VERSION 3

# A Common Cause for the Interaction of Material Particles, the Corpuscular Structure of Matter, and Quantum Phenomena.

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**Abstract**

With the term "Law of Selfvariations" we mean an exactly determined increase of the rest mass and electric charge of material particle. In this article we present the basic theoretical investigation of the law of selfvariations. We arrive at the central conclusion that the interaction of material particles, the corpuscular structure of matter, and the quantum phenomena can be justified by the law of Selfvariations. We predict a unified interaction between material particles with a unified mechanism (Unified Selfvariations Interaction, USVI). Every interaction is the result of three clearly distinct terms with clearly distinct consequences in the USVI. We predict a wave equation, whose special cases are the Maxwell equations, the Schrödinger equation, and the related wave equations. We determine a mathematical expression for the total of the conservable physical quantities, and we calculate the curent density 4-vector. The corpuscular structure and wave behaviour of matter and their relation emerge clearly, and we give a calculation method for the rest masses of material particles. We prove the «internal symmetry» theorem which justifies the cosmological data. From the study we present, the method for the further investigation of the Selfvariations and their consequences also emerges.

**Keywords**: Particles and Fields, Quantum Physics, Cosmology.

## 1. Introduction

In the present article we attempt to give an axiomatic foundation of theoretical Physics based on three axioms: The principle of the conservation of the four-vector of momentum, the equation of the Theory of Special Relativity for the rest mass of the material particles, and the law of Selfvariations.

With the term “Law of Selfvariations” we mean an exactly determined increase of the rest mass and electric charge of material particle. It is consistent with the principles of conservation of energy, momentum, angular momentum and electric charge. It is also invariant under the Lorentz-Einstein transformations.

The most immediate consequence of the law of Selfvariations is that the energy, the momentum, the angular momentum, and the electric charge of material particles are distributed in the surrounding spacetime (when the material particle is electrically charged).

In order for the value of the electric charge to increase in absolute value, the electron, in some way, should 'emit' a positive electric charge in the space-time environment. Otherwise, the conservation of the electric charge is violated. Similarly, the increase of the rest mass of the material particle involves the “emission” of negative energy as well as momentum in the space-time surrounding the material particle (spacetime energy-momentum, STEM). The law of Selfvariations describes quantitatively the interaction of material particles with the STEM.

Every material particle interacts both with the STEM emitted by itself due to the selfvariations, and with the STEM originating from other material particles. The material particle and the STEM with which it interacts, comprise a dynamic system which we called “generalized particle”. We study this continuous interaction in the present article. For the formulation of the equations the following notation is used:

$W =$ the energy of the material particle

$\mathbf{J} =$ the momentum of the material particle

$m_0 =$ the rest mass of the material particle

$E =$ the energy of the STEM interacting with the material particle

$\mathbf{P} =$ the momentum of the STEM interacting with the material particle

$E_0 =$ the rest energy of the STEM interacting with the material particle

With the above symbolism, the law of Selfvariations for the rest mass is given by equations

$$\begin{aligned} \frac{\partial m_0}{\partial t} &= -\frac{b}{\hbar} E m_0 \\ \nabla m_0 &= \frac{b}{\hbar} \mathbf{P} m_0 \end{aligned} . \tag{1.1}$$

in every system of reference $0(\mathrm{t}, x, y, z)$. , $\hbar$ is Planck's constant, $b$ constant, $b \neq 0, b \in \mathbb{C}$ and

$$\nabla = \begin{pmatrix} \dfrac{\partial}{x} \\ \dfrac{\partial}{y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} .$$

The the findings resulting from the law of Selfvariations will be referred to as "the Theory of Selfvariations" (TSV). Initially, we present the TSV in inertial frames of reference.

## 2. The basic study of the internal structure of the generalized particle

We consider a material particle with rest mass $m_0 \neq 0$. That is, we consider a generalized particle. The rest mass $m_0$ and the rest energy $E_0$ given by equations (2.1) and (2.2) respectively according to special relativity [1-4]

$$m_0^2 c^4 = W^2 - c^2 \mathbf{J}^2 \tag{2.1}$$

$$E_0{}^2 = E^2 - c^2 \mathbf{P}^2 . \tag{2.2}$$

We now denote the four-vectors

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix} \quad (2.3)$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iw}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix} \quad (2.4)$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_x \\ P_y \\ P_z \end{bmatrix} \quad (2.5)$$

where $c$ **is** the *vacuum velocity* of *light* and $i$ is the imaginary unit, $i^2 = -1$.

Using this notation, equations (1.1), (2.1) and (2.2) are written in the form of equations (2.6), (2.7) and (2.8)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k m_0, k = 0,1,2,3 \quad (2.6)$$

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 + {m_0}^2 c^2 = 0 \quad (2.7)$$

$$P_0^2 + P_1^2 + P_2^2 + P_3^2 + \frac{E_0^2}{c^2} = 0. \quad (2.8)$$

After differentiating equation (2.7) with respect to $x_k, k = 0,1,2,3$ we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} + m_0 c^2 \frac{\partial m_0}{\partial x_k} = 0$$

and with equation (2.6) we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} + \frac{b}{\hbar} P_k m_0^2 c^2 = 0$$

and with equation (2.7) we obtain

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k \left( J_0^2 + J_1^2 + J_2^2 + J_3^2 \right) = 0$$

$$\begin{aligned} & J_0 \left( \frac{\partial J_0}{\partial x_k} - \frac{b}{\hbar} P_k J_0 \right) + J_1 \left( \frac{\partial J_1}{\partial x_k} - \frac{b}{\hbar} P_k J_1 \right) \\ & + J_2 \left( \frac{\partial J_2}{\partial x_k} - \frac{b}{\hbar} P_k J_2 \right) + J_3 \left( \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k J_3 \right) = 0, k = 0,1,2,3 \end{aligned} . \tag{2.9}$$

We now symbolize

$$\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar} P_k J_i = \lambda_{ki}, \ k,i = 0,1,2,3 . \tag{2.10}$$

With this notation, equation (2.9) can be written in the form

$$J_0 \lambda_{k0} + J_1 \lambda_{k1} + J_2 \lambda_{k2} + J_3 \lambda_{k3} = 0, \ k = 0,1,2,3 . \tag{2.11}$$

We now need the $4 \times 4$ matrix $T$ as given by equation

$$T = \begin{bmatrix} \lambda_{00} & \lambda_{01} & \lambda_{02} & \lambda_{03} \\ \lambda_{10} & \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{20} & \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{30} & \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} . \tag{2.12}$$

With this notation, equation (2.11) can be written in the form

$$TJ = 0 . \tag{2.13}$$

We now prove the following relationship

$$\frac{\partial P_i}{\partial x_k}=\frac{\partial P_k}{\partial x_i}\vee m_0=0\vee\left\{\frac{\partial P_i}{\partial x_k}=\frac{\partial P_k}{\partial x_i}\wedge m_0=0\right\}. \tag{2.14}$$
$$k\neq i, k, i=0,1,2,3$$

**Proof.** Indeed, by differentiating equation (2.6) with respect to $x_i$, $i=0,1,2,3$ we get

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and using the identity

$$\frac{\partial}{\partial x_i}\left(\frac{\partial m_0}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)$$

we get

$$\frac{\partial}{\partial x_k}\left(\frac{\partial m_0}{\partial x_i}\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$

and with equation (2.6) we have

$$\frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_i m_0\right)=\frac{b}{\hbar}\frac{\partial}{\partial x_i}(P_k m_0)$$
$$P_i\frac{\partial m_0}{\partial x_k}+m_0\frac{\partial P_i}{\partial x_k}=P_k\frac{\partial m_0}{\partial x_i}+m_0\frac{\partial P_k}{\partial x_i}$$

and with equation (2.6) we have

$$P_i\frac{b}{\hbar}P_k m_0+m_0\frac{\partial P_i}{\partial x_k}=P_k\frac{b}{\hbar}P_i m_0+m_0\frac{\partial P_k}{\partial x_i}$$
$$m_0\left(\frac{\partial P_i}{\partial x_k}-\frac{\partial P_k}{\partial x_i}\right)=0$$

from which we obtain equation (2.14). □

We now prove the following theorem:

**Theorem** 2.1 ‘’For every $k,i,\nu=0,1,2,3$ the following equation holds

$$\frac{\partial\lambda_{ki}}{\partial x_\nu}-\frac{b}{\hbar}P_\nu\lambda_{ki}=\frac{\partial\lambda_{\nu i}}{\partial x_k}-\frac{b}{\hbar}P_k\lambda_{\nu i}\text{.’’} \tag{2.15}$$

**Proof.** Indeed, by differentiating equation (2.10) with respect to $x_v$, $v = 0,1,2,3$ we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with identity

$$\frac{\partial}{\partial x_v}\left(\frac{\partial J_i}{\partial x_k}\right) = \frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right)$$

we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial}{\partial x_k}\left(\frac{\partial J_i}{\partial x_v}\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

and with equation (2.10) we have

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial}{\partial x_k}\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}\frac{\partial}{\partial x_k}\left(P_v J_i\right) - \frac{b}{\hbar}\frac{\partial}{\partial x_v}\left(P_k J_i\right)$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} + \frac{b}{\hbar}J_i\frac{\partial P_v}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v} - \frac{b}{\hbar}J_i\frac{\partial P_k}{\partial x_v}$$

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v} + \frac{b}{\hbar}J_i\left(\frac{\partial P_v}{\partial x_k} - \frac{\partial P_k}{\partial x_v}\right)$$

and with equation (2.14) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\frac{\partial J_i}{\partial x_k} - \frac{b}{\hbar}P_k\frac{\partial J_i}{\partial x_v}$$

and with equation (2.10) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\left(\frac{b}{\hbar}P_k J_i + \lambda_{ki}\right) - \frac{b}{\hbar}P_k\left(\frac{b}{\hbar}P_v J_i + \lambda_{vi}\right)$$

and we finally have

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{\partial \lambda_{vi}}{\partial x_k} + \frac{b}{\hbar}P_v\lambda_{ki} - \frac{b}{\hbar}P_k\lambda_{vi}.$$

which is equation (2.15). □

## 3. Physical quantities $\lambda_{ki}$, $k, i = 0,1,2,3$ and the conservation principles of energy and momentum

The physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ are related to the conservation of energy and momentum of the generalized particle. This investigation we will present in this section. We prove the following theorem:

**Theorem** 3.1 ′′If the generalized particle conserves its momentum along the axes $x_i, i = 0,1,2,3$, that is

$$J_i + P_i = c_i = \text{constant} . \tag{3.1}$$

then the following equation holds

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{\hbar}\left(c_k P_i - c_i P_k\right) \tag{3.2}$$

for every $k, i = 0,1,2,3,\ k \neq i.$ ′′

**Proof.** Combining equations (2.14) and (3.1) we obtain

$$\frac{\partial}{\partial x_k}\left(c_i - J_i\right) = \frac{\partial}{\partial x_i}\left(c_k - J_k\right)$$

$$\frac{\partial J_i}{\partial x_k} = \frac{\partial J_k}{\partial x_i}$$

and with equation (2.10) we get

$$\frac{b}{\hbar} P_k J_i + \lambda_{ki} = \frac{b}{\hbar} P_i J_k + \lambda_{ik}$$

$$\lambda_{ki} - \lambda_{ik} = \frac{b}{\hbar}\left(J_k P_i - J_i P_k\right)$$

which is equation (3.2). The rest of equations (3.2) are derived taking into account equation (3.1). (3.2). Equation (3.2) holds for $k \neq i,\ k, \mathrm{i} = 0,1,2,3$, since equation (2.14), from which equation (3.2) results is an identity $k = i$ and gives no information in this case. □

We now prove the following theorem:

**Theorem 3.2. TSV theorem for the symmetry of indices:**

´´If the generalized particle conserves its momentum along the axes $x_i$ and $x_k$ with $k \neq i$, the following equivalences hold

1. $\lambda_{ik} = \lambda_{ki} \Leftrightarrow J_k P_i = J_i P_k \Leftrightarrow c_i J_k = c_k J_i \Leftrightarrow c_k P_i = c_i P_k$ . (3.3)

2. $\lambda_{ik} = -\lambda_{ki} \Leftrightarrow \lambda_{ki} = \frac{b}{2\hbar}\left(J_k P_i - J_i P_k\right) = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right) = \frac{b}{2\hbar}\left(c_k P_i - c_i P_k\right)$ . (3.4)

$k, i = 0,1,2,3, \; k \neq i$ .´´

**Proof.** The theorem is an immediate consequence of equation 3.2. □

We now consider the four-vector $C$, as given by equation

$$C = J + P = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}. \tag{3.5}$$

When the generalized particle conserves its momentum along every axis, then the four-vector $C$ is constant. Also, we denote $M_0$ the total rest mass of the generalized particle, as given by equation

$$C^T C = c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2\, \mathrm{c}^2 . \tag{3.6}$$

where $C^T$ is the adjoint of the column vector $C$.

For reasons that will become apparent later in our study, we give the following definitions: We name the symmetry $\lambda_{ik} = \lambda_{ki}, \; k \neq i, \; k, i = 0,1,2,3$ internal symmetry, and the symmetry $\lambda_{ik} = -\lambda_{ki}, \; k \neq i, \; k, i = 0,1,2,3$ external symmetry. We now prove the following theorem:

**Theorem 3.3. Internal Symmetry Theorem:**

´´ If the generalized particle conserves its momentum in every axis, the following hold:

1. $\lambda_{ik} = \lambda_{ki}$ for every $k, i = 0,1,2,3$ $\Leftrightarrow$ $J$, $P$ and $C$ are parallel

$$\Leftrightarrow P = \Phi J \text{ where } \Phi \in \mathbb{C},\ \Phi \neq 0. \tag{3.7}$$

2. For $\Phi = -1$ the following equation holds:

$$E_0 = \pm m_0 c^2 \tag{3.8}$$

3. For $\Phi \neq -1$ the following equations hold:

$$\Phi = K \exp\left[-\frac{b}{\hbar}\left(c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3\right)\right] \tag{3.9}$$

$$m_0 = \pm \frac{M_0}{1+\Phi} \tag{3.10}$$

$$E_0 = \mp \frac{M_0 c^2 \Phi}{1+\Phi} \tag{3.11}$$

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3 \tag{3.12}$$

$$P_i = \frac{\Phi c_i}{1+\Phi}, \mathrm{i} = 0,1,2,3 \tag{3.13}$$

where $K$ is a dimensionless constant physical quantity.

4. We have $\lambda_{ik} = \lambda_{ki}$ for every $k, \mathrm{i} = 0,1,2,3$

$$\Leftrightarrow \tag{3.14}$$

$\lambda_{ki} = 0$ for every $k, \mathrm{i} = 0,1,2,3$ . ´´

**Proof.** Equivalence (3.7) results immediately from equivalence (3.3). For $\Phi = 0$ from the last of equivalence (3.7) we obtain $P = 0,$ which is impossible, since in this case the Selfvariations of the rest mass $m_0 \neq 0$, do not exist, as seen from equation (2.6). Therefore, $\Phi \neq 0.$ For $\Phi = -1$ from the last of equivalence (3.7) we obtain $P = -J$ and from equations (2.7) and (2.8) we obtain

$$E_0^2 = m_0^2 c^4$$

which is equation (3.8).

For $\Phi \neq -1$ from the last of equivalence (3.7) we obtain $P_i = \Phi J_i$ for every $i = 0,1,2,3$ and with equation (3.1) $J_i + P_i = c_i$ we initially obtain equations (3.12) and (3.13). Then, combining equations (2.7) and (3.12) we get

$$m_0^2 c^2 + \frac{1}{(\Phi+1)^2}\left(c_0^2 + c_1^2 + c_2^2 + c_3^2\right) = 0$$

and with equation (3.6) we obtain equation

$$m_0^2 c^2 - \frac{M_0^2 c^2}{(\Phi+1)^2} = 0 \qquad (3.15)$$

and we finally have

$$m_0 = \pm \frac{M_0}{1+\Phi}$$

which is equation (3.10). Similarly, combining equations (2.8) and (3.13) we obtain equation (3.11). We now prove that function $\Phi$ is given by equation (3.9).

Differentiating equation (3.15) with respect to $x_v$, $v = 0,1,2,3$ and considering equation (2.6) we obtain

$$\frac{2b}{\hbar} P_v m_0^2 c^2 + \frac{2M_0^2 c^2}{(\Phi+1)^3} \frac{\partial \Phi}{\partial x_v} = 0$$

and with equation (3.15) we have

$$\frac{b}{\hbar} P_v \frac{M_0^2 c^2}{(\Phi+1)^2} + \frac{M_0^2 c^2}{(\Phi+1)^3} \frac{\partial \Phi}{\partial x_v} = 0$$

$$\frac{\partial \Phi}{\partial x_v} = -\frac{b}{\hbar} P_v (\Phi+1)$$

and with equation (3.13) for $i = v$ we arrive at equation

$$\frac{\partial \Phi}{\partial x_v} = -\frac{b}{\hbar} c_v \Phi, \ \ v = 0,1,2,3. \tag{3.16}$$

By integration of equation (3.16) we obtain

$$\Phi = K \exp\left[ -\frac{b}{\hbar}\left( c_0 X_0 + c_1 X_1 + c_2 X_2 + c_3 X_3 \right) \right]$$

where $K$ is the integration constant, which is equation (3.9).

Combining equations (2.10), (3.12) and (3.13) for $k = 0,1,2,3$ we obtain

$$\lambda_{ki} = \frac{\partial}{\partial x_k}\left( \frac{c_i}{1+\Phi} \right) - \frac{b}{\hbar} \frac{\Phi c_k}{1+\Phi} \frac{c_i}{1+\Phi}$$

$$\lambda_{ki} = -\frac{c_i}{\left(1+\Phi\right)^2} \frac{\partial \Phi}{x_k} - \frac{b}{\hbar} \frac{\Phi c_k c_i}{\left(1+\Phi\right)^2}$$

and with equation (3.16) for $v = k$ we obtain

$$\lambda_{ki} = \frac{c_i}{\left(1+\Phi\right)^2} \frac{b}{\hbar} c_k \Phi - \frac{b}{\hbar} \frac{\Phi c_k c_i}{\left(1+\Phi\right)^2}$$

$\lambda_{ki} = 0$. □

According to the previous theorem, internal symmetry is equivalent to the parallelism of the four-vectors $J, P$. Starting from this conclusion we can determine the physical content of the internal symmetry.

In an isotropic space the spontaneous emission of generalized photons by the material particle is isotropic. Due to the linearity of the Lorentz-Einstein transformations, this isotropic emission has as a consequence the parallelism of the four-vectors $J, P$ [5] ( par. 5.3). Thus, the theorem of internal symmetry 3.3 holds for the spontaneous emission of generalized photons by the material particle due to Selfvariations .

In the following paragraphs, we will make clear that the internal symmetry refers to a spontaneous internal increase of the rest mass and the electrical charge of the material particles, independent of any external causes. The consequences of this increase is the cosmological data, as we'll see in Paragraph 11. Also, the internal symmetry is associated with Heisenberg's uncertainty principle.

We start the investigation of the internal symmetry with the proof of the following theorem:

**Theorem 3.4.** First theorem of the TSV for the internal symmetry**:** ′′If the generalized particle conserves its momentum along every axis, and the symmetry $\lambda_{ik} = -\lambda_{ki}$ holds for every $\mathrm{k} \neq \mathrm{i}, k, i = 0,1,2,3$, then:

$$1.\quad \begin{aligned} &c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = 0 \\ &c_i J_{vk} + c_k J_{iv} + c_v J_{ki} = 0 \\ &c_i P_{vk} + c_k P_{iv} + c_v P_{ki} = 0 \end{aligned} \tag{3.17}$$

for every $i \neq v, v \neq k, k \neq i,\ \ k, i, v = 0,1,2,3$.

$$2.\quad \frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} - \frac{bc_v}{2\hbar}\lambda_{ki} = -\frac{b}{\hbar} J_v \lambda_{ki} + \frac{bc_v}{2\hbar}\lambda_{ki} \tag{3.18}$$

for every $k \neq i, k, i, v = 0,1,2,3$.

$$3.\quad \lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} = 0\,. \,'' \tag{3.19}$$

**Proof.** From equivalence (3.4) we obtain

$$\lambda_{ki} = \frac{b}{\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3\,. \tag{3.20}$$

Considering equation (3.20) we get

$$c_i\lambda_{vk} + c_k\lambda_{iv} + c_v\lambda_{ki} = \frac{b}{2\hbar}\left[c_i\left(c_k J_v - c_v J_k\right) + c_k\left(c_v J_i - c_i J_v\right) + c_v\left(c_i J_k - c_k J_i\right)\right] = 0$$

Thus, we get the first of equations (3.17). Similarly, from the other two equalities of equivalence (3.4) we obtain the second and the third equation of (3.17). Since $k \neq i$ in equivalence (3.4), the physical quantities $\lambda_{vk}, \lambda_{iv}, \lambda_{ki}$ in equations (3.17) are defined for $v \neq k, i \neq v, k \neq i, k, i, v = 0,1,2,3$ .

Differentiating equation (3.20) with respect to $x_v, v = 0,1,2,3$ we obtain

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left( c_i \frac{\partial J_k}{\partial x_v} - c_k \frac{\partial J_i}{\partial x_v} \right)$$

and with equation (2.10) we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[ c_i \left( \frac{b}{\hbar} P_v J_k + \lambda_{vk} \right) - c_k \left( \frac{b}{\hbar} P_v J_i + \lambda_{vi} \right) \right]$$
$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{2\hbar}\left[ \frac{b}{\hbar} P_v \left( c_i J_k - c_k J_i \right) + c_i \lambda_{vk} - c_k \lambda_{vi} \right]$$
$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \frac{b}{2\hbar}\left( c_i J_k - c_k J_i \right) + \frac{b}{2\hbar}\left( c_i \lambda_{vk} - c_k \lambda_{vi} \right)$$

and with equation (3.20) we obtain

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} + \frac{b}{2\hbar}\left( c_i \lambda_{vk} - c_k \lambda_{vi} \right)$$

and with the first of equations (3.17) we obtain

$$c_i \lambda_{vk} - c_k \lambda_{vi} = -c_v \lambda_{ki}$$

we get

$$\frac{\partial \lambda_{ki}}{\partial x_v} = \frac{b}{\hbar} P_v \lambda_{ki} - \frac{b c_v}{2\hbar} \lambda_{ki}$$

which is equation (3.18). The second equality in equation (3.18) emerges from the substitution

$$P_v = c_v - J_v, v = 0,1,2,3$$

according to equation (3.5).

Taking into account equation (3.20) we obtain

$$\lambda_{01}\lambda_{32} + \lambda_{02}\lambda_{13} + \lambda_{03}\lambda_{21} =$$
$$\frac{b^2}{4\hbar^2}\left[ \left( c_1 J_0 - c_0 J_1 \right)\left( c_2 J_3 - c_3 J_2 \right) + \left( c_2 J_0 - c_0 J_2 \right)\left( c_3 J_1 - c_1 J_3 \right) + \left( c_3 J_0 - c_0 J_3 \right)\left( c_1 J_2 - c_2 J_1 \right) \right] = 0$$

after the calculations. □

In the next paragraphs we investigate the external symmetry.

## 4. The Unified Selfvariations Interaction (USVI)

According to the law of selfvariations every material particle interacts both with the generalized photons emitted by itself due to the selfvariations, and with the generalized photons originating from other material particles. In the second case, an indirect interaction emerges between material particles through the generalized photons. Generalized photons emitted by one material particle interact with another material particle. Through this mechanism the TSV predicts a unified interaction between material particles. The individual interactions only emerge from the different, for each particular case, physical quantity $Q$ which selfvariates, resulting in the emission of the corresponding generalized photons.In this paragraph we study the basic characteristics of the USVI. We suppose that for the generalized particle the conservation of energy-momentum holds, hence the equations of the preceding paragraph also hold. For the rate of change of the four-vector $\frac{1}{m_0}J$ we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=-\frac{J_i}{m_0^2}\frac{\partial m_0}{\partial x_k}+\frac{1}{m_0}\frac{\partial J_i}{\partial x_k}$$

and with equations (2.6) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=-\frac{J_i}{m_0^2}\frac{b}{\hbar}P_k m_0+\frac{1}{m_0}\left(\frac{b}{\hbar}P_k J_i+\lambda_{ki}\right)$$

and we finally obtain

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{m_0}\right)=\frac{\lambda_{ki}}{m_0},\ \ k,i=0,1,2,3\,. \qquad (4.1)$$

According to equation (4.1), when $\lambda_{ki}\neq 0$ for at least two indices $k,i,\ \ k,i=0,1,2,3,$ the kinetic state of the material particle is disturbed. According to equivalence (3.14) in the internal symmetry it is $\lambda_{ki}=0$ for every $k,i=0,1,2,3.$ Therefore, in the internal symmetry the material particle maintains its kinetic state. In an isotropic space we expect that the spontaneous emission of generalized photons by the material particle cannot disturb its kinetic state. Consequently, the internal symmetry concerns the spontaneous emission of generalized photons by the material particle in an isotropic space.

In contrast, in the case of the external symmetry it can be $\lambda_{ki} \neq 0$ for some indices $k,i,\ \ k,i = 0,1,2,3$. Therefore, the external symmetry must be due to generalized photons with which the material particle interacts, and which originate from other material particles. The distribution of generalized photons depends on the position in space of the material particle relative to other material particles. This leads to the destruction of the isotropy of space for the material particle. The external symmetry factor will emerge in the study that follows.

The initial study of the Selfvariations concerned the rest mass and the electric charge. The study we have presented up to this point allows us to study the Selfvariations in their most general expression.

We consider a physical quantity $Q$ which we shall call selfvariating "charge $Q$ ", or simply charge $Q$, unaffected by every change of reference frame, therefore Lorentz-Einstein invariant, and obeys the law of Selfvariations, that is equation

$$\frac{\partial Q}{\partial x_k} = \frac{b}{\hbar} P_k Q,\ \ k = 0,1,2,3. \tag{4.2}$$

In equation (4.2) the momentum $P_k, \mathrm{k} = 0,1,2,3$, i.e. the four-vector $P$, depends on the selfvariating charge $Q$. Two material particles carrying a selfvariating charge of the same nature interact with each other when the STEM emitted by the charge $Q_1$ of one of them interacts with the charge $Q$ of the other. In this particular case, we denote $Q$ the charge of the material particle we are studying.

The rest mass $m_0$ is defined as a quantity of mass or energy divided by $c^2$, which is invariant according to the Lorentz-Einstein transformations. The 4-vector of the momentum $J$ of the material particle is related to the rest mass $m_0$ through equation (2.7). The charge $Q$ contributes to the energy content of the material particle and, therefore, also contributes to its rest mass. Furthermore, the charge $Q$ modifies the 4-vector of momentum $J$ of the material particle and, therefore, contributes to the variation of the rest mass $m_0$ of the material particle. Consequently, for the change of the four-vector $J$ of the material particle due to the charge $Q$, the four-vector $P$ of equation (2.10) enters into equation (4.2). The consequences of this conclusion become evident when we calculate the rate of change of the four-vector $\frac{1}{Q}J$.

**Theorem 4.1** Second theorem of the TSV for the external symmetry

´´The rate of change of the four-vector $\frac{1}{Q}J$ due to the Selfvariations of the charge $Q$ is given by equation

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q}, \quad k,i=0,1,2,3 \qquad (4.3)$$

For $k \neq i$ the physical quantities $\frac{\lambda_{ki}}{Q}$ are given by

$$\frac{\lambda_{ki}}{Q}=za_{ki}, \ \mathrm{k} \neq \mathrm{i}, \mathrm{k}, \mathrm{i}=0,1,2,3 \qquad (4.4)$$

where z is the function

$$z=\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]. \qquad (4.5)$$

For the constants $a_{ki}$ the following equations hold

$$\begin{aligned} c_i a_{vk}+c_k a_{iv}+c_v a_{ki}&=0\\ J_i a_{vk}+J_k a_{iv}+J_v a_{ki}&=0\\ P_i a_{vk}+P_k a_{iv}+P_v a_{ki}&=0 \end{aligned} \qquad (4.6)$$

for every $i \neq v, v \neq k, k \neq i, \ i,k,v=0,1,2,3$.

$$\alpha_{ik}=-\alpha_{ki}, k \neq i, k, i=0,1,2,3 \qquad (4.7)$$

$$\alpha_{01}\alpha_{32}+\alpha_{02}\alpha_{13}+\alpha_{03}\alpha_{21}=0 \text{.”} \qquad (4.8)$$

**Proof.** In order to prove the theorem, we take

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=-\frac{J_i}{Q^2}\frac{\partial Q}{\partial x_k}+\frac{1}{Q}\frac{\partial J_i}{\partial x_k}$$

and with equations (4.2) and (2.10) we get

$$\frac{\partial}{\partial x_k}\left(\frac{J_i}{Q}\right)=\frac{\lambda_{ki}}{Q},$$

which is equation (4.3).

Equations (4.2) and (2.10) hold for every $k,\mathrm{i}=0,1,2,3.$ Therefore, equation (4.3) also holds for every $k,i=0,1,2,3.$. For $k\neq i,\ \ k,i=0,1,2,3$ and $v=0,1,2,3$ equation (3.18) holds and, since $Q\neq 0$, we obtain

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\frac{b}{\hbar}P_vQ\lambda_{ki}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

and with equation (4.2) we get

$$Q\frac{\partial\lambda_{ki}}{\partial x_v}=\lambda_{ki}\frac{\partial Q}{\partial x_v}-\frac{bc_v}{2\hbar}Q\lambda_{ki}$$

$$\frac{\partial}{\partial x_v}\left(\frac{\lambda_{ki}}{Q}\right)=-\frac{bc_v}{2\hbar}\frac{\lambda_{ki}}{Q}$$

and integrating we obtain

$$\frac{\lambda_{ki}}{Q}=a_{ki}\exp\left[-\frac{b}{2\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right],$$

where $a_{ki},k\neq i,k,i=0,1,2,3$ are the integration constants, and with (4.5) we get equation (4.4). Equations (4.6) are derived from the combination of equations (3.17) and (4.4), taking into account that $zQ\neq 0$. Equation (4.7) is derived from the combination of equation $\lambda_{ik}=-\lambda_{ki},k\neq i,k,i=0,1,2,3$ with equation (4.4). Simirarly, equation (4.8) is derived from the combination of equations (3.19) and (4.4). □

We will also use equation

$$\frac{\partial z}{\partial x_k}=-\frac{bc_k}{2\hbar}z,k=0,1,2,3 \tag{4.9}$$

which results immediately from equation (4.5).

For $k = i, k, i = 0,1,2,3$ equation (4.4) does not hold. So we define the physical quantities $\Phi_k$ and $T_k$ as given by equation

$$\Phi_k = zT_k = z\alpha_{kk} = \frac{\lambda_{kk}}{Q}, k = 0,1,2,3. \qquad (4.10)$$

Taking into account the notation of equation (4.10) the main diagonal of matrix $T$ of equation (2.12) is given from matrix $\Lambda$

$$\Lambda = \frac{1}{Q}\begin{bmatrix} \lambda_{00} & 0 & 0 & 0 \\ 0 & \lambda_{11} & 0 & 0 \\ 0 & 0 & \lambda_{22} & 0 \\ 0 & 0 & 0 & \lambda_{33} \end{bmatrix} = \begin{bmatrix} \Phi_0 & 0 & 0 & 0 \\ 0 & \Phi_1 & 0 & 0 \\ 0 & 0 & \Phi_2 & 0 \\ 0 & 0 & 0 & \Phi_3 \end{bmatrix} = \begin{bmatrix} zT_0 & 0 & 0 & 0 \\ 0 & zT_1 & 0 & 0 \\ 0 & 0 & zT_2 & 0 \\ 0 & 0 & 0 & zT_3 \end{bmatrix}. \qquad (4.11)$$

We now define the three-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, as given by equations (4.12) and (4.13) respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} ic\lambda_{01} \\ ic\lambda_{02} \\ ic\lambda_{03} \end{pmatrix} \qquad (4.12)$$

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = \frac{1}{Q}\begin{pmatrix} \lambda_{32} \\ \lambda_{13} \\ \lambda_{21} \end{pmatrix} \qquad (4.13)$$

Vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ contain all of the physical quantities $\lambda_{ki}$ for $k \neq i, k, i = 0,1,2,3$ since $\lambda_{ik} = -\lambda_{ki}$.

Combining equations (4.12) and (4.13) with equation (4.4), the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are written in the form of equations (4.14) and (4.15), respectively

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = icz\begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \qquad (4.14)$$

$$\mathbf{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} = z \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} . \tag{4.15}$$

We write equation (2.10) in the form

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + \lambda_{ki}, \mathrm{k}, \mathrm{i} = 0,1,2,3. \tag{4.16}$$

The rate of change of the momentum of the material particle equals the sum of the two terms in the right part of equation (4.16). For $k = 0$, and since $x_0 = ict$, equation (83) gives the rate of change of the particle momentum with respect to time $t$, i.e. the physical quantity we call "force". By using the concept of force, as defined by Newton, we also have to use the concept of velocity. For this reason we symbolize $\mathbf{u}$ the velocity of the material particle, as given by equation

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} . \tag{4.17}$$

Also, we define the 4-vector of the four-vector $u$, as given by equation

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} ic \\ u_x \\ u_y \\ u_z \end{bmatrix}. \tag{4.18}$$

We now prove the following theorem:

**Theorem 4.2.** ´´The rates of change with respect to time $t\left(x_0 = ict\right)$ of the four-vectors $J$ and $P$ of the momentum of the generalized particle carrying charge $Q$ are given by equations

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J - \frac{i}{c} Q\Lambda u - \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} \tag{4.19}$$

$$\frac{dP}{dx_0} = -\frac{dQ}{Qdx_0} J + \frac{i}{c} Q\Lambda u + \frac{i}{c} Q \begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} + \mathbf{u}\times\boldsymbol{\beta} \end{bmatrix} .'' \tag{4.20}$$

**Proof.** The matrix $\Lambda$ is given in equation (4.11). By $\mathbf{u}\times\boldsymbol{\beta}$ we denote the outer product of vectors $\mathbf{u}$ and $\boldsymbol{\beta}$.

We now prove the first of equations (4.19):

$$\frac{d}{dt}\left(\frac{J_0}{Q}\right) = \frac{\partial}{\partial t}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial y}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial z}\left(\frac{J_0}{Q}\right)$$

and using the notation of equation (2.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\partial}{\partial x_0}\left(\frac{J_0}{Q}\right) + u_1 \frac{\partial}{\partial x_1}\left(\frac{J_0}{Q}\right) + u_2 \frac{\partial}{\partial x_2}\left(\frac{J_0}{Q}\right) + u_3 \frac{\partial}{\partial x_3}\left(\frac{J_0}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_0}{Q}\right) = ic\frac{\lambda_{00}}{Q} + u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} - \frac{i}{c}\left(u_1\frac{\lambda_{10}}{Q} + u_2\frac{\lambda_{20}}{Q} + u_3\frac{\lambda_{30}}{Q}\right)$$

$$\frac{d}{dx_0}\left(\frac{J_0}{Q}\right) = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{1}{Q}\frac{dJ_0}{dx_0} - \frac{J_0}{Q^2}\frac{dQ}{dx_0} = \frac{\lambda_{00}}{Q} + \frac{i}{c}\left(u_1\frac{\lambda_{01}}{Q} + u_2\frac{\lambda_{02}}{Q} + u_3\frac{\lambda_{03}}{Q}\right)$$

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Qdx_0} J_0 + \lambda_{00} + \frac{i}{c}\left(u_1\lambda_{01} + u_2\lambda_{02} + u_3\lambda_{03}\right)$$

and with equations (4.10) and (4.12) we have

$$\frac{dJ_0}{dx_0}=\frac{dQ}{Qdx_0}J_0+Q\Phi_0-\frac{i}{c}Q\left(\frac{i}{c}u_1\alpha_1+\frac{i}{c}u_2\alpha_2+\frac{i}{c}u_3\alpha_3\right)$$

which is the first of equations (4.19) since

$$-\frac{i}{c}Q\Phi_0 u_0=-\frac{i}{c}Q\Phi_0 ic=Q\Phi_0.$$

We prove the second of equations (4.19) and we can similarly prove the third and the fourth: $\frac{d}{dt}\left(\frac{J_x}{Q}\right)=\frac{\partial}{\partial t}\left(\frac{J_x}{Q}\right)+u_1\frac{\partial}{\partial x}\left(\frac{J_x}{Q}\right)+u_2\frac{\partial}{\partial y}\left(\frac{J_x}{Q}\right)+u_3\frac{\partial}{\partial z}\left(\frac{J_x}{Q}\right)$

and using the notation of equations (2.3) and (2.4) we obtain

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=\frac{ic\partial}{\partial x_0}\left(\frac{J_1}{Q}\right)+u_1\frac{\partial}{\partial x_1}\left(\frac{J_1}{Q}\right)+u_2\frac{\partial}{\partial x_2}\left(\frac{J_1}{Q}\right)+u_3\frac{\partial}{\partial x_3}\left(\frac{J_1}{Q}\right)$$

and with equation (4.3) we get

$$\frac{icd}{dx_0}\left(\frac{J_1}{Q}\right)=ic\frac{\lambda_{01}}{Q}+u_1\frac{\lambda_{11}}{Q}+u_2\frac{\lambda_{21}}{Q}+u_3\frac{\lambda_{31}}{Q}$$

$$\frac{d}{dx_0}\left(\frac{J_1}{Q}\right)=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{1}{Q}\frac{dJ_1}{dx_0}-\frac{J_1}{Q^2}\frac{dQ}{dx_0}=-\frac{iu_1}{c}\frac{\lambda_{11}}{Q}+\frac{\lambda_{01}}{Q}-\frac{iu_2}{c}\frac{\lambda_{21}}{Q}+\frac{iu_3}{c}\frac{\lambda_{13}}{Q}$$

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{iu_1}{c}\lambda_{11}+\lambda_{01}-\frac{iu_2}{c}\lambda_{21}+\frac{iu_3}{c}\lambda_{13}$$

and with equations (4.10), (4.12) and (4.13), we obtain

$$\frac{dJ_1}{dx_0}=\frac{dQ}{Qdx_0}J_1-\frac{i}{c}Q\Phi_1-\frac{i}{c}Q\alpha_1-\frac{i}{c}Q\left(u_2\beta_3-u_3\beta_2\right)$$

which is the second of equations (4.19). Equation (4.20) results from the combination of equations (4.19) and (3.5). □

Using the symbol $\mathbf{J}$ for the momentum vector of the material particle

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

and taking into account equations (2.3) and (2.4) and (4.11) the set of equations (4.19) can be written in the form

$$\begin{aligned} \frac{dW}{dt} &= \frac{dQ}{Qdt} W + Qc^2 \Phi_0 + Q\mathbf{u} \cdot \boldsymbol{\alpha} \\ \frac{d\mathbf{J}}{dt} &= \frac{dQ}{Qdt} \mathbf{J} + Q \begin{pmatrix} \Phi_1 u_1 \\ \Phi_2 u_2 \\ \Phi_3 u_3 \end{pmatrix} + Q\left(\boldsymbol{\alpha} + \mathbf{u} \times \boldsymbol{\beta}\right) \end{aligned} . \quad (4.21)$$

Equations (4.21) are a simpler form of equation (4.19) with which are equivalent.

The rate of change of the four-vector $J$ of the momentum of the material particle is given by the sum of the three terms in the right part of equation (86). The USVI and its consequences for the material particle depend on which of these terms is the strongest and which is the weakest.

The first term expresses a force parallel to four-vector $J$ which is always different than zero due to the Selfvariations. As we will see next, the second term is related to the curvature of spacetime. The third term on the right of equation (4.19) is known as the Lorentz force, in the case of electromagnetic fields. In many cases a term or some of the terms on the right of equation (4.19) are zero, with the exception of the first term which is always different than zero.

From equation (4.19) we conclude that the pair of vectors $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ expresses the intensity of the field of the USVI according to the paradigm of the classical definition of the field potential. From equation (2.10) we derive tha physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ have units (dimensions) of $kg \cdot s^{-1}$. Thus, from equation (4.12) we derive that if $Q$ is the rest mass, the intensity $\boldsymbol{\alpha}$ has unit of $m \cdot s^{-2}$. If $Q$ is the electric charge, the intensity $\boldsymbol{\alpha}$ has unit of $N \cdot C^{-1}$. Now we will prove that for field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ the following equations (4.22) hold.

**Theorem 4.3.** ´´ For the vector pair $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ the following equations hold:

$$\nabla \cdot \boldsymbol{\alpha} = -\frac{icbz}{2\hbar}\left(c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03}\right) \qquad \text{(a)}$$

$$\nabla \cdot \boldsymbol{\beta} = 0 \qquad \text{(b)}$$

$$\nabla \times \boldsymbol{\alpha} = -\frac{\partial \boldsymbol{\beta}}{\partial t} \qquad \text{(c)}$$

(4.22)

$$\nabla \times \boldsymbol{\beta} = -\frac{bz}{2\hbar}\begin{pmatrix} c_0\alpha_{01} + c_2\alpha_{21} + c_3\alpha_{31} \\ c_0\alpha_{02} + c_2\alpha_{12} + c_3\alpha_{32} \\ c_0\alpha_{03} + c_2\alpha_{13} + c_3\alpha_{23} \end{pmatrix} + \frac{\partial \boldsymbol{\alpha}}{c^2\partial t}\,. \qquad '' \qquad \text{(d)}$$

**Proof.** Differentiating equations (4.14) and (4.15) with respect to $x_k$, $k = 0,1,2,3$ and considering equation (4.9), we obtain equations

$$\frac{\partial \boldsymbol{\alpha}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\alpha} \qquad (4.23)$$

$$\frac{\partial \boldsymbol{\beta}}{\partial x_k} = -\frac{bc_k}{2\hbar}\boldsymbol{\beta}\,. \qquad (4.24)$$

From equations (4.23) and (4.24) we can easily derive equations (4.22). Indicatively, we prove equation (4.22b). From equation (4.15) we obtain

$$\nabla \cdot \boldsymbol{\beta} = \alpha_{32}\frac{\partial z}{\partial x_1} + \alpha_{13}\frac{\partial z}{\partial x_2} + \alpha_{21}\frac{\partial z}{\partial x_3}$$

and with equation (4.9) we get

$$\nabla \cdot \boldsymbol{\beta} = -\frac{bz}{2\hbar}\left(c_1\alpha_{32} + c_2\alpha_{13} + c_3\alpha_{21}\right)$$

and with the first of equations (4.6) for $(i,v,k) = (1,3,2)$ we get

$$\nabla \cdot \boldsymbol{\beta} = 0$$

The first of equations (4.6) should be taken into account for the proof of the rests of equations of (4.22). □

Considering equations (4.22) we define the scalar quantity $\rho$ and the vector quantity $\mathbf{j}$, as given by equations

$$\begin{aligned}
&\rho=\sigma\nabla\cdot\boldsymbol{\alpha}=-\sigma\frac{icbz}{2\hbar}\left(c_1a_{01}+c_2a_{02}+c_3a_{03}\right)\\
&\mathbf{j}=\sigma\frac{c^2bz}{2\hbar}\begin{pmatrix}-c_0a_{01}-c_2\,a_{21}+c_3a_{13}\\-c_0a_{02}+c_1\,a_{21}-c_3a_{32}\\-c_0a_{03}-c_1\,a_{13}+c_2a_{32}\end{pmatrix}
\end{aligned}\qquad (4.25)$$

where $\sigma\neq 0$ is a constant. We now prove that for the physical quantities $\rho$ and $\mathbf{j}$ the following continuity equation holds:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j}=0\,. \qquad (4.26)$$

**Proof.** : From the first of equations (4.25) we obtain

$$\begin{aligned}
&\rho=\sigma\nabla\cdot\boldsymbol{\alpha}\\
&\frac{\partial\rho}{\partial t}=\sigma\frac{\partial}{\partial t}\left(\nabla\cdot\boldsymbol{\alpha}\right)\\
&\frac{\partial\rho}{\partial t}=\nabla\cdot\left(\sigma\frac{\partial\boldsymbol{\alpha}}{\partial t}\right)
\end{aligned}$$

and with the second of equations (4.25) and equation (4.22d) we get

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}=\nabla\cdot\left(\sigma c^2\nabla\times\boldsymbol{\beta}-\mathbf{j}\right)\\
&\frac{\partial\rho}{\partial t}=-\nabla\cdot\mathbf{j}
\end{aligned}$$

which is equation (4.26). □

According to equation (4.26), the physical quantity $\rho$ is the density of a conserved physical quantity $q$ with current density $\mathbf{j}$. The conserved physical quantity $q$ is related to field $\left(\boldsymbol{\alpha},\boldsymbol{\beta}\right)$ through equations (4.22).We will revert to the issue of sustainable physical quantities in the next paragraphs.

The density $\rho$ and the current density $\mathbf{j}$ have a rigidly defined internal structure as derived from equations (4.25).

We now consider the four-vector of the current density $j$ of the conserved physical quantity $q$, as given by equation

$$j = \begin{bmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{bmatrix} = \begin{bmatrix} i\rho c \\ j_x \\ j_y \\ j_z \end{bmatrix} \tag{4.27}$$

and the $4\times4$ matrices $M$

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & 0 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & 0 \end{bmatrix}. \tag{4.28}$$

Using matrix $M$ equations (4.25) can be written in the form of equation

$$j = \frac{\sigma c^2 bz}{2\hbar} MC\,. \tag{4.29}$$

From equations (4.22bc) we conclude that the potential is always defined in the $(\boldsymbol{\alpha},\boldsymbol{\beta})$ - field of the USVI. That is, the scalar potential

$$V = V(t,x,y,z) = V(x_0,x_1,x_2,x_3)$$

and the vector potential $\mathbf{A}$

$$\mathbf{A} = \mathbf{A}(t,x,y,z) = \mathbf{A}(x_0,x_1,x_2,x_3) = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

are defined through the equations

$$\boldsymbol{\beta} = \nabla \times \mathbf{A}$$
$$\boldsymbol{\alpha} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\nabla V - \frac{ic\partial \mathbf{A}}{\partial x_0} \quad .$$

We can introduce in the above equations the gauge function $f$. That is, we can add to the scalar potential $V$ the term

$$-\frac{\partial f}{\partial t} = -\frac{ic\partial f}{\partial x_0}$$

and to the vector potential $\mathbf{A}$ the term

$$\nabla f$$

for an arbitrary function $f$

$$f = f(t, x, y, z) = f(x_0, x_1, x_2, x_3)$$

without changing the intensity $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of the field. The proof of the above equations is known and trivial and we will not repeat it here. For the field potential of the USVI the following theorem holds:

**Theorem 4.4.** ′′In the $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-field of USVI the pair of scalar-vector potentials $(V, \mathbf{A})$ is always defined through equations

$$\begin{aligned} \boldsymbol{\beta} &= \nabla \times \mathbf{A} \\ \boldsymbol{\alpha} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = ic\nabla A_0 - \frac{ic\partial \mathbf{A}}{\partial x_0} \quad . \end{aligned} \tag{4.30}$$

The four-vector $A$ of the potential

$$A = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{iV}{c} \\ A_x \\ A_y \\ A_z \end{bmatrix} \tag{4.31}$$

is given by equation

$$A_i = \begin{cases} \dfrac{2\hbar}{b}\dfrac{\alpha_{ki}}{c_k} z + \dfrac{\partial f_k}{\partial x_i}, \text{for } i \neq k \\ \dfrac{\partial f_k}{\partial x_i}, \text{for } i = k \end{cases} \tag{4.32}$$

where $c_k \neq 0, k, i = 0,1,2,3$ and $f_k$ is the gauge function´´

**Proof.** Equations (4.30) are equivalent to equations (4.22b, c) as we have already mentioned. The proof of equation (4.32) can be performed through the first of equations (4.6). The mathematical calculations do not contribute anything useful to our study, thus we omit them. You can verify that the potential of equation (4.32) gives equations (4.14) and (4.15) through equations (4.30) taking also into account the first of equations (4.6). □

From equation (4.32) the following four sets of the potentials follow:

$$\begin{aligned} & c_0 \neq 0 \\ & A_0 = \frac{\partial f_0}{\partial x_0} \\ & A_1 = \frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3} \end{aligned} \tag{4.33}$$

$$\begin{aligned} & c_1 \neq 0 \\ & A_0 = \frac{2\hbar z}{b}\frac{\alpha_{10}}{c_1} + \frac{\partial f_1}{\partial x_0} \\ & A_1 = \frac{\partial f_1}{\partial x_1} \\ & A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_1}{\partial x_2} \\ & A_3 = \frac{2\hbar z}{b}\frac{\alpha_{13}}{c_1} + \frac{\partial f_1}{\partial x_3} \end{aligned} \tag{4.34}$$

$$
\begin{aligned}
&c_2 \neq 0\\
&A_0 = \frac{2\hbar z}{b}\frac{\alpha_{20}}{c_2} + \frac{\partial f_2}{\partial x_0}\\
&A_1 = \frac{2\hbar z}{b}\frac{\alpha_{21}}{c_2} + \frac{\partial f_2}{\partial x_1}\\
&A_2 = \frac{\partial f_2}{\partial x_2}\\
&A_3 = \frac{2\hbar z}{b}\frac{\alpha_{23}}{c_2} + \frac{\partial f_2}{\partial x_3}
\end{aligned}
\tag{4.35}
$$

$$
\begin{aligned}
&c_3 \neq 0\\
&A_0 = \frac{2\hbar z}{b}\frac{\alpha_{30}}{c_3} + \frac{\partial f_3}{\partial x_0}\\
&A_1 = \frac{2\hbar z}{b}\frac{\alpha_{31}}{c_3} + \frac{\partial f_3}{\partial x_1}\\
&A_2 = \frac{2\hbar z}{b}\frac{\alpha_{32}}{c_3} + \frac{\partial f_3}{\partial x_2}\\
&A_3 = \frac{\partial f_3}{\partial x_3}
\end{aligned}
\tag{4.36}
$$

Indicatively, we calculate the components $\alpha_1$ and $\beta_1$ of the intensity $(\mathbf{\acute{a}},\mathbf{\hat{a}})$ of the USVI field from the potentials (4.35). From the second of equations (4.30) we obtain

$$\alpha_1 = ic\left(\frac{\partial A_0}{\partial x_1} - \frac{\partial A_1}{\partial x_0}\right)$$

and with equations (4.33) we get

$$\alpha_1 = ic\left[\frac{\partial}{\partial x_1}\left(\frac{\partial f_0}{\partial x_0}\right) - \frac{\partial}{\partial x_0}\left(\frac{2\hbar z}{b}\frac{\alpha_{01}}{c_0} + \frac{\partial f_0}{\partial x_1}\right)\right]$$

$$\alpha_1 = -ic\frac{2\hbar}{b}\frac{\alpha_{01}}{c_0}\frac{\partial z}{\partial x_0}$$

and with equation (4.9) we get

$$\alpha_1 = icz\alpha_{01}$$

that is we get the intensity $\alpha_1$ of the field, as given by equation (4.14).

From the first of equations (4.30) we have

$$\beta_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$$

and with equations (4.33) we get

$$\beta_1 = \frac{\partial}{\partial x_2}\left(\frac{2\hbar z}{b}\frac{\alpha_{03}}{c_0} + \frac{\partial f_0}{\partial x_3}\right) - \frac{\partial}{\partial x_3}\left(\frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2}\right)$$

$$\beta_1 = \frac{2\hbar}{b}\frac{\alpha_{03}}{c_0}\frac{\partial z}{\partial x_2} - \frac{2\hbar}{b}\frac{\alpha_{02}}{c_0}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we get

$$\beta_1 = -\frac{c_2\alpha_{03}}{c_0} z + \frac{c_3\alpha_{02}}{c_0} z$$

and considering that $\alpha_{02} = -\alpha_{20}$, we get

$$\beta_1 = -\frac{z}{c_0}\left(c_2\alpha_{03} + c_3\alpha_{20}\right). \tag{4.37}$$

From the first of equations (4.6) for $(i,v,k) = (2,0,3)$ we obtain

$$c_2 a_{03} + c_3 a_{20} + c_0 a_{32} = 0$$
$$c_2 a_{03} + c_3 a_{20} = -c_0 a_{32}$$

and substituting into equation (4.37), we see that

$$\beta_1 = z\alpha_{32}$$

that is, we get the intensity $\beta_1$ of the field, as given by equation (4.15).

The gauge functions $f_k, \mathrm{k} = 0,1,2,3$ in equations (4.33)-(4.36) are not independent of each other. For $c_k \neq 0$ and $c_i \neq 0$ for $k \neq i, k,i = 0,1,2,3$ equation (4.38) holds

$$f_k = f_i + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{ki}}{c_k c_i}, c_k c_i \neq 0, k \neq i,\ k,i = 0,1,2,3. \tag{4.38}$$

The proof of equation (4.38) is through the first of equations (4.6). The proof is lengthy and we omit it. Indicatively, we will prove the third of equations (4.33) from the third of equations (4.34) for $k=1$ and $i=0$ in equation (4.38).

For $c_0 \neq 0$ and $c_1 \neq 0$ both equations (4.33) and equations (4.34) hold. From equation (4.38) for $k=1$ and $i=0$ we get equation

$$f_1 = f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0 c_1} \ . \tag{4.39}$$

From the third of equations (4.33) and equation (4.39) we get

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial}{\partial x_2}\left( f_0 + \frac{4\hbar^2 z}{b^2}\frac{\alpha_{10}}{c_0 c_1}\right)$$

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} + \frac{4\hbar^2}{b^2}\frac{\alpha_{10}}{c_0 c_1}\frac{\partial z}{\partial x_2}$$

and with equation (4.9) we obtain

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{12}}{c_1} + \frac{\partial f_0}{\partial x_2} - \frac{2\hbar z}{b}\frac{c_2 \alpha_{10}}{c_0 c_1}$$

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0 \alpha_{12} - c_2 \alpha_{10}\right) + \frac{\partial f_0}{\partial x_2}$$

and since $\alpha_{10} = -\alpha_{01}$, we get equation

$$A_2 = \frac{2\hbar z}{b c_0 c_1}\left(c_0 \alpha_{12} + c_2 \alpha_{01}\right) + \frac{\partial f_0}{\partial x_2} \tag{4.40}$$

From the first of equations (4.6) for $(i,v,k) = (0,1,2)$ we obtain

$$c_0 a_{12} + c_2 a_{01} + c_1 a_{20} = 0$$
$$c_0 a_{12} + c_2 a_{01} = -c_1 a_{20}$$
$$c_0 a_{12} + c_2 a_{01} = c_1 a_{02}$$

and substituting into equation (4.40) we obtain equation

$$A_2 = \frac{2\hbar z}{b}\frac{\alpha_{02}}{c_0} + \frac{\partial f_0}{\partial x_2} \tag{4.41}$$

Equation (4.41) is the third of equations (4.33).

According to equation (4.38), if $c_k \neq 0$ for more than one of the constants $c_k, k = 0,1,2,3$, the sets of equations of potential resulting from equation (4.32) have in the end a gauge function. In the application we presented assuming $c_0 \neq 0$ and $c_1 \neq 0$ for a specific gauge function $f_0$ in equations (4.33), the gauge function $f_1$ in equations (4.34) is given by equation (4.39). □

We conclude the investigation of the potential of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of USVI by proving the following corollary:

**Corollary 4.1.** ´´In the external symmetry, the 4-vector $C$ of the total energy content of the generalized particle cannot vanish:

$$C=\begin{bmatrix}c_0\\c_1\\c_2\\c_3\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\\0\end{bmatrix}.^{\prime\prime} \tag{4.42}$$

**Proof.** Indeed, for $C=0$ we obtain $J=-P$ from equation (3.5). Therefore, the four-vectors $J$ and $P$ are parallel. According to equivalence (3.7) the parallelism of the four-vectors $J$ and $P$ is equivalent to the internal symmetry. Therefore, in the external symmetry it is $C\neq 0$. □

A direct consequence of these findings is that the potential of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ of USVI is always defined, as given from equation (4.42). This conclusion is derived from the fact that at least one of the constants $c_k, k\in\{0,1,2,3\}$ is always different than zero.

## 5. The conserved physical quantities of the generalized particle and the wave equation of the TSV

The generalized particle has a set of conserved physical quantities $q$ which we determine in this paragraph. At first, we generalize the notion of the field, as it is derived from the equations of theTSV. We prove the following theorem:

**Theorem 5.1.** ´´For the field $(\boldsymbol{\xi},\boldsymbol{\omega})$ of the pair of vectors

$$\boldsymbol{\xi}=ic\Psi\begin{pmatrix}a_{01}\\a_{02}\\a_{03}\end{pmatrix} \tag{5.1}$$

$$\boldsymbol{\omega} = \Psi \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.2}$$

where $\Psi = \Psi(x_0, x_1, x_2, x_3)$ is a function satisfying equation

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar}(\lambda J_k + \mu P_k)\Psi$$

$k = 0,1,2,3, \quad (\lambda, \mu) \neq (0,0), \quad \lambda, \mu \in \mathbb{C}$ are functions of $x_0, x_1, x_2, x_3$, the following equations hold

$$\begin{aligned} &\nabla \cdot \boldsymbol{\omega} = 0 \\ &\nabla \cdot \boldsymbol{\xi} = -\frac{\partial \boldsymbol{\omega}}{\partial t} \end{aligned} \tag{5.4}$$

The generalized particle has a set of conserved physical quantities $q$ with density $\rho$ and current density $\mathbf{j}$

$$\begin{aligned} &\rho = \sigma \nabla \cdot \boldsymbol{\xi} \\ &\mathbf{j} = \sigma c^2 \left( \nabla \times \boldsymbol{\omega} - \frac{\partial \boldsymbol{\xi}}{c^2 \partial t} \right) \end{aligned} \tag{5.5}$$

where $\sigma \neq 0$ are constants, for which conserved physical quantities the following continuity equation holds

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0 \,. \tag{5.6}$$

The four-vectors of the current density $j$ are given by equation

$$j = -\frac{\sigma c^2 b}{\hbar} \Psi M \left( \lambda J + \mu P \right) \,.'' \tag{5.7}$$

**Proof.** Matrix $M$ in equation (5.7) is given by equation (4.28). We denote $\mathbf{J}$ and $\mathbf{P}$ the three-dimensional momentums as given by equations

$$\mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} \tag{5.8}$$

$$\mathbf{P} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}. \tag{5.9}$$

For the proof of the theorem we first demonstrate the following auxiliary equations (5.10)-(5.15)

$$\mathbf{J} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.10}$$

$$\mathbf{P} \cdot \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0 \tag{5.11}$$

$$\mathbf{J} \times \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -J_0 \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.12}$$

$$\mathbf{P} \times \begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = -P_0 \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} \tag{5.13}$$

$$\mathbf{J} \times \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} J_2 a_{21} - J_3 a_{13} \\ J_3 a_{32} - J_1 a_{21} \\ J_1 a_{13} - J_2 a_{32} \end{pmatrix} \tag{5.14}$$

$$\mathbf{P} \times \begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = \begin{pmatrix} P_2 a_{21} - P_3 a_{13} \\ P_3 a_{32} - P_1 a_{21} \\ P_1 a_{13} - P_2 a_{32} \end{pmatrix} \tag{5.15}$$

In order to prove equation (5.10) we get

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = J_1 a_{32} + J_2 a_{13} + J_3 a_{21}$$

and with the second of equations (4.6) for $(i,v,k)=(1,3,2)$, we have

$$\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} = 0$$

Similarly, from the third of equations (4.6) we obtain equation (5.11). We now get

$$\mathbf{J}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = \begin{pmatrix} J_2 a_{03} - J_3 a_{02} \\ J_3 a_{01} - J_1 a_{03} \\ J_1 a_{02} - J_2 a_{01} \end{pmatrix} = \begin{pmatrix} J_2 a_{03} + J_3 a_{20} \\ J_3 a_{01} + J_1 a_{30} \\ J_1 a_{02} + J_2 a_{10} \end{pmatrix}$$

and with the second of equations (4.6) we obtain

$$\mathbf{J}\times\begin{pmatrix} a_{01} \\ a_{02} \\ a_{03} \end{pmatrix} = \begin{pmatrix} -J_0 a_{32} \\ -J_0 a_{13} \\ -J_0 a_{21} \end{pmatrix}$$

which is equation (5.12). Similarly, by considering the third of equations (4.6) we derive equation (5.13). Equations (5.14) and (5.15) are derived by taking into account equations (5.8) and (5.9).

Equations (5.4) are proven with the use of equations (5.10)-(5.15). We prove the first as an example. From equation (5.2) we obtain

$$\nabla\cdot\boldsymbol{\omega} = \nabla\Psi\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equation (5.3) we get

$$\nabla\cdot\boldsymbol{\omega} = \frac{b}{\hbar}\lambda\Psi\mathbf{J}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix} + \frac{b}{\hbar}\mu\Psi\mathbf{P}\cdot\begin{pmatrix} a_{32} \\ a_{13} \\ a_{21} \end{pmatrix}$$

and with equations (5.10) and (5.11) we obtain

$$\nabla \cdot \boldsymbol{\omega} = 0.$$

From equations (5.4) and (5.5), the continuity equation (5.6) results. The proof is similar to the one for equation (4.26). The proof of equation (5.7) is done with the use of equations (5.10)-(5.15), and equation (4.28). □

Field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ presented in the previous paragraph is a special case of the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$ for $\lambda = \mu = -\frac{1}{2}$. For these values of the parameteres $\lambda, \mu$ we obtain from equations (5.3)

$$\frac{\partial \Psi}{\partial x_k} = \frac{b}{\hbar}\left(-\frac{1}{2} J_k - \frac{1}{2} P_k\right)\Psi$$
$$\frac{\partial \Psi}{\partial x_k} = -\frac{b}{2\hbar}\left(J_k + P_k\right)\Psi$$

and with equation (3.5) we obtain

$$\frac{\partial \Psi}{\partial x_k} = -\frac{bc_k}{2\hbar}\Psi$$

and finally we obtain

$$\Psi = z = \exp\left[-\frac{b}{2\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]$$

and from equations (5.1),(5.2) and (4.14),(4.15) we obtain $\boldsymbol{\xi} = \boldsymbol{\alpha}$ and $\boldsymbol{\omega} = \boldsymbol{\beta}$ .

From equation (2.10) it emerges that the dimensions of the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$ are

$$[\lambda_{ki}] = kgs^{-1}, k, i = 0,1,2,3 .$$

Thus, from equations (4.12), (4.13) and (4.14), (4.15) we obtain the dimensions of the physical quantities $Q\alpha_{ki}, k, i = 0,1,2,3$. Furthermore, from equation (4.11) we obtain the dimensions of the physical quantities $T_k, k = 0,1,2,3.$ Thus, we get the following relationships

$$
\begin{aligned}
&[Q\alpha_{ki}] = kgs^{-1}, k \neq i, k, i = 0,1,2,3, \\
&[T_k] = kgs^{-1}, k = 0,1,2,3.
\end{aligned} \tag{5.16}
$$

Using the first of equations (5.16) we can determine the units of measurement of the $(\boldsymbol{\xi}, \boldsymbol{\omega})$-field for every selfvariating charge $Q$. When $Q$ is the electric charge, we can verify that the field units are $\left(V \cdot m^{-1}, \mathrm{T}\right)$. When $Q$ is the rest mass, the field units are $\left(m \cdot s^{-2}, s^{-1}\right)$. The dimensions of the field depend solely on the units of measurement of the selfvariating charge $Q$.

From equation (5.7) and taking into account that $\lambda, \mu \in \mathbb{C}$ we can define the dimensions of the physical quantities $q$ through the first of equations (5.16). For $\sigma = \varepsilon_0$, where $\varepsilon_0$ is the electric permeability of the vacuum, $q$ is a conserved physical quantity of electric charge. For $\sigma = \dfrac{\varepsilon_0}{e}$, where $e$ the constant value we measure in the lab for the electric charge of the electron, $q$ is a conserved physical quantity of angular momentum. For $\sigma = \dfrac{1}{4\pi G}$, where $G$ is the gravitational constant, $q$ is a conserved physical quantity of matter. Theorem 5.1 reveals the conserved physical quantities of the generalized particle.

One of the most important corollaries of the theorem 5.1 is the prediction that the generalized particle has wave-like behavior. We prove the following corollary:

**Corollary 5.1.** ´´For function $\Psi$ the following equation holds

$$
\begin{aligned}
&\sigma c^2 \alpha_{ki} \left( \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \\
&\sigma c^2 \alpha_{ki} \left( \nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} \right) = \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i}
\end{aligned} \tag{5.17}
$$

$k \neq i, \quad k, i = 0,1,2,3$.´´

**Proof.** To prove the corollary, considering that $x_0 = ict$ , we write equations (5.4) and (5.5) in the form

$$\begin{aligned}
&\nabla \cdot \boldsymbol{\xi} = -\frac{i}{\sigma c} j_0 \\
&\nabla \cdot \boldsymbol{\omega} = 0 \\
&\nabla \times \boldsymbol{\xi} = -\frac{ic\partial \boldsymbol{\omega}}{\partial x_0} \\
&\nabla \times \boldsymbol{\omega} = \frac{1}{\sigma c^2} \mathbf{j} + \frac{i\partial \boldsymbol{\xi}}{c\partial x_0}
\end{aligned} \tag{5.18}$$

We will also use the identity (162) which is valid for every vector $\boldsymbol{\alpha}$

$$\nabla \times \nabla \times \boldsymbol{\alpha} = \nabla(\nabla \cdot \boldsymbol{\alpha}) - \nabla^2 \boldsymbol{\alpha} \tag{5.19}$$

From the third of equations (5.18) we obtain

$$\begin{aligned}
&\nabla \times \nabla \times \boldsymbol{\xi} = -\nabla \times \left( \frac{ic\partial \boldsymbol{\omega}}{\partial x_0} \right) \\
&\nabla \times \nabla \times \boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}(\nabla \times \boldsymbol{\omega})
\end{aligned}$$

and using the identity (5.19) we get

$$\nabla(\nabla \cdot \boldsymbol{\xi}) - \nabla^2 \boldsymbol{\xi} = -\frac{ic\partial}{\partial x_0}(\nabla \times \boldsymbol{\omega})$$

and with the first and fourth of equations (5.18) we get

$$\nabla\left( -\frac{i}{\sigma c} j_0 \right) - \nabla^2 \boldsymbol{\xi} = \frac{\partial^2 \boldsymbol{\xi}}{\partial x_0^2} - \frac{i}{\sigma c} \frac{\partial \mathbf{j}}{\partial x_0}$$

and we finally get

$$\nabla^2 \boldsymbol{\xi} + \frac{\partial^2 \boldsymbol{\xi}}{\partial x_0^2} = \frac{i}{\sigma c}\left( \frac{\partial \mathbf{j}}{\partial x_0} - \nabla j_0 \right) \tag{5.20}$$

Working similarly from equation (5.18) we obtain

$$\nabla^2 \boldsymbol{\omega} + \frac{\partial^2 \boldsymbol{\omega}}{\partial x_0^2} = -\frac{1}{\sigma c^2} \nabla \times \mathbf{j} \tag{5.21}$$

Combining equations (5.20) and (5.21) with equations (5.1) and (5.2), we get

$$\alpha_{ki} \left( \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial x_0^2} \right) = \frac{i}{\sigma c} \left( \frac{\partial j_i}{\partial x_k} - \frac{\partial j_k}{\partial x_i} \right), \; k \neq i, \quad k, i = 0,1,2,3$$

which is equation (5.17). $\square$

Equation (5.17) can be characterized as **"the wave equation of the TSV"**. The basic characteristics of equation (5.17) depend on whether the physical quantity

$$F = \nabla^2 \Psi + \frac{\partial^2 \Psi}{\partial^2 x_0^2} = \nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} \tag{5.22}$$

is zero or not.

This conclusion is drawn through the following theorem:

**Theorem 5.2.** ´´For the generalized particle the following equivalences hold

$$\nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} = 0 \tag{5.23}$$

if and only if for each $k \neq i, \; k, i = 0,1,2,3$ it is

$$\frac{\partial j_i}{\partial x_k} = \frac{\partial j_k}{\partial x_i} \tag{5.24}$$

if and only if

$$\begin{aligned} &\nabla^2 \boldsymbol{\xi} - \frac{\partial^2 \boldsymbol{\xi}}{c^2 \partial t^2} = 0 \\ &\nabla^2 \boldsymbol{\omega} - \frac{\partial^2 \boldsymbol{\omega}}{c^2 \partial t^2} = 0 \end{aligned} \quad ,, \tag{5.25}$$

**Proof.** In the external symmetry there exists at least one pair of indices $(k,i), \; k \neq i, \; k, i \in \{0,1,2,3\}$, for which $\alpha_{ki} \neq 0$. Therefore, when equation (5.24) holds, then equation (5.23) follows from equation (5.17), and vice versa. Thus, equations (5.23) and (5.24) are equivalent. When equation (5.24) holds, then the right hand sides of equations (5.24) and (5.25) vanish, that is, equations (5.25) hold. The converse also holds, thus equations (5.24) and (5.25) are equivalent. Therefore, equations (5.23), (5.24), and (5.25) are equivalent. □

In case that $F=0$, that is in case that equivalences (5.23), (5.24) and (5.25) hold, we shall refer to the state of the generalized particle as the "generalized photon". According to equations (5.25), for the generalized photon the $(\boldsymbol{\xi},\boldsymbol{\omega})$-field is propagating with velocity $c$ in the form of a wave.

For the generalized photon, the following corollary holds:

**Corollary 5.2:** ′′ For the generalized photon, the four-vector $j$ of the current density of the conserved physical quantities $q$, varies according to the equations

$$\nabla^2 j_k - \frac{\partial^2 j_k}{c^2 \partial t^2} = 0,\ k = 0,1,2,3 .'' \tag{5.26}$$

**Proof.** We prove equation (5.26) for $k=0$, and we can similarly prove it for $k=1,2,3$. Considering equation (4.27), we write equation (5.6) in the form

$$\frac{\partial j_0}{\partial x_0} + \frac{\partial j_1}{\partial x_1} + \frac{\partial j_2}{\partial x_2} + \frac{\partial j_3}{\partial x_3} = 0 \tag{5.27}$$

Differentiating equation (5.27) with respect to $x_0$ we get

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_0}\left(\frac{\partial j_1}{\partial x_1}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_2}{\partial x_2}\right) + \frac{\partial}{\partial x_0}\left(\frac{\partial j_3}{\partial x_3}\right) = 0$$

$$\frac{\partial^2 j_0}{\partial x_0^2} + \frac{\partial}{\partial x_1}\left(\frac{\partial j_1}{\partial x_0}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial j_2}{\partial x_0}\right) + \frac{\partial}{\partial x_3}\left(\frac{\partial j_3}{\partial x_0}\right) = 0$$

and with equation (5.24) we get

$$\frac{\partial^2 j_0}{\partial x_0^2}+\frac{\partial}{\partial x_1}\left(\frac{\partial j_0}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial j_0}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(\frac{\partial j_0}{\partial x_3}\right)=0$$

$$\frac{\partial^2 j_0}{\partial x_0^2}+\nabla^2 j_0=0$$

which is equation (5.26) for $k=0$, since $x_0=ict$. □

The way in which equations (5.25) emerge in the TSV is completely different from the way in which the electromagnetic waves emerge in Maxwell's electromagnetic theory [6-10]. In Maxwell's theory, equations (5.25) emerge for $j=0$. In the TSV it is $j\neq 0$ due to the Selfvariations. Furthermore, according to the TSV, in the electromagnetic waves, the current density $j$ varies according to equation (5.26).

One of the most important conclusions of the theorem 5.1 is that it gives the degrees of freedom of the equations of the TSV. In equation (5.7) the parameters $\lambda,\mu\in\mathbb{C},(\lambda,\mu)\neq(0,0)$ can have arbitrary values or can be arbitrary functions of $x_0,x_1,x_2,x_3$. Therefore, the investigation of the TSV takes place through the parameters $\lambda$ and $\mu$ of equation (5.7).

If we set $(\lambda,\mu,b)=(1,0,i)$ in equation (5.7), we get equations

$$\begin{aligned}\nabla\Psi&=\frac{i}{\hbar}\mathbf{J}\Psi\\ \frac{\partial\Psi}{\partial x_0}&=\frac{i}{\hbar}J_0\Psi\end{aligned}\quad. \tag{5.28}$$

Taking into account that $x_0=ict$ and $J_0=\frac{iW}{c}$, we recognize in equations (5.28) the Schrödinger operators. Using the macroscopic mathematical expressions of the momentum $\mathbf{J}$ and energy $W$ of the material particle, we get the Schrödinger equation [11-15]. The Schrödinger equation is a special case of the wave equation of the TSV.

In Schrodinger's equations, we can slightly modify the three parameters $(\lambda,\mu,b)$. If we set $(\lambda,\mu,b)=(1,\alpha,i)$ in equation (5.7), where $\alpha$ the fine structure constant, and take into account equation (3.5), we get equations

$$\nabla\Psi = \frac{i}{\hbar}\left(\left(1-\alpha\right)\mathbf{J}+\alpha\mathbf{C}\right)\Psi$$
$$\frac{\partial\Psi}{\partial x_0} = \frac{i}{\hbar}\left(\left(1-\alpha\right)J_0+\alpha c_0\right)\Psi \quad . \qquad (5.29)$$

The fine structure constant in the TSV can have the following three forms

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 c\hbar}$$
$$\alpha = \frac{eQ}{4\pi\varepsilon_0 c\hbar} \qquad (5.30)$$
$$\alpha = \frac{Q^2}{4\pi\varepsilon_0 c\hbar}$$

in the electromagnetic interaction. We denote $e$ the constant value we measure in the lab for the electric charge of the electron . By $Q$ we denote the electron's selfvariating charge. The difference between the two physical quantities $e$ and $Q$ is due to ´´the internality of the Universe to the measurement procedure´´.The unit of measurement of the charge $Q$ is itself subject to the Selfvariations [5] (par. 4.9).

The combination of equation (5.28) with each of equations (5.29), as well as the Schrödinger equation (5.28), give the exact same results for the hydrogen atom. For the TSV, the investigation of physical reality is put on the following terms: "In the application of the TSV, and in every case except of the generalized photon, the determination of the parameters $\lambda$ and $\mu$ , is sought. This determination can be either theoretical or based on experimental data." The determination of the parameter $b$ of the law of Selfvariations is made from the boundary conditions of the differential equations of the TSV, to which we will not refer to in the present study.

## 6. The Lorentz-Einstein-Selfvariations Symmetry

In this paragraph we calculate the Lorentz-Einstein transformations of the physical quantities $\lambda_{ki}$ , $k,i=0,1,2,3.$ The part of spacetime occupied by the generalized particle can be flat or curved. TheLorentz-Einstein transformations give us information about this subject.

We consider an inertial frame of reference $O'(t', \mathrm{x}', \mathrm{y}', \mathrm{z}')$ moving with velocity $(u, 0, 0)$ with respect to another inertial frame of reference $O(t, \mathrm{x}, \mathrm{y}, \mathrm{z})$, with their origins $O'$ and $O$ coinciding at $t' = t = 0$. We will calculate the Lorentz-Einsteintransformations for the physical quantities $\lambda_{ki}$, $k, i = 0,1,2,3$. We begin with transformations (6.1) and (6.2)

$$
\begin{aligned}
\frac{\partial}{\partial t'} &= \gamma\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right) \\
\frac{\partial}{\partial x'} &= \gamma\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right) \\
\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y} \\
\frac{\partial}{\partial z'} &= \frac{\partial}{\partial z}
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
W' &= \gamma(W - uJ_x) & E' &= \gamma(E - uP_x) \\
J_x{}' &= \gamma\left(J_x - \frac{u}{c^2}W\right) & P_x' &= \gamma\left(P_x - \frac{u}{c^2}E\right) \\
J_y{}' &= J_y & P_y' &= P_y \\
J_z{}' &= J_z & P_z' &= P_z
\end{aligned}
\tag{6.2}
$$

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

We then use the notation (2.3), (2.4), (2.5) and obtain the transformations (6.3) and (6.4)

$$\frac{\partial}{\partial x_0{}'} = \gamma\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)$$
$$\frac{\partial}{\partial x_1{}'} = \gamma\left(\frac{\partial}{\partial x_1} + i\frac{u}{c}\frac{\partial}{\partial x_0}\right) \qquad (6.3)$$
$$\frac{\partial}{\partial x_2{}'} = \frac{\partial}{\partial x_2}$$
$$\frac{\partial}{\partial x_3{}'} = \frac{\partial}{\partial x_3}$$

$$J_0{}' = \gamma\left(J_0 - i\frac{u}{c}J_1\right) \qquad P_0' = \gamma\left(P_0 - i\frac{u}{c}P_1\right)$$
$$J_1{}' = \gamma\left(J_1 + i\frac{u}{c}J_0\right) \qquad P_1' = \gamma\left(P_1 + i\frac{u}{c}P_0\right) \qquad (6.4)$$
$$J_2{}' = J_2 \qquad P_2' = P_2$$
$$J_3{}' = J_3 \qquad P_3' = P_3$$

We now derive the transformation of the physical quantity $\lambda_{00}$. From equation (2.10) for $k = i = 0$ we get for the inertial reference frame $O'(t', \mathrm{x}', \mathrm{y}', \mathrm{z}')$

$$\lambda_{00}{}' = \frac{\partial J_0{}'}{\partial x_0{}'} - \frac{b}{\hbar}P_0'J_0{}'$$

and with transformations (6.3) and (6.4) we obtain

$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right)\left(J_0 - i\frac{u}{c}J_1\right) - \frac{b}{\hbar}\gamma^2\left(P_0 - i\frac{u}{c}P_1\right)\left(J_0 - i\frac{u}{c}J_1\right)$$
$$\lambda_{00}{}' = \gamma^2\left(\frac{\partial J_0}{\partial x_0} - i\frac{u}{c}\frac{\partial J_1}{\partial x_0} - i\frac{u}{c}\frac{\partial J_0}{\partial x_1} - \frac{u^2}{c^2}\frac{\partial J_1}{\partial x_1} - \frac{b}{\hbar}P_0J_0 + i\frac{u}{c}\frac{b}{\hbar}P_0J_1 + i\frac{u}{c}\frac{b}{\hbar}P_1J_0 + \frac{u^2}{c^2}\frac{b}{\hbar}P_1J_1\right)$$

and replacing physical quantities

$$\frac{\partial J_0}{\partial x_0}, \frac{\partial J_1}{\partial x_0}, \frac{\partial J_0}{\partial x_1}, \frac{\partial J_1}{\partial x_1}$$

from equation (2.10) we get

$$\lambda_{00}{}' = \gamma^2 (\frac{b}{\hbar} P_0 J_0 + \lambda_{00} - i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1$$
$$- \frac{u^2}{c^2}\lambda_{11} - \frac{b}{\hbar} P_0 J_0 + i\frac{u}{c}\frac{b}{\hbar} P_0 J_1 + i\frac{u}{c}\frac{b}{\hbar} P_1 J_0 + \frac{u^2}{c^2}\frac{b}{\hbar} P_1 J_1)$$

and we finally obtain equation

$$\lambda_{00}{}' = \gamma^2 \left( \lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right).$$

Following the same procedure for $k, \mathrm{i} = 0,1,2,3$ we obtain the following 16 equations (27) for the Lorentz-Einstein transformations of the physical quantities $\lambda_{ki}$ :

$$\begin{aligned}
\lambda_{00}{}' &= \gamma^2 \left( \lambda_{00} - i\frac{u}{c}\lambda_{01} - i\frac{u}{c}\lambda_{10} - \frac{u^2}{c^2}\lambda_{11} \right) \\
\lambda_{01}{}' &= \gamma^2 \left( \lambda_{01} + i\frac{u}{c}\lambda_{00} - i\frac{u}{c}\lambda_{11} + \frac{u^2}{c^2}\lambda_{10} \right) \\
\lambda_{02}{}' &= \gamma \left( \lambda_{02} - i\frac{u}{c}\lambda_{12} \right) \\
\lambda_{03}{}' &= \gamma \left( \lambda_{03} - i\frac{u}{c}\lambda_{13} \right) \\
\\
\lambda_{10}{}' &= \gamma^2 \left( \lambda_{10} - i\frac{u}{c}\lambda_{11} + i\frac{u}{c}\lambda_{00} + \frac{u^2}{c^2}\lambda_{01} \right) \\
\lambda_{11}{}' &= \gamma^2 \left( \lambda_{11} + i\frac{u}{c}\lambda_{10} + i\frac{u}{c}\lambda_{01} - \frac{u^2}{c^2}\lambda_{00} \right) \\
\lambda_{12}{}' &= \gamma \left( \lambda_{12} + i\frac{u}{c}\lambda_{02} \right) \\
\lambda_{13}{}' &= \gamma \left( \lambda_{13} + i\frac{u}{c}\lambda_{03} \right) \\
\\
\lambda_{20}{}' &= \gamma \left( \lambda_{20} - i\frac{u}{c}\lambda_{21} \right) \\
\lambda_{21}{}' &= \gamma \left( \lambda_{21} + i\frac{u}{c}\lambda_{20} \right) \\
\lambda_{22}{}' &= \lambda_{22} \\
\lambda_{23}{}' &= \lambda_{23}
\end{aligned} \qquad (6.5)$$

The first two of equations (6.5) is self-consistent when equation

$$\lambda_{00} = \lambda_{11} \tag{6.6}$$

Then by the second of equations (6.5) we obtain

$$\lambda_{01}{}' = \lambda_{01}\,.$$

According to equivalence (3.14) these transformations relate to the external symmetry, in which it holds that $\lambda_{ik} = -\lambda_{ki}$ for $i \neq k, i, k = 0,1,2,3$. Thus, we obtain the following transformations for the physical quantities $\lambda_{ki}, k, i = 0,1,2,3$

$$\begin{array}{ll} & \lambda_{01}{}' = \lambda_{01} \\ & \lambda_{02}{}' = \gamma\left(\lambda_{02} + i\dfrac{u}{c}\lambda_{21}\right) \\ \lambda_{00}{}' = \lambda_{00} & \\ \lambda_{11}{}' = \lambda_{11} & \lambda_{03}{}' = \gamma\left(\lambda_{03} - i\dfrac{u}{c}\lambda_{13}\right) \\ \lambda_{22}{}' = \lambda_{22} & \lambda_{32}{}' = \lambda_{32} \\ \lambda_{33}{}' = \lambda_{33} & \lambda_{13}{}' = \gamma\left(\lambda_{13} + i\dfrac{u}{c}\lambda_{03}\right) \\ & \lambda_{21}{}' = \gamma\left(\lambda_{21} - i\dfrac{u}{c}\lambda_{02}\right) \end{array} \tag{6.7}$$

Taking into account equations (4.4), (4.10) and that the physical quantity $zQ$ is invariant under the Lorentz-Einstein transformations, we obtain the following transformations for the constants $\alpha_{ki}, k \neq i, k, i = 0,1,2,3$ and the physical quantities $T_k, k = 0,1,2,3$

$$
\begin{array}{ll}
 & \alpha_{01}{}' = \alpha_{01} \\
 & \alpha_{02}{}' = \gamma\left(\alpha_{02} + i\dfrac{u}{c}\alpha_{21}\right) \\
T_0{}' = T_0 & \alpha_{03}{}' = \gamma\left(\alpha_{03} - i\dfrac{u}{c}\alpha_{13}\right) \\
T_1{}' = T_1 & \\
T_2{}' = T_2 & \alpha_{32}{}' = \alpha_{32} \\
T_3{}' = T_3 & \alpha_{13}{}' = \gamma\left(\alpha_{13} + i\dfrac{u}{c}\alpha_{03}\right) \\
 & \alpha_{21}{}' = \gamma\left(\alpha_{21} - i\dfrac{u}{c}\alpha_{02}\right)
\end{array}
\qquad (6.8)
$$

Equation (6.6) correlates the physical quantities $\lambda_{00}$ and $\lambda_{11}$ in the same inertial frame of reference. Taking into account equation (4.10) we obtain

$$T_0 = T_1$$

Thus, when transformations (6.8) hold, equation (6.9) also holds. Thus, we derive the following two corollaries.

**Corollary 6.1.** ´´ If the part of spacetime occupied by the generalized particle in external symmetry is flat, then

$$T_1 = T_0 .\text{´´} \qquad (6.9)$$

**Corollary 6.2.**´´If

$$T_1 \neq T_0 \qquad (6.10)$$

the part of spacetime occupied by the generalized particle cannot be flat, it is curved.´´

In the external symmetry it is $\alpha_{ki} \neq 0$ for at least on pair of indices $k,i \in \{0,1,2,3\}$ . Thus, in external symmetry it is $\alpha_{ki} = 0$ only for some pairs of indices $k,i \in \{0,1,2,3\}$ . The Lorentz-Einstein transformations reveal that in flat spacetime this cannot be arbitrary. Let's assume that it is

$$\alpha_{02} = 0$$

for every inertial frame of reference. Then, we obtain

$$\alpha_{02}{}' = 0$$

and with transformations (6.8) we obtain

$$\gamma\left(\alpha_{02} + i\frac{u}{c}\alpha_{21}\right) = 0$$

and since it is $\alpha_{02} = 0$ we obtain that it also holds

$$\alpha_{21} = 0\,.$$

Working similarly with all of the transformations (6.8) we end up with the following four sets of equations of external symmetry in the flat spacetime:

$$\begin{aligned}
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\
&\alpha_{02} \neq 0 \\
&\alpha_{03} \neq 0 \\
&\alpha_{32} \neq 0 \\
&\alpha_{13} \neq 0 \\
&\alpha_{21} \neq 0
\end{aligned} \tag{6.11}$$

$$\begin{aligned}
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\
&\alpha_{02} = 0 \\
&\alpha_{03} = 0 \\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\
&\alpha_{13} = 0 \\
&\alpha_{21} = 0
\end{aligned} \tag{6.12}$$

$$\begin{aligned}
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\
&\alpha_{02} \neq 0 \vee \alpha_{02} = 0 \\
&\alpha_{03} = 0 \\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\
&\alpha_{13} = 0 \\
&\alpha_{21} \neq 0 \vee \alpha_{21} = 0
\end{aligned} \qquad \text{(6.13}$$

$$\begin{aligned}
&\alpha_{01} \neq 0 \vee \alpha_{01} = 0 \\
&\alpha_{02} = 0 \\
&\alpha_{03} \neq 0 \vee \alpha_{03} = 0 \\
&\alpha_{32} \neq 0 \vee \alpha_{32} = 0 \\
&\alpha_{13} \neq 0 \vee \alpha_{13} = 0 \\
&\alpha_{21} = 0
\end{aligned} \tag{6.14}$$

The symmetry that equations (6.11)-(6.14) express will be referred to as the symmetry of the Lorentz-Einstein-Selfvarlations. These symmetries hold only in case that the part of spacetime occupied by the generalized particle is flat.

## 7. The Fundamental Study for The Corpuscular Structure of Matter. The Π-Plane.

The internal structure of the generalized particle is determined by the relations among the elements of the matrix $T$. The same holds for the rest mass $m_0$ of the material particle, the rest energy $E_0$ of STEM, with which the material particle interacts, and the total rest mass $M_0$ of the generalized particle. In this paragraph, we study this relation among the elements of the matrix $T$. We now prove the following theorem:

**Theorem 7.1.** ‘’ For the elements of the matrix $T$ matrix it holds that:

$$T_0T_1T_2T_3 + T_0T_1\alpha_{32}^2 + T_0T_2\alpha_{13}^2 + T_0T_3\alpha_{21}^2 + T_1T_2\alpha_{03}^2 + T_1T_3\alpha_{02}^2 + T_2T_3\alpha_{01}^2 = 0 \text{ .”} \tag{7.1}$$

**Proof.** We develop equation (2.13), obtaining the set of equations

$$\begin{aligned}
&J_0\lambda_{00} + J_1\lambda_{01} + J_2\lambda_{02} + J_3\lambda_{03} = 0 \\
&-J_0\lambda_{01} + J_1\lambda_{11} - J_2\lambda_{21} + J_3\lambda_{13} = 0 \\
&-J_0\lambda_{02} + J_1\lambda_{21} + J_2\lambda_{22} - J_3\lambda_{32} = 0 \\
&-J_0\lambda_{03} - J_1\lambda_{13} + J_2\lambda_{32} + J_3\lambda_{33} = 0
\end{aligned}$$

and from equations (4.4) and (4.10) we have

$$\begin{aligned}
&J_0zQT_0 + J_1zQ\alpha_{01} + J_2zQ\alpha_{02} + J_3zQ\alpha_{03} = 0 \\
&-J_0zQ\alpha_{01} + J_1zQT_1 - J_2zQ\alpha_{21} + J_3zQ\alpha_{13} = 0 \\
&-J_0zQa_{02} + J_1zQa_{21} + J_2zQ\mathrm{T}_2 - J_3zQa_{32} = 0 \\
&-J_0zQ\alpha_{03} - J_1zQ\alpha_{13} + J_2zQ\alpha_{32} + J_3zQT_3 = 0
\end{aligned}$$

and since it holds that $zQ \neq 0$ , we take the set of equations

$$
\begin{aligned}
&J_0T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\
&-J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} + J_3\alpha_{13} = 0 \\
&-J_0\alpha_{02} + J_1\alpha_{21} + J_2T_2 - J_3\alpha_{32} = 0 \\
&-J_0\alpha_{03} - J_1\alpha_{13} + J_2\alpha_{32} + J_3T_3 = 0
\end{aligned} \tag{7.2}
$$

The set of equations given in (7.2) comprise a $4\times4$ homogeneous linear system of equations with unknowns the momenta $J_0, J_1, J_2, J_3$. In order for the material particle to exist, the system of equations (7.2) must obtain non-vanishing solutions. Therefore, its determinant must vanish. Thus, we obtain equation

$$
\begin{aligned}
&T_0T_1T_2T_3 + T_0T_1\alpha_{32} + T_0T_2\alpha_{13} + T_0T_3\alpha_{21} + T_1T_2\alpha_{03} + T_1T_3\alpha_{02} + T_2T_3\alpha_{01} \\
&+(\alpha_{01}\alpha_{32} + \alpha_{02}\alpha_{13} + \alpha_{03}\alpha_{21})^2 = 0
\end{aligned}
$$

and with equation (4.8) we arrive at equation (7.1). □

We consider the $4\times4$ matrix $N$ matrix, given as:

$$
N = \begin{bmatrix} 0 & \alpha_{32} & \alpha_{13} & \alpha_{21} \\ -\alpha_{32} & 0 & -\alpha_{03} & \alpha_{02} \\ -\alpha_{13} & \alpha_{03} & 0 & -\alpha_{01} \\ -\alpha_{21} & -\alpha_{02} & \alpha_{01} & 0 \end{bmatrix} . \tag{7.3}
$$

Using the matrix $N$, we now write equation (4.6) in the form of

$$
\begin{aligned}
&NC = 0 \\
&NJ = 0\ . \\
&NP = 0
\end{aligned} \tag{7.4}
$$

We now prove Lemma 7.1:

**Lemma 7.1.** ‘’The four-vectors $C, J, P$ satisfy the set of equations

$$
\begin{aligned}
&N^2C = 0 \\
&N^2J = 0\ .\text{”} \\
&N^2P = 0
\end{aligned} \tag{7.5}
$$

**Proof.** We multiply the set of equations (7.4) by the left with the matrix $N$, and equations (7.5) follow. □

Using lemma 7.1 we prove theorem 7.2 :

**Theorem 7.2.** ‘’For $M \neq 0$ it holds that:

1. $MN = NM = 0$ (7.6)

2. $M^2 + N^2 = -\alpha^2 I$ (7.7)

$$\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \tag{7.8}$$

Here, $I$ is the $4 \times 4$ identity matrix.

3. For $\alpha \neq 0$ the matrix $M$ has two eigenvalues $\tau_1$ and $\tau_2$, with corresponding eigenvectors $v_1$ and $v_2$, given by:

$\tau_1 = i\alpha$

$$v_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix} \tag{7.9}$$

$\tau_2 = -i\alpha$

$$v_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 \\ \alpha_{03}\alpha_{13} - \alpha_{02}\alpha_{21} \\ \alpha_{01}\alpha_{21} - \alpha_{03}\alpha_{32} \\ \alpha_{02}\alpha_{32} - \alpha_{01}\alpha_{13} \end{bmatrix}. \tag{7.10}$$

4. For $\alpha \neq 0$ the matrix $N$ has the same eigenvalues with the matrix $M$, and two corresponding eigenvectors $n_1$ and $n_2$, given by:

$\tau_1 = i\alpha$

$$n_1 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} - \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix} \tag{7.11}$$

$\tau_2 = -i\alpha$

$$n_2 = \frac{1}{\alpha}\begin{bmatrix} 0 \\ \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{bmatrix} + \frac{i}{\alpha^2}\begin{bmatrix} \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 \\ \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{bmatrix}. \tag{7.12}$$

5. $\alpha^2 = \alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 + \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0$ (7.13)

$M^2C = 0$

6. $M^2J = 0$ ." (7.14)

$M^2P = 0$

**Proof.** The matrices $M$ and $N$ are given by equations (4.28) and (7.3). The proof of equations (7.6), (7.7), (7.9), (7.10), (7.11) and (7.12) can be performed by the appropriate mathematical calculations and the use of equation (4.8).

We multiply equation (7.7) from the left with the column matrices $C, J, P$ , and obtain

$$M^2C + N^2C = -\alpha^2 C$$
$$M^2J + N^2J = -\alpha^2 J$$
$$M^2P + N^2P = -\alpha^2 P$$

and from equations (7.5) we obtain

$$\begin{aligned} M^2C &= -\alpha^2 C \\ M^2J &= -\alpha^2 J \ . \\ M^2P &= -\alpha^2 P \end{aligned} \tag{7.15}$$

According to the set of equations (7.15) , and for $\alpha \neq 0$, the matrix $M^2$ has as eigenvalue $\alpha^2 \neq 0$ with corresponding eigenvector $\nu \neq 0$. From equations (7.15) it is evident that the four-vectors $C, J, P$ are parallel to the four-vector $\nu$, hence they are also parallel to each other. This is imposssible in the case of the external symmetry, according to Theorem 3.3. Therefore, $\alpha^2 = 0$ , so that the matrix $M^2$ does not have the four-vector $\nu$ as an eigenvector. Thus, we arrive at equation (7.13). Then, from equations (7.15) we arrive at equations (7.14), since it holds that $\alpha^2 = 0$ .□

The matrix $M^2$ , for $M \neq 0$ , is a $4 \times 4$ symmetric matrix. Furthermore, according to theorem 7.2 , it holds that $tr\left(M^2\right) = \alpha^2 = 0$. An immediate consequence of theorem 7.2 is corollary 7.1.

**Corollary 7.1.** ''For the matrix of the external symmerty $T$, not only one out of the physical quantities $\alpha_{ki} \neq 0, \mathrm{k} \neq \mathrm{i}, \mathrm{k}, \mathrm{i} \in \{0,1,2,3\}$ can be zero."

**Proof.** Let us suppose that the matrix $T$ has only one element (physical quantity), for which it holds that: $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$. From equation (4.28) we see that $M \neq 0$, and from equation (7.8) we obtain $\alpha^2 = \alpha_{ki}^2 \neq 0$. This cannot hold, according to equation (7.13).□

From theorem 7.2 corollary 7.2 follows:

**Corollary 7.2.** ''For the four-vector $j$ of the conserved physical quantities $q$ it holds that:

$$Mj = 0 \tag{7.16}$$

$$Nj = 0\text{.''} \tag{7.17}$$

**Proof.** We multiply equation (5.7) by matrix $M$ by the left and obtain

$$Mj = -\frac{\sigma c^2 b}{\hbar}\Psi\left(\lambda M^2 J + \mu M^2 P\right)$$

and with the second and the third of equations (7.14) we have

$$Mj = 0 .$$

We multiply the terms of equation (5.7) by the left with the matrix $N$ , and obtain

$$Nj = -\frac{\sigma c^2 b}{\hbar}\Psi NM\left(\lambda J + \mu P\right)$$

and with equation (7.6) we take

$$Nj = 0 . \square$$

In the equations of the TSV there appear sums of squares that vanish, as the corresponding ones appearing in equations (3.6) and (7.13). Writing these equations in a convenient manner, we can introduce into the equations of the TSV complex numbers. From equation (3.6) , and for $M_0 \neq 0$ , we obtain

$$\left(\frac{c_0}{M_0 c}\right)^2 + \left(\frac{c_1}{M_0 c}\right)^2 + \left(\frac{c_2}{M_0 c}\right)^2 + \left(\frac{c_3}{M_0 c}\right)^2 + 1 = 0$$

Therefore, the physical quantities

$$\frac{c_0}{M_0 c}, \frac{c_1}{M_0 c}, \frac{c_2}{M_0 c}, \frac{c_3}{M_0 c}$$

belong in general to the set of complex numbers $\mathbb{C}$ . This transformation of the equations of the TSV is not necessary. It suffices to remember that within the equations of the TSV there are sums of squares that vanish. We prove theorem 7.3, which intercorrelates together all of the elements of the matrix $T$ :

**Theorem 7.3.** ''In the external symmetry and for the elements of the matrix $T$ it holds that:

$$\begin{array}{l} T_i a_{\nu k} = 0 \\ i \neq \nu, \nu \neq k, k \neq i, i, \nu, k = 0,1,2,3 \end{array} \text{.''}$$

**Proof.** We differentiate the second equation of the set of equations (4.6)

$$J_i\alpha_{\nu k}+J_k\alpha_{i\nu}+J_\nu\alpha_{ki}=0$$
$$i\neq\nu,\nu\neq k,k\neq i,i,\nu,k=0,1,2,3$$

with respect to $x_j$, $j=0,1,2,3$. Considering equations (2.10) and (4.4), we have

$$\alpha_{\nu k}\left(\frac{b}{\hbar}P_jJ_i+zQ\alpha_{ji}\right)+\alpha_{i\nu}\left(\frac{b}{\hbar}P_jJ_k+zQ\alpha_{jk}\right)+\alpha_{ki}\left(\frac{b}{\hbar}P_jJ_\nu+zQ\alpha_{j\nu}\right)=0$$

$$\frac{b}{\hbar}P_j\left(J_i\alpha_{\nu k}+J_k\alpha_{i\nu}+J_\nu\alpha_{ki}\right)+zQ\left(\alpha_{\nu k}\alpha_{ji}+\alpha_{i\nu}\alpha_{jk}+\alpha_{ki}\alpha_{j\nu}\right)=0$$

and with the second equation of the set of equations (4.6), and taking into account that $zQ\neq 0$, we obtain

$$\begin{aligned}&\alpha_{\nu k}\alpha_{ji}+\alpha_{i\nu}\alpha_{jk}+\alpha_{ki}\alpha_{j\nu}=0\\&i\neq\nu,\nu\neq k,k\neq i,i,\nu,k,\ j=0,1,2,3\end{aligned}\qquad(7.19)$$

Inserting successively into equation (7.19) the values of the elements having as indices the triples

$(i,\nu,k)=(0,1,2)(0,1,3)(0,2,3)(1,2,3)$, and for $j=0,1,2,3$, we arrive at the set of equations

$$\begin{aligned}&T_0\alpha_{32}=0\\&T_0\alpha_{13}=0\\&T_0\alpha_{21}=0\\&T_1\alpha_{02}=0\\&T_1\alpha_{03}=0\\&T_1\alpha_{32}=0\\&T_2\alpha_{01}=0\\&T_2\alpha_{03}=0\\&T_2\alpha_{13}=0\\&T_3\alpha_{01}=0\\&T_3\alpha_{02}=0\\&T_3\alpha_{21}=0\end{aligned}\ .\qquad(7.20)$$

The set of equations (7.20) is equivalent to equation (7.18).□

Theorem 7.3 is one of the most powerful tools for investigating the external symmetry. This results from corollary 7.3 :

**Corollary 7.3.** ‘’For the elements of the matrix $T$ of the external symmetry the following hold:

1. For every $k\neq i,\nu\neq k,\nu\neq i,k,i,\nu\in\{0.1.2,3\}$ it holds that

$$\left.\begin{array}{l}\alpha_{ki} \neq 0 \\ k \neq i \\ \nu \neq k, i\end{array}\right\} \Rightarrow \mathrm{T}_{\nu} = 0 . \qquad (7.21)$$

2. If $\alpha_{ki} = 0$ for maximum up to two pairs of indices $(k,i), k \neq i, k, i \in \{0,1,2,3\}$, then all the elements of the main diagonal of the matrix $T$ vanish:

$$\left.\begin{array}{l}\alpha_{ki} = 0 \\ \alpha_{\nu j} = 0 \\ k \neq i, \nu, j \\ i \neq \nu, j \\ \nu \neq j \\ k, i, \nu, j \in \{0,1,2,3\}\end{array}\right\} \Rightarrow T_0 = T_1 = T_2 = T_3 = 0 .\text{''} \qquad (7.22)$$

**Proof.** Corollary 7.3 is an immediate consequence of theorem 7.3.□

From theorem 7.3 corollary 7.4 follows, regarding the elements of the main diagonal of the matrices of the external symmetry:

**Corollary 7.4.** ‘’The elements of the main diagonal of the matrix $T$ cannot all be different from zero.’’

**Proof.** If $T_{\nu} \neq 0$ for every $\nu \in \{0,1,2,3\}$, from equations (7.20) we obtain $\alpha_{ki} = 0$ for every set of indices $k \neq i, k, i = 0,1,2,3$, and from equation (7.1) we take

$$T_0 T_1 T_2 T_3 = 0$$

This cannot hold, since we assumed that $T_{\nu} \neq 0$ for every $\nu = 0,1,2,3$. Therefore, at least one element of the main diagonal of the matrix $T$ is equal to zero.

We present a second way for proving this result. In the case of $T_{\nu} \neq 0$ for every $\nu \in \{0,1,2,3\}$, we obtain from equations (7.20) that $\alpha_{ki} = 0$, for every $k \neq i, k, i = 0,1,2,3$.Thus, the matrix $T$ takes the form

$$T = \begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}.$$

From equation (2.13) we take

$$T_0 J_0 = T_1 J_1 = T_2 J_2 = T_3 J_3 = 0$$

Since we assumed that

$$T_0T_1T_2T_3 \neq 0$$

we obtain

$$J_0 = J_1 = J_2 = J_3 = 0$$

Thus, the material particle does not exist.□

We consider the three-dimensional vectors

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} \tag{7.23}$$

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \tag{7.24}$$

In the case of the $T$ matrices, where $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$, we define the vector $\boldsymbol{\mu} \neq \mathbf{0}$ as

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} \alpha_{02}\alpha_{21} - \alpha_{03}\alpha_{13} \\ \alpha_{03}\alpha_{32} - \alpha_{01}\alpha_{21} \\ \alpha_{01}\alpha_{13} - \alpha_{02}\alpha_{32} \end{pmatrix} \ . \tag{7.25}$$

Combining equations (5.1), (5.2) with equations (7.23) and (7.24) we obtain

$$\boldsymbol{\xi} = ic\Psi\,\mathbf{n} \tag{7.26}$$

$$\boldsymbol{\omega} = \Psi\boldsymbol{\tau} \ . \tag{7.27}$$

The field $\boldsymbol{\xi}$ stays parallel to the vector $\mathbf{n}$ and the field $\boldsymbol{\omega}$ stays parallel to the vector $\boldsymbol{\tau}$.

For every vector

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

as determined by the physical quantities of the TSV, we define the physical quantity

$$\|\,\boldsymbol{\alpha}\,\| = \left(\boldsymbol{\alpha}^T\boldsymbol{\alpha}\right)^{\frac{1}{2}} = \left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2\right)^{\frac{1}{2}} \tag{7.28}$$

Here, the matrix $\boldsymbol{\alpha}^T$ is the inverse matrix of the column matrix $\boldsymbol{\alpha}$.

From equations (7.23) and (7.24) we obtain

$$\boldsymbol{\tau}\cdot\mathbf{n}=\alpha_{01}\alpha_{32}+\alpha_{02}\alpha_{13}+\alpha_{03}\alpha_{21}$$

Also, from equation (4.8) we have

$$\boldsymbol{\tau}\cdot\mathbf{n}=0. \tag{7.29}$$

Therefore, the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other. Considering also equation (7.25), we see that the triple of the vectors $\{\boldsymbol{\mu},\mathbf{n},\boldsymbol{\tau}\}$ forms a right-handed vector basis.

From equation (7.13) we have

$$\alpha_{01}^2+\alpha_{02}^2+\alpha_{03}^2=-\left(\alpha_{32}^2+\alpha_{13}^2+\alpha_{21}^2\right)$$

and with equations (7.23), (7.24), and using the notation of equation (7.28), we obtain

$$||\,\mathbf{n}\,||^2=-||\,\boldsymbol{\tau}\,||^2$$

and finally we obtain

$$||\,\mathbf{n}\,||=\pm i\,||\,\boldsymbol{\tau}\,||. \tag{7.30}$$

From equation (7.25) we have

$$\boldsymbol{\mu}^2=\left(\mathbf{n}\times\boldsymbol{\tau}\right)^2$$

and since the vectors $\boldsymbol{\tau}$ and $\mathbf{n}$ are perpendicular to each other, we obtain from equation (7.29) that

$$\boldsymbol{\mu}^2=\mathbf{n}^2\boldsymbol{\tau}^2$$

and using the notation of equation (7.28) we have

$$||\,\boldsymbol{\mu}\,||^2=||\,\mathbf{n}\,||^2||\,\boldsymbol{\tau}\,||^2$$

$$||\,\boldsymbol{\mu}\,||=\pm\,||\,\mathbf{n}\,||\,||\,\boldsymbol{\tau}\,||$$

and from equation (7.30) we take

$$||\,\boldsymbol{\mu}\,||=\pm i\,||\,\mathbf{n}\,||^2=\mp\,||\,\boldsymbol{\tau}\,||^2. \tag{7.31}$$

In the case of the $T$ matrices, where $||\,\mathbf{n}\,||\neq\mathbf{0}$,and from equation(7.31), it follows that $||\,\boldsymbol{\tau}\,||\neq 0, ||\,\boldsymbol{\mu}\,||\neq 0$. In these cases we can define the set of unit vectors $\{\boldsymbol{\varepsilon}_1,\boldsymbol{\varepsilon}_2,\boldsymbol{\varepsilon}_3\}$, given by

$$
\begin{aligned}
&\boldsymbol{\varepsilon}_1 = \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \\
&\boldsymbol{\varepsilon}_2 = \frac{\mathbf{n}}{\|\mathbf{n}\|} \quad . \\
&\boldsymbol{\varepsilon}_3 = \frac{\boldsymbol{\tau}}{\|\boldsymbol{\tau}\|} \\
&\|\mathbf{n}\| \neq 0
\end{aligned}
\tag{7.32}
$$

The triple of the vectors $\{\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3\}$ forms also a right-handed orthonormal vector basis.

In the cases of the $T$ matrices, where $\boldsymbol{\tau} \neq \mathbf{0}$, we define as $\Pi$ the plane perpendicular to the vector $\boldsymbol{\tau} \neq \mathbf{0}$ . Furthermore, in the cases, for which it also holds $\mathbf{n} \neq \mathbf{0}$, we obtain from equation (7.25) that $\boldsymbol{\mu} \neq \mathbf{0}$.

In these cases the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ are perpendicular to the vector $\boldsymbol{\tau}$, as we obtain from equations (7.25) and (7.29). Therefore, the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$ belong to the plane $\Pi$, and they also form an orthogonal basis of this plane. We note that the vectors of the TSV, which eventually might belong to the plane $\Pi$, are given as a linear combination of the vectors $\mathbf{n}$ and $\boldsymbol{\mu}$. Therefore, the condition for $\boldsymbol{\tau} \neq \mathbf{0}$ is not sufficient, in order for the plane $\Pi$ to acquire a physical meaning. Also, we note that because of equation (7.13), the plane $\Pi$, when it is defined, is not a vector subspace of $\mathbb{R}^3$.

We prove theorem 7.4:

**Theorem 7.4.** ‘’In the case of the $T$ matrices, where $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$ and $\boldsymbol{\tau} \neq \pm\mathbf{n} \neq \mathbf{0}$, the vectors

$\mathbf{J}, \mathbf{P}, \mathbf{C}, \mathbf{j}, \nabla\Psi$ belong to the same plane $\Pi$.’’

**Proof.** From equations (4.6), and for the indices $(i,\nu,k) = (1,3,2)$, we obtain

$$
\begin{aligned}
&c_1\alpha_{32} + c_2\alpha_{13} + c_3\alpha_{21} = 0 \\
&J_1\alpha_{32} + J_2\alpha_{13} + J_3\alpha_{21} = 0 \\
&P_1\alpha_{32} + P_2\alpha_{13} + P_3\alpha_{21} = 0
\end{aligned}
$$

and from equations (5.8),(5.9) and (7.23) we take

$$
\begin{aligned}
&\boldsymbol{\tau} \cdot \mathbf{C} = 0 \\
&\boldsymbol{\tau} \cdot \mathbf{J} = 0 \\
&\boldsymbol{\tau} \cdot \mathbf{P} = 0
\end{aligned}
\tag{7.33}
$$

where

$$
\mathbf{C} = \mathbf{J} + \mathbf{P} \tag{7.34}
$$

as derived from equation (3.5). From equation (7.33) we obtain that the vectors $\mathbf{C}, \mathbf{J}, \mathbf{P}$, being perpendicular to vector $\boldsymbol{\tau}$, belong to the plane $\Pi$. From equation (5.3) and equations (5.8) and (5.9) we obtain

$$\nabla\Psi = \frac{b}{\hbar}\left(\lambda\mathbf{J} + \mu\mathbf{P}\right)$$

Therefore, the vector $\nabla\Psi$, as a linear combination of the vectors $\mathbf{J}, \mathbf{P}$, belongs to the plane $\Pi$. By developing the terms of equation (7.17), the first obtained equation is

$$\alpha_{32} j_1 + \alpha_{13} j_2 + \alpha_{21} j_3 = 0$$

and using equation (7.23) we have

$$\boldsymbol{\tau} \cdot \mathbf{j} = 0 \,. \qquad (7.35)$$

Therefore, the vector $\mathbf{j}$, being perpendicular to the vector $\boldsymbol{\tau}$, belongs to the plane $\Pi$. The set of vectors $\mathbf{J}, \mathbf{P}, \mathbf{C}, \mathbf{j}, \nabla\Psi$ vary according to the equations of the TSV, but they always stay on the same plane $\Pi$. □

From this study we can obtain a method about the determination of the four-vectors $\mathbf{J}, \mathbf{P}, \mathbf{C}$, as well as for the set of the rest masses $m_0, \frac{E_0}{c^2}, M_0$. This method is applied in the case the matrix $M$ does not vanish, that is $M \neq 0$. We shall refer to this method as the $SV-M$ method.

**The steps of the *SV-M* method:**

**Step 1**. We choose the object of our study, that is one of the matrices of the external symmetry $T$.

**Step 2**. We apply Theorem 7.3.

**Step 3**. We use equation (7.13).

**Step 4**. We use equation (2.13), or the equivalent equations (7.2).

**Step 5**. We use the second of the set of equations (4.6).

**Step 6**. We use the first of the set of equations (7.14).

**Step 7.** We use the first of the set of equations (4.6).

Step 8. We use equation (3.5).

As an example, we apply this method on the specific matrix $T$:

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix}. \tag{7.36}$$

Here, $\alpha_{01}\alpha_{21} \neq 0$.

From equations (7.20), and since $\alpha_{01} \neq 0$ and $\alpha_{21} \neq 0$, we obtain $T_0 = T_2 = T_3 = 0$, so that

$$T = zQ\begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & T_1 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7.37}$$

For $T_1 \neq 0$, we have $T_1 \neq T_0 = 0$. According to corollary 6.2 the portion of spacetime occupied by the generalized particle is curved. From equation (4.10) we obtain $\Phi_1 \neq 0$. Therefore, the second term of the second part of the second equation in the set of equations (4.21) does not vanish.

In the case the portion of spactime occupied by the generalized particle is flat, we obtain from corollary 6.1 that $T_1 = T_0$. Therefore, $T_0 = T_1 = T_2 = T_3 = 0$. In this case, and from equation (4.11), we obtain $\Lambda = 0$, and the second term of the second part of equation (4.19) vanishes.

From equation (7.13) we take

$$\alpha^2 = \alpha_{01}^2 + \alpha_{21}^2 = 0$$

$$\alpha_{21} = \pm i\alpha_{01}. \tag{7.38}$$

From equations (7.2) we obtain

$$\begin{aligned} &J_1\alpha_{01} = 0 \\ &-J_0\alpha_{01} + J_1T_1 - J_2\alpha_{21} = 0 \\ &J_1\alpha_{21} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{21} \neq 0$, we have that

$$\begin{aligned} &J_1 = 0 \\ &J_2 = -\frac{\alpha_{01}}{\alpha_{21}} J_0 \end{aligned}. \tag{7.39}$$

From the second of the set of equations (4.6), and for the indices $(i,\nu,k) = (3,0,1)$ we have

$$J_3\alpha_{01} + J_1\alpha_{30} + J_0\alpha_{13} = 0$$

and since

$$\alpha_{01} \neq 0, \alpha_{30} = -\alpha_{03} = 0, \alpha_{13} = 0$$

we obtain

$$J_3 = 0 \,. \tag{7.40}$$

From equations (7.39) and (7.40), and from equation (2.4), we have for the four-vector $J$

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ -\dfrac{\alpha_{01}}{\alpha_{21}} \\ 0 \end{bmatrix} = J_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix}. \tag{7.41}$$

For the second equality in equation (7.41) we applied the second equation of the set of equations (7.38).

From equations (4.29) and (7.37) we take

$$M = \begin{bmatrix} 0 & \alpha_{01} & 0 & 0 \\ -\alpha_{01} & 0 & -\alpha_{21} & 0 \\ 0 & \alpha_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7.42}$$

$$M^2 = \begin{bmatrix} -\alpha_{01}^2 & 0 & -\alpha_{01}\alpha_{21} & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_{01}\alpha_{21} & 0 & -\alpha_{21}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7.43}$$

From the first of the set of equations (7.14) we see that

$$M^2 C = 0$$

and with equations (3.5) and (7.43) we obtain

$$-a_{01}^2 c_0 - a_{01}a_{21}c_2 = 0$$
$$-a_{01}a_{21}c_0 - a_{21}^2 c_2 = 0$$

and taking into account that $\alpha_{01}\alpha_{21} \neq 0$, we obtain

$$c_2 = -\frac{\alpha_{01}}{\alpha_{21}} c_0 . \tag{7.44}$$

From the first equation from the system of equations (4.6), and for the indices $(i,\nu,k) = (0,1,2) = (0,1,3) = (0,2,3) = (1,2,3)$ we also obtain

$$\begin{aligned} c_0\alpha_{12} + c_2\alpha_{01} + c_1\alpha_{20} &= 0 \\ c_0\alpha_{13} + c_3\alpha_{01} + c_1\alpha_{30} &= 0 \\ c_0\alpha_{23} + c_3\alpha_{02} + c_2\alpha_{30} &= 0 \\ c_1\alpha_{23} + c_3\alpha_{12} + c_2\alpha_{31} &= 0 \end{aligned}$$

and taking into account the vanishing elements of the matrix $T$ we have

$$\begin{aligned} c_0\alpha_{21} + c_2\alpha_{01} &= 0 \\ c_3\alpha_{01} &= 0 \\ c_3\alpha_{12} &= 0 \end{aligned}$$

and since

$$\begin{aligned} j_0\alpha_{12} + j_2\alpha_{01} &= 0 \\ j_3\alpha_{01} &= 0 \\ j_3\alpha_{12} &= 0 \end{aligned}$$

we obtain

$$\begin{aligned} c_2 &= \frac{\alpha_{21}}{\alpha_{01}} c_0 . \\ c_3 &= 0 \end{aligned} \tag{7.45}$$

The first equation of the couple of equations (7.45) is equation (7.44), because of equation (7.38).

From equations (3.5) and (7.45) we obtain the four-vector $C$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ \frac{\alpha_{21}}{\alpha_{01}} c_0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \pm i c_0 \\ 0 \end{bmatrix} . \tag{7.46}$$

Combining equation (3.5)

$$P = C - J$$

with equations (7.41) and (7.46) we obtain the four-vector $P$

$$P=\begin{bmatrix} c_0-J_0 \\ c_1 \\ \mp i\left(c_0-J_0\right) \\ 0 \end{bmatrix}. \tag{7.47}$$

After having determined the four-vectors $J,P,C$, we can calculate the rest masses $m_0,\frac{E_0}{c^2},M_0$. From equations (2.7) and (7.41) we take

$$m_0=0. \tag{7.48}$$

From equations (2.8) and (7.47) we have

$$E_0=\pm\frac{ic_1}{c}. \tag{7.49}$$

From equations (3.6) and (7.46) we also have

$$M_0=\pm\frac{ic_1}{c}.$$

(7.50)

The calculation of the four-vector $j$ of the current density of the conserved physical quantities $q$ is accomplished with the help of corollary 7.2. This method is applied for $M\neq 0$, and is performed in two steps. We shall refer to this method as the $SV_q$ -method.

**The steps of the $SV_q$ - method:**

**Step 1.** We use equation (7.17), or the equivalent equations:

$$\begin{aligned} &j_i\alpha_{\nu k}+j_k\alpha_{i\nu}+j_\nu\alpha_{ki}=0 \\ &i\neq\nu,\nu\neq k,k\neq i,i,\nu,\mathrm{k}=0,1,2,3 \end{aligned} \tag{7.51}$$

**Step 2.** We use equation (7.16).

We apply the $SV_q$ -method on the matrix $T$ given by equation (7.37). From equation (7.51), and for $(i,\nu,k)=(0,1,2)=(0,1,3)=(0,2,3)=(1,2,3)$ , we obtain

$$\begin{aligned} &j_0\alpha_{12}+j_2\alpha_{01}+j_1\alpha_{20}=0 \\ &j_0\alpha_{13}+j_3\alpha_{01}+j_1\alpha_{30}=0 \\ &j_0\alpha_{23}+j_3\alpha_{02}+j_2\alpha_{30}=0 \\ &j_1\alpha_{23}+j_3\alpha_{12}+j_2\alpha_{31}=0 \end{aligned}$$

and taking into account the elements of the matrix $T$ we have

$$j_0\alpha_{12} + j_2\alpha_{01} = 0$$
$$j_3\alpha_{01} = 0$$
$$j_3\alpha_{12} = 0$$

and since $\alpha_{01} \neq 0, \alpha_{30} = -\alpha_{03} = 0, \alpha_{13} = 0$, we also obtain

$$\begin{aligned} j_2 &= \frac{\alpha_{21}}{\alpha_{01}} j_0 \\ j_3 &= 0 \end{aligned} . \tag{7.52}$$

The matrix $M$ is given by equation (7.42). Thus, from equations (4.27) and (7.16) we have

$$j_1\alpha_{01} = 0$$
$$-j_0\alpha_{01} - j_2\alpha_{21} = 0$$
$$j_1\alpha_{21} = 0$$

and since $\alpha_{01} \neq 0$ and $\alpha_{21} \neq 0$, we take

$$\begin{aligned} j_1 &= 0 \\ j_2 &= -\frac{\alpha_{01}}{\alpha_{21}} j_0 \end{aligned} . \tag{7.53}$$

The first equation of the couple of equations (7.52) and the second equation of the couple of equations (7.53) are identical due to equations (7.38). From equations (7.52) and (7.53) we obtain the four-vector $j$

$$j = j_0 \begin{bmatrix} 1 \\ 0 \\ \dfrac{\alpha_{21}}{\alpha_{01}} \\ 0 \end{bmatrix} = j_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix}. \tag{7.54}$$

We summarize the obtained information for the generalized particle of the matrix $T$ of equation (7.36):

$$J = J_0 \begin{bmatrix} 1 \\ 0 \\ \pm i \\ 0 \end{bmatrix} \qquad P = \begin{bmatrix} c_0 - J_0 \\ c_1 \\ \mp i\left(c_0 - J_0\right) \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} c_0 \\ c_1 \\ \pm c_0 \\ 0 \end{bmatrix} \qquad j = j_0 \begin{bmatrix} 1 \\ 0 \\ \mp i \\ 0 \end{bmatrix}$$

$m_0 = 0$, $E_0 = \pm\dfrac{ic_1}{c}$, $M_0 = \pm\dfrac{ic_1}{c}$

$T_1 \neq 0 \Rightarrow$ curved spacetime.

Flat spacetime $\Rightarrow T_1 = 0$.

From equations (5.7) and (7.41), (7.47) and (7.54) we have

$$j_0 = -\frac{\sigma c^2 b}{\hbar}\Psi\lambda c_1\alpha_{01} = \pm\frac{i\sigma c^2 b}{\hbar}\Psi\lambda c_1\alpha_{21} \tag{7.57}$$

for the chosen specific matrix $T$. Also, from equations (5.17) and (7.54) we obtain

$$\begin{aligned}
&\frac{\partial j_0}{\partial x_1} = -\sigma c^2 F\alpha_{01} = \pm i\sigma c^2 F\alpha_{21}\\
&\frac{\partial j_0}{\partial x_2} = \pm i\frac{\partial j_0}{\partial x_0}\\
&\frac{\partial j_0}{\partial x_3} = 0\\
&F = \nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2}
\end{aligned} \tag{7.58}$$

Equations (7.57) and (7.58) correlate the function $\Psi$ with the four-vector $j$ of the current density of the conserved physical quantities $q$. These equations hold for the chosen matrix $T$ defined in equation (7.37).

The presented method about the study of the generalized particle is possibly the simplest, but surely not the only one. The TSV stems from one equation, which generates an extremely complex network of equations. We present one method, which serves as a test for the self-consistency of the TSV. By using the same method we can also test the validity of the obtained equations, as we proceed from one set of equations of the TSV into another set of equations. We shall refer to this method as the $SV-T$-method.

The internal structure of every generalized particle depends on the matrix $T$, to which it corresponds. The $SV-T$ method can by analyzed according to the following steps:

The Steps of the $SV-T$ - Method:

We choose an equation $(E_1)$, which holds for the matrix $T$, and for which there exist at least two different components of the four-vector $J$, or one component and the rest mass $m_0$. By differentiating equation $(E_1)$ with respect to $x_k, k = 0,1,2,3$ we obtain a second equation $(E_2)$.

With the help of equation (2.10)

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i + \lambda_{ki} = \frac{b}{\hbar} P_k J_i + zQ\alpha_{ki}$$
$$k, i = 0,1,2,3$$

the constants $\alpha_{ki}, k, i = 0,1,2,3$ are introduced into equation $(E_2)$. Equation $(E_2)$ has to be compatible with the elements of the matrix $T$. In the case equation $(E_1)$ contains the term of the rest mass $m_0$ we apply equation (2.6)

$$\frac{\partial m_0}{\partial x_k} = \frac{b}{\hbar} P_k J_i m_0, k = 0,1,2,3 .$$

We apply this method for the specific matrix $T$ of equation (7.37). From equation (7.41) we obtain

$$J_2 = \pm i J_0 . \tag{7.59}$$

In this equation there appear the components $J_0, J_2$ of the four-vector $J$. We differentiate equation (7.59) with respect to $x_k, k = 0,1,2,3$, obtaining

$$\frac{b}{\hbar} P_k J_2 + zQ\alpha_{k2} = \pm i\left(\frac{b}{\hbar} P_k J_0 + zQ\alpha_{k0}\right)$$

and using equation (7.59) we have

$$zQ\alpha_{k2} = \pm izQ\alpha_{k0}$$

and since $zQ \neq 0$ we take

$$\alpha_{k2} = \pm i\alpha_{k0}, k = 0,1,2,3 . \tag{7.60}$$

In equation (7.60) we insert successively the indices $k = 0,1,2,3$

For $k = 0$ we obtain

$$\alpha_{02} = \pm i\alpha_{00} = \pm iT_0$$

which holds, since $\alpha_{02} = 0, T_0 = 0$.

For $k = 1$ we obtain

$$\alpha_{12} = \pm i\alpha_{10}$$

and since $\alpha_{10} = -\alpha_{01}$, we take

$$\alpha_{12} = \pm i\alpha_{01}$$
$$\alpha_{01}^2 + \alpha_{21}^2 = 0$$

which are equations (7.38).

For $k = 2$ we obtain

$$a_{22} = \pm i a_{20}$$
$$T_2 = \pm i a_{20}$$

which holds for the matrix $T$, since $a_{02} = 0, T_2 = 0$.

For $k = 3$ we have

$$\alpha_{32} = \pm i\alpha_{30}$$
$$\alpha = \mp i\alpha_{03}$$

which holds for the matrix $T$, since $\alpha_{32} = 0, \alpha_{03} = 0$.

For the chosen matrix it holds that $\boldsymbol{\tau} \neq \mathbf{0}$ and $\mathbf{n} \neq \mathbf{0}$ and $\boldsymbol{\tau} \neq \pm\mathbf{n} \neq \mathbf{0}$, therefore plane $\Pi$ is defined. From equations (7.32) we have

$$\boldsymbol{\varepsilon}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$
$$\boldsymbol{\varepsilon}_2 = \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix}. \qquad (7.61)$$
$$\boldsymbol{\varepsilon}_3 = \begin{pmatrix} 0 \\ 0 \\ i \end{pmatrix}$$

From equations (7.46) and (7.61) we have

$$\mathbf{C} = \begin{pmatrix} c_1 \\ \pm i c_0 \\ 0 \end{pmatrix} = \pm i c_0 \boldsymbol{\varepsilon}_1 + c_1 \boldsymbol{\varepsilon}_2 . \qquad (7.62)$$

In equations (7.62) the components $(\pm i c_0, c_1)$ of the vector $\mathbf{C}$ with respect to the vectorial basis $(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2)$ of the $\Pi$-plane are given. Considering that the vectors $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2$ are perpendicular to each other, we obtain from equation (7.62)

$$\boldsymbol{\varepsilon}_1 \cdot \mathbf{C} = \pm i c_0$$
$$\boldsymbol{\varepsilon}_2 \cdot \mathbf{C} = i c_1$$

and from equations (7.49) and (7.50) we have

$$\begin{aligned} &\boldsymbol{\varepsilon}_1 \cdot \mathbf{C} = \pm i c_0 \\ &\boldsymbol{\varepsilon}_2 \cdot \mathbf{C} = \pm M_0 = \frac{E_0}{c} \cdot \end{aligned} \tag{7.63}$$

From the first equation of the set of equations (7.63) we obtain the equivalence

$$\boldsymbol{\varepsilon}_1 \cdot \mathbf{C} = 0 \Leftrightarrow c_0 = 0 . \tag{7.64}$$

The total amount of energy $ic_0$ of the generalized particle vanishes, if and only if the vector $\mathbf{C}$ of the generalized particle is perpendicular to the vector $\boldsymbol{\varepsilon}_1$, therefore also parallel to the vector $\boldsymbol{\varepsilon}_2$. Similarly, from the second of the set of equations (7.63) we also obtain the equivalence

$$\boldsymbol{\varepsilon}_2 \cdot \mathbf{C} = 0 \Leftrightarrow M_0 = 0 \Leftrightarrow E_0 = 0 . \tag{7.65}$$

The rest masses $M_0$ and $\frac{E_0}{c^2}$ vanish, if and only if the vector $\mathbf{C}$ of the total momentum of the generalized particle is parallel to the vector $\boldsymbol{\varepsilon}_1$.

As a consequence of theorem 7.3, and for a large set of matrixes of the external symmetry, which contain many non-vanishing elements $\alpha_{ki} \neq 0, k, i = 0,1,2,3$, it holds that

$$T_0 = T_1 = T_2 = T_3 = 0 .$$

For these matrices we prove theorem 7.5:

**Theorem 7.5.** ‘’In the matrices $T$ of the external symmetry, for which all of the elements of the main diagonal vanish, the four-vectors $J$ and $j$ are parallel to each other.’’

**Proof.** From equations (2.12) and (4.4),(4.10) we have

$$T = zQ \begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ -\alpha_{02} & \alpha_{21} & T_2 & -\alpha_{32} \\ -\alpha_{03} & -\alpha_{13} & \alpha_{32} & T_3 \end{bmatrix} . \tag{7.66}$$

In the case of

$$T_0 = T_1 = T_2 = T_3 = 0$$

from equations (4.28) and (7.66) we obtain

$$T = zQM \ . \tag{7.67}$$

Combining equations (2.13), (7.17), and since $zQ \neq 0$, we have

$$MJ = 0$$

and taking into account the second equation of the set of equations (7.4) we take

$$\begin{matrix} MJ = 0 \\ NJ = 0 \end{matrix} \ . \tag{7.68}$$

Since equations (7.68) and (7.16) hold simultaneously, the four-vectors $J$ and $j$ are parallel to each other.□

An immediate consequence of theorem 7.5 is corollary 7.4 :

**Corollary 7.4.** ''For the cases of the symmetries, for which the matrix $T$ has all of its elements of the main diagonal equal to zero, there exists a function

$$V = V\left(x_0, x_1, x_2, x_3\right) \neq 0$$

satisfying the continuity equation

$$\nabla \cdot \left(\frac{\mathbf{J}}{V}\right) + \frac{\partial}{\partial t}\left(\frac{W}{V}\right) = 0 \ .\text{''} \tag{7.69}$$

**Proof.** From theorem 7.5 there exists a function

$$V = V\left(x_0, x_1, x_2, x_3\right) \neq 0$$

so that

$$J = Vj \ . \tag{7.70}$$

Equation (7.69) results by the combination of equations (4.27), (5.6) and (7.70), since

$$J_0 = \frac{iW}{c} \ .\square$$

We shall not present in the present work the physical content of equation (7.69).

Theorem 7.6 correlates the four-vector $J$ with the elements of the main diagonal of the matrix of external symmetry $T$ .

**Theorem 7.6.** ''For every matrix of external symmetry $T$ it holds that

$$T_0 J_0^2 + T_1 J_1^2 + T_2 J_2^2 + T_3 J_3^2 = 0 .\text{''} \tag{7.71}$$

**Proof.** Since the material particle exists, at least one component of the four-vector $J$ does not vanish. We prove the theorem for $J_0 \neq 0$. The proof for $J_i \neq 0, i = 1,2,3$ follows similar lines. For $J_0 \neq 0$, we obtain from equations (7.2)

$$J_0 T_0 + J_1 \alpha_{01} + J_2 \alpha_{02} + J_3 \alpha_{03} = 0$$

$$\alpha_{01} = \frac{1}{J_0}\left( J_1 T_1 - J_2 \alpha_{21} + J_3 \alpha_{13} \right)$$

$$\alpha_{02} = \frac{1}{J_0}\left( J_1 \alpha_{21} + J_2 T_2 - J_3 \alpha_{32} \right)$$

$$\alpha_{03} = \frac{1}{J_0}\left( -J_1 \alpha_{13} + J_2 \alpha_{32} + J_3 T \right)$$

and replacing the terms $\alpha_{01}, a_{02}, a_{03}$ in the first equation we obtain

$$J_0 T_0 + \frac{J_1}{J_0}\left( J_1 T_1 - J_2 \alpha_{21} + J_3 \alpha_{13} \right) + \frac{J_2}{J_0}\left( J_1 \alpha_{21} + J_2 T_2 - J_3 \alpha_{32} \right)$$
$$+ \frac{J_3}{J_0}\left( -J_1 \alpha_{13} + J_2 \alpha_{32} + J_3 T_3 \right) = 0$$

$$J_0^2 T_0 + J_1^2 T_1 - J_1 J_2 \alpha_{21} + J_1 J_3 \alpha_{13} + J_2 J_1 \alpha_{21} + J_2^2 T_2$$
$$- J_2 J_3 \alpha_{32} - J_3 J_1 \alpha_{13} + J_3 J_2 \alpha_{32} + J_3^2 T_3 = 0$$

$$T_0 J_0^2 + T_1 J_1^2 + T_2 J_2^2 + T_3 J_3^2 = 0 \ . \ \square$$

An immediate consequence of theorem 7.6 is corollary 7.5.

**Corollary 7.5.** ‘’For every matrix $T$ of the external symmetry the following hold:

1. $T_0 = T_1 = T_2 = T_3 \neq 0 \Rightarrow m_0 = 0$ (7.72)

2. $\left.\begin{matrix} T_0 = T_1 = T_2 = T_3 \\ m_0 \neq 0 \end{matrix}\right\} \Rightarrow T_0 = T_1 = T_2 = T_3 = 0$ ’’ (7.73)

**Proof.** For $T_0 = T_1 = T_2 = T_3$ we obtain from equation (7.71)

$$T_0 \left( J_0^2 + J_1^2 + J_2^2 + J_3^3 \right) = 0$$

and with equation (2.7) we have

$$T_0 m_0 c^2 = 0 \ . \qquad (7.74)$$

1. Since $T_0 \neq 0$, from equation (7.74) we have $m_0 = 0$.

2. Since $m_0 \neq 0$, from equation (7.74) we have $T_0 = 0$. Since $T_0 = T_1 = T_2 = T_3$, we obtain

$T_0 = T_1 = T_2 = T_3 = 0$. □

We calculate the number of the matrices of the external symmetry. This number is determined by theorem 7.3 and corollarys 7.1 and 7.4. Also, the matrices of the external symmetry are non-vanishing. Applying simple combinatorial rules, we see that altogether there exist

$$N_1 = 14$$

matrices of external symmetry, with $\alpha_{ki} = 0$ for every $k \neq i, k, i = 0,1,2,3$. These matrices contain non-vanishing elements only on their main diagonal. The number $N_2^{'}$ of the matrices with two elements, for which it holds that $\alpha_{ki} \neq 0, k \neq i, k, i \in \{0,1,2,3\}$ is

$$N_2^{'} = 27$$

With three elements, this number is

$$N_3^{'} = 23$$

With four elements, this number is

$$N_4^{'} = 15$$

With five elements, this number is

$$N_5 = 6$$

With six elements, this number is

$$N_6 = 1.$$

From equation (2.13) and the second of the set of equations (4.6) we can prove that some matrices belonging to aforementioned set of matrices give the four-vector $J = 0$, thus they are rejected. Therefore, we obtain

$$\begin{aligned}
N_1 &= 14 \\
N_2 &= N_2^{'} - 3 = 24 \\
N_3 &= N_3^{'} - 17 = 6 \\
N_4 &= N_4^{'} - 12 = 3 \\
N_5 &= 6 \\
N_6 &= 1
\end{aligned}$$

Thus, the total number $N_T$ of the set of matrices of external symmetry is

$$N_T = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 = 54 \tag{7.75}$$

The matrix $T = 0$ is unique

$$N_O = 1$$

and according to theorem 3.3 this matrix expresses the internal symmetry. Therefore, the total number of the matrices of the internal and external symmetry predicted by the Law of Selfvariations is

$$N_{OT} = N_O + N_T = 55 \tag{7.76}$$

There are 14 matrices among the $N_T = 55$ matrices which differ only in one element of their main diagonal, while they share the same four-vectors $J, P, C, j$. Thus, there exist

$$N_J = N_T - 14 = 40 \tag{7.77}$$

matrices of the external symmetry with different eigenvectors $J, P, C, j$.

We indicatively prove that the following matrix, given as

$$T = zQ \begin{bmatrix} T_0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & T_1 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & T_2 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & T_3 \end{bmatrix}$$

is not a matrix of the external symmetry.

Applying theorem 7.3 for the above matrix we see that

$$T = Q\Lambda$$

and therefore it takes the form

$$T = zQ \begin{bmatrix} 0 & \alpha_{01} & 0 & \alpha_{03} \\ -\alpha_{01} & 0 & -\alpha_{21} & \alpha_{13} \\ 0 & \alpha_{21} & 0 & 0 \\ -\alpha_{03} & -\alpha_{13} & 0 & 0 \end{bmatrix}$$

and with equation (2.13) we obtain

$$J_1\alpha_{01} + J_3\alpha_{03} = 0$$
$$-J_0\alpha_{01} - J_2\alpha_{21} + J_3\alpha_{13} = 0$$
$$J_1\alpha_{21} = 0$$
$$-J_0\alpha_{03} - J_1\alpha_{13} = 0$$

and since

$$\alpha_{01}\alpha_{03}\alpha_{13}\alpha_{21} \neq 0$$

we take that

$$J_0 = J_1 = J_2 = J_3 = 0$$

and from theorem 3.1 we obtain that $\alpha_{ki} = 0$ for every set of indices $k \neq i, k, i \in \{0,1,2,3\}$, which is impossible. In the case of $J_0 = J_1 = J_2 = J_3 = 0$ the material particle does not exist.

We present now a notation for the matrices of the external symmetry. In every matrix $T$ we use an upper and a lower index. As lower indices we use the couples $(k,i), k \neq i, k, i = 0,1,2,3$ of the constants $\alpha_{ki} \neq 0$, which do not vanish. These indices, which appear always in couples, are placed in the following order of the constants: $\alpha_{01}, \alpha_{02}, \alpha_{03}, \alpha_{32}, \alpha_{13}, \alpha_{21}$, which are not equal to zero. As upper indices we use the indices of the elements of the main diagonal, which are different from zero, in the following order: $T_0, T_1, T_2, T_3$. For example, the matrix $T$ given in equation (7.37) is denoted as $T^{1}_{0121}$.

With this notation, the $N_T = 54$ matrices of external symmetry are given from the six following sets $\Omega$:

$$\Omega_1 = \{T^0, T^1, T^2, T^3, T^{01}, T^{02}, T^{03}, T^{12}, T^{13}, T^{23}, T^{012}, T^{013}, T^{023}, T^{123}\}$$

$$\Omega_2 = \{T^0{}_{0102}, T_{0102}, T^0_{0103}, T_{0103}, T^0_{0203}, T_{0203}, T^3_{3213}, T_{3213}, T^2_{3221}, T_{3221}, T^1_{1321}, T_{1321},$$

$$T^1_{0113}, T_{0113}, T^1_{0121}, T_{0121}, T^2_{0232}, T_{0232}, T^2_{0221}, T_{0221}, T^3_{0332}, T_{0332}, T^3_{0313}, T_{0313}\}$$

$$\Omega_3 = \{T^0_{010203}, T_{010203}, T_{010232}, T_{010221}, T^3_{033213}, T_{033213}\} \tag{7.78}$$

$$\Omega_4 = \{T_{01023213}, T_{01033221}, T_{02031321}\}$$

$$\Omega_5 \{T_{0102033221}, T_{0102033213}, T_{0102031321}, T_{0102321321}, T_{0103321321}, T_{0203321321}\}$$

$$\Omega_6 = \{T_{010203321321}\}$$

The study of the matrix of external symmetry $T_{010203321321}$ with elements $\alpha_{ki} \neq 0, \forall k \neq i, k, i \in \{0,1,2,3\}$ is algebraically demanding.

We shall finish this paragraph by stating the elements of the specific matrix we have chosen:

$$T = T_{010203321321} = zQ \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & \mp\alpha_{03} & \pm\alpha_{02} \\ -\alpha_{02} & \pm\alpha_{03} & 0 & \mp\alpha_{01} \\ -\alpha_{03} & \mp\alpha_{02} & \pm\alpha_{01} & 0 \end{bmatrix} \tag{7.79}$$

$$\alpha_{01}\alpha_{02}\alpha_{03}\alpha_{32}\alpha_{13}\alpha_{21} \neq 0$$

$$\boldsymbol{\tau} = \pm\mathbf{n} \neq \mathbf{0} \tag{7.80}$$

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = \alpha_{32}^2 + \alpha_{13}^2 + \alpha_{21}^2 = 0 \tag{7.81}$$

$$J = J_2\begin{bmatrix} \mp\dfrac{a_{03}}{a_{01}} \\ -\dfrac{a_{02}}{a_{01}} \\ 1 \\ 0 \end{bmatrix} + J_3\begin{bmatrix} \pm\dfrac{a_{02}}{a_{01}} \\ -\dfrac{a_{03}}{a_{01}} \\ 0 \\ 1 \end{bmatrix} \qquad (7.82)$$

$$m_0 = 0\,. \qquad (7.83)$$

The USVI of this symmetry is given by equation

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0}J - zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -i\alpha_{01} & -\alpha_{02} & \alpha_{03} & 0 \\ -i\alpha_{02} & -\alpha_{03} & 0 & \alpha_{01} \\ -i\alpha_{03} & \alpha_{02} & -\alpha_{01} & 0 \end{bmatrix}\begin{bmatrix} -1 \\ \dfrac{iu_1}{c} \\ \dfrac{iu_2}{c} \\ \dfrac{iu_3}{c} \end{bmatrix}. \qquad (7.84)$$

Based on the theorems of the TSV we can study the whole set of the matrices of external symmetry. In the following paragraphs we present the study of two other matrices belonging to this set of matrices.

## 8. The Symmetry T=QΛ

In this paragraph we study the $T$ matrices, with all of their elements are equal to zero, except of the elements on their main diagonal. These matrices are of the form

$$T = Q\Lambda = zQ\begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix} \qquad (8.1)$$

using the notation of equation (4.11). From equations (4.28) ,(7.3) and (8.1) we take

$$\begin{aligned} M &= 0 \\ N &= 0 \end{aligned}. \tag{8.2}$$

The matrices $M$ and $N$ vanish; as a consequence the matrices of the symmetries $T = Q\Lambda$ share common properties, which we shall study in the following.

According to corollary 7.4 ,the diagonal elements of the matrices defined in equation(8.1) cannot all be different from zero simultaneously. Also, all of them cannot vanish simultaneously, since in the case of the external symmetry it holds that $T \neq 0$. Therefore, there is a number of

$$N_1 = \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 14$$

different matrices for which the relation $T = Q\Lambda$ holds.

In the symmetry $T = Q\Lambda$ at least one element of the matrix $Q\Lambda$ is different from zero, that is $Q\Lambda \neq 0$. Furthermore, it holds that $\alpha_{ki} = 0$ for every set of indices $k \neq i.k, i \in \{0,1,2,3\}$, therefore we obtain that $\boldsymbol{\alpha} = \mathbf{0}, \boldsymbol{\beta} = \mathbf{0}$, with the help of equations (4.14) and (4.15). Therefore, the USVI in the case of the symmetry $T = Q\Lambda$ is given by the set of equations

$$\begin{aligned} \frac{dJ}{dx_0} &= \frac{dQ}{Qdx_0} J - \frac{i}{c} Q\Lambda u \\ \frac{dP}{dx_0} &= -\frac{dQ}{Qdx_0} J + \frac{i}{c} Q\Lambda u \end{aligned} \tag{8.3}$$

This is a consequence of equations (4.19) and (4.20).

Another characteristic for the 14 in number kinds of the symmetry $T = Q\Lambda$ is the equality $\boldsymbol{\tau} = \mathbf{0}$, therefore the plane $\Pi$ is not defined. Similarly, the vectors $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3$ defined in the set of equations (7.32) are also not defined.

One fundamental characteristic of the symmetries $T = Q\Lambda$ is that the four-vector $j$ of the conserved physical quantites $q$ vanishes. Combining the first of the set of equations (8.2) with equation (5.7) we obtain

$$j = 0. \tag{8.4}$$

Therefore, in the part of spacetime occupied by the generalized particle, there does not exist any flow of conserved physical quantities $q$.

Another common characteristic is that the rest mass $m_0$ of the material particle can be also diferent from zero, that is

$$m_0 = 0 \vee m_0 \neq 0 \tag{8.5}$$

for all of the 14 in number matrices of the symmetry. The form of the four-vector $J$ is different, according to each particular matrix of the symmetry.

We calculate the four-vector of momentum $J$ of the matrix $T^{12}$. According to our notation we have

$$T^{12} = zQ\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{8.6}$$

$T_1T_2 \neq 0$

From equation (2.13), and since $T_1T_2 \neq 0$, $T_0 = T_3 = 0$, we obtain for the four-vector $J$

$$J = \begin{bmatrix} J_0 \\ 0 \\ 0 \\ J_3 \end{bmatrix}. \tag{8.7}$$

Combining equations (2.7) and (8.7), we obtain for the rest mass $m_0$ the equation that

$$-m_0^2c^2 = J_0^2 + J_3^2. \tag{8.8}$$

We apply now the $SV-T$ method :

We differentiate equation (8.8) with respect to $x_k, k = 0,1,2,3$ and taking into account equations (2.6), (2.10) and (4.4) we obtain

$$-\frac{b}{\hbar}P_k m_0^2c^2 = J_0\left(\frac{b}{\hbar}P_k J_0 + zQ\alpha_{k0}\right) + J_3\left(\frac{b}{\hbar}P_k J_3 + zQ\alpha_{k3}\right)$$

and from equation (8.8) we have

$$zQJ_0\alpha_{k0} + zQJ_3\alpha_{k3} = 0$$

and sincce $zQ \neq 0$, we take

$$J_0\alpha_{k0} + J_3\alpha_{k3} = 0, k = 0,1,2,3. \tag{8.9}$$

We insert successively the indices $k = 0,1,2,3$ into equation (8.9):

For $k=0$ we have

$J_0 T_0 + J_3 \alpha_{03} = 0$

which holds, since for the matrix $T^{12}$ it holds that $T_0 = \alpha_{03} = 0$.

For $k=1$ we have

$J_0 \alpha_{10} + J_3 \alpha_{13} = 0$

which also holds, since for the matrix $T^{12}$ we have that $\alpha_{10} = \alpha_{13} = 0$.

For $k=2$ we obtain

$J_0 \alpha_{20} + J_3 \alpha_{23} = 0$

which also holds, since for the matrix $T^{12}$ we have that $\alpha_{20} = \alpha_{32} = 0$.

For $k=3$ we also take

$J_0 \alpha_{30} + J_3 T_3 = 0$

which holds, since for the matrix $T^{12}$ we have $\alpha_{30} = T_3 = 0$.

According to equation 8.7, in $T^{12}$ symmetry it holds that $J_0 = 0$ or $J_3 = 0$, but it cannot hold that $J_0 = J_3 = 0$, since in this case the material particle cannot exist. Therefore, from equation (8.8) we conclude that

$$m_0 \neq 0 \vee \{m_0 = 0 \wedge J_3 = \pm i J_0\}. \qquad (8.10)$$

Simirarly, we can prove that other relations, corresponding to relation (8.10), hold for all matrices of symmetry $T = Q\Lambda$.

For the matrix $T^{12}$ it holds that $T_1 \neq T_0 = 0$. Therefore, the part of spacetime occupied by the generalized particle of the symmetry $T^{12}$ is curved, according to corollary 6.2.

Because of equation (8.4) the wave equation (5.17) holds identically $(0=0)$. Therefore, in the case of the symmetries $T = Q\Lambda$ we cannot extract any information about the wave behavior of matter. The only information we can obtain comes through the set of equations (8.3).

From equation (4.11) we obtain

$$\Lambda = z\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (8.11)$$
$$T_1 T_2 \neq 0$$

about the symmetry matrix $T^{12}$. From equations (8.7) and (8.11), and by the first of the set of equations (8.3), we obtain

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} J$$

and with equation (8.7) we have

$$\frac{dJ_0}{dx_0} = \frac{dQ}{Qdx_0} J_0$$
$$\frac{dJ_3}{dx_0} = \frac{dQ}{Qdx_0} J_3$$

abd finally, we obtain

$$\begin{aligned} &J_0 = \sigma_0 Q \\ &J_3 = \sigma_3 Q \\ &\sigma_0 \sigma_3 \neq 0, \\ &\sigma_0, \sigma_3 \, constants \end{aligned} \quad . \qquad (8.12)$$

Thus, the four-vector $J$ is given by equation

$$J = Q\begin{bmatrix} \sigma_0 \\ 0 \\ 0 \\ \sigma_3 \end{bmatrix}$$
$$\sigma_0 \sigma_3 \neq 0, \qquad (8.13)$$
$$\sigma_0, \sigma_3 \, constants$$

as derived from equation (8.7). Therefore, for the symmetry $T^{12}$ the momentum of the material particle is proportional to the charge $Q$. This feature is a common characteristic for all of the set of matrices of the symmetry $T = Q\Lambda$.

For $Q = m_0$, and from equation (8.13) we take

$$J_0 = m_0\sigma_0$$
$$J_3 = m_0\sigma_3$$

and with equation (8.8) we obtain

$$m_0^2\left(c^2+\sigma_0^2+\sigma_3^2\right)=0$$

and for $m_0 \neq 0$ we take

$$\begin{aligned}&\left(\frac{\sigma_0}{c}\right)^2+\left(\frac{\sigma_3}{c}\right)^2+1=0\\ &\frac{\sigma_0}{c},\frac{\sigma_3}{c}\in\mathbb{C}\\ &\sigma_0\sigma_3\neq 0\\ &Q=m_0\neq 0\end{aligned} \tag{8.14}$$

In the symmetries $T = Q\Lambda$ equations (2.10) obtain the form

$$\begin{aligned}&\frac{\partial J_i}{\partial x_k}=\frac{b}{\hbar}P_k J_i=\frac{b}{\hbar}\left(c_k-J_k\right)J_i\\ &k,i=0,1,2,3\end{aligned}. \tag{8.15}$$

The solution of these differential equations gives the four-vector

$$J = J\left(x_0,x_1,x_2,x_3\right) \tag{8.16}$$

and using equation (2.7) we also obtain the rest mass of the material particle

$$m_0 = m_0\left(x_0,x_1,x_2,x_3\right)\neq 0. \tag{8.17}$$

Therefore, the study of the symmetries $T = Q\Lambda$ can also be accomplished through equations (8.15). For the chosen symmetry $T^{12}$ the equations (8.15) obtain the form

$$\begin{aligned}&J_1=0\\ &J_2=0\\ &\frac{\partial J_0}{\partial x_k}=\frac{b}{\hbar}\left(c_k-J_k\right)J_0\ .\\ &\frac{\partial J_3}{\partial x_k}=\frac{b}{\hbar}\left(c_k-J_k\right)J_3\\ &k=0,1,2,3\end{aligned} \tag{8.18}$$

The study of the remaining 13 symmetries of $T = Q\Lambda$ is accomplished similarly to the study of the symmetry $T^{12}$ we have presented.

## 9. The Symmetry $T^0_{010203}$ and the Symmetry $T_{010203}$.

In this paragraph we study the generalized particle corresponding to the matrix

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & T_1 & 0 & 0 \\ -\alpha_{02} & 0 & T_2 & 0 \\ -\alpha_{03} & 0 & 0 & T_3 \end{bmatrix} \qquad (9.1)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

From theorem 7.3 we take have that for this matrix it holds

$$T_1 = T_2 = T_3 = 0$$

and thus it is written in the form

$$T = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \qquad (9.2)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

From the matrix given in equation (9.2) we obtain the symmetries

$$T = T^0_{010203} = zQ\begin{bmatrix} T_0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \qquad (9.3)$$

$$\alpha_{01}\alpha_{02}\alpha_{03}T_0 \neq 0$$

$$T = T_{010203} = zQ\begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix}. \qquad (9.4)$$

$$\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$$

First, we study the symmetry $T^{0}_{010203}$. For the symmetry $T^{0}_{010203}$ it holds that $M \neq 0$, so that we can apply the $SV-M$ -method. From equation (7.3) we have

$$\alpha_{01}^2 + \alpha_{02}^2 + \alpha_{03}^2 = 0. \tag{9.5}$$

From equations (7.2) we obtain

$$\begin{aligned} &J_0 T_0 + J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \\ &J_0\alpha_{01} = 0 \\ &J_0\alpha_{02} = 0 \\ &J_0\alpha_{03} = 0 \end{aligned}$$

and since $\alpha_{01}\alpha_{02}\alpha_{03} \neq 0$ and $T_0 \neq 0$ we have

$$\begin{aligned} &J_0 = 0 \\ &J_1\alpha_{01} + J_2\alpha_{02} + J_3\alpha_{03} = 0 \end{aligned}. \tag{9.6}$$

From the second equation of the set of equations (4.6), and for the indices $(i,\nu,\kappa) = (0,1,2) = (0,1,3) = (0,2,3) = (1,2,3)$ we obtain

$$\begin{aligned} &J_2\alpha_{01} - J_1\alpha_{02} = 0 \\ &J_3\alpha_{01} - J_1\alpha_{03} = 0 \\ &J_3\alpha_{02} - J_2\alpha_{03} = 0 \end{aligned}. \tag{9.7}$$

From equations (9.6),(9.7), and since it holds that $a_{01}a_{02}a_{03} \neq 0$, we have

$$\begin{aligned} &J_0 = 0 \\ &J_2 = \frac{\alpha_{02}}{\alpha_{01}} J_1 \\ &J = \frac{\alpha_{03}}{\alpha_{01}} J_1 \end{aligned}. \tag{9.8}$$

From equations (9.8) we obtain the four-vector $J$

$$J = J_1 \begin{bmatrix} 0 \\ 1 \\ \dfrac{\alpha_{02}}{\alpha_{01}} \\ \dfrac{\alpha_{03}}{\alpha_{01}} \end{bmatrix}. \tag{9.9}$$

From equations (4.28) and (9.3) we have

$$M = \begin{bmatrix} 0 & \alpha_{01} & \alpha_{02} & \alpha_{03} \\ -\alpha_{01} & 0 & 0 & 0 \\ -\alpha_{02} & 0 & 0 & 0 \\ -\alpha_{03} & 0 & 0 & 0 \end{bmatrix} \qquad (9.10)$$

$$M^2 = \begin{bmatrix} -\alpha_{01}^2 - \alpha_{02}^2 - \alpha_{03}^2 & 0 & 0 & 0 \\ 0 & -\alpha_{01}^2 & -\alpha_{01}\alpha_{02} & -\alpha_{01}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{02} & -\alpha_{02}^2 & -\alpha_{02}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{03} & -\alpha_{02}\alpha_{03} & -\alpha_{03}^2 \end{bmatrix}$$

and using equation (9.5) we obtain

$$M^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha_{01}^2 & -\alpha_{01}\alpha_{02} & -\alpha_{01}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{02} & -\alpha_{02}^2 & -\alpha_{02}\alpha_{03} \\ 0 & -\alpha_{01}\alpha_{03} & -\alpha_{02}\alpha_{03} & -\alpha_{03}^2 \end{bmatrix}. \qquad (9.11)$$

From the first equation of the set of equations (7.14) and from equation (9.11), after the respective calculations, we finally obtain

$$c_1\alpha_{01} + c_2\alpha_{02} + c_3\alpha_{03} = 0 \,. \qquad (9.12)$$

From the first equation of the set of equations (4.6), and for the indices $(i,\nu,\kappa) = (0,1,2) = (0,1,3) = (0,2,3) = (1,2,3)$ we obtain

$$\begin{aligned} c_0\alpha_{01} - c_1\alpha_{02} &= 0 \\ c_3\alpha_{01} - c_1\alpha_{03} &= 0 \;. \\ c_3\alpha_{02} - c_2\alpha_{03} &= 0 \end{aligned} \qquad (9.13)$$

From equations (9.12) and (9.13) we also take

$$\begin{aligned} c_1 &= \frac{\alpha_{01}}{\alpha_{02}} c_0 \\ c_2 &= c_0 \qquad . \\ c_3 &= \frac{\alpha_{03}}{\alpha_{02}} c_0 \end{aligned} \qquad (9.14)$$

From equations (9.14) we obtain the four-vector $C$

$$C = c_0 \begin{bmatrix} 1 \\ \dfrac{\alpha_{01}}{\alpha_{02}} \\ 1 \\ \dfrac{\alpha_{03}}{\alpha_{02}} \end{bmatrix}. \tag{9.15}$$

From equation (3.5) and equations (9.9) and (9.15) we obtain the four-vector $P$

$$P = \begin{bmatrix} c_0 \\ \dfrac{\alpha_{01}}{\alpha_{02}} c_0 - J_1 \\ c_0 - \dfrac{\alpha_{02}}{\alpha_{01}} J_1 \\ \dfrac{\alpha_{03}}{\alpha_{02}} c_0 - \dfrac{\alpha_{03}}{\alpha_{01}} J_1 \end{bmatrix}. \tag{9.16}$$

Since we know the four-vectors $J, P, C$ we can calculate the rest masses $m_0, \dfrac{E_0}{c^2}, M_0$. From equations (2.7) and (9.9) we take

$$-m_0^2 c^2 = J_1^2 \left[ 1 + \left( \frac{\alpha_{02}}{\alpha_{01}} \right)^2 + \left( \frac{\alpha_{03}}{\alpha_{01}} \right)^2 \right]$$

and using equation (9.5) we obtain

$$m_0 = 0\,. \tag{9.17}$$

From equations (2.8) and (9.16) we have that

$$E_0 = \pm i c c_0\,. \tag{9.18}$$

For the proof of equation (9.18) we also used equation (9.5). From equations (3.6) and (9.15) we take

$$M_0 = \pm \frac{i c_0}{c}\,. \tag{9.19}$$

The vector $\boldsymbol{\tau}$ vanishes

$$\boldsymbol{\tau} = \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Therefore, the plane $\Pi$ is not defined. For the same reason, it also holds that $\boldsymbol{\mu} = \mathbf{0}$. On the contrary, the vector $\mathbf{n}$ does not vanish

$$\mathbf{n} = \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{9.20}$$

From equations (9.9),(9.15) and (9.16) we see that the vectors $\mathbf{J}, \mathbf{P}, \mathbf{C}$ are parallel to the vector $\mathbf{n}$. We write the vectors $\mathbf{J}$ , $\mathbf{C}$, as given by equations (9.9), (9.15) and (9.20), in the form of

$$\mathbf{J} = \frac{J_1}{\alpha_{01}} \mathbf{n} \tag{9.21}$$

$$\mathbf{C} = \frac{c_0}{\alpha_{01}} \mathbf{n} \ .$$

The vector $\mathbf{C}$ is a constant vector aligned to the direction of the vector $\mathbf{n}$. From equivalence (3.4) we obtain

$$\lambda_{ki} = \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right), k \neq i, k, i = 0,1,2,3$$

and using equation (4.4) we also have

$$zQ\alpha_{ki} \frac{b}{2\hbar}\left(c_i J_k - c_k J_i\right)$$

and for the set of indices $k = 0, i = 0,1,2,3$ we obtain

$$zQ\alpha_{01} = \frac{b}{2\hbar}\left(c_1 J_0 - c_0 J_1\right)$$
$$zQ\alpha_{02} = \frac{b}{2\hbar}\left(c_2 J_0 - c_0 J_2\right)$$
$$zQ\alpha_{03} = \frac{b}{2\hbar}\left(c_3 J_0 - c_0 J_3\right)$$

and using equations (9.9) and (9.15) we take

$$zQ\alpha_{01} = -\frac{bc_0}{2\hbar} J_1$$
$$zQ\alpha_{02} = -\frac{bc_0}{2\hbar} J_2$$
$$zQ\alpha_{03} = -\frac{bc_0}{2\hbar} J_3$$

and solving with respect to $J_1, J_2, J_3$ we obtain

$$J_1 = -\frac{2\hbar}{bc_0} zQ\alpha_{01}$$
$$J_2 = -\frac{2\hbar}{bc_0} zQ\alpha_{02}$$
$$J_3 = -\frac{2\hbar}{bc_0} zQ\alpha_{03}$$

and using equation (9.20) we take

$$\mathbf{J} = -\frac{2\hbar}{bc_0} zQ\mathbf{n}$$

and taking into account that $J_0 = 0$, we have

$$\begin{aligned} &J_0 = 0 \\ &\mathbf{J} = -\frac{2\hbar}{bc_0} zQ\mathbf{n} \quad . \\ &J = -\frac{2\hbar zQ}{bc_0}\begin{bmatrix} 0 \\ \mathbf{n} \end{bmatrix} \end{aligned} \tag{9.22}$$

In equation (9.22) the function $z$ is given by equation (4.5). Equation (9.22) expresses the dependence of the four-vector $J$ on the charge $Q$ in the case of the external symmetry $T^0_{010203}$ .

For the matrix $T^0_{010203}$ it holds that $M \neq 0$. We therefore apply the $SV_q$ - method for determining the four-vector $j$. For the indices $(i,\nu,\kappa) = (0,1,2) = (0,1,3) = (0,2,3) = (1,2,3)$ appearing in equation (7.51), and by considering the elements of the matrix $T^0_{010203}$, we obtain

$$\begin{aligned} j_2 &= \frac{\alpha_{02}}{\alpha_{01}} j_1 \\ j_3 &= \frac{\alpha_{03}}{\alpha_{01}} j_1 \end{aligned} \quad . \tag{9.23}$$

From equations (4.27), (7.16) and (9.10) we take

$$\alpha_{01} j_1 + \alpha_{02} j_2 + \alpha_{03} j_3 = 0 \quad . \tag{9.24}$$

From equations (9.23) and (9.24) we obtain the four-vector $j$

$$j=\begin{bmatrix} j_0 \\ j_1 \\ \dfrac{\alpha_{02}}{\alpha_{01}}j_1 \\ \dfrac{\alpha_{03}}{\alpha_{01}}j_1 \end{bmatrix} . \tag{9.25}$$

From equations (9.25) and (9.20) we obtain the current density

$$\mathbf{j}=\frac{j_1}{\alpha_{01}}\mathbf{n} . \tag{9.26}$$

Therefore, the current density $\mathbf{j}$ has the same direction with the direction of the vector $\mathbf{n}$.

From the wave equation (5.17) and equations (9.25) and (9.20) we obtain

$$\begin{gathered} \sigma c^2 F\mathbf{n}=\frac{1}{\alpha_{01}}\frac{\partial j_1}{\partial x_0}\mathbf{n}-\nabla j_0 \\ \frac{1}{\alpha_{01}}\frac{\partial j_1}{\partial x_1}=\frac{1}{\alpha_{02}}\frac{\partial j_1}{\partial x_2}=\frac{1}{\alpha_{03}}\frac{\partial j_1}{\partial x_3} . \\ F=\nabla^2\Psi+\frac{\partial^2\Psi}{\partial x_0^2} \end{gathered} \tag{9.27}$$

From the second equation of the set of equations given in (9.27) we have

$$\nabla j_1=\left(\frac{\partial j_1}{\partial x_1},\frac{\partial j_1}{\partial x_1}\frac{\alpha_{02}}{\alpha_{01}},\frac{\partial j_1}{\partial x_1}\frac{\alpha_{03}}{\alpha_{01}}\right)$$

$$\nabla j_1=\frac{1}{\alpha_{01}}\frac{\partial j_1}{\partial x_1}\begin{pmatrix}\alpha_{01}\\ \alpha_{02}\\ \alpha_{03}\end{pmatrix}$$

and using equation (9.20) we take

$$\nabla j_1=\frac{1}{\alpha_{01}}\frac{\partial j_1}{\partial x_1}\mathbf{n} . \tag{9.28}$$

From equations (9.28) and (9.20) we have

$$\nabla\cdot\mathbf{j}=\frac{1}{\alpha_{01}^2}\frac{\partial}{\partial x_1}\left(\alpha_{01}^2+\alpha_{02}^2+\alpha_{03}^2\right)$$

and using equation (9.5) we take

$$\nabla\cdot\mathbf{j}=0 \tag{9.29}$$

Combining the continuity equation (5.6) with equation (9.29) we obtain

$$\frac{\partial j_0}{\partial x_0} = 0 \ . \tag{9.30}$$

Therefore, the charge density $j_0 = i\rho c$ does not depend on time for this case of the chosen external symmetry. Finally, after combining equations (5.3), (9.9) and (9.16) we have

$$\begin{aligned} &\frac{\partial \Psi}{\partial x_0} = \frac{bc_0\mu}{\hbar}\Psi \\ &\nabla\Psi = \frac{b}{\hbar}\left(\frac{\lambda-\mu}{\alpha_{01}}J_1 + \frac{\mu c_0}{\alpha_{02}}\right)\Psi\mathbf{n} \ . \\ &\lambda,\mu\in\mathbb{C},(\lambda,\mu)\neq(0,0) \end{aligned} \tag{9.31}$$

Let us remind us that the parameters $\lambda,\mu$ appearing in equation (9.31) express the two degrees of freedom of the TSV.

The portion of space-time occupied by the generalized particle is curved, since $T_0 \neq T_1 = 0$, according to corollary 6.2. Also, from the combination of equations (9.9), (4.19), and relations $T_1 = T_2 = T_3 = 0$, we obtain

$$\frac{dJ}{dx_0} = \frac{dQ}{Qdx_0} + zQ\begin{bmatrix} T_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \frac{i}{c}Q\begin{bmatrix} \frac{i}{c}\mathbf{u}\cdot\boldsymbol{\alpha} \\ \boldsymbol{\alpha} \end{bmatrix} \tag{9.32}$$

for the *USVI* of the external symmetry $T^0_{010203}$.

In symmetry $T_{010203}$ it holds that $T_0 = T_1 = T_2 = T_3 = 0$. From theorem 7.6 we could obtain that $J_0 \neq 0$, and the four-vector $J$ could take the form

$$\begin{aligned} &J = \begin{bmatrix} J_0 \\ J_1 \\ \frac{\alpha_{02}}{\alpha_{01}}J_1 \\ \frac{\alpha_{03}}{\alpha_{01}}j_1 \end{bmatrix} \\ &J_0 \neq 0 \end{aligned} \tag{9.33}$$

in the case of the symmetry $T_{010203}$. However, equation (9.33) is rejected. Following the same procedure, as the one for proving equation (9.17), from equation (9.33) we obtain

$$m_0^2c^2 = -J_0^2 \neq 0. \tag{9.34}$$

On the other hand, by applying the $SV-T$ method, we conclude that equation (9.34) cannot hold. Therefore, the symmetries $T^0_{010203}$ and $T_{010203}$ have the same four-vectors $J, P, C$ and $j$. Therefore, the symmetries $T^0_{010203}$ and $T_{010203}$ are identical. Their only difference lies in the vanishing or non-vanishing of the physical quantity $T_0$. As derived from equation (9.32) this differences bears consequences on the *USVI* of the two symmetries. The symmetry $T_{010203}$ symmetry belongs to the set of the $N_T - N_J = 14$ symmetries, according to the classification of the matrices of external symmetry, as we have presented it in paragraph 7.

## 10. The Generalized Particle of the Field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ and the Confinement Equation.

In this paragraph we study the generalized particle of the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$, for which the function $\Psi$ is known. This shall allow us to perform a concrete application of theorem (5.1).

For $\lambda = \mu = -\frac{1}{2}$ in equation (5.3) we obtain

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{b}{2\hbar}\left(J_k + P_k\right)\Psi, k = 0,1,2,3$$

and using equation (3.5) we take

$$\frac{\partial \Psi_k}{\partial x_k} = -\frac{bc_k}{2\hbar}\Psi, k = 0,1,2,3$$

and using the notation of equation (4.9) we have

$$\Psi = z = \exp\left[-\frac{b}{2\hbar}\left(c_0x \ + c_1x_1 + c_2x_2 + c_3x_3\right)\right]. \tag{10.1}$$

From equation (10.1) and equations (5.1), (5.2) and (4.14), (4.15), we obtain

$$\boldsymbol{\xi} = \boldsymbol{\alpha} = icz \begin{pmatrix} \alpha_{01} \\ \alpha_{02} \\ \alpha_{03} \end{pmatrix} = icz\mathbf{n}$$
$$\boldsymbol{\omega} = \boldsymbol{\beta} = z \begin{pmatrix} \alpha_{32} \\ \alpha_{13} \\ \alpha_{21} \end{pmatrix} = z\boldsymbol{\tau} \quad . \tag{10.2}$$

The field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is a special case of the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$ for $\lambda = \mu = -\frac{1}{2}$ .

The fact that the function $\Psi$ of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is known allows us to derive two impotant results about the total rest mass $M_0$ of the generalized partcle. From equation (10.1) we obtain

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = \frac{b^2}{4\hbar^2}\left(c_0^2 + c_1^2 + c_1^2 + c_1^2\right)$$

and with equation (3.6) we take

$$\nabla^2\Psi + \frac{\partial^2\Psi}{\partial x_0^2} = \nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = -\frac{b^2}{4\hbar^2} M_0^2 c^2 \Psi \ . \tag{10.3}$$

According to equation (10.3) and theorem 5.2 the generalized photon in the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ exists, if and only if

$$M_0 = 0 \, , \tag{10.4}$$

that is in the case the total rest mass of the generalized particle vanishes. For $M_0 \neq 0$ the generalized particle appears.

Setting $\lambda = \mu = -\frac{1}{2}$ in the equations of paragraph 5, we arrive at the equations of the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. For example, by setting $\lambda = \mu = -\frac{1}{2}$ into equation (5.7) we obtain

$$j = \frac{\sigma c^2 bz}{2\hbar} MC \, .$$

This is equation (4.9), as we have proved in paragraph 4, for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. On the other hand, equation (10.3) results only because the function $\Psi$ is known, as given by equation (10.1) for the field $(\boldsymbol{\alpha}, \boldsymbol{\beta})$.

By knowing the function $\Psi$ we can study the consequences for a material particle when it is confined within a constant volume $V$ . The physical quantity $q$ is conserved, therefore it remains constant within the volume $V$ occupied by the generalized particle. Therefore, it holds that

$$\begin{aligned}&\frac{dq}{dt}=\frac{icdq}{dx_0}=0\\&V=constant\end{aligned}. \qquad (10.5)$$

The total conserved physical quantity $q$ contained within the volume $V$ occupied by the generalized particle is

$$q=\int_V \rho dV \qquad (10.6)$$

Equation (10.6) holds independently, whether the volume $V$ varies or not. The density $\rho$ for the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ is given by the first of the system of equations (4.25)

$$\rho=-\sigma\frac{icbz}{2\hbar}\left(c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}\right). \qquad (10.7)$$

In the case of

$c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}=0$ ,

that is in the case of

$\mathbf{n}\cdot\mathbf{C}=0$ ,

as derived from equations (3.5) and (7.24), we obtain from equation (10.7) that $\rho=0$ . That is, for the field $(\boldsymbol{\alpha},\boldsymbol{\beta})$ the following equivalence hold

$$\rho=0\Leftrightarrow\mathbf{n}\cdot\mathbf{C}=0\Leftrightarrow c_1\alpha_{01}+c_2\alpha_{02}+c_3\alpha_{03}=0. \qquad (10.8)$$

In the case of $\rho\neq0$ , and from the combination of equations (10.6) and (10.7), we also have

$$q=-\sigma\frac{icb\left(\mathbf{n}\cdot\mathbf{C}\right)}{2\hbar}\int_V zdV . \qquad (10.9)$$

The integration in the second part of equation (10.9) is performed within the total volume $V$ occupied by the generalized particle. Therefore, in the case the volume $V$ is constant, the integral in the second part of equation (10.9) is independent of the quantiites $x_1=x,x_2=y,x_3=z$ . Therefore, in the case volume $V$ is constant, the physical quantity $q$ in equation (10.9) depends only on time.

Thus, by combining equations (10.5) and (10.9) for a constant volume $V$ , we obtain

$$\frac{\partial}{\partial t}\int_V zdV = 0 \quad . \qquad (10.10)$$
$$V = constant$$

The analysis of equation (10.10) for the general case is complicated. Therefore, in the present work, we shall refrain on the study only a simpler case. We shall study the case for which the total momentum **C** of the generalized particle is aligned on the direction of the $x$ - axis, that is for the case of $c_1 \neq 0, c_2 = c_3 = 0$. In this case we obtain from equation (3.6) that $M_0^2 c^2 = -c_0^2 - c_1^2$. Furthermore, it must also hold that $\rho \neq 0$, that is $c_1 \alpha_{01} \neq 0$, according to equivalence (10.8) Since it also holds that $c_1 \neq 0$, it must also hold that $\alpha_{01} \neq 0$. Therefore, the study of this particular case refers to the situation, where

$$\begin{aligned} & c_1 \neq 0 \\ & c_2 = c_3 = 0 \\ & \alpha_{01} \neq 0 \\ & M_0^2 c^2 = -c_0^2 - c_1^2 \end{aligned} \quad . \qquad (10.11)$$

We suppose that the generalized particle occupies the constant volume $V$ defined by the relations (10.12) in the chosen frame of reference $O(t, x_1 = x, x_2 = y, x_3 = z)$.

$$\begin{aligned} & \alpha \leq x_1 \leq \beta \\ & 0 \leq x_2 \leq L_2 \\ & 0 \leq x_3 \leq L_3 \\ & \alpha < \beta \\ & L = \beta - \alpha > 0 \\ & L_2, L_3 > 0, L_2, L_3 constants \end{aligned} \qquad (10.12)$$

For the quantities $\alpha, \beta$ it holds that

$$\frac{d\alpha}{dt} = \frac{d\beta}{dt} = u < c \qquad (10.13)$$

Here, $u$ is the velocity of the volume $V$ in the chosen frame of reference.

From equation (10.1), relations (10.11), and since $x_0 = ict$, we have

$$z = \exp\left(-\frac{icbc_0}{2\hbar}t\right)\exp\left(-\frac{bc_1}{2\hbar}x\right)$$

$$\int_V zdV = -\frac{2\hbar L_2 L_3}{bc_1}\exp\left(-\frac{icbc_0}{2\hbar}\right)\left[\exp\left(-\frac{bc_1\beta}{2\hbar}\right) - \exp\left(-\frac{bc_1\alpha}{2\hbar}\right)\right]. \qquad (10.14)$$

From equation (10.14) we see that equation equation(10.10) holds, if and only if

$$\exp\left(-\frac{bc_1\beta}{2\hbar}\right)-\exp\left(-\frac{bc_1\alpha}{2\hbar}\right)=0$$
$$\exp\left(-\frac{bc_1\left(\beta-\alpha\right)}{2\hbar}\right)=1$$

$$\exp\left(\frac{bc_1L}{2\hbar}\right)=1\,. \qquad (10.15)$$

Equation (10.15) holds only in the case the constant $b$ of the Law of Selfvariations is an imaginary number, $b=i\,||\,b\,||$. Therefore, we obtain

$$\cos\left(\frac{bc_1L}{2\hbar}\right)=1$$
$$\sin\left(\frac{bc_1L}{2\hbar}\right)=0$$
$$b=i\left\|b\right\|$$

and finally, we get

$$c_1=n\frac{4\pi\hbar}{L\,||\,b\,||},n=\pm1,\pm2,\pm3,... \qquad (10.16)$$

Combining equation (10.16) with the last of the set of equations (10.11) we take

$$M_0^2c^2=-c_0^2-n^2\,\frac{16\pi^2\hbar^2}{L^2\,||\,b\,||^2},n=1,2,3... \qquad (10.17)$$

Therefore, the momentum $c_1$ and the rest mass $M_0$ of the confined generalized particle is quantized.

In the case of the generalized photon, that is for $M_0=0$, and according to equation (10.17) we take

$$c_0=n\frac{i4\pi\hbar}{L\,||\,b\,||},n=\pm1,\pm2,\pm3,...$$
$$M_0=0 \qquad (10.18).$$

Combining equations (10.1),(10.16) and (10.18) we have

$$\begin{aligned}&\Psi = z = \exp\left[n\frac{4\pi i}{L}(ct-x)\right]\\&n = \pm1, \pm2, \pm3, \ldots \qquad .\\&M_0 = 0\end{aligned} \tag{10.19}$$

The function $\Psi$ expresses a harmonic wave of wavelenght $\lambda = \frac{L}{2n}$ propagating along the $x$ -axis.

We now calculate the equation corresponding to the equation (10.10) for the field $(\boldsymbol{\xi}, \boldsymbol{\omega})$, in general. The reason of not having calculated the general equation (in the case of the one spatial dimension) already in paragraph 5 is that the confinement relation of the generalized particle with the appearance of the quantization would not have become obvious.

From equations (4.27), (4.28) and (5.7), and since it holds that $j_0 = i\rho c$, we obtain:

$$\rho = \frac{icb}{\hbar}\Psi\left[\lambda\left(\alpha_{01}J_1 + \alpha_{02}J_2 + \alpha_{03}J_3\right) + \mu\left(\alpha_{01}P_1 + \alpha_{02}P_2 + \alpha_{03}P_3\right)\right]$$

Together with equations (5.8), (5.9) and (7.24) we also have

$$\rho = \frac{icb}{\hbar}\Psi\left(\lambda\mathbf{J}\cdot\mathbf{n} + \mu\mathbf{P}\cdot\mathbf{n}\right). \tag{10.20}$$

From equations (10.5), (10.20) for the generalized particle occupying a constant volume $V$ we obtain

$$\frac{\partial}{\partial t}\int_V \Psi\left(\lambda\mathbf{J}\cdot\mathbf{n} + \mu\mathbf{P}\cdot\mathbf{n}\right)dV = 0. \tag{10.21}$$

For $\lambda = \mu = -\frac{1}{2}$ equation (10.21) gives equation (10.10), after considering equations (3.5) and (10.1).

For the internal symmetry $T = 0$ it holds that $M = 0$, and from equation (5.7) we obtain $j = 0$. Equation (10.5) degenerates into identity, $0 = 0$, therefore the confinement equation (10.21) does not hold. The same holds also in the case of all of the external symmetries $T = \Lambda$, as a result of equation (8.4). Hence, the confinement equation (10.21) holds for the generalized particle of the $N_J - N_1 = 26$ in number external symmetries.

## 11. The Cosmological Data as a Consequence of the Theorem of Internal Symmetry

The theorem 3.3., that is the theorem of internal symmetry, predicts and justifies the cosmological data. We present the relevant study in this paragraph.

The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe today has taken place a long time interval ago. At the moment of the emission the rest mass and the electric charge of the material particles had smaller values than the corresponding ones measured in the laboratory, “now”, on Earth, due to the manifestation of the Selfvariations. These consequences, resulting from this difference, are recorded in the cosmological data. The cosmological data have a microscopic, and not a macroscopic, cause.

Due to the Selfvariations of the rest masses of the material particles the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model (SCM). The gravitational interaction cannot cause neither the collapse, nor the expansion of the Universe, since it decreases on a cosmological scale according to the factor $\frac{1}{1+z}$. The gravitational interaction exercised on our galaxy by a far-distant astronomical object with redshift $z=9$ is only the $\frac{1}{10}$ of the expected one. The Universe is static and flat, according to the law of Selfvariations.

For a non- moving particle, that is for $J_1 = J_2 = J_3 = 0$, from equation (3.12) we get that $c_1 = c_2 = c_3 = 0$ and from equation (3.9) we obtain

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0\right)$$

and since $x_0 = ict$, we take

$$\Phi = K \exp\left(-\frac{icc_0}{\hbar} t\right)$$

and from equation (3.10) we obtain

$$m_0 = m_0(t) = \pm \frac{M_0}{1 + K \exp\left(-\frac{icc_0}{\hbar} t\right)} \quad . \tag{11.1}$$

The rest mass $m_0$ of the material particle is a function of time $t$ .

We now denote by $k$ the constant

$$k = -\frac{icc_0}{\hbar} \tag{11.2}$$

We also denote by $A$ the time-dependent function

$$A = A(t) = -K \exp(kt) = -\Phi . \tag{11.3}$$

Following this notation, equation (11.1) is written as

$$m_0 = m_0(t) = \pm \frac{M_0}{1-A} \quad . \tag{11.4}$$

From equation (11.3) we have

$$\frac{dA}{dt} = \dot{A} = kA \quad . \tag{11.5}$$

for the expression of the parameter $A = A(t)$. Similarly, using the above notation equation (3.11) is written as

$$E_0 = E_0(t) = \mp \frac{M_0 c^2 A}{1-A} \quad . \tag{11.6}$$

We consider an astronomical object at distance $r$ from Earth. The emission of the electromagnetic spectrum of the far-distant astronomical objects we observe “now”on Earth has taken place before a time interval $\delta t = t - \frac{r}{c}$ . From equation (11.3) we have that the parameter $A$ obtained the value

$$A = A(r) = A(t) exp\left(-k\frac{r}{c}\right)$$

and from equation (11.4) we have

$$m_0(r) = \pm \frac{M_0}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \tag{11.7}$$

Similarrly, from equation (11.6) we take

$$E_0(r) = \mp \frac{M_0 c^2 A\exp\left(-k\frac{r}{c}\right)}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \tag{11.8}$$

From equations (11.4) and (11.7) we also have

$$m_0(r) = m_0 \frac{1-A}{1 - A\exp\left(-k\frac{r}{c}\right)} \quad . \tag{11.9}$$

We can prove that for the electric charge $q$ of the material particles a similar equation holds, analogous to equation (11.7). From equation (4.2) it can be shown that for the electric charge $q$ of the material particles, an equation corresponding to equation (11.9) holds, that is the following equation

$$q(r) = q\frac{1-B}{1-B\exp\left(-k_1\dfrac{r}{c}\right)} \ . \tag{11.10}$$

The fine structure constant $\alpha$ is defined as

$$\alpha = \frac{q^2}{4\pi c\hbar} \ . \tag{11.11}$$

and using equation (11.10) we obtain

$$\alpha(r) = \alpha\left(\frac{1-B}{1-B\exp\left(-k_1\dfrac{r}{c}\right)}\right)^2 \ . \tag{11.12}$$

The wave length $\lambda$ of the linear spectrum is inversely proportional to the factor $m_0 q^4$, where $m_0$ is the rest mass and $q$ is the electric charge of the electron. If we denote by $\lambda_0$ the wavelength of a photon emitted by an atom "now"on Earth, and by $\lambda$ the same wavelength of the same atom received "now" on Earth from the far-distant astronomical object, the following relation holds:

$$\frac{\lambda}{\lambda_0} = \frac{m_0 q^4}{m_0(r)q^4(r)}$$

and from equations (11.9) and (11.10) we obtain

$$\frac{\lambda}{\lambda_0} = \frac{1-A\exp\left(-k\dfrac{r}{c}\right)}{1-A}\left(\frac{1-B\exp\left(-k_1\dfrac{r}{c}\right)}{1-B}\right)^4 \ . \tag{11.13}$$

For the redshift

$$z = \frac{\lambda-\lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

the redshift of the astronomical object from equation (11.13) is given as

$$z = \frac{1-A\exp\left(-k\dfrac{r}{c}\right)}{1-A}\left(\frac{1-B\exp\left(-k_1\dfrac{r}{c}\right)}{1-B}\right)^4 - 1 \, . \tag{11.14}$$

Equation (11.14) can also be written as

$$z = \frac{1 - Aexp\left(-k\frac{r}{c}\right)}{1 - A}\left(\frac{\alpha(r)}{\alpha}\right)^2 - 1. \qquad (11.15)$$

after considering equation (11.12).

From the cosmological data, and from measurements conducted on Earth, we know that the variation of the fine structure constant is extremely small. Therefore, from equation (11.15), we obtain with extremely accurate approximation

$$z = \frac{1 - A\exp\left(-k\frac{r}{c}\right)}{1 - A} - 1$$

$$z = \frac{A}{1 - A}\left(1 - e^{-\frac{kr}{c}}\right). \qquad (11.16)$$

Equation (11.16) holds with great accuracy. The variation of the fine structure constant is so small, so that any of its contribution into the redshift is overlapped by the same contributions from the far-distant astronomical objects, due to Doppler's effect.

For small distances $r$, we obtain from equation (11.16)

$$z = \frac{A}{1 - A}\left(1 - 1 + \frac{kr}{c}\right)$$

$$z = \frac{kA}{c(1 - A)}r$$

and comparing this with Hubble's law

$$cz = Hr$$

we take

$$\frac{kA}{1 - A} = H \qquad (11.17)$$

where H is Hubble's parameter.

From equation (11.17) we have

$$\frac{dH}{dt} = \dot{H} = \frac{k\dot{A}(1 - A) + kA\dot{A}}{(1 - A)^2}$$

$$\dot{H} = \frac{k\dot{A}}{(1-A)^2}$$

and with equation (11.5) we obtain

$$\dot{H} = \frac{k^2 A}{(1-A)^2}$$

and from equation (11.17) we take

$$\dot{H} = \frac{H}{A} \quad . \tag{11.18}$$

Also, from equation (11.17) we have that

$$\frac{kA}{1-A} > 0$$

From these equations we list three possible combinations among the constant $k$ and the parameter $A$:

$$\begin{array}{l} k > 0 \wedge 0 < A < 1 \\ k < 0 \wedge A < 0 \\ k < 0 \wedge A > 1 \end{array} \quad . \tag{11.19}$$

The cosmological data are justified from all of these three combinations. Some differences are predicted for some rates of change, and for the case of the extremely large distances, in the very ealry Universe, which stay beyond the detection limits of our current observational instruments. In the following, we shall conduct our study for the first of these cases. At the end of the paragraph we shall present a comparison among these.

From equations (2.4), (2.5), (3.5) and (11.2) we obtain

$$k = \frac{W+E}{\hbar} .$$

Therefore, the sign of the constant $k$ depends on the sign of the constant sum $W+E$. For $k > 0$, we take from equation (11.16)

$$\lim_{r\to\infty} z = \frac{A}{1-A} \tag{11.20}$$

The redshift takes an upper limit, depending on the value of the parameter $A$.

From equations (11.16) and (11.5), after the calculations, we finally obtain

$$\frac{dz}{dt} = \dot{z} = \frac{kA}{(1-A)^2}\left(1-e^{-\frac{kr}{c}}\right)$$

and with equation (11.17) we have

$$\dot{z} = \frac{H}{1-A}\left(1-e^{-\frac{kr}{c}}\right) . \qquad (11.21)$$

From equation (11.21) we obtain for the first and the third of the cases listed in (11.19) that it holds that $\dot{z} > 0$, whereas for the second we have $\dot{z} < 0$. According to equation (11.18), the same also holds for the rate of change of Hubble's parameter $H$

In the first case of equations (11.19) redshift has an upper limit, according to equation (11.20), hence we obtain

$$z < \frac{A}{1-A}$$

and since it holds that $1-A > 0$ , we take

$$\frac{z}{1+z} < A$$

and since $A < 1$ , it holds that

$$\frac{z}{1+z} < A < 1. \qquad (11.22)$$

From inequality (11.22) we take

$$A \to 1^- \ . \qquad (11.23)$$

We prove that, as $A \to 1^-$ , equation (11.16) tends to the familiar expression of Hubble's law $cz = Hr$.

We set $x = \frac{1-A}{A}$ , hence $x \to 0^+$ for $A \to 1^-$, while from equation (11.17) we obtain $k = xH$. Then, equation (11.6) can be written in the form

$$z = \frac{1}{x}\left[1-\exp\left(-\frac{Hr}{c}\right)\right].$$

Thus, we have

$$\lim_{A\to 1^-} z = \lim_{x\to 0^+} \frac{1}{x}\left[1-\exp\left(-x\frac{Hr}{c}\right)\right] = \frac{Hr}{c}.$$

From relation (11.5) we also take that

$$\frac{dA}{dt} = kA > 0$$

for the first of the cases listed in (11.19). Therefore, the parameter $A$ increases with the passage of time. According to the previous proof, equation (11.16) tends in the limit to the expression of Hubble's law.

Combining equations (11.9) and (11.16) we obtain

$$m_0(z) = \frac{m_0}{1+z} \ . \tag{11.24}$$

Equation (11.24) bears many consequences for distances at a cosmological scale.

According to equation (11.24) , the gravitational interaction among two astronomical objects is lower than expected by a factor of $\frac{1}{1+z}$. Redshift $z$ is defined by their relative distance $r$, as given by equation (11.16), that is the redshift an observer would measure on an astronomical object as observing another astronomical object.

In the case of the solar system, or for a galaxy, or for a cluster of galaxies, equation (11.24) bears no consequences. At these scales of distances it holds that $z = 0$. But, we can search for the consequences at these scales from another equation.

From equation (11.4) we have

$$\frac{dm_0}{dt} = \dot{m}_0 = \pm \frac{M_o \dot{A}}{(1-A)^2}$$

and with quation (11.4) we take

$$\dot{m}_0 = m_0 \frac{\dot{A}}{1-A}$$

and from equation (11.5) we have

$$\frac{\dot{m}_0}{m_0} = \frac{kA}{1-A}$$

and finally, from equation (11.17) we obtain

$$\frac{\dot{m}_0}{m_0} = H \ . \tag{11.25}$$

Equation (11.25) refers to the mass $m_0 = m_0(t)$. Therefore, its consequences can be searched within the region of our galaxy, or within the limits of our solar system. We note that probably the value of Hubble's parameter $H$ is smaller than the corresponding one used today. We shall not present this analysis in the present work. In every case, the experimental confirmation of equation (11.25) can be performed by very sensitive observational instruments. Also, during the process of the measurements, equation (4.19) of the USVI should also be considered.

Equation (11.24) has important consequences for the distances at the cosmological scale. At these distances, the gravitational interaction decreases rapidly, and after a particular distance, practically vanishes. Furthermore, this interaction played a crucial role for the creation of all of the large scale structures in the Universe.

As we shall see, the state of the very early Universe differs only slightly from the state of the vacuum. The gravitational interaction strengthens with the passage of time, as the rest masses of the material particles increase. Furthermore, its strength depends on distance, as given both from the law of universal attraction, and from equation (11.24), for distances at the cosmological scale. These two factors, which are not considered in the evaluation of the cosmological data based on the SCM, played a decisive role in the creation of the large scale structures in the Universe we observe it today.

From the equations $E = mc^2$ and (11.24) we obtain

$$E(z) = \frac{E}{1+z} \tag{11.26}$$

in every case of the transformation of mass into energy. The production of energy in the Universe is mainly accomplished by the fusion of hydrogen, and by the nuclear reactions. Therefore, the energy produced in the past, in the far-distant astronomical objects, acquired smaller values than the corresponding ones produced today in our galaxy, and by the same physical pocesses. This fact has two immediate cosnequences.

The first consequence is that equation (11.16) holds also for the redshift $z_\gamma$ of the radiation $\gamma$ ,given as

$$z_\gamma \frac{A}{1-A}\left(1-e^{-\frac{kr}{c}}\right). \tag{11.27}$$

The other consequence refers to the luminosity distance $D$ of the far-distant astronomical objects. The general decrease of the quantity of the produced energy in the past, due to equation (11.26), has as a consequence an also general decrease of the luminosity of the far-distant astronomical objects. From the defintion of the luminosity distance $D$ we can easily prove that

$$D = r\sqrt{1+z} \tag{11.28}$$

This relation gives the dependence between the real distance $r$ of the astronomical object and the distance $D$ measured based on its luminosity. The luminosity distance $D$ is measured to be always much larger than the real distance of the astronomical object. The real distance $r$ of the far distant astronomical object is given by equation

$$r = \frac{c}{k}\ln\left(\frac{A}{1-z\left(1-A\right)}\right) \tag{11.29}$$

as obtained from equation (11.16). The measurement of the distance according to equation (11.29) can be accomplished, as far as we know the values of the constant $k$ and the parameter $A$. Generally, because of equation (11.17), it suffices to know two out of the set of the parameters $k, A, H$.

The atomic ionization energy, as well as the atomic excitation energies $X_n$ is proportional to the factor $m_0 q^4$, where $m_0$ denotes the rest mass of the electron, and $q$ its electric charge. We have that

$$\frac{X_n\left(r\right)}{X_n} = \frac{m_0\left(r\right)}{m_0}\left(\frac{q\left(r\right)}{q}\right)^4$$

$$\frac{X_n\left(r\right)}{X_n} = \frac{m_0\left(r\right)}{m_0}\left(\frac{\alpha\left(r\right)}{\alpha}\right)^2$$

and since

$$\frac{\alpha\left(r\right)}{\alpha} \approx 1$$

we obtain

$$\frac{X_n\left(r\right)}{X_n} = \frac{m_0\left(r\right)}{m_0}$$

and from equation (11.24) we have

$$\frac{X_n\left(r\right)}{X_n} = \frac{X_n\left(z\right)}{X_n} = \frac{1}{1+z}$$

$$X_n\left(r\right) = X_n\left(z\right) = \frac{X_n}{1+z} \quad . \tag{11.30}$$

From equation (11.30) we conclude that the atomic ionization and excitation energies decrease with the increase of the redshift. This fact bears some consequences about the ionization degree of the atoms in the far distant astronomical objects.

The number of the excited atoms of a gas in a state of thermodynamic equilibrium is given by Boltzmann's equation

$$\frac{N_n}{X_1} = \frac{g_n}{g_1} \exp\left(-\frac{X_n}{KT}\right) \tag{11.31}$$

Here, $N_n$ denotes the number of the atoms being at the energy level with quantum number $n$, $X_n$ stands for the ionization energy, as measured from the ground state to the energy level of quantum number $n$, and $K = 1.38\times10^{-23}\,JK^{-1}$ is Boltzmann's constant. Also, $T$ denotes the temperature given in Kelvin degrees, and $g_n$ is the multiplicity degree of the energy level of number $n$, that is, the number of energy levels into which level $n$ splits in the presence of a magnetic field.

Combining equations (11.30) and (11.31) we have

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} \exp\left(-\frac{X_n}{KT(1+z)}\right).$$

(11.32)

For the hydrogen atom, and for $n = 2, X_2 10.5eV = 16.4\times10^{-19}\,J, g_1 = 2, g_2 = 8$, on the surface of the Sun, where $T \sim 6000K$, equation (11.32) states that only one out of $10^8$ hydrogen atoms is in the $n = 2$ state. Analogously, from equation (11.33) we have that, for $z = 1$, $\frac{N_2}{N_1} = 2.2\times10^{-4}$, for $z = 2$, $\frac{N_2}{N_1} = 5.8\times10^{-3}$, and for $z = 5$, $\frac{N_2}{N_1} = 0.15$.

Considering equation (11.20), we obtain from equation (11.30)

$$X_n(r \to \infty) = X_n(1 - A). \tag{11.33}$$

Considering relations (11.22) and (11.23), we conclude that the atomic ionization and excitation energies tend to vanish in the state of the very early Universe. The Universe underwent from an ionization phase during the initial steps of its evolution.

The laboratory value for Thomson's scattering coefficient is given by equation

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3}\frac{q^4}{m_0^2 c^4} \tag{11.34}$$

Here, $m_0$ denotes the rest mass of the electron, and $q$ denotes its electric charge. Thus, we obtain

$$\frac{\sigma_T(z)}{\sigma_T} = \left(\frac{m_0}{m_0(z)}\right)\left(\frac{\alpha(z)}{\alpha}\right)^2$$

and since $\alpha(z) \sim \alpha$, we take

$$\frac{\sigma_T(z)}{\sigma_T} = \left(\frac{m_0}{m_0(z)}\right)^2$$

From equation (11.24) we also obtain

$$\frac{\sigma_T(z)}{\sigma_T} = (1+z)^2 . \tag{11.35}$$

The Thomson's scattering coefficient refers to the scattering of photons with small energies $E$. In the case of photons with large values of energy $E$, the scattering of the photons is determined by the Klein-Nishina scattering coefficient, given by

$$\sigma = \frac{3}{8}\sigma_T \frac{m_0}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right] \tag{11.36}$$

for the laboratory value, "now", on Earth and by

$$\sigma(z) = \frac{3}{8}\sigma_T(z)\frac{m_0(z)c^2}{E(z)}\left[\ln\left(\frac{2E(z)}{m_0(z)c^2}\right)+\frac{1}{2}\right] \tag{11.37}$$

for an astronomical object with redshift $z$.

From equations (11.24) and (11.26) we take

$$\frac{m_0(z)}{E(z)} = \frac{m_0}{E}$$

Thus, from equation (11.37) we obtain

$$\sigma(z) = \frac{3}{8}\sigma_T(z)\frac{m_0c^2}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right]$$

and with equation (11.35) we have

$$\frac{\sigma(z)}{\sigma} = \frac{\sigma_T(z)}{\sigma_T} = (1+z)^2.$$

(11.38)

From equation (11.38) we conclude that the Thomson and Klein-Nishina coefficients incease in the same manner with the increase of the redshift . Considering equation (11.20), we have

$$\frac{\sigma(r\to\infty)}{\sigma} = \frac{\sigma_T(r\to\infty)}{\sigma_T} = \frac{1}{(1-A)^2} . \tag{11.39}$$

Considering equations (11.22) and (11.23) we conclude that the Thomson and Klein-Nishina scattering coefficients acquired enormous values in the very early Universe. In this initial state the Universe was completely opaque. From this initial phase of the Universe originates the detected Cosmic Microwave Background Radiation (CMBR) observed today.

The theorem of the internal symmetry predicts that the initial state of the Universe was a state described as the "state of a vacuum", being at a temperature of $T = 0K$. Because of the Selfvariations the Universe evolved into the current state. This evolution stays in accordance with the fact that the CMRB corresponds to a black-body temperature of $T \sim 2.73K$.

Combining equations (3.11) and (11.3), we take for the laboratory value

$$J_i = \frac{c_i}{1 - A(t)}, i = 0,1,2,3$$

We also take for the value at the astronomical object located at a distance $r$

$$J_i(r) = \frac{c_i}{1 - A\left(t - \frac{r}{c}\right)} = \frac{c_i}{1 - A\exp\left(-\frac{kr}{c}\right)}, i = 0,1,2,3 .$$

Combining these equations with equation (11.9) we obtain

$$\frac{J_i(r)}{J_i} = \frac{m_0(r)}{m_0}$$

and with equation (11.24) we have

$$\frac{J_i(z)}{J_i} = \frac{1}{1+z}$$

$$J_i(z) = \frac{J_i}{1+z}, i = 0,1,2,3. \qquad (11.40)$$

From Heisenberg's Uncertainty Principle, and for the $x_1 = x$ axis, we take for the laboratory value

$$J_1 \Delta x \sim \hbar$$

and for the corresponding value of a far-distant astronomical object

$$J_1(z)\Delta x(z) \sim \hbar$$

From these relations, we obtain

$$J_1(z)\Delta x(z) = J_1 \Delta x$$

and from equation (11.40) we also have

$$\Delta x(z) = (1+z)\Delta x. \tag{11.41}$$

From equation (11.41) we conclude that the uncertainty $\Delta x(z)$ of the position of the material particle increases with the increase of the redshift. Furthermore, as the redshift decreases, that is as the Universe evolved towards its current state, the uncertainty of the postion of the material particles decreased.

From equations (11.41) and (11.20) we obtain

$$\Delta x(r \to \infty) = \frac{\Delta x}{1-A}. \tag{11.42}$$

Considering equations (11.22) and (11.23) we conclude that in the very early stage of the Universe there existed an enormous degree of uncertainty for the positions of the material particles. The same holds also for the Bohr radius. The TSV stays in accordance with the Uncertainty Principle. In the forthcoming paragraph we shall see that the uncertainty in the positions of the material particles is another consequence of theorem (3.3).

From equation (11.30) we obtain that, as the Universe evolved towards its current state, the ionization energies increased. This prediction is of general validity, for the negative potential energies of all kind, which are responsible for holding together the composite material particles. From equation (11.24) we obtain

$$\Delta m_0(z)c^2 = \frac{\Delta m_0 c^2}{1+z} \tag{11.43}$$

for the amount of energy $\Delta m_0 c^2$, that is for the mass deficit, resposible for holding together the particles constituting the atomic nuclei. Equations (11.30) and (11.43) obtain the same form. Therefore, the energy $\Delta m_0 c^2$ increased during the evolution of the Universe towards its present state.

Material particles, such as the electron, which are considered to be elementary, can in reality be composed by other, more fundamental particles. Our inability to decompose them into their constituting particles could arise from the strengthening of the binding energies of the particles composing them. The mass $M_0$ of equation (3.10) might very probably be the only true elementary rest mass, out of which the masses of all the other material particles are composed of.

From equations (11.24) and (11.20) we obtain

$$m_0(r \to \infty) = m_0(1-A) \neq 0. \tag{11.44}$$

Considering relations (11.22) and (11.23), we conclude that in the initial state of the Universe the rest masses of the material particles tend to zero

$$m_0(r \to \infty) = m_0(1-A) \to 0 . \tag{11.45}$$

From equation (11.8) we have

$$E_0(r \to \infty) = 0 . \tag{11.46}$$

From equations (11.7) and (11.8) we obtain

$$m_0(r)c^2 + E_0(r) = M_0 c^2 \tag{11.47}$$

and from equations (11.44) and (11.46) we take

$$m_0(1-A) = M_0 . \tag{11.48}$$

Working simirarly, we obtain from equations (11.7) and (11.8), in the case of $k<0$

$$\begin{aligned} & m_0(r \to \infty) = 0 \\ & E_0(r \to \infty) = \mp M_0 c^2 . \\ & k<0 \end{aligned} \tag{11.49}$$

Equation (11.48), which results from equation (11.47), and holds for $k>0$, is equation (11.4). If we assume that $r \to R < \infty$ for the cases where $k<0$, then equation (11.47) holds identically.

From equation (11.16), and for $k<0$, we obtain

$$z(R) = \frac{A}{1-A}\left(1-\exp\left(-\frac{kR}{c}\right)\right). \tag{11.50}$$

$$\begin{aligned} & \lim_{r\to\infty} z = \infty \\ & k<0 \end{aligned} \tag{11.51}$$

These relations hold indepedently on the case of $A<0$ or of $A>1$. One difference already determined about these two cases is expressed by the relations:

$$\begin{aligned} & A<0 \\ & k<0 \\ & \dot{z}<0 \wedge \dot{H}<0 \\ \\ & A>1 \\ & k<0 \\ & \dot{z}>0 \wedge \dot{H}>0 \end{aligned} \tag{11.52}$$

Using the notation

$$x = \frac{1-A}{A} \tag{11.53}$$

from equations (11.16) and (11.17) we obtain

$$z = \frac{1}{x}\left[1 - \exp\left(-x\frac{Hr}{c}\right)\right]. \tag{11.54}$$

If $k < 0, A < 0$, we obtain from equation (11.5)

$$\frac{dA}{dt} = kA > 0$$

Therefore, with the passage of time, the parameter $A$ increases, tending to zero

$$\begin{array}{l} A \to 0^- \\ A < 0 \quad . \\ k < 0 \end{array} \tag{11.55}$$

In that case, we obtain from equation (11.53)

$$\lim_{A \to 0^-} x = -\infty$$

and from equation (11.54) we also have

$$\lim_{x \to -\infty} z = \lim_{x \to -\infty} \frac{1}{x}\left[1 - \exp\left(-x\frac{Hr}{c}\right)\right] = 0. \tag{11.56}$$

Therefore, in the case of $k < 0, A < 0$, the redshift shall become unobservable in a theoretically infinite time.

In the case of $k < 0, A > 1$, we obtain from equation (11.5)

$$\frac{dA}{dt} = kA < 0$$

and hence, with the passage of time the parameter $A$ decreases, tending to become one

$$\begin{array}{l} A \to 1^+ \\ A > 0 \quad . \\ k < 0 \end{array}$$

(11.57)

In this case, we derive from equation (11.53)

$$\lim_{A \to 1^+} x = 0^-$$

and from equation (11.54) we also have

$$\lim_{x\to 0^-} z = \lim_{x\to 0^-} \frac{1}{x}\left[1-\exp\left(-x\frac{Hr}{c}\right)\right] = \frac{Hr}{c}. \tag{11.58}$$

Therefore, in the case of $k<0, A>1$. With the passage of time equation (11.16) becomes eventually the Hubble's law. Finally, in the cases where $k<0$ equation (11.29) can be written in the form

$$r = -\frac{c}{k}\ln\left(\frac{A}{1-z(1-A)}\right). \tag{11.59}$$

$$\kappa < 0$$

We have studied the case of $J_1 = J_2 = J_3 = 0$ in equation (3.12) in order to bypass the consequences on the redshift produced by the proper motion of the electron. Thus, from equation (3.6) we obtain

$$M_0 = \pm i c_0$$

From equation (11.2) we also have

$$M_0 = \pm\frac{k\hbar}{c^2}. \tag{11.60}$$

From equation (11.17) we obtain that the constant $k$ obtains an extremely small value. Therefore, the same holds and for the rest mass $M_0$, as a resu lt of equation (11.60).

From equation (11.5) we conclude that the parameter $A$ varies only very slightly with the passage of time. The age of the Universe is correlated at a greater degree with the value of the parameter $A$ we measure today, and less with Hubble's parameter $H$.

All of the presented consequences of theorem (3.3) are recorded within the cosmological data [16-26]. For the confirmation of the predictions of the theorem for the initial state of the Universe the improvement of our observational instruments is demanded.

In the observations conducted for distances of cosmological scales, we observe the Universe as it was in the past. That is, we observe directly the consequences of the Selfvariations. We do not possess this possibility for the distances of smaller scales. The cosmological data are the result of the immediate observation of the Selfvariations and their consequences.

## 12. Other Consequences of the Theorem of the Internal Symmetry

The consequences of the theorem of the internal symmetry cover a wider spectrum, than the one already stated for the cosmological data. In these, the consequences of the dependence of the function $\Phi$ on time $x_0 = ict$ are recorded. The function $\Phi$, according to equation (3.9), is a function of the set of the coordinates and also of the constants { $x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3$ }, and given as

$$\Phi = \Phi\left(x_0, x_1, x_2, x_3, c_0, c_1, c_2, c_3\right) = K \exp\left[\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right)\right]. \tag{12.1}$$

As in the previous paragraph, we refrain our study in the case of $\Phi \neq -1$, as included in theorem 3.3.. This case is equivalent with the relation: $C \neq 0$.

quations (3.10) and (3.13) express the rest mass $m_0$ of the material particle and the rest energy $E_0$ as a function of $\Phi$

$$m_0 = \pm \frac{M_0}{1+\Phi}. \tag{12.2}$$

$$E_0 = \mp \frac{M_0 c^2}{1+\Phi}. \tag{12.3}$$

$$E_0 + m_0 c^2 = M_0 c^2. \tag{12.4}$$

In these equations the only constant is the rest mass $M_0$ of the generalized particle.

The rest mass $m_0$ of the generalized particle depends on the position $A\left(x_0, x_1, x_2, x_3\right)$ of the generalized particle in the chosen system of coordinates $O\left(x_0, x_1, x_2, x_3\right)$. Also, the rest mass $m_0$ of the material particle depends on the set of constants { $c_0, c_1, c_2, c_3$ }, by regarding the following argument: A generalized particle with rest mass $M_0$ can be at point $A\left(x_0, x_1, x_2, x_3\right)$ in many different states, depending on the values of the constants $c_0, c_1, c_2, c_3$. There is an infinite number of four-vectors $C$, hence an infinite nuber of the generalized particle states at point $A\left(x_0, x_1, x_2, x_3\right)$, for which equation (3.6) holds

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = -M_0^2 c^2. \tag{12.5}$$

We now deduce corollary 12.1 of theorem 3.3.

**Corollary 12.1.** ‘’ The only constant physical quantity for a material particle is its total rest mass $M_0$. The evolution of the Universe, or of a system of particles, or of one particle, does not depend only on time. Its evolution is determined by the Selfvariations, as this manifestation is expressed through the function $\Phi$.’’

**Proof.** Corollary 12.1 is an immediate consequence of theorem 3.3. □

According to corollary 12.1, each material particle is uniquely defined from the rest mass $M_0$ of equations (12.2) and (12.3).

From equations (2.4), (2.5) and (3.5) , and since it holds that $x_0 = ict$ , we can write the function $\Phi$ in the form

$$\Phi = \Phi(t, x_1, x_2, x_3) = K \exp\left(\frac{b}{\hbar}\left[-(W+E)t + c_1 x_1 + c_2 x_2 + c_3 x_3\right]\right) \tag{12.6}$$

with the sum $W + E = -icc_0$ being constant. This equation gives $\Phi$ as a function of time $t$ , instead of the variable $x_0 = ict$ .

In the afterword we present the reasons, according to which the TSV strenghtens at an important degree the Theory of Special Relativity [27-28]. In contrast, the theorem of internal symmerty highlights a fundamental difference between the TSV and the Theory of General Relativity. According to equations (12.1) and (12.2), the physical quantity, which is being introduced into the equations of the TSV and remains invariant in repsect to all of the systems of reference, is the quantity given by

$$\delta = \frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3\right) \in \mathbb{C} \ . \tag{12.7}$$

Therefore, the TSV studies the physical quantity $\delta$ , and not, the also invariant with respect to all systems of reference, physical quantity of the four-dimensional arc length

$$dS^2 = (dx_0)^2 + (dx_1)^2 + (dx_2)^2 + (dx_3)^2 \ . \tag{12.8}$$

This arc length is studied by the Theory of General Relativity. The study of $dS^2$ can be interpreted in the manner that the Theory of General Relativity is a macroscopic theory. On the contrary, in the TSV a differentiation between the levels of the macrocosm and the microcosm does not exist. In equations (3.12), and for the energy and the momentum of the material particle,

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3 \tag{12.9}$$

the concept of velocity does not exist. With the exception of equations (4.19) and (4.20), within the totality of the equations of the TSV we already presented, the concept of velocity does not enter. As we will see in the following, theorem 3.3 which justifies the cosmological data predicts the the uncertainty of the postion-momentum of the material particles. The difference among these two theories is highlighted in a concrete manner by the comparison of equations (12.7) and (12.8). In the first, spacetime appers together with the four-vector $C$ . The second equation refers only to spacetime.

We present an example which highlights the diffrences among these two theories. It is the famous Twin Paradox. We consider that the reader is familiar with this thought experiment, as well as the result of the Theory of General Relativity [29]. The Theory of General Relativity predicts correctly the time difference in the time duration counted by the two twins. On the other hand, according to corollary 12.1, this time difference does not suffice for providing a difference in the evolution of the twins. The twins have the same generalized particles, which acquire the same rest masses $M_0$, at the time they meet together. At the beginning and at the end of the travel the two twins are identical. Einstein drives the wrong conclusion, not because the Theory of General Relativity is wrong, but because he regards that this time difference implies and a different evolution of the twins. But, this is not a characteristic of the Theory of General Relativity. This is a common characteristic of all the physical theories preceding the TSV.

At this point let me commentate. Einstein refers to this thought experiment as the "Twin Paradox", and not as a consequence of the Theory of General Relativity. According to my opinion, Einstein understood that something was missing from the Theory of General Relativity. To this pont advocates and his peristance for determining the cause of the quantum phenomena.

We consider a generalized particle with rest mass $M_0$ at a particular point $A(x_0, x_1, x_2, x_3)$ of spacetime. According to equations (12.1), (12.2) and (12.9), the material particle of the generalized particle can be found in an infinite number of different states, as these are defined by the four-vector $C$. If we consider that the four-vector varies, and that equation (12.5) holds always, since we refer to a concrete generalized particle, we obtain from equation (12.9)

$$\frac{\partial J_i}{\partial c_k} = \frac{1}{1+\Phi} - \frac{c_i}{(1+\Phi)^2}\frac{\partial \Phi}{\partial c_k}$$

and with equation (12.1) we have

$$\frac{\partial J_i}{\partial c_k} = \frac{1}{1+\Phi} - \frac{b}{\hbar} x_k \frac{\Phi c_i}{(1+\Phi)^2}, \mathrm{k}, \mathrm{i} = 0,1,2,3. \tag{12.10}$$

Equation (12.10) gives the variation of the four-vector $J$, as the four-vector $C$ varies, with the generalized particle being positioned at the same point $A(x_0, x_1, x_2, x_3)$ of spacetime. For $i = k$, and from equation (12.10) we obtain

$$\frac{\partial J_k}{\partial c_k} = \frac{1}{1+\Phi} - \frac{b}{\hbar} x_k \frac{\Phi c_k}{(1+\Phi)^2}, k = 0,1,2,3. \tag{12.11}$$

From equation (12.6) we have that if $K, b \in \mathbb{R}$, then $\Phi \in \mathbb{R}$. In this case, we can derive the consequences for the material particle, according to if the rates of change of the quantities

$$\frac{\partial J_i}{\partial c_k}, \frac{\partial J_k}{\partial c_k}$$

are positive, negative, or zero, respectively. We prove corollary 12.2 of theorem 3.3.

**Corollary 12.2.** "For $K,b \in \mathbb{R}$ , the following hold:

1. $\frac{\partial J_k}{\partial c_k} > 0 \Leftrightarrow \frac{\partial P_k}{\partial c_k} < 0 \Leftrightarrow x_k J_k < \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_k < \frac{\hbar}{b}, k = 0,1,2,3$ (12.12)

2. $\frac{\partial J_k}{\partial c_k} < 0 \Leftrightarrow \frac{\partial P_k}{\partial c_k} > 0 \Leftrightarrow x_k J_k > \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_k > \frac{\hbar}{b}, k = 0,1,2,3$ (12.13)

3. $\frac{\partial J_k}{\partial c_k} = 0 \Leftrightarrow \frac{\partial P_k}{\partial c_k} = 0 \Leftrightarrow x_k J_k = \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_k = \frac{\hbar}{b}, k = 0,1,2,3$ (12.14)

4. $\frac{\partial J_i}{\partial c_k} > 0 \Leftrightarrow \frac{\partial P_i}{\partial c_k} < 0 \Leftrightarrow x_k J_i < \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_i < \frac{\hbar}{b}, k \neq i, ki \neq 0, k,i = 0,1,2,3$ (12.15)

5. $\frac{\partial J_i}{\partial c_k} < 0 \Leftrightarrow \frac{\partial P_i}{\partial c_k} > 0 \Leftrightarrow x_k J_i > \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_i > \frac{\hbar}{b}, k \neq i, ki \neq 0, k,i = 0,1,2,3$ (12.16)

6. $\frac{\partial J_i}{\partial c_k} = 0 \Leftrightarrow \frac{\partial P_i}{\partial c_k} = 0 \Leftrightarrow x_k J_i = \frac{\hbar}{b\Phi} \Leftrightarrow x_k P_i = \frac{\hbar}{b}, k \neq i, k,i = 0,1,2,3$'' (12.17)

**Proof.** We first consider that $\Phi > 0$, hence $\frac{\Phi}{1+\Phi} > 0$ .These relations do not constraint the study of the general case. We just suppose positive momentum $J_i P_i, i = 0,1,2,3$ , as resulting from equations (3.12) and (3.13). From equation (12.11) we obtain

$$\frac{\partial J_k}{\partial c_k} > 0 \Leftrightarrow 1 - \frac{b}{\hbar}\frac{\Phi c_k}{1+\Phi} > 0$$

and with equations (3.12) and (1.13) we obtain the set of equivalences given in (12.12). Equivalence

$$\frac{\partial J_k}{\partial c_k} < 0 \Leftrightarrow \frac{\partial P_k}{\partial c_k} > 0$$

results from equation (3.5). The proof of the other equivalences is performed similarly, by considering also equation (12.10). In the equivalences (12.15) and (12.16) the relation among the indices $ki \neq 0$ is equivalent to relations

$$(k,i) \neq (0,1),(1,0),(0,2),(2,0),(0,3),(3,0).$$

In these cases it holds that

$$\frac{\partial J_i}{\partial c_k} \in \mathbb{C} - \mathbb{R}$$

As a consequence, equivalences (12.15) and (12.16) become meaningless. □

AS the four-vector $C$ varies in such a way, so that the four-dimensional momentum of the material particle decreases, and equivalently the four dimensional momentum of the material particle increases in the surrounding spacetime of the material particle, there exists an uncertainty in its position and momentum. This uncertainty is expressed by Heisenberg's Uncertainty Principle [30]. On the other hand, when the four-vector $C$ varies in such a way, so that its four dimensional momentum decreases in the surrounding part of spacetime, equevalences (12.12) hold. In the case the four-vector $C$ varies in such a way, so that the four dimensional momentum of the material particle remains unchanged, or equivalently the same holds also for the four dimensional momentum in the surrounding part of spacetime, then equivalences (12.14) hold. Relationships (12.12)-(12.17) are the only constraints that the equations of TSV impose on the position of the material particle. This relationships relate to the position-momentum product.

In every measurement that we perfom in the laboratory, we alter the state of the material particle, that is the four-vector $C$. For that reason, the consequences of the position-momentum uncertainty are very intense in the laboratory.

We observe that the equivalences (12.15)-(12.17) do not correspond to a principle of the physical theories of the former century. The theorem of internal symmetry, as well as the two degrees of freedom appearing in equations (5.3) and (5.7), foundain on a novel basis the manipulation of quantum information.

From equations (3.12) and (3.13) we obtain

$$J_k + P_k = c_k, k = 0,1,2,3$$

hence, we obtain

$$\frac{\partial J_k}{\partial c_k} + \frac{\partial P_k}{\partial c_k} = 1, k = 0,1,2,3. \qquad (12.18)$$

Finally, we note that within the TSV the equivalences (12.12)-(12.17) are not considered as a hypothetical principle, but as a consequence of the theorem (3.3) of the internal symmetry.

## 13. Afterword

As an afterword we make some general comments about the TSV. Having concluded our study, it is clear that the whole network of the equations of the TSV stems from combination of the axiom of Selfvariations, as given in equation (4.2) with the principle of conservation of the four-vector of momentum, and equation (2.7). The principle of conservation of the four-vector of momentum has been derived and tested empirically, from the experimental data. Equation (2.7) is probably derived by the other two axioms. The TSV bases axiomatically the science of theoretical Physics with only three axioms. As far as I know, this is a minimum number of axioms, including the axiomatization of many mathematical or physical theories. Equation (2.7) is derived by the Theory of Special Relativity. For this reason, we begin our comments from the relation between the TSV and the Theory of Special Relativity.

The Theory of Special Relativity imposes constraints on the mathematical formulation of the physical laws. All mathematical expressions of the physical laws must remain invariant with respect to the Lorentz-Einstein transformations. The TSV imposes further constraints on the mathematical formulation of the physical laws. If we denote by $L$ the set of equations remaining invariant according to the Lorentz-Einstein transformations, and by $S$ the set of equations compatible with the Law of Selfvariations, it holds that $S \neq L$, with $S \subset L$.

One indicative example refers to the Lienard-Wiechert electromagnetic potentials. These potentials were proposed by Lienard and Wiechert in 1899 and give the correct form of the electromagnetic field and the electromagnetic radiation for an arbitrarily moving electric charge. After the formulation of the Theory of Special Relativity by Einstein in 1905, it has been proved that the Lienard-Wiechert potentials remain invariant under the Lorentz-Einstein transformations. After the formulation of the TSV it has been proved that they are not compatible with the Selfvariations. The TSV replaces the Lienard-Wiechert electromagnatic potentials with the macroscopic potentials of the TSV, which give exactly the same field, as the one produced by the Lienard-Wiechert potentials. The macroscopic potentials of the TSV are compatible with the Selfvariations, and with the Lorentz-Einstein transformations ($S \subset L$). Another characteristic of the macroscopic potentials of the TSV is the following:

"We can consider that the Selfvariations are manifested, or that the electric charge is constant, and obtain exactly the same field. This is another expression of the "internality of the Universe during the process of measurement". "

For the derivation of the Lorentz-Einstein transformations we consider two observers exchanging signals with velocity $c$. If the observers move with equal velocities to each other, the Lorentz-Einstein transformations result. If the observes exhange signals with a velocity different than $c$, for example acoustic signals, we derive another set of transformations, which are wrong. Einsten's answer was that, generally, we choose the exchange of signals propagating with $c$ by the obtained result, that is because in this way we derive the correct form of transformations.

At this point, the TSV reinforces greatly the Theory of Special Relativity. Among the material particles one constant exchange of generalized photons exists, which propagates in the macrocosm with velocity $c$. According to the TSV, the exchange of signals with velocity $c$ is not just an assumption undertaken in order to derive the Lorentz-Einstein transformations, but constitutes a continuous physical reality.

The Selfvariations of the rest masses are realized if and only if they are balanced by a corresponding emission of negative energy (STEM) in the surrounding space of the material particle, so that the conservation of energy-momentum holds. This energetic content of spacetime is expressed by the four-vector $P$ given in equation (2.5). Microscopically, the energy of STEM is expressed by equation (11.9), which is a result of equation (3.12), that is of the theorem of internal symmetry. This is expected, since the internal symmetry expresses exactly the spontaneous realization of the Selfvariations. An analogous situation occurs and for the electric charge, and also for every self-variating charge $Q$. The spontaneous emmision of negative energy (STEM) in spacetime bears two fundamental consequences.

STEM has as a consequence the continuous exhange of information among the material particles. If the Universe is finite, with finite age, there still exist regions which have not exchanged information, as a result of the finite speed of propagation of STEM. This shall be accomplished in the future time. With the passage of time every region of the Universe interacts with an even increasing part of the rest of the Universe. Now, acccording to equation (11.43), and going back in time, the uncertainty of the position of the material particles shall tend to infinity. Regions of the Universe, which shall interact through the STEM in the future time, have already interacted through the material particles in the past time. The aforementioned argument holds also in the case the Universe is infinite, and has an infinite age. The only diffrence is that all of its points have interacted throught the STEM, as well. Thus, we infer the following fundamental conclusion of the TSV: “The Universe acts as one object”.

The second consequence of STEM is the indirect dynamical interaction of the material particles (USVI). When I performed the differentiation for calculating the rate of change of the momentum of a material particle, there were some concrete data: The Law of Selfvariations predicts a cohesive mechanism for all of the interactions. Therefore, after the differentiation somehow the Lorentz force should be derived, and, in some way, the relation between the USVI with the curavature of spacetime, according to the work of Einstein on the theory of the gravitational interaction. Now we know that these two terms are contained within equation (4.19). We also know that the USVI is accompanied always by a particle, corresponding to the matrix of the external symmetry of the interaction.

For every selfvariating charge $Q$ there exist $N_J = 40$ matrices of external symmetry with different four-vectors $J, P, C, j$. Therefore, there exist $N_T = 54$ channels of interaction for each charge $Q$, and each one of these is accompanied by its own particle. In paragraph 8 we proved that in the $N_1 = 14$ symmetries of $T = Q\Lambda$ it can hold that $m_0 \neq 0$. The rest $N_T - N_1 = 40$ external symmetries have a rest mass of $m_0 = 0$.

We can also remark some features of the equations (2.10), (2.13) and (4.6). Equation (2.10) cannot be derived without the axiom of Selfvariations. The fundamental physical quantites $\lambda_{ki}, k, i = 0.1, 2, 3$ cannot be derived from the theories of Physics of the former century. That is, if we assumed that the Selfvariations were not manifested, then there would be no possible way for these quantities to be defined. From these equations comes the whole network of the equations of the TSV, including equations (2.13) and (4.6).

Equations (2.13) and (4.6) express the USVI, and furthermore correlate the corpuscular and the wave behavior of matter. The propeties of the wave function $\Psi$, as well as of the four-vector $j$ of the conserved physical quantities, are stated exactly by these equations. There exist the four laws of Maxwell exactly because the first equation of the set of equation (4.6) is decomposed into four partial equations. The theorems of paragraph 7, which define the corpuscular structure of matter, are just the consequences of equation (2.10). The same holds and for the theorem of internal symmerty and its consequences, which also result form equations (2.10).

With the $SV-T$ method we can inspect the self-consistency of the whole netwrok of the equations we presented. The TSV is a closed and self-consistent physical theory.

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