A prime number based strategy to label graphs and represent its structure as a single numerical value

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Abstract: We present a simple theoretical strategy to represent using a single numerical value "A" called the prime vertex labeling Adjacency value product, all structural information encoded in a graph. This strategy has the potential to allow us to reconstruct the graph in its entirety based on a single number. To do so we assume that we have access to a large list of prime numbers which are infinite in number. This method will allow us to store graph backbone as a numerical value for retrieval and re-use and may also allow development of algorithms that exploit this representation feature as shortcut to address graph isomorphism.

Results: A graph is made up of vertices and edges that link the vertices. Graphs are represented either as adjacency lists or adjacency matrices. Prime number/co-prime based labeling of graph vertices has been discussed in the past (Vaidya and Kanani, 2010). However its potential to represent all the structural information in a graph as a single numerical value has not been discussed before likely because of the large values associated with its full expansion. However we will introduce it in this paper because it might open up the possibility of other applications.

Our method relies on labeling each of the vertices of a given graph using a distinct prime number chosen from the infinite sequence of primes 2,3,5,7,11,………….

Then we can create an adjacency list for each vertex. The adjacency list for a vertex lists all vertices that are connected to it. Therefore based on the adjacency list for any arbitrary vertex which is connected to k other vertices we have (k+1) distinct primes associated with that vertex. One prime represents the vertex itself and "k" other primes represent its connected vertices.

For each vertex represented by prime p_x connected to "k" other vertices represented by distinct primes $p_a,p_b,p_c\ldots\ldots p_k$, we calculate the prime vertex labeling adjacency value for a vertex a_x, where

$a_x = ((p_x)^\wedge(p_a p_b p_c\ldots\ldots p_k))$

Similarly calculate the prime vertex labeling adjacency value for each of the other vertices.

The prime vertex labeling adjacency value product "A" = product of adjacency values of all "n" vertices of the graph= $(a_1a_2a_3\ldots\ldots a_n)$ and represents the graph for that particular prime labeling.

Therefore given the value of any product "A", it should be possible to reconstruct the graph represented by A.

The same graph when labeled differently using the same set of primes will give an entirely different value except when the vertex labeling are swapped between a pair of vertices which are connected to the same set of vertices and therefore are indistinguishable. We will prefer to use sequence of primes beginning 2 for labeling since it will allow us to easily obtain the factors if we are given an Adjacency value product. This method will give an even value for A. However we may also modify the technique and use non-sequential primes as long as they allow us to label each vertex distinctly and may or may not result in an even value for A depending upon the inclusion or exclusion of 2 in the prime labeling.

We also propose that any random number when factored into its prime factors if allows us to derive a precise graph labeled by distinct prime numbers according to our strategy, may be called a "graphical number" and if it encompasses a continuous sequence of primes beginning 2, is a "perfect graphical number".

A two vertex graph may be labeled by primes 2,3 and represented as $(2)^3(3)^2=72$. Thus 72 represents the smallest perfect graphical number.

eg. A graph that is shaped like a triangle that has prime vertices 2,3,5 would be represented by prime vertex labeling individual adjacency values 2^{15}, 3^{10}, 5^6 and the prime vertex adjacent labeling value product $A=(2)^{3.5}(3)^{2.5}(5)^{2.3}$ or $A=(2)^{15}(3)^{10}(5)^6$ Either of the above two formats may be used depending upon the application.

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References:

Vaidya S.K. and Kanani K.K. (2010) Prime labeling for some cycle related graphs