $x, x^1 \in B$.
Step 6: Verify that $\bigcap M_j = M_0 \neq \{0\}$.
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Boolean-$l$-Algebra

Rao has introduced the notions of Boolean-$l$-algebra and lattice ordered groups. In [8] he proved A is a Boolean-ring if and only if A is a Boolean-$l$-algebra such that $x \leq a$ implies $x \bigwedge (a-x) = 0$. He has established that the class of Boolean-$l$-algebra is a subclass of DRI semigroups also. Let $A = I_0$. Now to construct a set B as follows.
B contains a unique minimal Boolean-$l$-algebra $I_0$ contained in all other non-zero Boolean-$l$-algebras. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.3.
Step 1: Consider a Boolean-ring A
Step 2: Let $A = I_0$, be a Boolean-$l$-algebra
Step 3: Let $I_i, i = 0,1,2,3,\ldots$ be supersets of $I_0$.
Step 4: Let $B = \bigcup I_i$.
Step 5: Choose the sets $I_j$ from $I_i$’s subject to for all $i_{j1}, i_{j2} \in I_j$ such that $i_{j1} \leq i_{j2}$ implies $i_{j1} \bigwedge (i_{j2} - i_{j1}) = 0$.
Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$.
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Brouwerian Algebra

Rao has established that the class of Brouwerian algebras. Brouwerian algebras being a subclass of Boolean-$l$-algebras. If $(B; -)$ is a Boolean-ring then $(B; \cdot)$ is a Boolean-$l$-algebra if and only if B is a Brouwerian such that that $x \leq a$ then $a = x \bigvee (a-x)$.

Let A be a Boolean – ring. Let $A = M_0$. Now to construct a set B as follows.
B contains a unique minimal Brouwerian algebra contained in all other non-zero Brouwerian algebras.
According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.
Algorithm 3.4.
Step 1: Consider a Boolean-ring \( A \)
Step 2: Let \( A = M_0 \)
Step 3: Let \( M_0, i = 0,1,2,3,\ldots \) be the supersets of \( M_0 \).
Step 4: Let \( B = \bigcup M_i \)
Step 5: Choose the sets \( M_j \) from \( M_i \)'s subject to for all \( x \) and \( a \in B \) such that \( x \leq a \) then \( a = x \cup (a-x) \).
Step 6: Verify that \( \bigcap M_j = M_0 \neq \{0\} \)
Step 7: If step (6) is true, then we write \( B \) is a Smarandache-boolean-near-ring.

Compatibility: A subset \( A \) of Boolean-near-ring \( B \) is said to be compatibility \( a \sim b \) if \( ab^2 = a^2b \). Let \( A = I_0 \). Now to construct a set \( B \) as follows. \( B \) contains a unique minimal compatibility \( I_0 \) contained in all other non-zero compatibilities. According to Pilz [4, Theorem (1.60 (d))], \( B \) is Boolean-near-ring. Now by definition, \( B \) is a Smarandache-boolean-near-ring.

Algorithm 3.5.
Step 1: Consider a Boolean-ring \( A \)
Step 2: Let \( A = I_0 \), be a compatibility
Step 3: Let \( I_0, i = 0,1,2,3,\ldots \) be the supersets of \( I_0 \).
Step 4: Let \( B = \bigcup I_i \)
Step 5: Choose the sets \( I_j \) from \( I_i \)'s subject to for all \( a, b \in A \) such that \( ab^2 = a^2b \in I_j \)
Step 6: Verify that \( \bigcap I_j = I_0 \neq \{0\} \)
Step 7: If step (6) is true, then we write \( B \) is a Smarandache-boolean-near-ring.

Maximal Set: Let \( B \) be a Boolean-near-ring and let \( A = \ldots, a, b, c, \ldots \) be a set of pairwise compatible elements of an associate ring \( R \). Let \( A \) be maximal in the sense that each element of \( A \) is compatible with every other element of \( A \) and no other such elements may be found in \( R \). Then \( A \) is called maximal compatible set or a maximal set. Let \( A = I_0 \). Now to construct a set \( B \) as follows. \( B \) contains a unique minimal maximal set \( I_0 \) contained in all other non-zero maximal sets. According to Pilz [4, Theorem (1.60 (d))], \( B \) is Boolean-near-ring. Now by definition, \( B \) is a Smarandache-boolean-near-ring.

Algorithm 3.6.
Step 1: Consider a Boolean-ring \( A \)
Step 2: Let \( A = I_0 \), be a maximal set
Step 3: Let \( I_i, i = 0,1,2,3,\ldots \) be the supersets of \( I \)
Step 4: Let \( B = \bigcup I_i \)
Step 5: Choose the sets \( I_j \) from \( I_i \)'s subject to for all \( a, b \in I_j \) such that \( a \vee b = a + b - 2ab = (a \cup b) - (a \cap b) \) and \( a \wedge b = a^0b = ab^0 = a \cap b \in I_j \), for all \( a, b \in I_j \)
Step 6: Verify that \( \bigcap I_j = I_0 \neq \{0\} \)
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Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Given two numbers $m > n \geq 1$, a ring B is said to be $(m,n)$-Boolean if \( x^m = x^n \), for all $x$ in $B$. Let $A = I_0$. Now to construct a set $B$ as follows. $B$ contains a unique minimal Polynomial identity $I_0$ contained in all other non-zero Polynomial identities. According to G. Pilz [4, Theorem (1.60 (d))], $B$ is Boolean-near-ring. Now by definition, $B$ is a Smarandache-boolean-near-ring.

Algorithm 3.7.

Step 1: Consider a Boolean-ring $A$
Step 2: Let $A = I_0$
Step 3: Let $I_i, i = 0,1,2,3,\ldots$ be the supersets of $I_0$
Step 4: Let $B = \bigcup I_i$
Step 5: Choose the sets $I_j$ from $I_i$'s subject to for all $m, n \in B$ and for all $x \in B$ such that $x^m = x^n \in I_j$
Step 6: Verify that $\bigcap I_j = I_0 \neq \{0\}$
Step 7: If step (6) is true, then we write $B$ is a Smarandache-boolean-near-ring.

Polynomial Identity: Let $m$ and $n$ be two positive integers such that \( x^{2^m+2^n} = x, \) for all $x$ in $B$. Let $A = M_0$. Now to construct a set $B$ as follows. $B$ contains a unique minimal Polynomial identity $M_0$ contained in all other non-zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], $B$ is Boolean-near-ring. Now by definition, $B$ is a Smarandache-boolean-near-ring.

Algorithm 3.8.

Step 1: Consider a Boolean-ring $A$
Step 2: Let $A = M_0$
Step 3: Let $M_i, i = 0,1,2,3,\ldots$ be the supersets of $M_0$
Step 4: Let $B = \bigcup M_i$
Step 5: Choose the sets $M_j$ from $M_i$'s subject to for all two positive integers $m$ and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^m+2^n} = x, \in M_j$
Step 6: Verify that $\bigcap M_j = M_0 \neq \{0\}$
Step 7: If step (6) is true, then we write $B$ is a Smarandache-boolean-near-ring.

Polynomial Identity: Let $m$ and $q$ be two fixed positive integers and \( x^{2^{m+q}+2^{2^n}} = x, \) for all $x$ in $B$. Then $B$ is known as a Smarandache-boolean-near-ring.

Let $A = P_0$. Now to construct a set $B$ as follows. $B$ contains a unique minimal Polynomial identity $P_0$ contained in all other non-zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], $B$ is Boolean-near-ring. Now by definition, $B$ is a Smarandache-boolean-near-ring.
Algorithm 3.9.
Step 1: Consider a Boolean-ring A
Step 2: Let A = P₀
Step 3: Let Pᵢ, i = 0, 1, 2, 3, …… be the supersets of P₀.
Step 4: Let B = \bigcup Pᵢ
Step 5: Choose the sets Pⱼ from Pᵢ’s subject to for all two positive integers m and q such that \( x^{m+q} = x \), \( x \) ∈ Pⱼ and for all \( x \) ∈ Pⱼ
Step 6: Verify that \( \bigcap P = P₀ \neq \{0\} \)
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let \( m \) and \( n \) be two positive integers such that \( x^{2m+n} + 2^n = x \), for all \( x \) in B. Let A = M₀.

Now to construct a set B as follows. B contains a unique minimal Polynomial identity \( M₀ \) contained in all other non-zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.10.
Step 1: Consider a Boolean-ring A
Step 2: Let A = M₀
Step 3: Let Mᵢ, i = 0, 1, 2, 3, …… be the supersets of M₀
Step 4: Let B = \bigcup Mᵢ
Step 5: Choose the sets Mⱼ from Mᵢ’s subject to for all two positive integers m and n ∈ B and for all \( x \) ∈ Mⱼ such that \( x^m = x^n \) and \( x^{2m+n} = x \), \( x \) ∈ Mⱼ
Step 6: Verify that \( \bigcap Mⱼ = M₀ \neq \{0\} \)
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let \( m \) and \( n \) be two positive integers such that \( x^{2m+n} + 2^n = x \), for all \( x \) in B. Let A = M₀. Now to construct a set B as follows. B contains a unique minimal Polynomial identity \( M₀ \) contained in all other non-zero Polynomial identities. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.11.
Step 1: Consider a Boolean-ring A
Step 2: Let A = M₀
Step 3: Let Mᵢ, i = 0, 1, 2, 3, …… be the supersets of M₀
Step 4: Let B = \bigcup Mᵢ
Step 5: Choose the sets $M_j$ from $M_i$'s subject to for all two positive integers $m$ and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{m+n+2^n}} = x$, $\in M_j$.

Step 6: Verify that $\bigcap M_j = M_0 \neq \{0\}$.

Step 7: If step (6) is true, then we write $B$ is a Smarandache-boolean-near-ring.

**Polynomial Identity:** Let $m$ and $n$ be two positive integers such that $x^{2^{m+n}+2^n} = x$, for all $x \in B$. Let $A = M_0$. Now to construct a set $B$ as follows. $B$ contains a unique minimal Polynomial identity $M_0$ contained in all other non-zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], $B$ is Boolean-near-ring. Now by definition, $B$ is a Smarandache-boolean-near-ring.

**Algorithm 3.12.**

1. Consider a Boolean-ring $A$.
2. Let $A = M_0$.
3. Let $M_i$, $i = 0, 1, 2, 3, \ldots$ be the supersets of $M_0$.
4. Let $B = \bigcup M_i$.
5. Choose the sets $M_j$ from $M_i$'s subject to for all two positive integers $m$ and $n \in B$ and for all $x \in M_j$ such that $x^m = x^n$ and $x^{2^{m+n}+2^n} = x$, $\in M_j$.
6. Verify that $\bigcap M_j = M_0 \neq \{0\}$.
7. If step (6) is true, then we write $B$ is a Smarandache-boolean-near-ring.

**Polynomial Identity:** Let $B$ be a Boolean-near-ring and let $m$, $q$ and $r$ be fixed positive integers with $r < m+1$ such that $x^{2^{m+q}+2^r} = x$, for all $x$ in $B$ and $x^{2^r} = x$, then $B$ is Smarandache-Boolean-near-ring. Let $A = M_0$. Now to construct a set $B$ as follows. $B$ contains a unique minimal Polynomial identity $M_0$ contained in all other non-zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], $B$ is Boolean-near-ring. Now by definition, $B$ is a Smarandache-boolean-near-ring.

**Algorithm 3.13.**

1. Consider a Boolean-ring $A$.
2. Let $A = M_0$.
3. Let $M_i$, $i = 0, 1, 2, 3, \ldots$ be the supersets of $M_0$.
4. Let $B = \bigcup M_i$.
5. Choose the sets $M_j$ from $M_i$'s subject to for all two positive integers $m$, $q$ and $r$ be three fixed positive integers with $r < m+1$ and for all $x \in M_j$ such that $x^{2^{(m+q)+2^r}} = x$, and $x^{2^r} = x$, $\in M_j$.
6. Verify that $\bigcap M_j = M_0 \neq \{0\}$.
Step 7: If step (6) is true, then we write $B$ is a Smarandache-Boolean-near-ring.

REFERENCES

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