

# Particle Physics and Cosmology in the Microscopic Model

**Bodo Lampe**

II. Institut für theoretische Physik der Universität Hamburg  
Luruper Chaussee 149, 22761 Hamburg, Germany



## **Abstract**

This review summarizes the results of a series of recent papers[1, 2, 3, 4, 5], where a microscopic structure underlying the physics of elementary particles has been proposed. The 'tetron model' relies on the existence of an internal isospin space, in which an independent physical dynamics takes place. This idea is critically re-considered in the present work. As becomes evident in the course of discussion, the model not only describes electroweak phenomena but also modifies our understanding of other physical topics, like gravity, the big bang cosmology and the nature of the strong interactions.

# 1 Basic Ideas

The Standard Model of elementary particles is very successful on the phenomenological level. The outcome of (almost) any particle physics experiment can be predicted accurately within this model, and where not, by some straightforward extension. For example, one may introduce right handed neutrinos to account for tiny neutrino masses[6].

Nevertheless, it is widely believed that the SM is only an effective low-energy theory valid below a certain energy scale, which is supposed to be larger than 1 TeV. This view is based on the fact that the SM has many unknown parameters with hitherto unexplained hierarchies. Furthermore, there is one rather mysterious component, the so-called Higgs field, needed for the spontaneous symmetry breaking (SSB) to take place in the model.

In recent papers a microscopic model has been developed[1, 2, 3, 4, 5], whose central assumption is the existence of a 3-dimensional *internal tetrahedral structure* attributed to each point of Minkowski space, in which an independent physical dynamics takes place.

Under this assumption spacetime originally is 6+1 dimensional, and at the time when the tetrahedrons are formed, it fibers into internal space and Minkowski space as  $R^3 \times R^{3,1}$ .

The sites  $i = 1, 2, 3, 4$  of a tetrahedron in  $R^{6+1}$  are populated by spinor fields  $\psi$ , called [tetrans](#). The tetron on site  $i$  will be denoted  $\psi_i$ .

The fundamental spinor representation in  $R^{6+1}$  is of dimension 8. It decomposes as[38]

$$8 \rightarrow (1, 2, 2) + (2, 1, 2) = ((1, 2) + (2, 1), 2) \quad (1)$$

under the fibration  $SO(6, 1) \rightarrow SO(3, 1) \times SO(3)^1$ .

---

<sup>1</sup>Here representations of  $SO(3, 1) \times SO(3)$  are denoted by a set of 3 numbers  $(a, b, c)$ , where  $(a, b)$  are representations of the Lorentz group and  $c$  is the dimension of a  $SO(3)$ -representation. For example,  $c=2$  corresponds to a non-relativistic Pauli spinor in internal space, whose 2 spin orientations are identified with the  $SU(2)$  flavors U and D. It should be noted that  $(1,2,2)$  and  $(2,1,2)$  are complex conjugate with respect to each other, so one is the antiparticle representation of the other.

Eq. (1) means that each tetron is an isospin doublet  $\psi = (U, D)$  of 3+1 dimensional Dirac fermions U and D. One may write it as a 2-index object  $\psi_\alpha^a$ , where  $\alpha = 1, 2, 3, 4$  is the Dirac index and  $a = 1, 2$  the internal index. The internal spin will be called isospin.

Using the triplet  $\vec{\tau}$  of internal Pauli matrices an isospin (pseudo)vector

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi \quad (2)$$

may be defined for any tetron  $\psi$ . It fixes a direction in the internal space and, up to an overall constant, can be interpreted as the internal angular momentum vector of the tetron  $\psi$ .

Since the tetrons are Dirac fermions on Minkowski space,  $\vec{Q}$  can be written in terms of creation and annihilation operators of a tetron ( $a^\dagger$  and  $a$ ) and an antitetron ( $b^\dagger$  and  $b$ ) as

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi = a^\dagger \vec{\tau} a - b^\dagger \vec{\tau} b \quad (3)$$

For the calculation of the quark and lepton masses the chiral iso-vectors

$$\vec{S} := \vec{Q}_L = \frac{1}{2} \psi^\dagger (1 - \gamma_5) \vec{\tau} \psi \quad \vec{T} := \vec{Q}_R = \frac{1}{2} \psi^\dagger (1 + \gamma_5) \vec{\tau} \psi \quad (4)$$

turn out to be of particular importance. For simplicity of notation they are called  $\vec{S}$  and  $\vec{T}$  in the following. Obviously, they fulfill  $\vec{Q} = \vec{S} + \vec{T}$ .

In fig. 1 the *local* ground state of the model is drawn, a configuration with 4 tetrons on the 4 sites of a tetrahedron, their isospin vectors  $\vec{Q}$  pointing in radial directions away from the origin. These internal vectors fulfill the commutation relations of a system of decoupled internal angular momenta. In other words, they play the role of angular momentum observables corresponding to rotations of the internal  $R^3$  space.

While the coordinate symmetry is  $S_4$ , the arrangement of isospin vectors in fig. 1 respects the Shubnikov symmetry[9, 11, 12]

$$G_4 := A_4 + S(S_4 - A_4) \quad (5)$$

where  $A_4(S_4)$  is the (full) tetrahedral symmetry group and S the internal time reversal operation that changes the direction of internal spin vectors. This is equivalent to saying that S interchanges the role of the internal spinors in the following way

$$S : (U, D) \rightarrow (-D^*, U^*) \quad (6)$$

Note the arrangement fig. 1 does not respect S or internal parity  $P_{in}$ , but only the product  $SP_{in}$ . Furthermore it is chiral, the configuration with opposite internal chirality being given when the isospin vectors would point inwards instead of outwards. As will be shown in (2.1.20), this internal chirality is dynamically related to the  $V - A$  nature of the q/l interactions.

As shown later in (2.4.20) and figs. 4 and 5, the definition (5) of the Shubnikov group has to be modified in a relativistic environment containing antiparticles to

$$G_4 := A_4 + CP_{au}T(S_4 - A_4) \quad (7)$$

where C is the charge conjugation operator of a Dirac field and  $P_{au}$  the ('external') parity transformation in physical space. Since the elements of  $S_4 - A_4$  contain an implicit factor of internal parity  $P_{in}$ , the symmetry (7) actually corresponds to CPT invariance in the full of  $R^{6+1}$ , cf. (2.4.20). Furthermore, the concept of an internal time S is dispensable here, so instead of S ordinary time reversal T may be used in (7).

As for the *global* ground state the set of all tetrahedrons forms a flat 3-dimensional crystal structure within the original  $R^6$ , similar to what is shown in fig. 2. This structure may be called a hyper-crystal. It is our world, the space in which all physical processes take place. Actually it will turn out to resemble an elastic or even a fluid system, so that it may as well be called a hyper-plastics or, within the Lorentz covariant cosmological framework to be developed later, the discrete micro-elastic spacetime continuum, the 'DMESC'.

Contrary to what is drawn, the tetrahedrons extend into internal space alone, not into physical space. In other words, physical space is *defined* to be the 3-dimensional subspace of  $R^6$  orthogonal to the 3 dimensions spanned by the aligned tetrahedrons.

In addition to the coordinate alignment of tetrahedrons, there is also an alignment of isospins of neighboring tetrahedrons in fig. 2. Before the appearance of this structure the internal spins U and D, which are the building blocks of the isospin vectors, can freely rotate, and thus there is an internal spin SU(2) symmetry group. In ordinary magnetism this group is usually called Heisenberg's SU(2); in sections (2.4.16)ff it will be shown how this is related to the Standard Model SU(2)<sub>L</sub> gauge group.

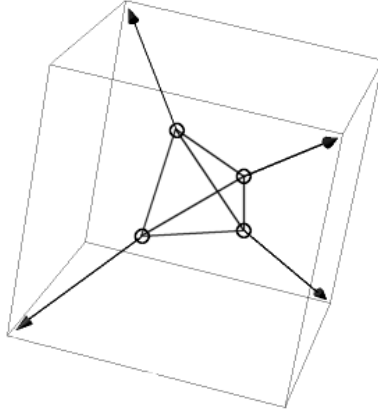


Figure 1: The local ground state of the model, living in a 3-dimensional isospin space called the 'fiber'. Shown are the tetron locations (open circles) and the 4 ground state isospin vectors  $\langle \vec{Q}_i \rangle$ , whose excitations will be identified with the spectrum of quarks and leptons. The origin of coordinates is taken to be the center of the tetrahedron, and is identical to the base point of the fiber in Minkowski space. The tetrahedron itself has the tetrahedral group  $S_4$  as point group symmetry. However, due to the pseudovector property of the isospin vectors the whole system has the Shubnikov point symmetry (5). The Shubnikov group is chiral, the configuration with opposite chirality being given when the 4 isospin vectors would point inwards instead of outwards. Before the formation of the chiral tetrahedron, the internal spins U and D, which according to (2) are the building blocks of the isospin vectors, can freely rotate and thus there is an internal spin  $SU(2)$  symmetry group, which however is broken to  $G_4$  when the chiral tetrahedron is formed. Note there are actually 2 tetrahedrons in this figure, one with respect to the internal coordinates (tetron locations) and the other one with respect to isospin vectors, and both tetrahedrons are 'aligned', in the sense that the coordinate vectors and the isospin vectors point into the same (radial) direction. This 'alignment' of coordinate and isospin vectors within one fiber has to be distinguished from the alignment of isospin vectors with respect to the isospins of neighboring tetrahedrons, as shown in fig. 2. The latter forms the basis for the electroweak phase transition, while the coordinate alignment of neighboring tetrahedrons is relevant for crystal formation at big bang temperatures.

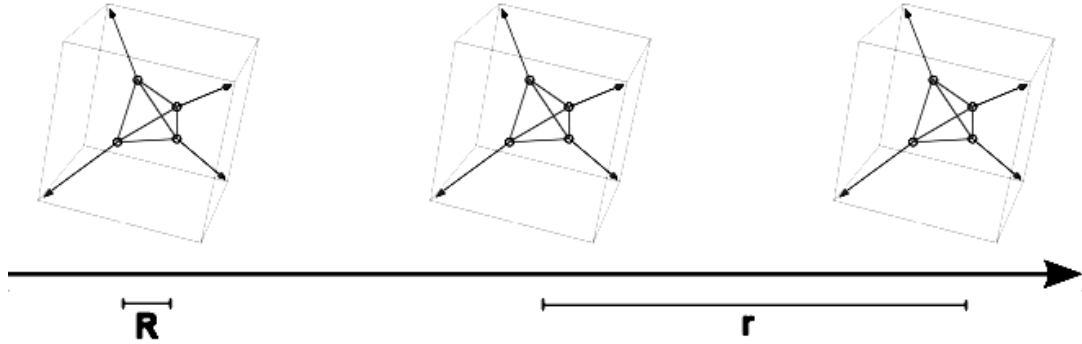


Figure 2: The global ground state of the model after the electroweak SSB consists of an aligned system of chiral tetrahedrons over physical space (the latter represented by the long arrow).  $R$  is the internal magnitude of one tetrahedron and  $r$  the distance between two of them. The figure is a bit misleading, not only because the tetrahedrons do not have an extension into physical space, but also the relative magnitudes are not correctly drawn. While  $r$  and  $R$  are tiny (of the order of the Planck length), the tetrahedrons formed by the isospin vectors are much larger, of the order of the Fermi scale (in inverse energy units). Actually there are 2 kinds of alignment in this figure: the alignment of neighboring coordinate tetrahedrons and the alignment of isospin vectors in neighboring tetrahedrons. The isospin vector alignment is associated to the electroweak symmetry breaking, because at temperatures above the Fermi scale (before the SSB) the isospins in each tetrahedron are oriented randomly (not shown) and there is a corresponding *local*  $SU(2)$  symmetry which gets broken when the isospin vectors align. The figure also shows how the universe looks like in the tetron model. It is a 3-dimensional 'monolayer' of internal tetrahedrons whose average distances are given by the Planck length  $\langle r \rangle = L_P$ . Gravity is due to the elasticity of the coordinate bonds between the tetrahedrons and corresponds to tiny deviations from this average in the vertical or horizontal direction. Finally, the coordinate alignment of the tetrahedrons is related to crystal formation at the big bang. Cosmic inflation is due to the sudden release of crystallization energy. These latter issues will be discussed in detail on pages 14-22 and in section 2.5.

An important point is that the  $SU(2)$  transformations are local symmetries in the sense that the isospin vectors can be rotated independently over each point of Minkowski space. The group gets broken to  $G_4$  when the internal arrangement fig. 1 is formed. It may be given the index L, because this arrangement is chiral and because there is a dynamical relation between internal and external chirality, as explained in (2.1.20).

The mixing with the electromagnetic  $U(1)$  symmetry has not been introduced at this point. This omission will be clarified later in (2.1.7) and (2.1.11), together with the tetron model interpretation of the electroweak mixing angle.

One wants to interpret the 3 generations of quarks and leptons as isospin wave excitations of the internal isospin structure. These excitations will be called [mignons](#). They behave as quasi-particles while they travel through Minkowski space and can be classified according to representations of  $G_4$ , as shown below.

$G_4$  is a finite group which remains intact to the lowest energies. As shown in [9] it has only 1- and 3-dimensional representations. To generate all possible excitations describing the quarks and leptons one has to consider the vibrations of  $\vec{S} = \vec{Q}_L$  and  $\vec{T} = \vec{Q}_R$  for each of the 4 tetrans separately, cf. (2.3.1) and (2.4.16).

The isospin vibrational excitations are described by deviations  $\delta$  from the ground state fig. 1, i.e.

$$\vec{S}_i = \langle \vec{S}_i \rangle + \delta \vec{S}_i \quad \vec{T}_i = \langle \vec{T}_i \rangle + \delta \vec{T}_i \quad (8)$$

or, more precisely, by certain linear combinations of them – the eigenmodes of the isospin Hamiltonian to be discussed later in (14) and (15).

The resulting 24 mignon states can be arranged in six singlet and six triplet representations  $A_{\uparrow,\downarrow}$  and  $T_{\uparrow,\downarrow}$  of  $G_4$  to yield precisely the multiplet structure of the 24 fermion states of the 3 generations, not less and not more:

$$\begin{aligned} A_{\uparrow}(\nu_e) + A_{\uparrow}(\nu_{\mu}) + A_{\uparrow}(\nu_{\tau}) &+ T_{\uparrow}(u) + T_{\uparrow}(c) + T_{\uparrow}(t) + \\ A_{\downarrow}(e) + A_{\downarrow}(\mu) + A_{\downarrow}(\tau) &+ T_{\downarrow}(d) + T_{\downarrow}(s) + T_{\downarrow}(b) \end{aligned} \quad (9)$$

The SM quantum numbers can be recovered from this spectrum in the following way:

–the  $\uparrow$  representations can be obtained from the  $\downarrow$  ones by the transformation

$\delta\vec{S} \leftrightarrow \delta\vec{T}$  for any of the tetrons, i.e. by interchanging left and right. As shown in (2.4.16) this is precisely what is needed for a weak isospin transition on the level of mignons.

–singlet and triplet Shubnikov states have a different U(1) charge. The corresponding symmetry can be interpreted as gauged tetron or  $B - L$  number. Details will be given in (2.1.7) and (2.2.3). The mixture with the photon and the appearance of the Weinberg angle will be discussed in (2.1.11).

–the 3 states within each triplet  $T$  in (9) are always degenerate, because  $G_4$  remains unbroken. The relation between those triplets and the QCD color triplets of quarks will be further discussed in (2.3.28).

Actually, to obtain the quark and lepton spectrum (9) a discrete structure is compelling only in internal space, not in physical space. Looking at fig. 2, one could try to come along with a continuous model of Minkowski space, i.e. with  $r \rightarrow 0$ . However, it is tempting to assume  $r \neq 0$ , i.e. that there is a sort of lattice underlying spacetime, with spacings so small that Lorentz symmetry is effectively maintained for all available energies.

Details of this idea will be discussed after (18) and in section 2.5, where it will be shown that the lattice must be (i) elastic and (ii) a Planck lattice, otherwise it would contradict (i) cosmological observations and (ii) Einstein's principle of equivalence[15, 16]. Due to quantum fluctuations it may be a foam[7] or a spin network[8] – although in the tetron model there is no a priori necessity to quantize gravity, cf. (2.5.39).

Can the aligned structure fig. 2 be understood heuristically? The answer is yes, if one assumes that the arrangement of isospin vectors follows similar rules than that of spin vectors in a magnetic environment, cf. (2.2.12). What matters are value and sign of (internal) exchange integrals  $J$  of tetron wave functions as a function of the distance between 2 tetrons, because these integrals will appear as couplings in the Heisenberg isospin Hamiltonian (14).

The behavior of isospins in fig. 2 can then be understood via the so-called Bethe-Slater curve shown in fig. 3. If the tetrons are part of one tetrahedron, their distance is small  $\sim R$  and according to the figure  $J$  is negative. This corresponds to anti-ferromagnetic behavior and leads to the formation of the frustrated structure fig. 1



with symmetry  $A_4 + S(S_4 - A_4)$ , because the spin vectors try to avoid each other as far as possible.

In contrast, if the internal spin vectors belong to different tetrahedrons, the distance of the corresponding tetrons is somewhat larger, of order  $r$ , and  $J$  is positive. This corresponds to ferromagnetic behavior and leads to the isospin alignment of neighboring tetrahedrons fig. 2.

Due to the tetrahedral 'star' structure fig. 1 it is appropriate to change the notion of isospin. Usually in an (anti)ferromagnetic environment, the spin vectors align into the + or - orientation of the  $z$ (=magnetization) direction, and the corresponding Pauli spinors are given by  $U = (1, 0)$  and  $D = (0, 1)$ . In the present case the (iso)magnetic structure is defined by isospin vectors either pointing outwards or inwards in the radial direction. Correspondingly, the isospinors  $U$  and  $D$  are to be understood as 'radial' spinors[22]

$$\begin{aligned} U_\star &= \sqrt{\frac{1}{3}}Y_1^0 U - \sqrt{\frac{2}{3}}Y_1^1 D = \cos \frac{\vartheta}{2} U + \sin \frac{\vartheta}{2} e^{i\frac{\varphi}{2}} D \\ D_\star &= \sqrt{\frac{2}{3}}Y_1^{-1} U - \sqrt{\frac{1}{3}}Y_1^0 D = \sin \frac{\vartheta}{2} e^{-i\frac{\varphi}{2}} U - \cos \frac{\vartheta}{2} D \end{aligned} \quad (10)$$

where  $Y_l^m$  denote the spherical harmonics and  $\vartheta$  and  $\varphi$  are the angles of the radial vector w.r.t. some cartesian coordinate system. These new spinors are radial in the sense that they reproduce the unit vector in polar coordinates

$$\vec{e}_r = U_\star^\dagger \vec{r} U_\star = -D_\star^\dagger \vec{r} D_\star \quad (11)$$

Furthermore they are normalized in such a way that

$$U_\star^\dagger U_\star + D_\star^\dagger D_\star = U^\dagger U + D^\dagger D \quad (12)$$

The iso-spinor state corresponding to an isospin vector pointing outwards is denoted by  $U_\star$ . According to figs. 1 and 2 it is the building block of the hyper-crystal in its ground state. As shown in sections 2.1 and (2.2.6) it is unpolarized and its electric charge vanishes.

Note that this presentation is equivalent to the 'universal'  $z$ -axis approach[70, 71] used in the actual mass calculations[2]. Although according to (11)  $D_\star$  has as much to do with  $U$  as it has with  $D$ , I will often leave out the star index in the following

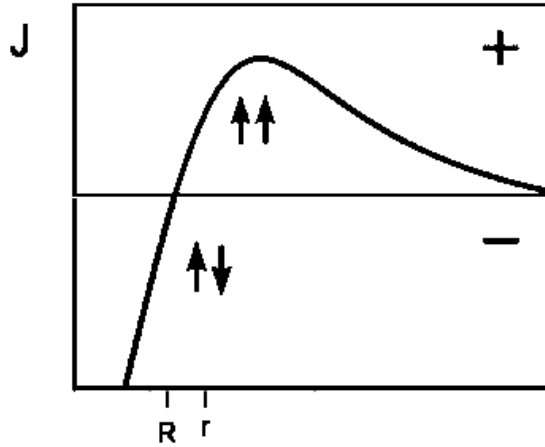


Figure 3: Bethe-Slater curve: the exchange integral coupling  $J$  as a function of the distance between 2 tetrons.

for reasons of simplicity and understand that always  $U_*$  and  $D_*$  are meant. I will include the star only when this is needed for clarity, e.g. in (2.1.8).

In the tetron model the SM SSB arises from the 'ferromagnetic' alignment of isospin vectors in neighboring tetrahedrons. As shown in (2.1.8) and (2.3.11), the corresponding order parameter is given by a non-vanishing vacuum expectation value

$$\langle \bar{U}_* U_* \rangle \neq 0 \quad (13)$$

In other words, there is a pairing active comparable to the formation of Cooper pairs in a superconductor, and excitations of this tetron-antitetron pairing will appear as the physical Higgs field and the electroweak bosons. *This is the way the SM Higgs mechanism is realized on the microscopic level.*

In [2] the masses of the mignons (9) have been calculated, and the observed hierarchy in the quark and lepton spectrum as well as the hierarchy in the CKM and non-hierarchy in the PMNS matrix elements has been reproduced. As described above, mignon masses can be identified with the eigenfrequencies of the vibrations of the isospin vectors  $\vec{S}$  and  $\vec{T}$ . These eigenfrequencies get contributions both from inner- and from inter-tetrahedral interactions.

Firstly, the *inner*-tetrahedral interactions are responsible for the frustrated tetrahe-

dral configuration fig. 1, i.e. for the structure of the local vacuum. They are small distance contributions and relatively simple to treat because they can be described by an internal Heisenberg Hamiltonian for one tetrahedron alone, with corresponding internal spin vector excitations. The most general form of this Hamiltonian is

$$H_H = -J_{SS} \sum_{i \neq j=1}^4 \vec{S}_i \vec{S}_j - J_{TT} \sum_{i \neq j=1}^4 \vec{T}_i \vec{T}_j - J_{ST} \sum_{i,j=1}^4 [\vec{S}_i \vec{T}_j + \vec{T}_i \vec{S}_j] - K_{ST} \sum_{i=1}^4 \vec{S}_i \vec{T}_i \quad (14)$$

where the couplings  $J$  are internal exchange energy densities characteristic for the internal Heisenberg interactions. By introducing  $K_{ST}$ , I have allowed that the coupling  $\vec{S}_i \vec{T}_j$  is different within a site ( $i = j$ ) than outside of it ( $i \neq j$ ).

Using (14) and (8) one is led to e.o.m. for  $\delta \vec{S}_i$  and  $\delta \vec{T}_i$  which can be solved in a similar way as the e.o.m. for magnons in solid state physics. On this basis the contributions from (14) to the eigenfrequencies of the 24 eigenmodes were calculated in [2].

Secondly, the *inter*-tetrahedral interactions are based on the parallel (=‘ferromagnetic’) alignment of isospins between different tetrahedrons fig. 2. Their leading effect turns out to be a contribution of order  $O(\Lambda_F)$  solely to the top quark mass[2]. Physically speaking, this interaction handicaps the specific eigenmode describing the top quark, because this mode disturbs the SSB alignment in the strongest possible way.

Mathematically, the effect can be described by adding terms to the inner-tetrahedral Heisenberg interaction with a normal ferromagnetic plus a Dzyaloshinskii-Moriya (DM) component[24]. The sum of the 2 components will yield a quasi-democratic mass matrix[21] which in leading order only contributes a term of order  $\Lambda_F$  to the top-quark mass and nothing to the masses of the other quarks and leptons.

More in detail, the Hamiltonian for the SSB interactions of neighboring tetrahedrons can be derived from the W-mass term of the SM Lagrangian. By considering a  $SU(2)_L$  gauge transformation, which removes the longitudinal components of the W-bosons from the Higgs part of the SM Lagrangian, one obtains

$$H_{SSB} = \frac{\mu^2}{4\Lambda_F} \sum_{i,j=1}^4 [\vec{S}_i \vec{S}'_j + i (\vec{S}_i \times \vec{D}_{ij}) \vec{S}'_j] \quad (15)$$

to be added to the Heisenberg Hamiltonian (14).  $\mu$  is the mass parameter of the SM Higgs potential. Only terms involving the left handed isospin vectors  $\vec{S} = \vec{Q}_L$  appear, as follows from (2.1.20) in accordance with the  $V - A$  structure of the weak interactions. In (15) the factor  $\vec{S}'_j$  denotes the left handed isospin vector of an adjacent tetrahedron. The precise relationship between (14) and (15) on one side and the SM Lagrangian terms on the other will be worked out in (2.1.14).

Eq. (15) contains a ferromagnetic interaction plus the additional DM term which is due to the non-abelian nature of the W-bosons. The overall normalization of the DM term is dictated by  $SU(2)_L$  gauge invariance, while the relative values of the DM couplings  $\vec{D}_{ij}$  are fixed by the internal  $A_4 + S(S_4 - A_4)$  symmetry[2].

Quite in general, a DM component stands for a tendency to form a rotational structure (instead of the ordinary ferromagnetic alignment of neighboring tetrahedrons depicted in fig. 2) simply because the DM term tends to rotate the spin vectors instead of aligning them. In the present case it appears as a consequence of the non-abelian nature of  $SU(2)$ . Therefore the DM term can be interpreted quite naturally, namely by the fact that a non-vanishing  $SU(2)_L$  gauge field induces a curvature of the fiber bundle formed by the system of all tetrahedrons, and the DM term simply takes care of this curvature effect to effectively maintain the aligned structure.

This argument is supported by the fact that the gauge transformation inherent in (15) leads to the SM Lagrangian in the so-called 'unitary gauge'. This point is analyzed in detail in (2.3.15).

Using (14) and (15) one can derive the e.o.m. for the isospin vectors. With the usual ansatz  $\sim \exp(i\omega t)$  one obtains a  $24 \times 24$  eigenvalue problem. The eigenvalues  $\omega$  correspond to the quark and lepton masses and were calculated in [2] as a function of the exchange couplings  $J_{SS}$ ,  $J_{TT}$  etc. In that paper it was explicitly verified that the corresponding 24 eigenstates can be arranged into 6 singlets and 6 triplets as predicted by the Shubnikov symmetry analysis (9), i.e. as 6 leptons and  $6 \times 3$  quarks. Each triplet (quark flavor) consists of 3 states with degenerate eigenvalues, because the Shubnikov symmetry  $A_4 + S(S_4 - A_4)$  is unbroken at low energies.

The dominant contribution from (15) gives the top quark a mass of the order of the Fermi scale while leaving the other quark and lepton masses unchanged. As detailed in (2.4.21), (2.4.22) and [2], b, c, s and  $\tau$  get their masses mainly from (14).

In contrast, there are no contributions from (14) and (15) to u, d, e and neutrino masses. These 10 excitations remain massless on this level. To obtain their masses one has to include additional small torsional interactions[2].

The masses of the neutrinos are particularly suppressed because the 3 neutrino modes correspond to the vibrations of the 3 components of the total internal angular momentum vector

$$\vec{\Sigma} := \sum_{i=1}^4 (\vec{S}_i + \vec{T}_i) = \sum_{i=1}^4 \vec{Q}_i = \sum_{i=1}^4 \psi_i^\dagger \vec{\tau} \psi_i \quad (16)$$

Whenever this quantity is conserved

$$d\vec{\Sigma}/dt = 0 \quad (17)$$

the neutrino masses will strictly vanish ( $\omega = 0$ ). In fact, the interactions considered so far, i.e. (14) and (15), conserve total internal angular momentum. Therefore, they fulfill (17) and give no contribution to the neutrino masses. Further details can be found in (2.4.12)-(2.4.15) and in [2].

The general solution to the eigenproblem given above does not only yield the energy eigenvalues but (via the corresponding eigenvectors) can also be used to accommodate the CKM and PMNS mixing matrices[2]. The mass eigenstates are the states corresponding to the energy eigenvalues, while the interaction eigenstates naturally correspond to the original vectors  $\vec{S}_i$  and  $\vec{T}_i$ , cf. (2.4.10).

Within this framework one can understand[2] why the CKM elements turn out to be small, whereas the PMNS matrix elements are naturally large: the lepton eigenstates (roughly given by  $\vec{S} \pm \vec{T}$ ) are 'far away' from  $\vec{S}$  and  $\vec{T}$ , while the up- and down-type quark eigenstates are relatively small deformations of  $\vec{S}$  and  $\vec{T}$ , respectively. Due to the dominant contribution from (15) the top quark triplet state has the smallest mixing matrix elements with other quarks, because it corresponds to the vibration of  $\vec{\Sigma}_L := \sum_i \vec{S}_i$  to an accuracy of less than 1%.

In summary, the present model describes the physical world as a huge ordered crystal of internal 'molecules', each molecule of tetrahedral form and arranged in such a way that the internal Heisenberg spin symmetry is spontaneously broken. As shown below, this approach not only provides a nice microscopic understanding of particle physics phenomena but in addition substantially supplements our understanding of

the inflationary big bang cosmology. In effect, it gives the phase transitions in the early universe a microscopic meaning.

To comprehend this fact, it is appropriate to redevelop the full history of the early universe within the assumptions of the tetron model: before the 'big bang' there were the free tetrans  $\psi$  floating around as a Fermi gas in  $R^{6+1}$  space at extremely high pressure and temperature. While the universe was cooling down, 3 fundamental transitions occurred:

**I. the formation of tetrahedral 'molecules'** from tetrans at very high temperature of order  $\Lambda_R$ , where the scale  $R$  is roughly given by the extension of one molecule. Although this process is not a phase transition in the strict sense it has certainly released a large amount of energy which has amplified the initial temperature of the universe.

Note that with 4 molecular sites each molecule 'fills' only 3 of the 6 spatial dimensions.

**II. the formation of the 'hyper-crystal'** from tetrahedrons takes place at somewhat lower temperatures  $T \sim \Lambda_r$ , where  $r$  is the 'lattice spacing', i.e. is roughly given by the distance between 2 tetrahedrons. This alignment of all tetrahedral structures is a coordinate alignment and to be distinguished from the isospin vector alignment (item III) describing the electroweak phase transition. It puts all 3-dimensional molecular structures in parallel thus separating an internal 3-dimensional space from the rest. In other words, the crystal expands into a 3+1-dimensional subspace of  $R^{6+1}$ , while the tetrahedrons extend into what becomes the 3 internal dimensions.

Since II corresponds to the process, in which our 3+1 dimensional universe was born, it may rightfully be called the big bang. As a crystallization process it is a first order phase transition associated with the sudden release of a large amount of energy. As will be explained later, the coordinate interactions among the tetrahedrons are of elastic type. Under this condition the outcome of phase transition II is not a crystal in the strict sense, and one may *as well call it a condensation of a hyper-plastics instead of a crystallization*, cf. (2.5.9). In any case the release of crystallization/condensation energy naturally drives an inflationary expansion of the system and the corresponding metric. Therefore, within the framework of

the tetron model, the big bang and the beginning of inflation are more or less identical. As argued in (2.5.9) and further below in this section, the characteristic scale  $\Lambda_r$  can be identified to be of the order of the Planck scale  $\Lambda_P$ .

**III. the arrangement of isospins** at temperatures of order  $\Lambda_F$ . Above those temperatures the isospin vectors fluctuate randomly with an associated internal 'Heisenberg'  $SU(2)$  symmetry, but at  $\Lambda_F$  they arrange into the chiral isomagnetic structure figs. 1 and 2. At that point the so far freely rotatable internal spins get ordered and the  $SU(2)$  is broken to the Shubnikov group (5). Note that it is a local symmetry group, because isospins can be rotated separately over each point of the Minkowski base space.<sup>2</sup>

It could happen that there is no internal coordinate order right after the crystallization, in the sense that the coordinates of tetrans in neighboring tetrahedrons are aligned.<sup>3</sup> In that case there is no global internal tetrahedral symmetry of the hypercrystal right after the condensation. However, since internal coordinate alignment is a prerequisite for the isomagnetic alignment (= electroweak transition) III, this has to be caught up later at the Fermi temperature, i.e. it takes place at about the same time as the isospin alignment.

This possibility will be called scenario C in later discussions, and in fact it has several benefits. For example, III automatically becomes a first order phase transition (cf. 2.3.16) with an associated second inflationary process that removes domain walls (cf. 2.3.24) from the visible parts of the universe. Furthermore, it is easier to understand that the electroweak phase transition is really spontaneous (cf. 2.3.15) and that the ground state fig. 1 is assumed by both the left- and the right-handed isospin vectors  $\vec{Q}_{Li}$  and  $\vec{Q}_{Ri}$  (cf. 2.4.16).

Since II happens after I, i.e. at lower temperature, one naturally expects  $\Lambda_R$  larger

---

<sup>2</sup>The relation between the internal Heisenberg  $SU(2)$  and weak isospin  $SU_L(2)$  is clarified in sections (2.4.16)ff. A more detailed description of phase transition III is given in (2.3.15) and (2.3.11). The mixing with the electromagnetic  $U(1)$  symmetry will be included in (2.1.7) and (2.1.11).

<sup>3</sup>I am not talking about a regular crystal structure in physical space, which for an elastic system is missing anyhow. Instead I am talking about the alignment of the *tetron* coordinates as depicted in fig. 2. See also the discussion at the end of (2.3.15) where the assumption of internal coordinate order is completely given up.

than  $\Lambda_r$  (i.e.  $R < r$ ) in agreement with fig. 2 and the Bethe-Slater curve fig. 3. As argued in (2.3.17) both scales are  $\geq \Lambda_P$  and much larger than the scale  $\Lambda_F$  where the isospins align. Note that while  $\Lambda_F$  approximately corresponds to the critical point of transition III, the values of the exchange integrals  $J$  and therefore the iso-magnetic behavior are determined at distances  $r$  and  $R$ , cf. (2.3.19).

To describe II in the framework of the Landau approach to phase transitions one should consider density fluctuations  $D \exp(i\vec{p}\vec{x})$  within the gaseous assembly of tetrahedral 'molecules' and use  $D$  as the order parameter of the phase transition.

For an ordinary crystal these fluctuations can be identified with phonons; in the general relativistic (GR) framework of a spacetime continuum at least some of them correspond to gravitational waves. This will become clearer below in this section, where elastic deformations of the crystal will be identified with metrical changes. More information about gravitons in the microscopic model can be found in (2.5.39) and (2.5.40).

Since the density perturbation adds to the uniform density of the tetrahedron gas, there is no symmetry under changing sign of the density wave, and so the Landau free energy expansion allows for a cubic term

$$\Delta F = \alpha(T - T_c)D^2 + \beta D^3 + \gamma D^4 \quad (18)$$

where  $T_c \sim \Lambda_r$  is the critical temperature for phase transition II.

Accepting the idea of an elastic spacetime continuum, the coordinate phase transition resembles a gas→liquid transition rather than a crystallization process. This even more, because the tetron particle density  $D$  is an order parameter characteristic for a condensation. While during a crystallization process the lattice symmetry plays an important role, in the hyper-crystal the bonds are elastic and there is no lattice symmetry at all in the Minkowski base space. When calling our universe a hyper-'crystal' and identifying the big bang with its 'crystallization', it should therefore be kept in mind that it shares more properties with a liquid or a deformable plastics than with a real crystal. This topic will be taken up in (2.5.10).

The appearance of the cubic term in (18) is characteristic for a first order phase transition where a second minimum, which develops in the potential when the temperature is lowered, for some time remains higher than the minimum at  $D = 0$  of



the gas phase, and furthermore the two minima are separated by a potential wall. When the temperature drops below the critical value, there is a discontinuity which is not present in second order transitions.

The latent heat associated with this discontinuity is released very suddenly and can be used to explain the extreme acceleration needed for cosmic inflation[13, 14]. As shown in (2.5.23) it provides all the necessary ingredients for the inflationary process to start and to eventually stop, once the condensation of tetrahedrons is completed and the energy is exhausted. Most of the initial molecular energy has then been transferred to the elastic energy of the crystal. However, some of it survives in the form of tetron-antitetron excitations (gauge bosons) and is converted into mignons (quarks and leptons), as the temperature decreases further.

In place of quarks and leptons, whose existence is tied to the isospin ordering fig. 2, shortly after crystallization other excitations are more important, like the internal coordinate vibrations discussed in (2.3.25) and [3], or excitations of the tetron-antitetron bonds discussed in (2.1.8) and (2.1.9). Most prominent among the latter are the gauge bosons and the visible and dark scalars of the 2HDM sector as described in (2.1.14) and (2.5.35).

Because of the dominance of the electroweak bosons this cosmological era is often called 'radiation dominated' or 'electroweak'. At temperatures far above the Fermi scale all these excitations are effectively massless states transforming under the local  $SU(2) \times U(1)$  symmetry, and they dominate the universe all the way down to the electroweak SSB (=isospin alignment). More details about the tetron model view on this era can be found in (2.1.12), (2.1.13), (2.3.8) and (2.3.9).

As well known, in general relativity a non-vanishing energy momentum tensor leads to a curvature of the spacetime continuum. Many authors have interpreted this on the basis of metric elasticity[83, 89, 90, 92, 91], and some of them have speculated that gravity forces might be explainable from a microscopic structure which in some sense is analogous to the atomic structures responsible for material elasticity in low-energy physics. In such a framework, GR is equivalent to an elastic continuum and the Einstein equations are not a fundamental but only an effective description of the microscopic dynamics, only valid at distances much larger than the Planck length.

Taking this point of view one avoids the main two problems of general relativity:

(i) the problems of quantization - as discussed in (2.5.39) and  
(ii) the existence of singular solutions - because in the micro-elastic interpretation a solution to the Einstein equation makes no sense at distances of the order of one lattice spacing  $L_P$ , where the discrete nature of the hyper-crystal become apparent. In the following I will describe some details of this approach and adapt it to the requirements of the tetron model. The fundamental dynamical quantity in general relativity is the metric which defines the distance between 2 spacetime points. It can be calculated e.g. from the transition function between arbitrary local coordinates  $x_\mu$  on the manifold and local Lorentz coordinates  $\xi^\alpha$  of an inertial system via

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\beta}{\partial x^\nu} \quad (19)$$

where  $\eta_{\alpha\beta}$  is the Minkowski metric valid in the inertial frame. Using this definition one can go further ahead and write down the curvature/field strength and the Einstein equations.

In the micro-elastic interpretation a gravitational field induces a deformation in the medium, i.e. a displacement of the internal tetrahedrons within physical space from  $x_\mu$  to  $x'_\mu$ . This corresponds to a modification of the metric

$$g'_{\rho\sigma} = g_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\rho} \frac{\partial x^\nu}{\partial x'^\sigma} \quad (20)$$

and corresponding changes in the curvature tensor.

Since GR is locally Lorentz invariant, the reader may wonder, what the physical meaning of deformations in the time direction is. In the spatial directions it is rather clear that the distances  $\sim L_P$  between neighboring tetrahedrons get modified when a gravitational field is applied. In the time direction it is the 'hopping time', which gets modified, i.e. the time a photon or some other quasi-particle needs to travel(=be emitted, run, get absorbed) from one tetrahedron to its neighbor.

Using these ideas one can understand many features of the Einstein theory. For example, the energy released during crystallization immediately blows up the distance between the tetrahedrons thus inflating the volume of the DMESC in accordance with inflationary cosmology. The initial crystallization energy is also responsible for the subsequent FLRW expansion of the universe. More details about these issues

are given in section 2.5. The tetron model view on dark energy can be found in (2.5.36).

Another example is the interpretation of the Newtonian limit. For a spherically symmetric configuration the metric can be given via the line element

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)(cdt)^2 - \left(1 - \frac{2\phi}{c^2}\right)(d\vec{x})^2 \quad (21)$$

and the Newtonian limit is defined by  $|\phi| \ll c^2$ . For a point mass  $M$  the gravitational potential far away from the source is given by  $\phi = -GM/|\vec{x}|$ . The square root of the coefficients

$$\sqrt{g_{00}} = 1 + \frac{\phi}{c^2} \quad \sqrt{g_{xx}} = 1 - \frac{\phi}{c^2} \quad (22)$$

give the general relativistic time dilation and length contraction, respectively. In the tetron model these effects are interpreted in the following way:

-The gravitational potential of the point source  $M$  modifies the average distance  $L_P$  between 2 neighboring tetrahedrons by a factor  $1 + \phi/c^2$ . As a consequence, any measured length of a physical object is modified by this factor.

-The gravitational potential of the point source  $M$  modifies the average hopping time that is needed by a hyper-crystal excitation to move from one tetrahedron to the next by a factor  $1 - \phi/c^2$ . This applies in particular for the hopping time  $T_P$  of a photon defined in (25). As a consequence, any measured time interval between physical events is modified by this factor.

It is clear then that the gravitational potential also can be understood to modify the speed of light  $c = L_P/T_P$  by a factor  $1 + 2\phi/c^2$ .

More details on the status of GR in the tetron model, as well as on FLRW, gravitational waves and the interpretation of the Newton limit will be given in section 2.5. In general one has to use the ADM formalism[46] or the approach by Carter et al.[47, 48] to describe a general relativistic elastic system which includes arbitrary transformations of the time coordinate. I have chosen to restrict myself to the special cases (21) and (91), because it makes the presentation much simpler and the arguments more transparent.

Furthermore, I have been intentionally vague about which version of GR must actually be chosen. There are generalizations like teleparallel, Poincare or Einstein-Cartan gravity where in addition to Lorentz transformations 4-dimensional translations are gauged. This leads to torsion in addition to curvature as dynamical

quantity[85, 96, 97]. Due to lack of experimental information on torsion and of full knowledge of the tetron dynamics it is difficult to say whether one needs a model which describes dislocations or disclinations[91] of tetrahedrons in a flat hyper-crystal or whether 'true' curvature effects are involved, in the sense that the tetrahedrons in fig. 2 are not only shifted by tiny amounts in the horizontal but also in the internal, i.e. vertical direction.

Personally, I give preference to the latter interpretation, because it complements Einstein's original idea of a Riemannian curvature by a physically intuitive micro-picture. One simply has to assume that there are elastic inter-tetrahedral *coordinate* interactions in addition to the isomagnetism describing the phenomena of particle physics. These elastic interactions can lead to a buckling and bulging of the 3-dimensional DMESC within the full  $R^{6+1}$  and can therefore be described by a curvature tensor. Curvature in the time coordinate is included as described above and then patched with the spatial curvature in a Lorentz covariant way a la [47, 48] by considering a matter manifold which is orthogonal to space-time. More details about this can be found in (2.5.27). Note that while the inter-tetrahedral coordinate interactions are elastic, the tetrahedral 'molecules' are rigid bodies which align in their internal spaces.

In any case, the behavior of the gravitational field is determined by the form of the gravitational action  $S_G$ . Since the equations of motion should be of second order in field derivatives,  $S_G$  must be at most quadratic in torsion and curvature

$$S_G = -\frac{c^4}{16\pi G} \int d^4x \sqrt{\det(g)} [\mathfrak{R} + O(\mathfrak{R}^2, \mathfrak{T}^2)] \quad (23)$$

where  $G$  is the Newton constant and  $\mathfrak{R}$  the curvature scalar. The explicit structure of  $S_G$ [62] is not given here because it is rather complicated, containing the leading term (formally identical to  $\mathfrak{R}$  appearing in the Einstein-Hilbert action) plus 3 independent terms quadratic in torsion and 6 quadratic in curvature, plus possibly the cosmological constant. It can be derived from an analysis which demands consistency with the principle of equivalence and the existence of second order e.o.m. and is an example of a generalized 'f(R,T)' gravity theory[86, 88].

All in all 11 independent coupling constants[62] appear in (23). This large number of free parameters is in accord with the idea that the complete description of gravity

must be quite complicated, because it is not more than an effective theory for an elastic system of microscopic entities (the internal tetrahedrons) that fill Minkowski space.

The tetron model allows to extend the view beyond this effective theory, to yield a new picture of material existence. According to this model, the world falls apart into 2 rather disparate pieces:

-firstly, *the realm of quasi-particles* like quarks, leptons, Higgs bosons and gauge fields. Since all these excitations fulfill Lorentz invariant wave equations, any phenomenon and signal propagation in this sphere is necessarily limited by the speed of light.

-secondly, *the realm of tetron matter*, i.e. of aligned tetrahedrons and of the hyper-crystal with its elastic/metrical structure.

Since the relevant scales  $\Lambda_P \gg \Lambda_F$  are so vastly different, these two spheres do not have much in common. We ourselves exist in the sphere of quasi-particles and can perceive anything coming from the tetron sector only if suitable devices of ordinary matter are patched in between. Gravity, for example, which originally corresponds to a shift of tetrahedron locations on the DMESC, becomes visible in our physical world only due to the reaction with suitable conglomerations of quasi-particles. This is discussed in more detail in (2.5.40), where also the role of gravitational waves is elucidated.

Since they are an independent form of matter, tetrons and tetrahedrons can propagate with velocities larger than  $c$ . Usually, this is not relevant because they are fixed by bindings within the hyper-crystal. However, the appearance of superluminal metrical velocities shortly after the big bang can be interpreted as bound tetrahedrons moving at larger than the speed of light, cf. (2.5.41).

If the DMESC was an ordinary crystal, one could speculate about the existence of an absolute rest system. Since it is elastic, there is only an approximate rest system at any given cosmic time, which according to the arguments given in (2.5.16) can be identified with the comoving coordinates used in cosmology to describe the Hubble flow of galaxies.

At first sight the existence of such a system seems to contradict special relativity - a well established concept which I do not want to question. Indeed, in the tetron

model all normal material objects are quasi-particle waves fulfilling Lorentz covariant wave equations. As such they cannot distinguish an absolute rest system, i.e. they naturally fulfill Einstein's principle of equivalence. On the other hand, the rest system fig. 2 is made of tetrons, and since it is merely the carrier of those quasi-particles, it is impossible to experimentally perceive it in Michelson Morley type of experiments. More details about this issue are given in (2.5.14)ff.

A particular consequence of tetron cosmology is that the lattice spacing  $r$  in fig. 2 is not fixed, but corresponds to an average distance between the tetrahedrons. In addition it varies with time (temperature) during cosmic expansion, simply because the properties of an elastic continuum depend on thermodynamic variables like temperature, pressure etc. The temperature dependence of  $c$  and  $G$  will be moderate, because most of it is contained in the energy momentum dependence of the Einstein equations.

For reasons explained in (2.5.6),  $r$  is to be identified with a time dependent Planck length, i.e. one has

$$L_P(t) = \langle r \rangle \quad (24)$$

with  $1.6 \times 10^{-35}$  m being its present value. There is clearly a relation of this quantity to the scale factor  $a(t)$  of the FLRW universe (91) because cosmic expansion is connected to a timely increase in  $L_P$ .

By definition, the Planck length is constructed from  $c$ ,  $G$  and  $h$  as one of 3 dimensional quantities which - in the absence of SM interactions - describe all the basic properties of space[m], time[s] and matter[kg]

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \quad T_P = \sqrt{\frac{\hbar G}{c^5}} \quad M_P = \sqrt{\frac{\hbar c}{G}} \quad (25)$$

One may invert these relations to obtain

$$c = \frac{L_P}{T_P} \quad (26)$$

$$\hbar = \Lambda_P T_P \quad (27)$$

$$\kappa = \frac{L_P}{\Lambda_P} \quad (28)$$

where  $\Lambda_P = M_P c^2$  is the Planck energy and  $\kappa = G/c^4$  the Einstein constant.

According to (23),  $\kappa$  is the coupling of choice in GR. As shown below, it has a rather intuitive meaning in the micro-elastic approach, and this statement actually is true

for all 3 quantities (26)-(28), because they reflect the physics of space, time and matter, as inherited in the quantities  $c$ ,  $h$  and  $G$ :

(i) since  $L_P$  is the average distance between 2 tetrahedrons, then  $c = L_P/T_P$  makes  $T_P$  the 'hopping' time it takes for a photon quasi-particle to hop from one tetrahedron to the next. The question why  $T_P$  is the time characteristic for the whole physical system of quasi-particles and valid even for gravitational waves is answered in (2.5.40).

(ii) since  $\Lambda_P \sim \Lambda_r$  is the binding energy of a tetrahedron in the DMESC, Planck's constant  $\hbar = \Lambda_P T_P$  reflects the action of the binding energy during the characteristic time, cf. (2.5.6).

(iii) finally  $\kappa = L_P/\Lambda_P$  gives the disclination of a tetrahedron in the DMESC per unit energy, i.e. applying an energy  $\Lambda_P$  to the tetrahedron will displace it by an amount  $L_P$ . In other words the gravitational coupling quantifies the elasticity of the ground state tetrahedron material, i.e. its reaction to any kind of mass/energy influx, as described by the Einstein equations. Formally, one may associate a Lamé constant (shear modulus)  $\mu$  to the tetrahedral material[63] and relate it to Einstein's constant via

$$\kappa = \frac{1}{L_P^2 \mu} \quad (29)$$

with  $\mu \approx 10^{112} kg m^{-1} s^{-2}$ . The weakness of gravity thus corresponds to an extremely large stiffness of the DMESC, which in turn is related to the rather strong forces among tetrahedrons, cf. (2.2.12), (2.3.19) and (2.3.21).

Using the value of  $\mu$  it is also possible to calculate the average present-day tetrahedral density in the universe

$$\rho_T = \frac{\mu}{c^2} \approx 10^{95} \frac{kg}{m^3} \quad (30)$$

again a very large value about 121 orders of magnitude larger than the density of ordinary (=mignon) matter  $\rho_M \approx 10^{-26} kg/m^3$ .

Eq. (29) may be inverted to obtain

$$\mu = \frac{\Lambda_P}{L_P^3} \quad (31)$$

According to this formula, the Lamé parameter can be interpreted as an energy density, which is of the order of the energy density of the vacuum arising in quantum field theories, if instead of renormalization one applies the Planck scale as a cutoff for

divergent integrals[64]. This result is no accident, because the vacuum of quantum field theories is the zero point energy of all quantum oscillators, and in the framework of the tetron model is determined by the vacuum state of aligned tetrahedrons fig. 2, cf. (2.5.29).

According to the tetron model,  $h$ ,  $c$  and  $G$  are derived and (moderately) temperature dependent<sup>4</sup> material properties of the DMESC, not valid outside of it, the only fundamental force (valid over full  $R^{6+1}$ ) being the unknown interaction (2.2.11) among the tetrons. According to the above  $h$ ,  $c$  and  $G$  are determined by the

- average 'lattice spacing'  $L_P$  between tetrahedrons,
- the tetrahedral density  $\rho_T$
- and by the elastic modulus  $\mu$ .

Therefore they are in principle calculable from the fundamental force among tetrons.

The same is true for the constants of particle physics appearing in the SM lagrangian, i.e. for the Higgs parameters, the fine structure constant  $\alpha$  (electric charge  $e$ ) and the weak mixing angle. While the latter will be derived in (2.1.12) and (2.1.6) from the fact that the photon is of  $D_\star$ -tetron content only,  $\alpha$  can be interpreted on a similar level as  $h$ ,  $c$  and  $G$ , simply because the photon - whose coupling defines  $\alpha$  - in the tetron model is not a fundamental but a quasi-particle confined to the hyper-crystal. Finally, the Higgs parameters can be traced back to isomagnetic exchange interactions of tetrons, as described in the first half of this section.

More details of the tetron model meaning of these quantities will be given in sections (2.5.6) for  $h$ , (2.5.40) for  $c$ , (2.3.21) for  $\alpha$ , (2.1.13) for the Higgs potential parameters and (2.4.10) and (2.4.21) for the Yukawa couplings. Some rudimentary ideas about the form of the fundamental tetron interaction can be found in (2.2.11).

The known particles [quarks, leptons and gauge bosons, cf. (2.5.32), (2.1.2) and (2.1.3)] are interpreted as intrinsic excitations of the hyper-crystal and as such will extend over at least one lattice spacing  $r$ . Therefore measurements involving physical particles can never be more accurate than  $L_P = \langle r \rangle$ . As discussed in (2.5.6), this

---

<sup>4</sup>It has been claimed that specifying the timely evolution of these dimensional 'constants' is meaningless[53], because the standard rulers also change with time, and that the only thing that counts in the definition of worlds are the values of the dimensionless constants. This claim has been rightfully refuted for various reasons by many authors, see [54].



corresponds to Heisenberg's uncertainty principle and can be used to fix the value of  $\hbar$  as

$$\hbar = \frac{c^3}{G} \langle r \rangle^2 \quad (32)$$

Similar for the speed of light: since the photon is an excitation of the hyper-crystal, a temperature dependence of  $c$  is to be expected and calculable from crystalline parameters, like that of the speed of sound in an everyday elastic medium. A simple argument will now be given why this is not detectable in present day experiments. The point is that photons on a lattice with spacing  $L_P$  have a dispersion

$$c(k) = \frac{2c_0}{L_P k} \left| \sin \frac{kL_P}{2} \right| \approx c_0 + O(kL_P)^2 \quad (33)$$

i.e. for wavelengths  $\lambda = 2\pi/k$  much larger than  $L_P$  the speed of light is constant to a very good approximation. Even with the hardest and 'oldest' cosmic gamma rays observed so far deviations from  $c_0$  cannot be tested.

Note that (33) relies on the existence of an equilibrium state and therefore does not control the behavior of  $c$  at the time of inflation when the hyper-crystal was formed under non-equilibrium circumstances.

Finally it should be mentioned that (33) is not valid in the tetron model near the boundary of the Brioullin zone, because it is derived assuming a fundamental photon wave which can acquire arbitrarily small wavelengths – which in the tetron model is not possible because photons are lattice excitations and cannot be produced with wavelengths  $\leq L_P$ . For further discussion on this point see the end of (2.5.6).

## 2 Questions and Answers

In this section a list of questions and answers is presented which arise in connection with the tetron model. Open problems will be specially marked and the more important ones reviewed in the summary section 3.

### 2.1 Questions about the Gauge Sector

According to section 1 the universe is interpreted as a discrete fiber bundle over Minkowski space  $R^{3+1}$  with fibers given by the iso-magnetic tetrahedrons fig. 1.

The electroweak gauge fields are to be interpreted as connections in that fiber space, i.e. they help to define what parallel alignment in between different fibers means.

### 2.1.1 How can such a model lead to a local gauge theory?

The internal 3-dimensional space which hosts the tetrahedrons is naturally endowed with a  $SU(2)_L \times U(1)_F$  symmetry:

- the  $SU(2)$  factor arises from the rotational symmetry of the internal spin vectors before their alignment. Some details have already been explained in section 1. The question how this symmetry is related to the weak isospin of quarks and leptons will be analyzed in (2.4.16)ff. The reason for why it receives the index L in the tetron model is explained in (2.1.20).

- the  $U(1)$  factor corresponds to tetron number conservation, which on the level of quarks and leptons translates into the  $B - L$  quantum number, cf. (2.1.7).

These groups act as *local* symmetries, because their elements can be chosen different for different points of the Minkowski base space. Connections can be defined for the  $SU(2)_L \times U(1)_F$  bundle, which are to be interpreted as gauge bosons. As shown in (2.1.11), there is a mixing of the  $U(1)_F$  field with  $W_z$ , with mixing angle equal to the Weinberg angle. After the mixing the corresponding local gauge fields are given by the observed  $W^\pm$ , Z and  $\gamma$ .

### 2.1.2 Are the electroweak bosons and the Higgs field composite?

The answer is yes, but one should specify how this works out in detail.

- The most straightforward possibility is that they are composites of mignon-antimignon pairs. However, as will be seen in (2.3.28), such a pairing is more appropriate to describe chiral symmetry breaking in QCD. Furthermore, such a construction would make the top-quark content dominate the boson sector of the SM, similar to top condensate models. Since  $m_t \gg m_W$ , this is usually not considered a convincing scenario.

- Secondly, one could be tempted to insist they are fundamental objects, because they are connections of the basic  $SU(2) \times U(1)$  fiber bundle in the sense of differential geometry. As such they could have been induced by curvature dynamics of the

full  $\mathbb{R}^{6+1}$  geometry. I do not think this is a very attractive option, because the flat  $\mathbb{R}^{6+1}$  knows nothing about the dynamics within the 'curved' hyper-crystal bundle. Furthermore, the Higgs field as a necessary add-on to account for the SSB does not have a simple interpretation in the pure differential geometric framework.

–Thirdly, they could be tetron-antitetron bound states traveling freely through the hyper-crystal. However, according to the picture developed in (2.2.4) and (2.2.5), tetrans are so strongly bound within the hyper-crystal with binding energies  $\sim \Lambda_P$ , that they cannot be split off, not even in pairs. This requirement is also dictated by the no-dissipation concept, cf. (2.5.32). Namely, one has to take care that such pairs do not leave the hyper-crystal and dissipate into  $\mathbb{R}^{6+1}$ , because otherwise energy would not be conserved inside of it.

–I adhere to the idea that they must be *excitations* of tetron-antitetron bonds, i.e. arise from the tetron interactions in the crystal. While mignons are defined on one tetrahedral fiber, the gauge bosons involve a system of 2 neighboring tetrahedrons. As excitations they thus consist in a global precession of the isospin 3-bein of one fiber with respect to the 3-bein of the neighboring fiber. There are 3 types of such precessions corresponding to the 3 internal Euler angles defining the d.o.f. of the differential geometric  $SU(2)$  connection among the fibers and thus to the 3 weak gauge bosons.

As a consequence, the gauge bosons form and travel solely inside the hyper-crystal and cannot exist outside of it. 'Traveling' of such a pairing excitation is meant in the sense of a quasi-particle, i.e. the excitation hops from one tetron-antitetron pair to another, while the tetrans themselves stick to their place in the crystal. It is possible to imagine it as a density wave bilinear in  $\bar{\psi}$  and  $\psi$  that travels through the crystal. This must be distinguished, however, from the density fluctuations involving  $\psi^\dagger$  and  $\psi$  of a single tetrahedron which are coined phinons in (2.3.25) and (2.1.14).

### **2.1.3 Can a stable and massless particle like the photon be an excitation?**

First of all, note that masslessness of the photon is protected by the  $U(1)$  gauge symmetry. As long as this symmetry holds, the photon remains massless, whether composite or not.

Furthermore, the masslessness of the photon implies its stability.

The answer to the question is 'no' in the QED7 model advocated in [1], where the ordinary photon is part of a '7-dimensional' photon, responsible for the iso-magnetic interactions. But yes within the 'no-dissipation' hypothesis advocated in this article, cf. (2.5.32). The latter has the advantage that energy is conserved for all processes inside the crystal, so no compactification of internal spaces is needed. The only objects which are not excitations are the tetrons, the building blocks of the crystal. These, however, are bound with energies  $> 10^{10}$  GeV.

The photon being an internal excitation cannot be scattered away from the hyper-crystal. Since according to (2.2.3) one has  $Q(U) = 0$ , the photon is a  $\bar{D} - D$  excitation of D-tetrons, conveniently abbreviated as

$$A_\mu \sim e Q(D) \bar{D} \gamma_\mu D \quad (34)$$

As discussed after (11), in the SSB phase actually radial isospinors should be used

$$A_\mu \sim e Q(D_\star) \bar{D}_\star \gamma_\mu D_\star \quad (35)$$

Furthermore, it must be noted that the no-dissipation hypothesis (2.5.32) has rather challenging consequences. If the photon is not a fundamental particle, it is difficult to believe that the Lorentz symmetry, valid inside the hyper-crystal with the known value of  $c$ , is a fundamental property of the original full  $R^{6+1}$  spacetime. The Lorentz structure as we know it, comes into being only when the crystal is formed and holds only inside of it. This point is further elucidated in (2.5.3), (2.5.14)ff and (2.5.40).

#### 2.1.4 What happens on the microscopic level when a mignon and an antimignon annihilate into an electroweak gauge boson?

Assume the 2 mignons are in 2 neighboring tetrahedrons. When the gauge bosons are formed, the mignon vibrations disappear and are replaced by an internal excitation of a bound  $\bar{\psi}-\psi$  pair involving 2 tetrons from the neighboring tetrahedrons. This is in contrast to [1] where these pairs were assumed to be made from free tetrons floating around. The latter idea has been abandoned because the binding energy of a tetron in the crystal is too large, of order  $\Lambda_P$ , and it would furthermore allow energy in the form of  $\bar{\psi}-\psi$  pairs to dissipate away from the hyper-crystal.

The excitations of bound pairs of tetrons behave trivially under  $A_4 + S(S_4 - A_4)$  Shubnikov transformations, i.e. the information about the discrete tetrahedral structure is washed out, because mignon and antimignon compensate each other in that respect. What remains is the transformation property under  $SU(2)_L \times U(1)_F$ . Since  $\psi$  is an isospin doublet, the product of  $\psi$  and  $\bar{\psi}$  leads to  $2 \otimes 2 = 3 + 1$ , i.e. a triplet (the weak bosons) and a singlet (the B-L photon).

These serve as connections in the fiber space. As such they are useful to define, what alignment of adjacent tetrahedrons means, cf. the discussion after (15).

### 2.1.5 Why can mignon couplings be understood as gauge couplings?

The mignons are dynamical sections in the  $SU(2)_L \times U(1)_F$  fiber bundle described above. In order to keep up gauge invariance they are naturally endowed with gauge couplings to the connections. As for the couplings of the fundamental tetron fields  $\psi = (U, D)$  one may consult (2.2.3).

### 2.1.6 What is the meaning of the initial $U(1)_F$ symmetry?

On the tetron level  $F$  is tetron number, on the mignon level it is  $B - L$ .

### 2.1.7 How do the electric charges of mignons arise?

According to (2.1.20) parity violation of the weak interaction follows from the internal chirality of the tetrahedral 'star' configuration fig. 1. This implies that there are no separate  $W_R$  bosons and that all  $V + A$  couplings to mignons necessarily vanish, cf. (2.1.18). Still it is possible to formally introduce a right handed isospin quantum number via  $I_3 = I_{3R} + I_{3L}$  (with vanishing coupling  $g_R$  due to the parity violating effect).

Furthermore,  $F = B - L = B + \bar{L}$  is the appropriate fermion number to choose for mignons (9), with  $F(l) = -1$  for leptons and  $F(q) = 1/3$  for quarks. The mixing among the neutral gauge bosons can then be described by introducing the unbroken generator  $Q$  as

$$Q = I_3 + \frac{F}{2} \tag{36}$$

so that

$$Q(u) = \frac{1}{2} + \frac{F(q)}{2} \quad Q(d) = -\frac{1}{2} + \frac{F(q)}{2} \quad (37)$$

$$Q(\nu) = \frac{1}{2} + \frac{F(l)}{2} \quad Q(e) = -\frac{1}{2} + \frac{F(l)}{2} \quad (38)$$

### 2.1.8 What is the tetron content of the Higgs field and of the SM vev?

To answer this question, the same idea is used which has led to the photon equation (34), namely that all observed scalars and vector bosons arise from correlations between tetrons and antitetrons of neighboring tetrahedrons, cf. questions (2.1.2) and (2.1.4).

One of these correlations is directly related to the electroweak SSB and is called the Higgs particle. Since it is to support the radial alignment of isospinors in fig. 2 responsible for the SSB, it can be identified as

$$H \sim \bar{U}_* U_* \quad (39)$$

where  $U_*$  is the 'radial' iso-spinor introduced in (11) corresponding to an isospin vector  $\vec{Q} = U_*^\dagger \vec{\tau} U_*$  pointing outward as in fig. 1. The point is that the content of the Higgs particle is in one-to-one correspondence with the vev needed to stabilize the alignment of isospins in fig. 2, and isospin vectors pointing outward correspond to radial spinors  $U_*$  while those pointing inwards correspond to  $D_*$ .

According to these considerations the SM Higgs doublet  $\Phi$  must be of the form

$$\Phi \sim \tau_2 (\bar{U}_{R*} Q_{L*})^{\dagger T} \sim \begin{pmatrix} -\bar{D}_*(1 + \gamma_5) U_* \\ \bar{U}_*(1 + \gamma_5) U_* \end{pmatrix} \quad (40)$$

i.e. not as in ordinary  $SU(2)_L \times SU(2)_R$  symmetric NJL theories[32, 33] but formally similar to top-color models[26] – provided the use of radial isospinors is understood. The implication of (40) on the vev and on the NJL structure inherent in the SM will be discussed in (2.1.14), (2.1.15) and (2.3.11). For use in those sections I include here the definition

$$\tilde{\Phi} := i\tau_2 \Phi^* \sim \bar{U}_{R*} Q_{L*} \quad (41)$$

### 2.1.9 What is the tetron content of the weak gauge bosons?

The answer to this question depends on whether one is talking about the ordered or about the symmetric phase. In the symmetric phase, e.g. shortly after the hyper-crystal was formed, there is only the coordinate tetrahedron but no tetrahedral 'star'-configuration of isospin vectors as in fig. 1. Therefore radial spinors (11) should play no role. The photon is given by (34) and the  $U(1)_F$  tetron number gauge boson by

$$B_\mu \sim g' F(\psi) [ \bar{U}\gamma_\mu U + \bar{D}\gamma_\mu D ] \quad (42)$$

where  $g'$  is the  $U(1)_F$  gauge coupling. Similar formulas hold for the  $SU(2)$  gauge bosons, cf. (2.1.11).

### 2.1.10 How does the transition between $(U_\star, D_\star)$ and $(U, D)$ work?

or in other words, what happens to the gauge bosons at the electroweak phase transition?

In the ordered phase, i.e. at temperatures below  $\Lambda_F$ , equations like (34) and (42) formally keep their validity,  $U$  and  $D$  naturally being replaced by  $U_\star$  and  $D_\star$ . The phase transition is the point, at which Higgs and W bosons attain their non-zero masses, and this is accompanied by a redefinition of what is meant by the isospins  $U$  and  $D$ . While in the symmetric phase  $U$  and  $D$  are defined by spanning the  $SU(2)$  isospin space, in the ordered phase one switches to an effective coordinate system spanned by  $U_\star$  and  $D_\star$ .

### 2.1.11 $\gamma$ -Z mixing and the value of the Weinberg angle in the tetron model

In (2.2.3) it is shown that  $F(\psi) = -1$  and  $Q(D) = -1$ . Using this input one can directly infer from (34) and (42) that the weak mixing angle at the unification/crystallization point  $\Lambda_r$  must be 45 degrees, i.e.  $\sin^2(\theta_w) = 1/2$ . The form of the Z-boson is

$$Z_\mu \sim -\frac{e}{\sin(\theta_w) \cos(\theta_w)} [ I_3(U) \bar{U}\gamma_\mu U + I_3(D) \bar{D}\gamma_\mu D + Q(D) \sin^2(\theta_w) \bar{D}\gamma_\mu D ] \quad (43)$$

which at  $\Lambda_r$  reduces to  $Z \sim \bar{U}\gamma_\mu U$ , i.e. at the unification point the Z consists only of U-tetrons. Left- and right-handed tetrons are not distinguished in these relations, because the SU(2) gauge bosons a priori contain lefthanded as well as righthanded tetrons. It is only the internal chirality of the configuration fig. 1 that prevents the  $V + A$  component to become active, cf. (2.1.17), (2.1.20), (36) and [1].

In the next subsection (2.1.12) the prediction  $\sin^2(\theta_w) = 1/2$  at  $\Lambda_r$  will be shown to agree with the present experimental value provided one uses 3 ingredients: (i) the evolution of the SM beta function as given in [65], (ii) eq. (44) and (iii) a value of the unification scale relatively close to Planck scale.

### 2.1.12 A 'unification scale' in the framework of the tetron model

In the tetron model the natural electroweak unification scale is given by the energy  $\Lambda_r$  at which the hyper-crystal is formed from tetrahedral 'molecules' via the phase transition II. As argued in sections 1 and (2.5.6) this scale corresponds to the average distance between 2 tetrahedrons in fig. 2 and is naturally of the order of the Planck scale. As shown in (2.1.11), at  $\Lambda_r$  the value of the Weinberg angle must be 45 degrees. This corresponds to a relation between the  $U(1)$  and  $SU(2)_L$  gauge couplings

$$g'(\Lambda_r) = g(\Lambda_r) \tag{44}$$

Note that (44) goes beyond the SM because the gauge group  $SU(2)_L \times U(1)_F$ , even in the form of a U(2) group, is not simply connected and therefore no relation between the values of  $g$  and  $g'$  is predicted within the SM. In contrast, in the tetron model a prediction is possible and given by (44). This is based on the observation that the original U(1) gauge symmetry is tetron number and that the photon according to (34) should be of D-content only.

Using the SM beta functions[65] one can extrapolate  $g$  and  $g'$  from their measured values at  $m_Z$  to ultrahigh energies in order to see for which values of  $\Lambda_r$  eq. (44) can be satisfied. Since there is no diminishing factor 3/5 in (44) like in typical GUT models[99],  $\Lambda_r$  comes out to be nearly equal to the Planck scale instead of  $\Lambda_{GUT} \approx 10^{15}$  GeV. Within the present model, this is a rather convenient result, because it allows to identify the electroweak unification scale with the crystallization temperature.



It must be stressed that this is only an order of magnitude result for the electroweak unification scale, because one may rightfully ask, whether the SM beta functions are really applicable up to such high energies, or whether they get appreciable corrections from other crystal excitations like the 2HDM Higgs partners discussed in (2.1.14), or from phinons and isospin density waves which appear at higher energies, cf. (2.3.25). Furthermore, as discussed in sections 1 and 2.5, the Planck scale is expected to be moderately cosmic time dependent in the tetron model.

### **2.1.13 Connection between the tetron model unification scale and the scales relevant for the standard cosmological model**

In the usual cosmological terminology there is the scale at which inflation ends, and this scale is usually identified as the temperature below which the radiation dominated epoqe starts. This era can be described as an equilibrium state of effectively massless electroweak gauge bosons.

In the tetron model, inflation is associated with the release of latent heat at crystal formation time. The end of inflation is the time when crystallization(=the inflation period) has finished, and the unification of the electromagnetic and the weak interactions is naturally interpreted as happening at this point. It is the time at which our 3+1 dimensional universe started to exist. According to the analysis in (2.1.12) and (2.5.6) this roughly corresponds to Planck scale energies, and therefore in the present model the electroweak era starts already at the Planck scale.

### **2.1.14 A relation between the isospin interactions (14)+(15) of the tetron model and the SM Lagrangian**

To explain this in detail, one first has to notice that while the mignon vibrators are supposed to 'live' within one tetrahedron, the Higgs excitations according to the philosophy discussed in (2.1.2) extend over two of them.

In accordance with this observation, two types of internal vectors should be distinguished:

(i) isospin vectors  $\vec{Q}$  of type (2) and (4) which are the carriers of the isospin waves (mignons). They correspond to the internal angular momentum of tetrans *within*

one tetrahedron and are the 'charges' of the conserved internal Noether currents. Their excitation spectrum leads to quark and lepton color and flavor.

(ii) fields like the gauge bosons or the  $\vec{\pi}$  component of the Higgs doublet, which involve  $\bar{\psi}$  instead of  $\psi^\dagger$ . Together with the vev  $\langle \bar{\psi}\psi \rangle \neq 0$  and the Higgs particle  $H = \bar{\psi}\psi - \langle \bar{\psi}\psi \rangle$  they are important for the pairing process between tetrons and antitetrons of neighboring tetrahedrons which in the tetron model is responsible for the SSB.

To understand this in more detail consider the SM Higgs potential with one doublet

$$V_{SM}(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 = -\frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 \quad (45)$$

where  $\sigma = \Lambda_F + H$ . This potential naturally describes the alignment of neighboring tetrahedrons and anti-tetrahedrons in fig. 2, although in that figure not the  $\vec{\pi}_i$  are drawn but the  $\vec{Q}_i$ . The point to note is that two of the  $\vec{\pi}_i$  are in parallel iff all the corresponding  $\vec{Q}_i$  are. Therefore the pairing force  $\sim -\mu^2\vec{\pi}_i\vec{\pi}_j$  implied by (45) exactly corresponds to a 'ferromagnetic exchange coupling' of strength  $\mu^2$  in the SSB interaction (15).

There is one drawback in this argument, and this concerns the number of d.o.f. While the SM Higgs doublet only has 4 real d.o.f., the isospin vibrators in the form of  $\vec{Q}_L$  and  $\vec{Q}_R$  contain 8. According to (4) these can be given as

$$\psi^\dagger\psi \quad \psi^\dagger i\gamma_5\vec{\tau}\psi \quad \psi^\dagger i\gamma_5\psi \quad \psi^\dagger\vec{\tau}\psi \quad (46)$$

I have included  $\psi^\dagger\psi$  and  $\psi^\dagger i\gamma_5\psi$  in this list albeit their vibrations do not correspond to mignons, but to phinons, cf. (2.3.25).

The expressions (46) are adapted to the  $SU(2)_L \times SU(2)_R$  symmetric limit. In more general cases, when this symmetry does not hold, the listing reads

$$U^\dagger U \quad D^\dagger D \quad U^\dagger D \quad D^\dagger U \quad U^\dagger\gamma_5 U \quad D^\dagger\gamma_5 D \quad U^\dagger\gamma_5 D \quad D^\dagger\gamma_5 U \quad (47)$$

By comparing with the Higgs doublet (40) one sees that half of the d.o.f. are missing in (40). To account for the other half one should add a second scalar doublet to the dynamics, e.g. in the form<sup>5</sup>

$$\Phi' \sim \bar{D}_R Q_L \sim \begin{pmatrix} \bar{D}(1 - \gamma_5)U \\ \bar{D}(1 - \gamma_5)D \end{pmatrix} \quad (48)$$

---

<sup>5</sup>The radial star indices (11) in these expressions are left out for simplicity.

This corresponds to adding a pseudo-scalar iso-scalar particle  $\eta$  and a scalar iso-vector triplet  $\vec{v}$  to the theory.

Together,  $\Phi$  and  $\Phi'$  form the basis for an extended SM with 2 Higgs doublets. Such models are usually abbreviated as 2HDM, and have been extensively discussed in the literature[67, 68, 69].

The argument about the ferromagnetic alignment induced by the negative mass term  $-\mu^2\vec{\pi}\vec{\pi}$  in the potential can be extended to the 2HDM model where the potential contains a term  $\sim \vec{v}\vec{v}$  in addition. This term, however, must not give an appreciable contribution to the SSB interaction (15), because otherwise the b-quark mass would come out to be of order  $\Lambda_F$ . In other words,  $\Phi'$  must not take part in the SSB; the 'mass term'  $\sim \vec{v}\vec{v}$  has to have a positive coefficient and correspondingly

$$\langle\Phi'\rangle = 0 \tag{49}$$

for the second Higgs doublet and therefore no q/l mass contributions from  $\Phi'$ .

This feature smartly agrees with the property of the inert version[68, 69] of the 2HDM model. It is interesting to note that in that model the  $\eta$  or the  $v_z$  (depending on which mass is smaller) is a serious dark matter candidate. For further details see (2.1.15) and (2.5.35).

### 2.1.15 What precisely is the argument in favor of the inert version of the 2HDM model?

The most general quark Yukawa Lagrangian in a 2HDM model is given by

$$-L_Y = \bar{q}_L(\Gamma\Phi + \Gamma'\Phi')d_R + \bar{q}_L(\Delta\tilde{\Phi} + \Delta'\tilde{\Phi}')u_R + c.c. \tag{50}$$

where 3x3 matrices of Yukawa couplings  $\Gamma$ ,  $\Gamma'$ ,  $\Delta$  and  $\Delta'$  in family space have been introduced. The resulting quark mass matrices are then given by

$$M_d = \Gamma\langle\Phi\rangle + \Gamma'\langle\Phi'\rangle \quad M_u = \Delta\langle\Phi\rangle^* + \Delta'\langle\Phi'\rangle^* \tag{51}$$

Unfortunately, the diagonalization of  $M_u$  and  $M_d$  does not in general diagonalize the quark Higgs Yukawa interactions implied by (50), and this leads to the problem that unwanted FCNCs are present in the most general 2HDM model[67]. This is usually

handled by the ad hoc introduction of an additional  $Z_2$  symmetry. For example, one may demand the 2HDM Lagrangian to be invariant under the transformation

$$\Phi' \rightarrow -\Phi' \quad (52)$$

In this case all Yukawa couplings involving  $\Phi'$  drop out, and all quarks and leptons couple solely to  $\Phi$ . Furthermore, symmetry under (52) forbids mixing terms  $\sim \Phi^\dagger \Phi' + c.c.$  in the 2HDM Higgs potential so that the Higgs field with vanishing vev can be unambiguously taken to be  $\Phi'$  in accordance with the representation (48).

In the tetron model one has explicit representations (40) and (48) for  $\Phi$  and  $\Phi'$  and may therefore ask whether the physical origin of the  $Z_2$  symmetry can be understood. It is easily seen from (40) and (48) that (52) corresponds to the transformation

$$D_R \rightarrow -D_R \quad (53)$$

among tetrons  $D_R$ . This kind of symmetry naturally arises in the tetron model, because mignon interactions with the  $D_R$  field in the effective tetron Lagrangian do not appear. The point is that according to fig. 1 the system's ground state is composed of tetrons U alone, and not of D. Since quarks and leptons are excitations of the ground state, it is thus understandable that their couplings to D are strongly suppressed. This reasoning applies only to  $D_R$  and not to  $D_L$ , because parity is broken in the tetron model already at hyper-crystal formation time, so that  $D_L$  couplings are present because  $D_L$  appears in an isodoublet with  $U_L$ .

Note in this and the previous question one is always talking about the radial isospinors  $U_\star$  and  $D_\star$  instead of U and D.

### 2.1.16 What is the tetron content of the additional scalar particles in the 2HDM?

This question is most easily answered in the  $SU(2)_L \times SU(2)_R$  symmetric limit of the model. There are 5 observable Higgs scalars in the 2HDM model given by

$$H \sim \bar{\psi}\psi \quad \eta \sim \bar{\psi}i\gamma_5\psi \quad \vec{v} \sim \bar{\psi}\vec{\tau}\psi \quad (54)$$

leading to a quadratic part of the potential[33]

$$-\mu^2\Phi^\dagger\Phi - \mu'^2\Phi'^\dagger\Phi' \sim G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + G'[(\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2] \quad (55)$$

where the terms with coupling  $G$  correspond to the quadratic term  $\sim \mu^2$  in the SM Higgs potential (45) and the terms with coupling  $G'$  to a quadratic term for the second Higgs doublet.

Although it is true that the vibrators  $\vec{Q}_{L,R}$  are chosen in a  $SU(2)_L \times SU(2)_R$  manner (4) in accordance with the discussion in (2.4.7), it is better to calculate (55) based on the representations (40) and (48) for the doublets  $\Phi$  and  $\Phi'$ . Anyway, due to the possibility of mixing, e.g. between  $\eta$ , H and  $v_z$ , the actually observed scalar excitations will in general be linear combinations of these states.

### 2.1.17 How can the chiral nature of the weak bosons be ensured?

The iso-magnetic tetrahedral structure in fig. 1 violates internal parity, the state with opposite internal parity being given by a system where the 4 internal spin vectors show inwards instead of outwards. As shown in [1] and (2.1.20) this internal parity violation triggers the violation of external parity needed for the weak interactions, provided the interaction among tetrons stems from a common interaction in the full  $R^{6+1}$  space, cf. (56) and (57).

### 2.1.18 Are there $SU(2)_R$ gauge fields $\vec{W}_R$ in addition to $SU(2)_L$ ?

No. Z and W are originally connections of an  $SU(2)$  bundle. According to the discussion in (2.1.20) it is only the formation of the chiral structure fig. 1 together with the  $R^{6+1}$  origin of the interaction which forces them to couple to left-handed mignons only. Without the internal chirality fig. 1 the weak interactions would be vectorlike.

### 2.1.19 If there is no $\vec{W}_R$ , why is there $\eta$ ?

The 2HDM model (2.1.14) naturally accompanies the vibrations of  $\vec{Q}_L$  and  $\vec{Q}_R$ . 2HDM models do not need a  $\vec{W}_R$ -field[67].

**2.1.20 Is there a tetron interaction which gives rise to such iso-magnetic structures? Can the parity violation of the weak interactions be explained from first principles?**

Before I start to discuss this question, note it is not about the fundamental coordinate forces which are responsible for the formation of (tetrahedral molecules and) the hyper-crystal at scale  $\Lambda_r$ , but only about the isomagnetic forces relevant for the isospin vector alignment at the Fermi scale. Of course, there is a connection between the 2 issues - to be explained in (2.2.11) and (2.2.12).

The model advocated in this paper consists of 6+1 dimensional spinor fields  $\psi$  ('tetrans'), which form tetrahedral structures and a hyper-crystal a la fig. 2. In this world quarks and leptons propagate through spacetime as quasi-particles made of isospin precessions. The Higgs and the observed vector bosons are excitations of tetron-antitetron bonds, and the system as a whole naturally gives rise to the  $SU(2)_L \times U(1)_F$  gauge symmetric SM. For the appearance of the strong interactions see (2.3.28), and the role of gravity is discussed in section 2.5.

The isomagnetic tetron interactions are claimed to derive from the octonion structure which is naturally inherent in a 6+1 dimensional space. The octonions form the unique non-associative, non-commutative and normed division algebra in 8 dimensions, and their imaginary units provide for 7 of the 21 generators of  $SO(6,1)$ . They are closely related to the Dirac matrices  $\Gamma_\mu$  in 6+1 dimensions[99]. A 6+1 dimensional vector current  $\bar{\psi}\Gamma_\mu\psi$  arises more or less directly from the product of two octonions corresponding to the spinors  $\bar{\psi}$  and  $\psi$ .

When the hyper-crystal is formed and  $R^{6+1}$  decomposes into Minkowski and internal space, the  $\Gamma_\mu$  split into  $SO(3,1)$  Dirac matrices  $\gamma_\mu$  and a remainder according to[99, 40]

$$\Gamma_{1-7} = (\gamma_{1-4}, \gamma_5 \tau_{x,y,z}) \quad (56)$$

where x, y and z denote the internal coordinates. This splitting has its physical origin in the coordinate interactions of the tetrans, which lead to the formation of the hyper-crystal, and mathematically it parallels the splitting of an octonion into 2 quaternions.

Starting from (56) one can try to derive the parity violation of the weak interaction. The important point to note is the appearance of the product  $\gamma_5 \vec{\tau}$  in the internal

part of (56). In principle, the presence of such a coupling corresponds already to a parity violating behavior, both in internal and Minkowski space, because  $\gamma_5$  signals axial behavior in Minkowski space and  $\vec{\tau}$  does the same job for the non-relativistic internal fiber.

According to (56), any 6+1 dimensional vector coupling  $\bar{\psi}\Gamma_\mu\psi$  reduced to internal space will induce such a term. However, for this to actually become perceivable, an additional appropriate 'chiral situation' has to be provided, again both in internal and Minkowski space. In Minkowski space this can be achieved, for example, by using polarized beams or if there is a second vertex with a  $\gamma_5$ -coupling in the Feynman diagram of the process.

An analogous requirement must be met in the internal space. In other words, a configuration with a handedness must be present, in order to pick up a non-vanishing contribution from the axial coupling, and this in the case at hand is given by the local chiral ground state structure fig. 1.

As a matter of fact, the non-relativistic circumstances of the internal  $R^3$  space make it a similar situation as one has in optical activity of molecules, where in addition to a circularly polarized photon there must be a handed molecule in order to produce a non-vanishing effect. According to (56) a 4-tetron interaction of two vector currents will induce among others a term

$$\vec{\pi}\vec{\pi} \sim [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \quad (57)$$

which agrees with the quadratic term of the Higgs potential (45) responsible for the alignment of isospin vectors and on the level of isospin vectors reproduces the Heisenberg ansatz (14). As discussed in connection with fig. 3 and in (2.2.12), the sign of the coupling must be anti-ferromagnetic for inner- and ferromagnetic for inter-tetrahedral distances. The latter is in accord with the negative mass term of the Higgs potential.

An important question is whether there is a renormalizable interaction in 6+1 dimensions which can accommodate the iso-magnetic properties described here. This will partly be answered in (2.2.11).

## 2.2 Questions about the fundamental Fermion $\psi$

According to (1) the tetron matter fields  $\psi$  which form the sites of the ground state fig. 2 transform as a spinor 8 under  $SO(6,1)$  and decompose into an isospin doublet when the hyper-crystal is formed. In this section some further properties of the tetrans are elucidated.

### 2.2.1 What is the use of introducing an additional level of matter?

There are several good reasons to do so:

1. the existence of 3 families of quarks and leptons with altogether 24 states (plus the corresponding mass and mixing values) strongly suggests that they are not truly elementary objects.

2. a material origin for the observed internal symmetry groups is highly desirable. Traditionally, they are pasted into the theory as purely abstract groups, representing a rather static behavior of the internal spaces. This line of thinking started with Heisenberg's invention of isospin  $SU(2)$ , included color  $SU(3)$  and ended with the (SUSY) GUT groups. The present model works differently, color and isospin being obtained by extending spacetime by 3 internal spatial dimensions in which an independent dynamics takes place.

3. spontaneous symmetry breaking is introduced in the SM in a more or less ad hoc way by adding a scalar field to a system which otherwise is made up solely of fermions and gauge bosons. This is similar in spirit to the Ginzburg-Landau model for superconductors, extending it to a relativistic and local non-abelian symmetry. However, as well known from many branches of physics, a material background is required for a phase transition and SSB to occur. For example, in superconductivity the scalar field is provided by electrons bound as Cooper pairs.

In the tetron model the breaking of  $SU(2)_L$  is associated to the alignment of the (material) internal spins over Minkowski space as shown in fig. 2[1]. For the interpretation of the Higgs field as a tetron-antitetron correlation see (2.1.8).

### 2.2.2 What is so interesting about the 8 of $SO(6,1)$ ?

This question is discussed in (2.1.20) and (2.5.11).



### 2.2.3 What are the couplings / charges of the tetrons?

After the hyper-crystal is formed a tetron  $\psi$  decomposes into its isospin components U and D. This fact fixes the weak charges

$$I_3(U) = +\frac{1}{2} \quad I_3(D) = -\frac{1}{2} \quad (58)$$

Using (36) one finds

$$Q(U) - Q(D) = 1 \quad F(\psi) = Q(U) + Q(D) \quad (59)$$

where F is a U(1) charge and given by tetron(=fermion) number. Therefore it must be the same for both types of tetrons, i.e.

$$F(\psi) = F(U) = F(D) \quad (60)$$

Tetrons do not have a color charge because they are not involved in interactions of triplets of the Shubnikov group, cf. (2.3.28). Therefore, it is appropriate to normalize F in analogy with leptons instead of quarks, i.e. to put

$$F(\psi) = -1 \quad (61)$$

Eq. (59) then leads to

$$Q(U) = 0 \quad Q(D) = -1 \quad (62)$$

In other words, the normalization (61) is equivalent to defining the U direction in iso-spinor space to be the one which is electrically neutral. This is in accord with the fact that there is no U component in the representation (34) of the photon and that the symmetry breaking ground state and the Higgs vev (2.3.11) are composed of U tetrons only (or actually  $U_*$ ).

Looking at (61) one may suspect that the result (62) is just a question of normalization and therefore cannot have much physical impact. However, there are only 2 possibilities for the tetron number of one tetrons, +1 or -1. The first choice leads to  $Q(D)=0$ , the other one to  $Q(U)=0$ . This means the only freedom one has is which tetron one wants to call U and which one is called D. In this paper the electrically neutral tetron is called U and gives the dominant contribution to the Higgs particle (39).

#### 2.2.4 How large is the mass/binding energy of a tetrahedron within the hyper-crystal. Can it be ionized?

Not with experimental means. The binding energies are extremely large, of the order of  $\Lambda_r \sim \Lambda_P$ , cf. (2.5.6) and the discussion after (18).

#### 2.2.5 How large is the mass of a single tetron?

Difficult to say. If gauge bosons and Higgs scalars would be  $\bar{\psi}\text{-}\psi$  bound states, the natural guess for  $m_\psi$  would be in the range of 40 to 60 GeV. However, in truth the bosons of the SM are not bound states but correlations of the  $\bar{\psi}\text{-}\psi$  bonds within the hyper-crystal. Tetrons are even more tightly bound within the tetrahedrons than the tetrahedrons are within the hyper-crystal, cf. (2.2.4).

#### 2.2.6 What is the spin/helicity of a single tetron within the internal tetrahedral ground state fig. 1?

Figures 1 and 2 contain all necessary isospin information for the hyper-crystal ground state. Since the tetrons U and D are ordinary Dirac fermion in 3+1 dimensions, one may also ask what their spin direction within the hyper-crystal is.

First of all, the total spin of all tetrons within one tetrahedron should add up to zero, because otherwise the vacuum state (i.e. the unexcited hyper-crystal) would be polarized. I do not know exactly how strong the limits are, but I am quite sure that a polarized vacuum is not a desirable option.

Now assuming the spins add up to zero, there are 2 options:

–they do so in a similar fashion as isospins do in fig. 1, i.e. because the sum of spin vectors over all tetrahedral sites vanishes.

–the spins from the left-handed and the right-handed isospin vectors  $\langle \vec{Q}_L \rangle$  and  $\langle \vec{Q}_R \rangle$  compensate each other on each site separately.

As a byproduct of the considerations in (2.4.16), it can be shown that the second option is fulfilled, because the spins from the tetron and the antitetron contributions in fig. 4 compensate each other.

### 2.2.7 Are there $\gamma_5$ anomalies in the tetron model, which could possibly make it inconsistent?

There are no anomalies in the fundamental theory of tetrans, because there are no  $\gamma_5$  couplings. Such couplings arise only in the effective description (the Standard Model) due to the existence of the iso-chiral tetrahedron fig. 1, cf. (2.1.20). On the level of the effective theory the familiar anomaly cancellations among the quarks and leptons apply.

### 2.2.8 \*How can a crystal system out of tetrans and anti-tetrans be stable?\*

In other words: why is there no annihilation between its particle and antiparticle components?

One observation is that according to (34) the tetrans  $U_\star$  making up the ground state of the hyper-crystal have vanishing electric charge and therefore cannot annihilate into a photon.

Actually, this is not really a fair argument, since we are talking here about tetrans and not about their excitations. Being the fundamental form of matter, tetrans anyhow are not expected to annihilate into quasi-particles like the photon or the weak gauge bosons. Writing down (34), (39) and (43) refers to *excitations* of tetron-antitetron pairs rather than the tetrans themselves.

Apart from this argument, I see two further alternatives to answer the question:

–the hyper-crystal in its ground state consists only of tetrans and not of antitetrans, i.e. the vacuum expectations values discussed in (2.1.8) and (2.3.11) do not have an antitetron contribution. Antiparticles would then only arise within the hyper-crystal on the level of excitations. Although at the moment I do not completely understand all the implications, this could modify our understanding of baryogenesis, an issue to be discussed in (2.2.9). However, because of the way weak isospin is constructed in (2.4.16), with the necessity of a tetron-antitetron pair on each tetrahedral site, I do not consider this a reasonable option.

–I adhere to the possibility that no field exists into which tetrans and antitetrans annihilate. This can happen because in the original  $SO(6,1)$  there is no notion of an antiparticle, but tetron and antitetron comprise into one single representation 8

according to (1).

Within the scenario developed in sections (2.5.14)ff, the hyper-crystal including its constitutive tetron matter is a non-relativistic object with a privileged rest system, while only the quasi-particles are of relativistic nature and the familiar  $SO(3,1)$  Lorentz structure emerges but on the level of excitations, one should start with a non-relativistic spinor representation  $4$  of  $SO(6)$ . Although antitetrons  $\bar{4}$  exist in addition to this  $4$ , annihilation processes usually do not occur in a non-relativistic environment, cf. (2.2.11) or [44, 45].

Both tetrons and antitetrons decompose into a spin and an isospin doublet according to  $4 \rightarrow (2, 2)$ , i.e.

$$\psi = (U_{\uparrow}, U_{\downarrow}, D_{\uparrow}, D_{\downarrow}) \quad (63)$$

when the hyper-crystal is formed. In other words, the crystallization induces a symmetry breaking

$$SU(4) \rightarrow SU(2) \times SU(2) \quad (64)$$

in 6 dimensions where  $SU(4)$  is the covering group of  $SO(6)$  and the two  $SU(2)$  factors correspond to 3-dimensional rotations in internal and physical space, respectively.

### 2.2.9 \*The baryon asymmetry in the light of the tetron model\*

Naively one could think that baryon asymmetry is due to a statistical fluctuation shortly after the big bang which makes the observable part of the universe 'baryonic' while other parts are predominantly 'antibaryonic'. However, the experts seem to agree that such an asymmetry would have long been washed out, e.g. by sphaleron effects from the non-perturbative sector of the SM. Instead they prefer to locate baryogenesis at temperatures near  $\Lambda_F$ [55]. In that kind of approach, it is then noted that the SM can only partly explain baryogenesis, mainly because of lack of 'enough' CP violation in the CKM sector. Additional new physics not too far above the Fermi scale is needed to remedy the situation.

What can be learned from the tetron model about this issue? First of all, within the tetron approach the SM is an effective theory, and it is not clear whether its equations are valid beyond the perturbative regime, i.e. whether the sphaleron argument is

	...1234...	...1423...	...1243...
	family 1	family 2	family 3
	$\tau, b_{1,2,3}$	$\mu, s_{1,2,3}$	$e, d_{1,2,3}$
$\nu$	$\overline{1234}$	$\overline{2314}$	$\overline{3124}$
$u_1$	$\overline{2143}$	$\overline{3241}$	$\overline{1342}$
$u_2$	$\overline{3412}$	$\overline{1423}$	$\overline{2431}$
$u_3$	$\overline{4321}$	$\overline{4132}$	$\overline{4213}$

Table 1: List of elements of  $A_4$  ordered in 3 families. Permutations with a 4 at the last position form a  $A_3$  subgroup of  $A_4$  and may be thought of giving the set of lepton states.

really true. Secondly, the low energy limit of the tetron model is a 2HDM model rather than the SM. Since the amount of CP violation is generally larger in 2HDM models than in the SM, baryogenesis can in principle be explained more easily[56]. Thirdly, the points put forward in (2.2.8) should be taken into account, and one could try to understand baryogenesis completely from the tetron level perspective, i.e. analyze whether the predicted CP violation[2] is enough to induce the necessary imbalance between baryons and antibaryons.

### 2.2.10 A simple memo to understand the role of the permutation group in the ordering of quarks and leptons

Since the 24-dimensional representation (9) of  $G_4$  is faithful, one can assign the elements of  $A_4$  to the fermions of the 3 families as shown in table 1. It should be noted, however, that this is only a heuristic assignment. Actually one has to consider linear combinations of permutation states[3].

### 2.2.11 \*What is the form of the fundamental tetron interaction?\*

As argued in [1], rather weak many-particle correlations are sufficient to understand the frustrated arrangement of isospins and obtain the masses of quarks and leptons. The fact that the weak bosons  $W^\pm$ , Z and H exhibit very small lifetimes agrees with

this argument and seems to indicate, that the phenomena of particle physics do not need a super-strong interaction.

The formation of the hyper-crystal at big bang temperatures might also be describable by many-particle correlations, in a similar way as ordinary crystal structures are mainly due to electrostatic interactions. The same would then probably be true for the initial formation of the tetrahedral molecules.

To make the analogy with non-relativistic QED complete, in [1] the existence of an internal 'photon' as part of a 6+1 dimensional U(1) gauge theory has been assumed. However, in contrast to the tetron isospin interactions, the tetron coordinate interactions involve binding energies that are enormous (of the order of the Planck scale), and since binding energies are a measure of the couplings involved, at this point we face the existence of some strong binding interaction, which holds the tetrahedral crystal and molecules together and which according to (29) is inversely proportional to the weakness of the gravitational force, cf. (2.2.12), (2.3.19) and (2.3.21).

Furthermore, there are some good arguments against the QED<sub>6+1</sub> model of [1]:

- the ordinary photon interpreted as an excitation cannot be part of the 6+1 dimensional 'photon' field, cf. (2.1.2) and (2.1.3),
- the 6+1 dimensional U(1) field must be very heavy in order that energy is not dissipated away from the hyper-crystal, cf. (2.5.32).

For these reasons I will follow here a more general approach and consider various possible effective interactions among tetrans as to whether they yield the correct low energy phenomena.

One could consider, for example, 4-tetron contact interactions. The SM together with the tetron description of its bosonic sector [eqs. (34), (39), (40), (48) and (57)] actually set important constraints on the form of such contact Lagrangian. These can be read off e.g. from the W-mass  $m_W^2 W_\mu W^\mu$  or the  $\vec{\pi}\vec{\pi}$  term in the SM Lagrangian (45) and, in a second step, must be adapted for use in a 6+1 dimensional Lagrangian framework.

In doing so, there is a scale factor to be considered when going from a 6+1 dimensional fermion  $\Psi$  to a doublet  $\psi = (U, D)$  of two 3+1 dimensional spinors bound in the hyper-crystal, because  $\Psi$  has energy dimension 3 whereas U and D have energy dimension 3/2. I will write

$$\Psi = \Lambda_R^{3/2} \psi \tag{65}$$

where  $\Lambda_R$  is the scale corresponding to squeezing the internal dimensions to a mono-layer of tetrahedrons of thickness  $R$ , cf. fig. 2.

Consider, as an example, a contact Lagrangian formed by two 7-dimensional vector currents

$$L_{6+1} = \frac{1}{\Lambda_R^5} [\bar{\Psi}\Gamma_\mu\Psi][\bar{\Psi}\Gamma^\mu\Psi] \quad (66)$$

where  $\Lambda_R$  determines the coupling, because the Lagrangian is to describe the formation of the internal tetrahedrons at big bang times. The appearance of the fifth power has dimensional reasons and is related to the fact that a propagator of any particle exchanged behaves differently in 6+1 dimensions than in 3+1 – namely as  $r^{-4}$  instead as  $r^{-1}$ .

$\Gamma_\mu$ ,  $\mu = 0, \dots, 6$  are the Dirac matrices in 6+1 dimensions. They are seven  $8 \times 8$  matrices which act on the 8 components of the tetron spinor  $\Psi \sim (U, D)$ . When the hyper-crystal with its 3+1 dimensional 'surface' structure is formed, they break up according to [99]

$$\Gamma_{0-7} = (\gamma_{0-3}, \vec{\tau}\gamma_5) \quad (67)$$

where according to the arguments in [1] and (2.1.20) the product  $\vec{\tau}\gamma_5$  is one of the ingredients necessary for weak parity violation.

The 7-dimensional Lagrangian is related to a 4-dimensional one via

$$L_{3+1} = \Lambda_R^3 L_{6+1} = \frac{1}{\Lambda_R^2} [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \quad (68)$$

This is because identifying  $\int d^3x$  with the volume filled by an internal tetrahedron one has

$$S = \int d^7x L_{6+1} = \int d^4x \frac{L_{6+1}}{\Lambda_R^3} \equiv \int d^4x L_{3+1} \quad (69)$$

The result (68) formally has the same structure as (57). More precisely, the  $\vec{\pi}\vec{\pi}$  term in (45) can be rewritten as

$$V_{SM} = \dots + \frac{\mu^2}{\Lambda_P^4} [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \quad (70)$$

where  $\Lambda_P$  is the Planck scale.

To obtain (70) I have identified

$$\vec{\pi} = \frac{1}{\Lambda_P^2} \bar{\psi}\vec{\tau}\gamma_5\psi \quad (71)$$

In a traditional technicolor model the TC scale (=mass of the TC gauge bosons exchanged) would appear in (71). Here we identify it with the scale  $\Lambda_P$  set by the Planck lattice constant of the hyper-crystal, cf. the discussion in (2.3.17).

Unfortunately, the couplings in (68) and (70) do not have much in common. While (68) is true at the big bang (=crystallization point), (70) holds after the electroweak (=isomagnetic) SSB.

Another disadvantage of using an approach with contact interactions like (70) is that it ignores the Galilean nature of the hyper-crystal to be discussed in (2.5.3) and (2.5.40). According to that point of view, tetrons and tetron matter inside the hyper-crystal should not be described by d'Alembert type of wave equations, but a non-relativistic framework should better be used.

Another aspect of this kind of reasoning is that a Lorentz structure in  $SO(6,1)$ , if it exists at all, would presumably involve a limiting speed  $c_6$  different from the speed of light valid within the hyper-crystal - simply because the photon is an excitation restricted to the hyper-crystal. Furthermore, Huygens' principle is not valid in 6 spatial dimensions. The wave fronts of d'Alembert do not stay sharp. These are the reasons why one should be hesitant to extend (68) and the other equations from the hyper-crystal to the full 6+1 dimensional spacetime and why in the following a non-relativistic approach is pursued to describe the fundamental tetron interactions. It is relatively simple to write down non-relativistic 4-tetron contact terms in 6+1 dimensions, e.g.

$$L_{nr} = \frac{1}{\Lambda_R^5} [\Psi^\dagger \lambda^a \Psi][\Psi^\dagger \lambda^a \Psi] \quad (72)$$

where  $\lambda^a$ ,  $a=1, \dots, 15$  are the generators of  $SO(6)$ .  $\Psi$  in this equation is formally given by (65), but instead of the tetron field  $\psi$  (= a relativistic spinor 8 of  $SO(6,1)$ ) the non-relativistic 4 of  $SO(6)$  should be taken, as defined in (63).

In this framework bound states are most conveniently analyzed using a generalization of Schrödinger's equation to 6 dimensions. Spin and isospin effects can in principle be included by extending this to a 6 dimensional equation of Pauli type. However, for many aspects of the dynamics like

- the coordinate formation of tetrahedrons,
- the big bang crystallization process and
- the shifts of tetrahedrons inside the elastic hyper-crystal (giving rise to the gravi-



tational interactions)

it is sufficient to average over spins and isospins, so that one can do without the Pauli terms and use the 6+1 dimensional version of the Schrödinger equation. Even the exchange couplings responsible for the iso-magnetic interactions (14) relevant for particle physics can be calculated with the scalar potential of the Schrödinger equation alone. Only when polarization effects matter, one should extend this to a 6+1 dimensional analog of the Pauli equation.

### **2.2.12 An intuitive understanding of the isomagnetic interactions in the hyper-crystal**

In section 1/fig. 3 some heuristic arguments have been given as how to understand the isospin vector configurations figs. 1 and 2 in a similar way as magnetically ordered states in solid state physics. In (2.3.2) the Pauli principle is used to prove that the tetrahedral configuration of tetrons is extremely stable, and in (2.5.1) a simple formula is given for its energy. These arguments will be extended here in order to show that the global configuration fig. 2 is by far the lowest energy state of a many tetron system.

Much of the effect can be understood by the fermion property of tetrons alone, i.e. without considering the detailed form of the fundamental tetron interaction (2.2.11). One may, for example, exploit the behavior of the tetrons under identical particle exchange. Since anti-tetrons are different particles, one needs to consider only half of the isospinors, let's say the ones with tetrons and without anti-tetrons, i.e. the lefthanded isovectors  $\vec{Q}_L$ . The total wave function should be antisymmetric under tetron exchange. Since the ordinary spins are all 'parallel' (all left-handed), the ordinary-spin part of the wave function is symmetric. Concerning the spatial and the isospin part there are then 2 possibilities: either the isospin part is antisymmetric under tetron exchange and the spatial part is symmetric or vice versa. I want to argue that inside a tetrahedral 'molecule' the first and for the 'crystal' binding between tetrahedrons the second possibility is realized:

(i) As compared to the inner-molecular forces, the inter-molecular coupling between 2 tetrahedrons is relatively weak, and to some extent even elastic, though with a large stiffness (29). This binding corresponds to a spatial part of the wave function

which is antisymmetric under tetron exchange, because under such a condition the wave function becomes small in the middle of the tetron-tetron bond, an effect which usually runs under the name 'Fermi hole' or 'exchange correlation hole'. The Pauli principle then demands that the isospin part of the wave function is symmetric, which corresponds to an aligned, i.e. 'iso-ferromagnetic' configuration. This means nothing else than aligned isospin vectors  $\vec{Q}_L$  in neighboring tetrahedral molecules as shown in fig. 2 and needed for the electroweak symmetry breaking.

(ii) Inside the tetrahedrons the isospin part of the wave function is antisymmetric under tetron exchange. This corresponds to an anti-aligned, i.e. 'iso-antiferromagnetic' configuration and leads to a frustrated configuration of isospin vectors  $\vec{Q}_L$  as in fig. 1. As a consequence, the spatial part of the wave function must be symmetric, and this implies that the forces inside a tetrahedron are very strong. In the language of molecular orbital theory they are 'sigma bonds', and the strengthening effect is often referred to as 'Fermi heap'.<sup>6</sup>

### **2.2.13 \*Why do the 'molecules' formed by tetrans have a tetrahedral coordinate structure?\***

In principle this question can be answered by analyzing the fundamental tetron interaction and showing that among the 'molecules' composed of n tetrans the ones with n=4 are energetically most favored. Subsection (2.5.31) about the growth of the hyper-crystal and (2.5.11) about the octonion origin of the interactions are recommended to read in this connection.

### **2.2.14 Why not use an internal molecular model instead of a crystal?**

According to (2.5.3) Lorentz invariance can be approximately established for small enough lattice spacings (of order  $L_P$ ). Nevertheless, some readers may find it difficult to imagine the world as an irregular elastic crystal, with every site occupied by 4 tetrans. So why not use a model, where excitations of tetrahedral 'molecules' in an

---

<sup>6</sup>In a chemical bond the Fermi heap allows both electrons to be localized in the internuclear region, thus shielding the positively charged nuclei from their respective electrostatic repulsion. It is a matter of speculation whether in the present case in addition to tetrans some kind of 'nuclei' are needed to form the tetrahedral molecules.

otherwise empty space are used to explain the  $q/l$  spectrum? The molecules would extend into internal dimensions and have a frustrated anti-ferromagnetic structure as in fig. 1. Even an explanation of the SSB as a re-arrangement within the molecules happening below a certain temperature is feasible.

However, with such a picture one would run into all the known problems of classic composite models[31]. The strongest counter argument certainly is, how one and the same molecule can sometimes have a mass larger than 100 GeV and sometimes be as light as neutrinos.

### 2.2.15 Why not use a tetrahedral lattice in ordinary space, without any internal dimensions?

Since higher dimensions have never been observed experimentally, it is important to critically scrutinize their introduction. In this subsection I follow the idea that the tetrahedrons extend into ordinary space and only *mimic* the existence of internal symmetries by forming encapsulated, neutral and ordered tetrahedral systems as in figs. 1 and 2 in which mignons can be excited just as in the picture with internal dimensions. As before, the extension of the tetrahedrons would have to be tiny, of order  $L_P$ .

The spin- $\frac{1}{2}$  nature of the mignons would be ensured by the spin- $\frac{1}{2}$  nature of the tetrons just as described in (2.4.2), while the 'internal' quantum numbers arise from the relative angular momentum of the tetrons inside the tetrahedron, i.e. from the Shubnikov symmetry. The whole system including gravity would look similar to a Cosserat continuum, where in addition to the Cosserat deformations describing gravity[91] there are interactions among the encapsulated tetron spins with corresponding spin wave excitations.

There are then several advantages of this approach as compared to the model with internal d.o.f.:

- there is no problem (2.5.32) with the dissipation of energy into internal dimensions.
- the question (2.5.10) why there is no growth of the crystal into the internal dimensions, does not arise.
- parity violation of the weak interaction would work analogous to optical activity in molecules, without the necessity to recur to an octonion structure.

However, there is also a drawback. It is related to the fact that a rotation in physical space would not only flip the ordinary spin of a mignon, but also transform the 'internal' coordinates.

This argument seems to kill the idea. One can only come around this conclusion, if one assumes rather strange behavior of the tetrahedrons, e.g. that they are immersed into a spacetime medium in such a way that they can be rotated independently of spatial rotations, i.e. have an extremely large relaxation time against outside rotations.

It may be noted that color has only been observed in singlet states which would not be sensitive to rotations anyhow. As for internal  $SU(2)$ , weak isospin partners like the electron and its neutrino after the SSB are  $G_4$ -singlets, too. Weak isospin transitions in the tetron model are constructed rather indirectly as transition between excitations of  $\vec{Q}_L$  and  $\vec{Q}_R$ , cf. the discussion in (2.4.16), and this construction can in principle be taken over to the scenario discussed in this subsection.

As before, the electroweak bosons would be related to  $U(1) \times SU(2)$  gauge transformations referring to tetron number and the rotations of the tetron spin vectors, respectively.

### **2.2.16 How large is the internal extension $R$ of one tetrahedron? How large is the average spacing $\langle r(t) \rangle$ between 2 tetrahedrons?**

In most parts of the paper the following scenario is considered:  $R$  is smaller than  $r$ , and  $r$  is of the order of the Planck length. This is a reasonable assumption because  $R$  is the scale of tetrahedral 'molecule' formation, which took place at higher temperatures than the hyper-crystallization, i.e. before our universe was born. Furthermore, the single internal tetrahedrons are rigid and strongly bound objects, while the condensed system of tetrahedrons is elastic and its 'lattice spacing'  $r$  grows with cosmic time.

Since  $\langle r(t) \rangle$  was identified with the Planck length  $L_P$  in (24), and is thus related to the uncertainties of quantum theory, the introduction of a length  $R$  smaller than  $L_P$  may look problematic. However, as discussed in (2.5.6) and in connection with (32), the quantum nature of matter arose when the hyper-crystal was formed during the big bang (via crystallization), i.e. *after* the time of tetrahedral molecule formation.

### **2.2.17 Why should the distance between 2 adjacent tetrahedrons be identified with the Planck scale?**

One may ask whether there is the possibility, that  $r$  and  $R$  are much larger than the Planck length. In that case tetrons would have nothing to do with general relativity, and one could forget all reasoning about gravity and cosmology presented in this work. Only the particle physics sections would apply, and  $r$  and  $R$  would be scales like appear in technicolor models. However, the arguments in (2.5.6) indicate that the most consistent picture is obtained by choosing  $r \approx L_P$ .

### **2.2.18 Is the quantum theory of angular momentum used in (14)-(15) applicable in case $R < L_P$ ?**

Yes, because the isomagnetic interactions responsible for the Standard Model physics take place within the hyper-crystal and at energies much smaller than  $\Lambda_P$  and  $\Lambda_r$ .

## **2.3 Questions about the local tetrahedral Structure and the Nature of the SSB**

To obtain the correct mass spectrum for quarks and leptons, not only the internal geometry but also other features of the model have to be fixed.

### **2.3.1 Are the tetrahedrons formed by 4 or 8 tetrons, i.e. how many vibrators are needed on each lattice site?**

A priori one may consider several options:

(i) the 4-tetron option: naively, one would think that 4 tetrons can give rise to only  $4 \times 3 = 12$  excitations of their isospin vectors  $\vec{Q}$ . However, there are 2 independent vibrators  $\delta\vec{Q}_L$  and  $\delta\vec{Q}_R$  for each tetron, and for 4 tetrons this gives the desired  $2 \times 4 \times 3 = 24$  states of eq. (9). For this picture to work the ground state values  $\langle\vec{Q}_L\rangle$  and  $\langle\vec{Q}_R\rangle$  must contain antitetron contributions. This is discussed in (2.4.16) and actually brings option (i) close to option (ii).

(ii) the 8-tetron option: here  $\vec{Q}_{Li}$  and  $\vec{Q}_{Ri}$  are carried by 2 different particles approximately occupying the same tetrahedral site  $i$ , as depicted in figs. 4 and 5. As

discussed in (2.4.16), it is most appropriate to assume this to be a particle and an antiparticle. Since these are sitting very close together, the system keeps its Shubnikov symmetry (7), and all arguments concerning the mignon spectrum remain unchanged.

The reason why the options (i) and (ii) are equivalent, can also be seen in the following way: according to (1) a tetron which transforms as 8 under  $SO(6,1)$  decomposes into a particle (1,2) and an antiparticle (2,1) under the Lorentz group  $SO(3,1)$ , and these are just the d.o.f. required in fig. 4.

When assuming a hyper-crystal with an originally Galilean structure as in (2.5.14)ff, the tetron and antitetron correspond to a 4 and  $\bar{4}$  of  $SU(4)$ , cf. (63). Within the above philosophy this is merely restating the fact that one may consider the particle and antiparticle contributions to the isospin vectors separately.

(iii) in case of 8 tetrons there is another option which however will be abandoned for reasons discussed below: namely one could consider unpolarized isospin vectors  $\vec{Q}_{1-8}$ , which appear in pairs  $\vec{Q}_i$  and  $\vec{Q}_{i+1}$  on each tetrahedral site  $i=1-4$ , again to be interpreted as tetrons  $i$  and  $i+1$  to be very close to each other (with a tiny but non-vanishing internal distance  $d_8$ ), i.e. more tightly bound than to the others. Mathematically it corresponds to a coordinate ground state symmetry  $A_4 \times Z_2$  instead of  $S_4$ . Assuming ground state isospins  $\langle \vec{Q}_i \rangle = \langle \vec{Q}_{i+1} \rangle$  on each site to be parallel, the isomagnetic ground state will again be symmetric under  $A_4 + S(S_4 - A_4)$ . It is then straightforward to see that a spectrum of the same form (9) as before is obtained from the vibrations of the  $\vec{Q}_{1-8}$ .

This scenario is clearly distinguished from those (i and ii) with chiral isospin vectors. First of all, the origin of isospin of mignons (quarks and leptons) is different in the 2 cases. According to (2.4.16) the transition  $L \leftrightarrow R$  can be chosen to accompany an isospin transformation. In contrast in the case with  $\vec{Q}_{1-8}$ , the  $Z_2$  exchange ( $i \leftrightarrow i + 1$ ) has to provide for an isospin transition.

In addition, there is another, stronger disadvantage of using the approach with  $\vec{Q}_{1-8}$ , because the intimate connection between left-handed vibrators and the top-quark gets lost, cf. (15) and (2.4.21), i.e. the understanding why  $m_t$  is of the order of the SSB scale while all other quarks and leptons, in particular the b-quark, have much smaller masses, cf. (2.4.22).

### 2.3.2 Why is the tetrahedral 'molecule' so stable?

The shortcut answer is that the tetrahedron is the 'helium' of the tetron model. One can use the Pauli principle to understand this point. While in the case of helium, two  $SU(2)$  spinors arrange antiparallel to form the most stable and abundant element in the universe, for tetrons the spinor representation 8 of  $SO(6,1)$  is relevant. The most stable configuration corresponds to a 'shell' filled with 8 tetrons all with *different*  $SO(6,1)$  quantum numbers.

### 2.3.3 Can there be a tetron and an antitetron on one and the same tetrahedral site?

If we accept the idea that there are 2 independent fermions on each tetrahedral site (a tetron and an antitetron, cf. (2.3.1) and figs. 4 and 5), then strictly speaking they cannot exist on exactly the same spot. In other words, there must be a tiny nonvanishing distance  $d$  between them. Since according to (2.4.16) and fig. 4 the tetron-antitetron pairing is to generate the weak isospin quantum number and in order to mimic the differential geometric structure of the  $SU(2)$  gauge bosons, it is a reasonable assumption, that this distance at least partly extends into physical space, not internal space. For obvious reasons one should furthermore have  $d \leq r, R$  where  $r$  and  $R$  were defined in fig.2 and identified as the Planck scale in sections 1 and (2.5.6).

### 2.3.4 Is the binding which makes up the Higgs field due to isospin or is it due to tetron coordinate interactions?

It is due to both. The Higgs particle (39) relies on the alignment of  $U_*$  isospinors and is therefore natural part of the iso-magnetic interactions. On the other hand, the Higgs (as well as all other scalar and vector fields) is an excitation of the tetron-antitetron bonds in the crystal, and therefore controlled by the coordinate interactions.

### **2.3.5 What are the unbroken symmetries of the model?**

The Shubnikov group  $G_4$  and  $U(1)_Q$ . The unbroken Shubnikov group has only singlet and triplet representations and leads exactly to the observed color and flavor spectrum of 3 families of quarks and leptons (9).

### **2.3.6 Can one calculate the Fermi scale, the Weinberg angle and W/Z and Higgs mass from first principles?**

The origin of the Weinberg angle was discussed in (2.1.11). The Fermi scale and the Higgs mass arise from iso-magnetic exchange and pairing interactions, as discussed in (2.1.8). Therefore if one would know the exact form of the fundamental tetron interaction (2.2.11), these quantities would be calculable from 6-dimensional exchange integrals.

### **2.3.7 Did gauge bosons exist at temperatures above the Fermi scale?**

Yes, they did. In the tetron model gauge bosons are particle-antiparticle correlations of crystally bound tetrans. They came into being shortly after the crystal was formed at temperature  $\Lambda_P$  and made up for the bulk of particles in the 'radiation dominated epoque', cf. (2.1.13).

### **2.3.8 Did quarks and leptons exist at temperatures above the Fermi scale?**

No, because from their very nature they require the existence of the iso-magnetically ordered state fig. 2. When the tetron gas cooled down and the hyper-crystal began to form, there was only the coordinate alignment of tetrahedrons but no alignment of isospin vectors. Therefore, at that stage, at temperatures above the Fermi scale, quarks and leptons did not exist, because they could not travel as quasi-particles through the isomagnetically disordered hyper-crystal. Only gravitons, phinons (2.3.25) and scalar and vector bosons (as excitations of the tetron-antitetron bonds) were present. This era is usually called radiation dominated, cf. (2.1.13).



### 2.3.9 Then why can quarks and leptons be produced at energies above the Fermi scale?

In collider experiments they can exist at energies much larger than 1 TeV because the alignment of isospins is stable much beyond  $\Lambda_F$  in the fully ordered hyper-crystal, i.e. in our universe. This is due to a collective hysteresis effect in which the crystal stabilizes itself by the concerted action of all aligned tetrahedrons to maintain the isomagnetic ordering. As a result, quarks and leptons can be produced and propagate normally, even in cases where energies locally exceed the critical temperature  $\Lambda_F$ .

### 2.3.10 Why is $\bar{\psi}$ involved in the order parameter and not $\psi^\dagger$ , whereas the total 'iso-magnetization' $\vec{\Sigma}$ eq. (16) is defined just like in ordinary magnetic models?

Short answer to a long question: the ground state value of  $\vec{\Sigma} = \sum_{i=1}^4 \psi_i^\dagger \vec{\tau} \psi_i$  vanishes due to the tetrahedral arrangement of isospin vectors within any internal tetrahedron fig. 1, and is therefore not useful as an order parameter in the present case.

Furthermore, the definition of  $\vec{\Sigma}$  involves only tetrons of one single tetrahedron, whereas the particle physics SSB consists in the isospin alignment of two *neighboring* tetrahedrons.

### 2.3.11 Is the vev $\langle \bar{U}U + \bar{D}D \rangle$ or $\langle \bar{U}U \rangle$ or what?

The vev is given by  $\langle \bar{U}_* U_* \rangle$  in accordance with (39) where  $U_*$  is the 'radial' iso-spinor introduced in (11) corresponding to an isospin vector pointing outward as in fig. 1. Such a vev is precisely what is needed to stabilize the alignment of isospins in fig. 2.

### 2.3.12 Why not use some other order parameter which is closer related to the isomagnetic alignment than the Higgs vev?

First of all it must be noted that according to (2.3.11) the Higgs vev in the tetron model has a lot to do with the isomagnetic alignment.

Secondly, it is true that in general for a given phase transition different order parameters are possible. In the present case one may for example consider 2 neighboring tetrahedrons A and B with tetrons  $\psi_{Ai}$  and  $\bar{\psi}_{Bi}$ , and  $i = 1, 2, 3, 4$  counting the tetrahedral sites. Unfortunately, the aligned tetrahedral 'star' configuration of these 2 tetrahedrons not only implies  $\langle \vec{\Sigma}_{A,B} \rangle = 0$  for each tetrahedron separately, but also  $\langle \vec{\Xi} \rangle = 0$ , where  $\vec{\Xi}$  is defined as  $\vec{\Xi} = \sum_i \bar{\psi}_{Bi} \vec{\tau} \psi_{Ai}$ . This is because one can show that all vectors  $\bar{\psi}_{Bi} \vec{\tau} \psi_{Ai}$  of 2 adjacent tetrahedrons are parallel, iff the corresponding isospin vectors  $\vec{Q}_{Ai,Bi}$  are. Thus  $\vec{\Xi}$  is not a useful order parameter either.

### 2.3.13 So what is the microscopic interpretation of the Higgs particle?

As discussed in (2.1.2) the Higgs is neither fundamental nor a bound state of mignons, but an excitation of tetron-antitetron pairs which are themselves bound within the hyper-crystal.

### 2.3.14 Should one consider separate tetrahedrons for anti-tetrons, with isospin vectors pointing inward?

This question may seem justified, because anti-fermions usually react to magnetic forces with an opposite sign. However, using isospin vectors (2) one is treating the problem in a covariant way. As can be seen in (3), the isospin vectors contain particle as well as antiparticle contributions, and the antiparticle contributions have a negative sign. More details are given in (2.4.16), where a  $\langle \vec{Q}_R \rangle$  pointing inwards will be defined in terms of a charge conjugate tetron field. In this connection the reader may also consult (2.3.3) and (2.2.8).

### 2.3.15 Is there a difference between the SM SSB and a ferromagnet, apart from the fact that the SM SSB takes place in internal space? Is the symmetry breaking in the tetron model really spontaneous?

Both cases (ferromagnet and tetron structure) are similar in that at high energies / temperatures the directions of (iso)spins are oriented randomly with an associated SU(2) Heisenberg symmetry, and this defines the symmetric state.

In an uni-axial ferromagnet an accidental magnetization axis usually appears spontaneously, based on a thermodynamic potential

$$V_{FM}(\vec{M}) = -a\vec{M}^2 + b\vec{M}^4 \quad (73)$$

where  $\vec{M}$  is the total magnetization and the minimum of the potential is at  $\langle \vec{M}^2 \rangle = a/2b$ .

In the case at hand the crystallization process at scale  $\Lambda_r$  is accompanied by a coordinate alignment of all tetrahedrons, i.e. there is a spontaneous selection of one global internal coordinate system for all tetrahedrons. This coordinate alignment, however, happens prior to the alignment of isospins and has not much to do with it. When the temperature decreases towards  $\Lambda_F$ , the anti-ferromagnetic tetrahedral 'star' configurations fig. 1 appear where the isospin vectors within one tetrahedron avoid each other as far as possible. Note there is an infinite SU(2) symmetric set of such 'star' configurations just as in a ferromagnet there is an infinite set of possible magnetized states corresponding to all possible magnetization directions in  $R^3$ . The difference as compared to a ferromagnet is that not only the stars over one tetrahedron have to be included but also those over all the other tetrahedrons over Minkowski space, with their independent SU(2) degeneracies, and this makes the problem a local gauge symmetric one.

The SSB consists in the simultaneous selection of one among all the possible star configurations over all Minkowski base points – namely the one with  $\langle \Phi \rangle \sim (0, 1)$ . According to (40) this corresponds to a vev for the  $U_*$  isospinor component, i.e. the one with an isospin vector pointing outwards in the radial direction. The choice of  $(0, 1)$  - and of U - is notational convention and, in the framework of the gauge theory, corresponds to choosing a certain gauge (the so called unitary gauge). There is again a similarity to the situation in a ferromagnet where some axis is selected by the spontaneous magnetization, and the coordinate system is then 'gauged' in such a way that this axis is called the z-axis  $\sim (0, 0, 1)$ .

One may ask what role the coordinate alignment of tetrahedrons at the crystallization point  $\Lambda_r \sim \Lambda_P$  plays in this game, because it seems plausible that the state, where the tetrahedrons of coordinate and isospin both point in the same radial directions, is energetically preferred. (This geometry is in fact depicted in figures 1 and 2.)

A similar situation is sometimes encountered in ordinary uni-axial ferromagnets in cases when the coordinate backbone of the crystal prefers one specific magnetization direction, so that the ferromagnetic phase transition is not really spontaneous. This effect can be modeled by adding a tiny explicit symmetry breaking contribution to the potential (73) by hand. At high temperatures due to thermal fluctuations this structural / coordinate effect is not important. But it becomes relevant near the Curie temperature where it fixes the magnetization direction.

In the present case, however, this possibility needs no consideration. The reason is that  $\Lambda_P$  is so large as compared to  $\Lambda_F$ , that the granular internal coordinate structure is not noticed by the isospin vectors (nor by any human experiment). From the perspective of the isospin vectors it looks as if they are sitting on an internal coordinate structure which is rotationally invariant. They only feel the anti-ferromagnetic aversion towards their 3 fellows within one tetrahedron.

As a consequence of these considerations all 'star' configurations are energetically equivalent, and the symmetry breaking is spontaneous.

One can even go as far to say that there could be no coordinate alignment among the tetrahedrons at all. This would be in accord with the idea that the inter-tetrahedral coordinate (=gravitational) interactions are elastic and therefore can give rise to any relative coordinate orientation between neighboring tetrahedrons. The alignment of isospins could live with this option, because the only thing which matters for the SM SSB is that the 4 isospin vectors point in radial direction and build up the isomagnetically aligned multi-tetrahedral configuration, irrespective of what the coordinates of the tetrons are.

Further it may be noted that the question whether the SB is really spontaneous, is much easier to answer in what was called scenario C in section 1. In that case the electroweak phase transition III consists in a *simultaneous* alignment of coordinate and isospin vectors as shown in fig. 2. In other words the tetrahedral star configuration consists in a coordinate and an isospin star where the coordinate and the associated isospin vector always point into the same radial directions. The transition to the ordered state is then necessarily spontaneous because all rotated (coordinate + isospins) tetrahedral star configurations are energetically equivalent.

### 2.3.16 Is the electroweak phase transition first or second order?

Lattice calculations in the SM with one Higgs doublet give no definite answer to this question. The transition seems to be second order for  $m_H \lesssim 120\text{GeV}$ , while for  $m_H \gtrsim 130\text{GeV}$  one obtains a first order transition[49]. In the intermediate region it may be a cross-over[50]. By contrast, in 2HDM models the situation is clearer, because the electroweak phase transition turns out to be first order[51] and terms of order  $\sim \Phi^3 T$  arise in the temperature dependent Higgs potential.

Since the 2HDM model (2.1.16) arises as the low energy approximation of the tetron model, one may be content with this result, in particular because a first order transition is preferable for phenomenological reasons, cf. (2.3.24) and [50]. However, it should also be possible to directly determine the nature of the phase transition in the tetron model without recurring to an effective theory. To achieve this aim, a calculation in the framework specified in (2.3.15) and by figs. 1 and 2 should be performed. If one looks at fig. 2, such an isomagnetic alignment is normally expected to be of second order. However, first order magnetic transitions are also known, in particular in connection with deformable structures[52].

### 2.3.17 What are the relevant scales in the model?

Naively, there are only 2 scales: the Fermi scale  $\Lambda_F$  and the Planck scale  $\Lambda_P$ . The binding and crystallization process with coordinate alignment of tetrahedrons but erratic directions of isospin vectors corresponds to scales  $\Lambda_r$  and  $\Lambda_R$  both of order  $\Lambda_P$ , while  $\Lambda_F$  is the scale where the isospin vectors align.

On a more sophisticated level some other scales might seem reasonable:

–the formation of tetrahedral 'molecules' from a tetron gas in 6 dimensions and of the hyper-crystal from the 'molecules' may happen at different scales  $\Lambda_R$  and  $\Lambda_r(0)$  where  $t=0$  corresponds to big bang time when the hyper-crystal was formed. Furthermore, due to the elasticity of the DMESC, the crystal binding energy at that point was different than it is now:  $\Lambda_r(0) \gg \Lambda_r(now)$ . This corresponds to the fact that the average distance between neighboring tetrahedrons in the hyper-crystal has grown since the big bang:  $\langle r(0) \rangle \ll \langle r(now) \rangle$  where according to (24) we should identify  $\langle r(now) \rangle$  with the present value of the Planck length. In other words one

has  $\Lambda_r(\text{now}) = \Lambda_P$  and thus a hierarchy of scales

$$\Lambda_R \geq \Lambda_r(0) \gg \Lambda_r(\text{now}) = \Lambda_P \gg \Lambda_F \quad (74)$$

–it is conceivable that the crystallization and the coordinate alignment of tetrahedrons do not happen at the same temperature, i.e. there is  $\Lambda_P$  for the crystallization and another scale  $\Lambda_A$  for the coordinate alignment fulfilling  $\Lambda_P > \Lambda_A > \Lambda_F$ . However, this would imply another phase transition in the early universe for which there is no indication. As explained in section 1 and (2.3.15) it is best to assume that either  $\Lambda_A = \Lambda_P$  or  $\Lambda_A = \Lambda_F$  or that there is actually no coordinate alignment at all, only isospin alignment.

–there may be a separate scale  $\Lambda_d$  for the pairing interaction of 2 tetrons on one tetrahedral site, as discussed in (2.3.3), (2.4.16) and (2.3.1).

### 2.3.18 What is the geometrical meaning of these scales?

According to fig. 2,  $r$  and  $R$  can be interpreted as spacings in the discrete structure under consideration, or as lengths of certain tetron bonds. Namely,  $R$  is the fixed bond length of 2 tetrons within a tetrahedron, while  $r$  is the variable (elastic) bond length of 2 tetrahedrons in the hyper-crystal. The Fermi scale measures the 'length' of isospin vectors of the ground state fig. 1.

### 2.3.19 Why are the scales $\Lambda_R$ and $\Lambda_r$ so much larger than the quark and lepton masses? Why is gravity so weak as compared to the isomagnetic interactions? Why is the Planck scale so large in comparison to the Fermi scale?

These questions are variation of why gravity is so weak as compared to QED and the other particle physics interactions.

An important point to note is that the weakness of the gravitational forces and the strong coordinate forces among tetrons and tetrahedrons are related. The latter express themselves in the close packing and the associated extreme stiffness of the tetrahedrons in the hyper-crystal, which was deduced from the smallness of Newton's constant in the discussion after (29). In other words, it is the strong binding of tetrons together with the close-meshed packing of tetrahedrons which

makes the hyper-crystal react to mignon excitations with only tiny distortions. In the Newtonian approximation, for example, writing  $g_{00}$  in (22) as  $g_{00} = -1 - 2V$ , the gravitational potential is  $V \approx 10^{-39}$  at the surface of a proton and not larger than  $10^{-6}$  at the surface of the sun.

Not knowing the precise nature (2.2.11) of the fundamental interaction among tetrons, one can only say that gravity is some kind of remnant elastic interaction among the mignons, while the tetrons themselves are strongly bound to the hyper-crystal structure.

On the other hand, the energies  $\sim \Lambda_F$  involved in the isospin alignment are much smaller than the energies  $\Lambda_r \approx \Lambda_P$  needed for the coordinate formation of the crystal.  $R$  and  $r$  are the length scales at which the iso-magnetic exchange integrals  $J$  have to be calculated, i.e. they are on the *abscissa* of the Bethe-Slater curve fig. 3, while the values of  $J$  are drawn on the *ordinate* of fig. 3 and always  $\leq \Lambda_F$ . The reason why one can have  $J \ll \Lambda_P$  is explained in (2.3.20).

**2.3.20 Why are the exchange energies  $J \sim O(GeV)$  in (14) so much smaller than the tetron binding energies  $\sim O(E_P)$ ?**

Exchange couplings/integrals within a tetrahedron generically are of the form

$$J = \int d^3y_1 d^3y_2 f_1(\vec{y}_1) f_2(\vec{y}_2) V(\vec{y}_1 - \vec{y}_2) f_1(\vec{y}_2) f_2(\vec{y}_1) \quad (75)$$

where  $y_1$  and  $y_2$  denote internal coordinates and  $f_i$  the corresponding tetron wave functions. The value of  $J$  is the smaller, the smaller the overlap of the wave functions at different sites is. If the wave functions are very strongly concentrated, it is no problem to have  $J \ll E_P$ .

**2.3.21 If everything observed can eventually be derived from the fundamental interactions among tetrons, why are the forces of gravity so much weaker than particle physics effects?**

As shown in (2.3.19) and (2.2.12), gravity is only weak in connecting mignons, but strong in connecting tetrons and tetrahedrons. More specifically, an argument can be given, that both  $G$  and the fine structure constant  $\alpha$  arise as remnants from one and

the same fundamental tetron interaction. To understand this one should remember that the binding energy of a tetron in the hyper-crystal is given by the Planck energy  $\Lambda_P$ . Accordingly, an 'effective mass'  $M_P$  can be associated to a tetron bound in the hyper-crystal, and  $\Lambda_P$  can be thought to arise from a gravitational 'force'  $F_N = GM_P^2/r^2$  between two tetrons. On the other hand, it is shown in (2.2.3), that the value of the electric charge of a tetron in the hyper-crystal is  $e$ . Comparing the Coulomb force  $F_C = e^2/r^2$  with the gravitational effect  $F_N$ , one then obtains

$$F_C = \alpha F_N \tag{76}$$

This implies, that from the point of view of tetrons, gravity is anything but a weak force. On the contrary, the electric force is suppressed by a factor of  $\alpha$ . This suppression is due to the fact that while gravity more or less directly reflects the dynamics of the fundamental tetron interaction, in electromagnetism we see it mediated and weakened by photon exchange.

### **2.3.22 Do GUT theories play any role?**

No. It is difficult to imagine why there should be other SSBs in the tetron model besides those described in section 1. I see no reason for the proliferated Higgs sector characteristic for most GUT models.

### **2.3.23 Is there a unification of electroweak and strong couplings?**

No. In the tetron model the QCD forces are on a less fundamental footing than the electroweak interactions. For further details see (2.5.33) and (2.3.28). The existence of a unification scale for electromagnetism and the weak interactions has been discussed in (2.1.12).

### **2.3.24 What about domain walls?**

Phase transitions in physics are usually associated with the formation of domains. However, domain structures have never been observed in cosmology.

In the tetron model, cosmological domains either appear as separate universes, which according to (2.5.37) were created in  $R^6$  via a condensation process similar to the one



that has led to our own universe, or, if they are part of our own universe, they have long disappeared beyond our event horizon. To understand this in detail, one should first realize that one has to distinguish (i) domains arising at crystallization time (coordinate alignment of tetrahedrons) from (ii) those arising at the electroweak phase transition (tetrahedral alignment of isospins).

(i) In an ordinary crystal, one expects the appearance of domains with different values of the order parameter arising from concurrent nucleations of crystal germs in different points of space. In principle, this is also true for the  $R^{6+1}$  space under consideration, and one would expect domains, where the ordered coordinate tetrahedrons are rotated by some angle as compared to those shown in fig. 2. The reason why domains do not arise in the case at hand, has to do with the fact that the hyper-crystal grows into and occupies only a quasi 3-dimensional subspace of  $R^6$ . Therefore it intersects with other hyper-crystals from concurrent nucleation points, which grow into other 3-dimensional subspaces of  $R^6$ , in at most 1 point (because the intersection of 2 almost flat 3-dimensional submanifolds in  $R^6$  in general is just 1 point). This means the result of the other nucleations will be different hyper-crystals, i.e. they correspond to different worlds whose intersection with our universe consists of at most one point. At this point there will be a defect within the hyper-crystal structure, cf. (2.5.37).

(ii) In an ordinary ferromagnet, one expects the appearance of 'Weiss domains' with different ordering directions of spin vectors. In the case of isospin vectors such domains can in principle exist, too, and would differ by a global rotation of the isomagnetic tetrahedral 'star' configuration figs. 1 and 2. However, as discussed in (2.3.16), the phase transition is first order, i.e. associated with a sudden release of latent heat, which blows up the micro-elastic continuum, i.e. triggers an inflationary process which in turn shifts domain walls outside the visible part of the cosmos.<sup>7</sup>

---

<sup>7</sup>Models with inflation near the electroweak scale have been discussed extensively in the literature[72, 73, 74, 75].

### 2.3.25 Are there excitations of the hyper-crystal besides the known quarks, leptons and scalar and vector bosons?

Yes. An incomplete listing includes:

–phinons. They are the analogs of phonons in a solid and have been described under more general circumstances in [3]. In the case at hand there are 12 phinon states, that can be classified according to representations of the permutation group  $S_4$ . They travel as quasi-particles through the hyper-crystal in the same way as mignons do. Phinon masses are expected to be much larger than mignon (quark/lepton) masses. While the mignon spectrum is lying at and below  $\Lambda_F$ , the phinon spectrum is concentrated towards the crystallization energy  $\Lambda_P$ . Note, this is not a very accurate characterization, in view of the fact that neutrino masses are so tiny with respect to the Fermi scale. Note further, phinons are internal coordinate vibrations, and thus have to be distinguished from gravitational waves. An interesting question is whether mignon-phinon scattering is possible.

–isospin density waves: they are to be distinguished from phinons and from mignons. The vibrators in this case are similar to the isospin vectors (4), however without the factor of  $\vec{\tau}$ , i.e. given by  $\psi^\dagger(1 \pm \gamma_5)\psi$ . As far as I can judge these excitations correspond to a fourth family of fermions, i.e. a lepton-like and a quark-like isospin doublet, probably higher in mass, because they are not related to the other families by the  $Z_3$  family quantum number inherent in the Shubnikov group  $A_4 + S(S_4 - A_4)$ . In particular, the fourth 'neutrino' is expected to be much heavier than the known neutrinos, because its mass is not suppressed by internal angular momentum conservation, cf. (2.4.12).

–one should mention scalar fields other than the standard Higgs boson. They are the components of the second, inert Higgs doublet (48), the lightest among them being the most promising candidate for dark matter, cf. (2.5.35).

### 2.3.26 What about vibrations of $\bar{\psi}\gamma_\mu\vec{\tau}\psi$ , $\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi$ etc?

These are other examples of higher mass excitations of the hyper-crystal.

### 2.3.27 Could quark and leptons be phinons?

or in other words: what is the advantage of using mignons with Shubnikov symmetry  $A_4 + S(S_4 - A_4)$  over phinon excitations with symmetry group  $A_4 \times Z_2$  as advocated in [3]?

The answer is that many of the attractive features of mignons are absent, like the explanation of the Higgs mechanism, of why  $m_t \gg m_b$ , of tiny neutrino masses etc.

### 2.3.28 \*Is it possible to understand the dynamics of the strong interactions from within the tetron approach?\*

I don't have a final answer to this question. From its very construction the tetron model is concerned mainly with the symmetries and interactions of electroweak physics. The colors of quarks arise merely as a byproduct, because they are interpreted as the 3 d.o.f. of a Shubnikov triplet representation. It is therefore clear that QCD with a color gauge group and SU(3) color triplets does not directly arise in the tetron model.

In section 1 it was argued that the phase transitions of a 6+1 dimensional spacetime filled with a condensing tetron gas supplies all relevant physics for the early universe and it may even account to understand the forces of gravity. As for the latter the suggestion is that it may arise from elastic forces between tetrahedrons which are the remnants of the fundamental 6-dimensional tetron-tetron coordinate interactions (2.2.11) and induce curvature and/or torsion effects on Minkowski space. One would like to interpret the strong interactions in a similar spirit, namely starting with the paradigm that there are no other interactions in the universe besides the ones among tetrons.

Gluons cannot be part of the isomagnetic geometry, because the  $SU(2) \times U(1)$  bundle connection allows only for the 4 electroweak gauge bosons. One may note, however, that a Shubnikov invariant mignon-mignon interaction is to be expected among the triplets, transforming as

$$T \times T = A + A' + A'' + 2T \tag{77}$$

As shown in [5] this structure can be embedded into a  $SU_c(3)$  algebra where the color indices correspond to the 3 d.o.f. of the triplet representations in (9). Unfortunately,

as yet there is no proof that gluons and QCD gauge interactions can really arise from this line of reasoning.

### 2.3.29 Is color a part of isospin?

In the tetron model the color triplets of quarks transform according 3-dimensional representations of the Shubnikov group (5), and this symmetry group is defined in terms of transformations within the same internal space as weak isospin. This could lead one to suspect that color in some sense is part of isospin. However, this is not the case. While the Shubnikov group is unbroken to the lowest energies, isospin symmetry corresponds to the free rotations of isovectors on each tetron site separately and is completely broken in the ordered state.

### 2.3.30 Is there a connection between the QCD vacuum and the electroweak condensate?

Not really. Chiral symmetry breaking is the appearance of a non-vanishing quark condensate  $\langle \bar{u}u + \bar{d}d \rangle \approx -(0.1\text{GeV})^3$  which breaks  $SU(2)_L \times SU(2)_R$  to the diagonal isospin  $SU(2)_V$  group. The main difference as compared to the electroweak case is that all these groups are global, not local symmetries. Furthermore, the value of the quark condensate is much smaller than that of the Higgs condensate. While the former extends over the strong interaction scale  $\sim 1$  GeV, the latter is related to the Fermi scale.

According to (39) the Higgs condensate is related to  $U_*$ , i.e. the tetrahedral star configuration pointing outside as in fig. 1. In contrast, the quark condensate is a singlet w.r.t. the relevant quantum number (in this case color, while for the Higgs condensate it is isospin). In other words

$$\langle \bar{q}q \rangle = \langle \bar{q}_1 q_1 + \bar{q}_2 q_2 + \bar{q}_3 q_3 \rangle \quad (78)$$

does not correspond to a preferred direction or orientation in color space, but has the property of an internal color density.

## 2.4 Questions about the Quark and Lepton Mass Spectrum and the CKM and PMNS mixing Matrices

Quark and lepton masses can be calculated as excitation frequencies of mignons via a straightforward procedure where the results are obtained quite naturally by considering isomagnetic interactions among the tetrons of one or two tetrahedrons only. Analytic expressions for masses have been derived in [2] in terms of internal Heisenberg and Dzyaloshinskii-Moriya exchange couplings.

### 2.4.1 Using exchange couplings instead of Yukawas – isn't one replacing one set of unknown parameters by another set and one effective theory (the Standard Model) by another one (the internal Heisenberg model)?

No, because the internal couplings can in principle be calculated from first principles as exchange integrals over internal space, just as in ordinary magnetism the exchange couplings of the Heisenberg model are in principle calculable from exchange integrals of electronic wave functions over physical space. What one needs to know is the underlying 6+1 dimensional dynamics of tetron interactions, cf. (2.1.20), (2.2.11) and (2.2.12).

### 2.4.2 Why are mignons spin- $\frac{1}{2}$ particles?

Since they are constructed from excitations of 'bosonic' isospin operators (2), one could be led to believe that they are bosons, just like magnons in ordinary ferromagnets are bosonic quasi-particles.

However, it is important to distinguish the behavior in internal space from that in Minkowski space. While mignons transform as Shubnikov singlets A and triplets T (i.e. *not* as projective representations) w.r.t. internal space, it is not hard to see that they are Dirac fermions w.r.t. Minkowski space.

The point to note is that mignons are not bound states of tetrons but eigenmodes of their excitations. As such they are not tensor products but linear combinations of (fluctuations of) tetron fields  $\psi_a^\alpha$  (where  $a = 1, 2$  is the internal and  $\alpha = 0, 1, 2, 3$  the

Dirac index of the tetron). Since each mignon is a vibration of *one* isospin eigenmode, one concludes that it must be a Dirac particle w.r.t. the Minkowski base space. Using the 'bosonic' isospin vectors is merely a tool to separate the proper isospin triplet vibrations of  $\vec{Q} = \psi^\dagger \vec{\tau} \psi$  from singlet density fluctuations of  $\psi^\dagger \psi$ , cf. (2.3.25). Looked at from 'below', i.e. from the Minkowski base space, the tetron excitations are Dirac fermions. If such an excitation travels through the crystal as a quasi-particle, it can be either L or R, particle or anti-particle.

### **2.4.3 Why are there exactly the quark and lepton states of the 3 generations?**

The unbroken (Shubnikov) symmetry group has only singlet and triplet representations, and the 8 independent isospin vectors on a tetrahedron lead exactly to the flavor spectrum of 24 quarks and leptons as given in (9).

### **2.4.4 Are there other ground states than the tetrahedral one, which yield the appropriate quark and lepton spectrum (9)?**

No. I scanned other geometries with 8 iso-magnetic vibrators and found that for most systems mignons appear in 2-dimensional representations[9, 10], not useful for the q/l spectrum of particle physics. This applies in particular to the configuration (m2) described in (2.5.1).

### **2.4.5 Should one really use covariant isospin vectors (3) containing both particle and anti-particle contributions as vibrators?**

Yes, one should. It is important for the vanishing electric charge of the hyper-crystal, for the formation of gauge bosons out of tetron-antitetron pairs, and in general to maintain relativistic covariance as seen in (7) and figs. 4 and 5. Technically it is important for the mass calculations corresponding to mignon mass terms  $\langle 0|T(\bar{q}q)|0\rangle$ .

#### 2.4.6 Why not use only $\vec{Q} = \vec{Q}_L + \vec{Q}_R$ as vibrators instead of $\vec{Q}_L$ and $\vec{Q}_R$ separately?

In order to obtain the 3 families of quarks and leptons one would have to consider systems with 8 instead of 4 tetrons. This possibility has been discussed in (2.3.1). The main counterargument is that the  $SU(2)_L$  vibrators  $\vec{Q}_L$  are needed to inherit the nature of the SM SSB and the top mass in the model, cf. (15).

#### 2.4.7 Is it okay to start the mass calculations using chiral $SU(2)_L \times SU(2)_R$ symmetry quantities $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$ ?

Yes. This is just the way the SM works. Non-zero fermion masses are developed via SSB starting from a massless, i.e.  $SU(2)_L \times SU(2)_R$  chirally symmetric theory. The role of custodial  $SU(2)$  in the tetron model is discussed in (2.4.17).

#### 2.4.8 Can the simple Heisenberg interaction (14) really explain the full q/l mass spectrum with its extreme hierarchies?

No. As shown in [2] the masses of some of the fermions get contributions from other physical sources, namely

–the top mass is dominated by a contribution of order  $\Lambda_F$  which stems from the symmetry breaking *inter*-tetrahedral interactions (15). Physically it arises because the top quark corresponds to the 3 eigenmodes which 'disturb' the global ground state in the strongest possible way. This disturbance is also responsible for the hierarchy observed in the CKM matrix elements.

–only strange-, charm- and muon-mass are dominated by anti-ferromagnetic exchange couplings within one tetrahedron, and thus can be obtained from the *inner*-tetrahedral exchange couplings (14) alone.

–down-quark, up-quark and electron are left massless by the Heisenberg and DM interactions (14) and (15). They get their relatively small masses from energetically favored torsion contributions[2], i.e. vibrations of the internal spin vectors of the generic form  $d\vec{Q}/dt \sim \vec{Q}$ .

–neutrino masses are protected by internal angular momentum conservation, i.e. by the internal rotational symmetry, cf. (2.4.12). The way how they acquire their tiny

mass values is described in [2].

#### 2.4.9 Is there a mismatch between the internal Heisenberg interaction (14) for a single tetrahedron and the DM interaction (15) involving neighboring tetrahedrons?

This is a technical question concerning the calculation of quark and lepton masses presented in [2], and the answer is that technically one can treat the isospin vectors from the neighbors as if they were vectors of the original tetrahedron.

Denoting 2 neighboring tetrahedrons as primed and unprimed and starting with interactions of the form  $\sim \vec{S}_i \vec{S}'_j$ , one would expect the doubling of the number of eigenmodes. However, due to the symmetry between the 2 tetrahedrons, i.e. between the  $\vec{S}$  and  $\vec{S}'$ , the modes arrange in pairs of identical energy. In other words, the doubling of modes is a trivial one.

To understand this in more detail consider the first term (Heisenberg contribution) in the inter-tetrahedral interaction (15) and assume that the inner-tetrahedral distances are much smaller than the inter-tetrahedral ones. This is in accord with arguments given in (2.3.17), that after the long time of cosmic expansion one expects  $r \gg R$  in fig. 2. This assumption implies that the couplings  $J_{SSB}$  of an  $\vec{S}_i$  to all isospin vectors in the neighboring tetrahedron are identical (as anticipated in (15)) and one obtains

$$\frac{d\vec{S}_i}{dt} = J_{SSB} [\vec{S}_i^0 \times \sum_{j=1}^4 \vec{S}'_j] \quad (79)$$

where the superscript 0 denotes ground state values. A second set of equations is obtained for  $d\vec{S}'_i/dt$  by exchanging the role of the 2 tetrahedrons, i.e. the primed and unprimed quantities. An obvious set of solutions to these equations fulfills  $\vec{S}_i = \vec{S}'_i$ , i.e. the vibrations are completely in step, and one obtains the trivial doubling of modes mentioned above.

As for the DM part (second term) in (15) a similar argument can be given. Following the results in [2] this leads to the conclusion that the top quark receives the overwhelming mass contribution from the inter-tetrahedral interaction (15).

It is interesting to note that in a cosmological scenario where one would have  $r \ll R$



the mignon mass spectrum would turn out quite different because in that case it is natural to assume that the coupling of  $\vec{S}_i$  to  $\vec{S}'_i$  is much larger than to the other  $\vec{S}'_j$ .

#### 2.4.10 How are the CKM and PLMN matrix elements obtained in the tetron model?

In the SM the possibility of inter-family mixing arises because interaction eigenstates are distinguished from mass eigenstates. In the tetron model the situation is similar: on one hand, there are the original modes which correspond to small vibrations of the isospin vectors, and on the other the eigenmodes of the vibrations whose eigenfrequencies correspond to the quark and lepton masses. The original modes  $\delta\vec{Q}_{iL/R}$  are directly related to the SM interaction eigenstates because  $\vec{Q}_L$  according to (4) defines the isospin 'charge' of the left handed SU(2) current appearing in the Glashow theory.

Therefore, in order to obtain the CKM and PLMN matrices in the tetron model, one can make use of the results of the diagonalization procedure from the  $\delta\vec{Q}_{iL/R}$  to the mass/frequency eigenmodes. Since the mixing in the right-handed sector is not observable, only projections onto the  $\delta\vec{Q}_{iL}$  need to be treated.

#### 2.4.11 Why are interfamily interactions suppressed in the tetron model?

It is a prevalent problem in many composite models, to explain why - apart from CKM mixing effects - transitions between fermions of different families do not exist, e.g. why  $\mu \rightarrow e\gamma$  is forbidden.

In the tetron model this fact can be understood from the symmetry of states. Since the gauge bosons W, Z and  $\gamma$  (2.1.4) behave trivially under the Shubnikov group, the same must be true for the fermion-antifermion conglomerates, which get pair produced from them.

Analyzing products of representations of (82), it turns out that all inter-family conglomerates behave non-trivially under Shubnikov transformations. For example, a combination of a  $\mu^-$ - and an  $e^+$ -mignon transforms as a non-trivial singlet and therefore cannot become a photon.

It may be noted that the antiparticle of  $A'_{1+is}$  transforms as  $A''_{1-is}$ .

#### 2.4.12 How can the smallness of neutrino masses be understood?

Neutrinos are interpreted as internal Goldstone particles of the breaking of the tetronic isospin by the formation of the discrete structure fig. 1. The associated conserved Noether charge is given by the total internal angular momentum  $\vec{\Sigma}$  defined in (16) which implies the existence of 3 zero-frequency modes. This is analogous to how magnons are interpreted as Goldstone modes in ordinary magnetism, except that here one is considering physical processes in the internal spaces.

While in ordinary ferromagnets after magnetization a U(1) symmetry about the z-axis survives, in the given frustrated configuration fig. 1 all three SO(3) generators give rise to Goldstone bosons, to be identified as the internal magnons corresponding to the 3 neutrino species.

#### 2.4.13 Neutrinos are fermions. How can they be Goldstone modes?

One has to distinguish the dynamics in internal from that in physical space. In physical space the neutrinos are fermions, but they are Goldstone bosons w.r.t. the dynamics in internal space, because in internal space they are described by (bosonic) excitations of the total internal angular momentum  $\vec{\Sigma}$  defined in (16) which is the conserved quantity associated with the internal rotational symmetry.

As discussed before, none of the representations in (9) are projective representations of the Shubnikov group, so all quarks and leptons are 'bosonic' w.r.t. the internal dynamics, cf. (2.4.2) and (2.4.18).

#### 2.4.14 How do neutrinos obtain their tiny non-zero masses?

As Goldstone modes neutrinos are strongly protected to getting masses. However, as proven in [2] the observed non-zero neutrino masses can be generated on the phenomenological level by tiny torsional interactions which violate (17). These can also be used to accommodate appropriate PMNS mixing values. Physically the existence of such interactions is a signal for the activity in isospin space of small anisotropic forces. This is discussed in more detail in (2.5.21).

### 2.4.15 Are neutrinos Dirac or Majorana particles?

They are Dirac fermions. Like all other quark and lepton flavors they inherit this property from the tetrons, cf. (2.4.2).

### 2.4.16 How is weak isospin realized on the mignon level?

The first fact to note is that the weak isospin of the SM cannot be directly identified with the tetronic isospin  $SU(2)$ . As can be seen by calculating the masses of quarks and leptons from vibrations  $\delta\vec{S} = \delta\vec{Q}_L$  and  $\delta\vec{T} = \delta\vec{Q}_R$  of the ground state fig. 1, the isospin of mignons corresponds to an exchange of the roles of  $\vec{S}$  and  $\vec{T}$ , i.e. of the  $SU(2)_L$  and the  $SU(2)_R$  sector.

The most appropriate way to construct quark and lepton isospin is to assume the existence of a tetron  $\psi_L$  and an antitetron  $(\chi_R)^c = (\chi^c)_L$  on each tetrahedral site  $i=1,2,3,4$ , as depicted in figs. 4 and 5. Here the charge conjugation operator is defined as usual for a Dirac spinor  $F$ , i.e.  $F^c = C\gamma_0 F^*$ .

One can either interpret fig. 4 in such a way that there are 2 particles (a tetron and an antitetron) on each site, or by saying that one is considering the tetron and the antitetron contribution of a single field to the isospin vectors  $\vec{Q}_L$  and  $\vec{Q}_R$ , cf. the discussion in (2.5.17).

Although  $(\chi_R)^c$  is left-handed, it transforms under  $SU(2)_R$  and thus can be used to build a  $\vec{Q}_R$  vibrator. For the question why tetrons and antitetrons do not annihilate inside the hyper-crystal see (2.2.8).

I also tried an approach without antiparticles, i.e. with  $\chi_R$  instead of  $(\chi_R)^c$ , but have abandoned this for the following reasons:

–since tetrons are fermions, the wave function for each pair would have to respect the Pauli principle. While the spatial part must be symmetric (the argument is similar as for in the helium atom), the spin and the isospin part are both expected to be anti-symmetric singlets, and this would violate the Pauli principle. If one of the tetrons is an antiparticle, one does not run into this problem of two identical fermions.

– $A(\nu_e)$  and  $A(e)$ ,  $T(u)$  and  $T(d)$  etc would not be true isospin partners but components of Kramers doublets. The exchange  $\vec{Q}_L \leftrightarrow \vec{Q}_R$  induces an isospin exchange  $U \leftrightarrow D$  only if one antiparticle is involved. The reason is that charge conjugation

compensates for the internal time reversal active in a Kramers doublet. This will be explained in more detail in (2.4.19).

–finally, since the Higgs field and the gauge bosons have been interpreted as tetron-antitetron excitations, it seems a good idea to have antitetrans already appearing in the ground state.

The isospin vectors, whose vevs are depicted in fig. 4, are given by

$$\vec{Q}_L = \frac{1}{2} \psi_L^\dagger \vec{\tau} \psi_L \quad \vec{Q}_R = \frac{1}{2} (\chi_R)^{c\dagger} \vec{\tau} (\chi_R)^c \quad (80)$$

A transition between them is then necessarily accompanied by an exchange of isospins U and D via[84]

$$(U_L, D_L) \rightarrow (-D_R^c, U_R^c) \quad (81)$$

If one strips off the Dirac structure, (81) is identical to an internal time reversal (6), which was considered in connection with the Shubnikov transformations in section 1. One concludes that the Shubnikov symmetry (5) can be defined as in (7) by using charge conjugation and without introducing the concept of an internal time. For more details on the action of discrete symmetries like C, P and T on the tetron ground state see (2.4.20). It is interesting to note that the internal reflection operators which exchange the elements of  $A_4$  and  $S_4 - A_4$  comprise isospin transformations, charge conjugation as well as transitions between left and right.

To summarize, it has been shown that the transition between weak isospin partners like  $A_1(e)$  and  $A_1(\nu_e)$  can be obtained by exchanging  $\vec{Q}_{Li}$  and  $\vec{Q}_{Ri}$  on the tetrahedral sites  $i$ . In the actual calculation of mignon eigenstates, it turns out that the top-quark is predominantly an  $\vec{S} = \vec{Q}_L$  excitation while the b-quark is  $\vec{T} = \vec{Q}_R$ . On the other hand the neutrinos are given by vibrations of the conserved quantity  $\sum_i (\vec{S}_i + \vec{T}_i)$ , while charged leptons are approximately excitations of the  $\sum_i (\vec{S}_i - \vec{T}_i)$  combination.

The connection between the attributes 'left-handed' and 'up-type' (and similarly 'right-handed' and 'down-type') is of fundamental importance in the tetron model. It relies on the chiral structure of the internal ground state fig. 1 and the octonion induced form of the tetron interactions (2.1.20). Furthermore, it is at the heart of the tetron model explanation of weak parity violation (2.1.20) and of the large value of the top quark mass from (15) as compared to the other q/l masses, cf. (2.4.21).

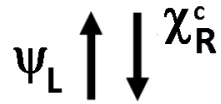


Figure 4: Ground state configuration of the tetron-antitetron pair on a single tetrahedral site. The vertical axis corresponds to the radial direction in the sense of (11), i.e. isospin U means outwards, D inwards. Drawn are the ground state values of isospin vectors  $\vec{Q}_L$  and  $\vec{Q}_R$  which in (80) are defined in terms of  $\psi_L$  and  $(\chi_R)^c$ , respectively. Considering the tetrahedron as a whole, the configuration is shown in fig. 5 and is chiral, both in internal and physical space. The internal chirality is flipped by  $U \leftrightarrow D$ , the external by  $L \leftrightarrow R$ . Note that  $(\chi_R)^c$  is left-handed, but transforms under  $SU(2)_R$ . Therefore and since the configuration shown is preferred over that of opposite chiralities through the sign of the interaction (68),  $(\chi_R)^c$  does not take part in the symmetry-breaking *inter-tetrahedral* interactions (15).

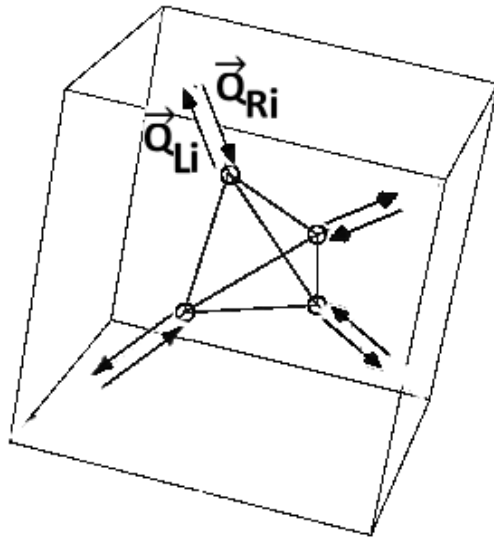


Figure 5: The local ground state of the tetron model with 8 internal spin vectors  $\vec{Q}_{Li}$  (pointing outwards) and  $\vec{Q}_{Ri}$  (pointing inwards),  $i=1,2,3,4$ , accounts for  $3 \times 8$  d.o.f. corresponding to 24 quarks and leptons according to (9). Due to the antiparticle nature of the isospin vectors pointing inwards, internal parity is maximally violated. For details on the action of discrete symmetries C, P and T on this state see 2.4.20.

### 2.4.17 The difference between tetron isospin and 'custodial' SU(2)

In the bosonic sector of the SM (i.e. without quarks and leptons) there is a global vectorlike SU(2) symmetry which remains intact after the SSB. This is usually called 'custodial SU(2)', and it should not be mixed up with the isospin SU(2) of the 3-dimensional internal space considered in the tetron model. While custodial SU(2) arises on the level of mignons, the tetron model SU(2) exists on the level of tetrans and comprises not only electroweak but also color and family symmetries. While the dominant corrections from custodial SU(2) breaking are  $\sim m_t - m_b$ , tetron SU(2) breaking terms generate the neutrino masses, cf. (2.4.12)ff. More details on the relation between tetron and electroweak(=mignon) SU(2) can be found in (2.3.29) and (2.4.16).

### 2.4.18 Do pairs of ordinary and true Shubnikov representations form isospin doublets?

No. To explain this point, I start with a remark, that 'true' Shubnikov representations have nothing to do with 'projective' representations mentioned above. While the latter are representations of the covering group, a true Shubnikov representation is a representation of  $G_4 = A_4 + S(S_4 - A_4)$  which is not a representation of  $A_4$ . True Shubnikov representations are labeled by an index  $s$  in the following. The Shubnikov group  $G_4$  under consideration has the property that for each ordinary representation  $D$  of  $G_4$  there is exactly one true Shubnikov representation  $D_s$ . If one puts an isospin along the z-direction, true Shubnikov representations are related to the excitations of the y-components of the isospin vectors, while  $Q_x$  and  $Q_z$  correspond to the ordinary representations.

Analyzing the representations appearing in (9) one finds that  $A$  and  $A_s$ ,  $T$  and  $T_s$  arise in the combinations

$$\begin{aligned} A(\nu_e) + A'_{1+is}(\nu_\mu) + A''_{1-is}(\nu_\tau) &+ T(u) + T_{1+is}(c) + T_{1-is}(t) + \\ A(e) + A'_{1+is}(\mu) + A''_{1-is}(\tau) &+ T(d) + T_{1+is}(s) + T_{1-is}(b) \end{aligned} \quad (82)$$

where the notation  $1 \pm is$  means that the second and third family are generated by excitations of  $Q_x \pm iQ_y$ , while the first family corresponds to excitations of  $Q_z$ . According to this result an isospin doublet is not given by a pair  $(A, A_s)$  or  $(T, T_s)$

of an ordinary representation and a Shubnikov representation. Instead, the rows  $A, A_{1+is}, A_{1-is}$  and  $T, T_{1+is}, T_{1-is}$  correspond to particles of the 1., 2. and 3. family. The representation (82) can be used to show that interfamily transitions are suppressed in the tetron model, cf. (2.4.11).

#### 2.4.19 A simple mathematical understanding of all the 24 Shubnikov states in (82)

First of all note that, although mignons are actually precessions of internal angular momentum vectors, I sometimes call them 'vibrations' or simply 'excitations' in this paper.

In this section the 24 mignon precessions are derived successively starting with the 3 mignons arising from a single tetron, then going over to 2 tetrons and ending with the 8 tetron configuration fig. 5. The results of the discussion will amount to the following statements:

- the existence of 3 families reflects the 3 dimensions of the internal space.
- weak isospin of quarks and leptons corresponds to isospin transformations of tetron and anti-tetron within one tetrahedral site.
- the color d.o.f. arise from the 4-fold structure of the tetrahedrons.

By slightly abusing notation, mignon excitations of a single tetron can be described as isospinors of the form

$$\psi = (\langle U \rangle + \delta U, \langle D \rangle + \delta D) = (1 + \delta U, \delta D) \quad (83)$$

$\langle U \rangle$  and  $\langle D \rangle$  are the isospinor values in the ground state. For the convenience of the following discussion,  $\langle U \rangle$  can be put to 1 and  $\langle D \rangle$  assumed to vanish.

$\delta U$  and  $\delta D$  contain a time dependence  $\sim \exp(i\omega t)$  where  $\omega$  is the mass/energy/frequency of the precession. The corresponding fluctuation of the isospin vector  $\vec{Q} = \psi^\dagger \vec{\tau} \psi$  is given by

$$\delta \vec{Q} = \vec{Q} - \langle \vec{Q} \rangle = \begin{pmatrix} \text{Re } \delta D \\ \text{Im } \delta D \\ \text{Re } \delta U \end{pmatrix} + O(\delta^2) \quad (84)$$

where  $\langle \vec{Q} \rangle = (0, 0, 1)$ .

It may be noted, that on the tetrahedron one has non-vanishing vevs  $\langle \vec{Q} \rangle \neq 0$

only for the single isospin vectors, whereas for the tetrahedron as a whole the total internal angular momentum vanishes in the ground state, i.e.

$$\sum_{i=1}^4 \langle \vec{Q}_i \rangle = 0 \quad (85)$$

and the only relevant vev becomes the Higgs vev as described in (2.3.10)-(2.3.11). Coming back to the single tetron states (84), the 3 vibrations  $\text{Re } \delta D$ ,  $\text{Im } \delta D$  and  $\text{Re } \delta U$  in the x, y and z-direction essentially correspond to the third, second and first family of quarks and leptons. The y-vibration is a true Shubnikov excitation in the sense of (2.4.18).

The color quantum number arises as soon as all four tetrahedral sites are taken into account. In place of one vibrator (83) one then has four, and arrives at one singlet (A) and one triplet (T). For more details about the origin of color in the tetron model see (2.3.28) and (2.3.29).

So far, we have ignored the fact that there are 2 vectors  $\vec{Q}_L$  and  $\vec{Q}_R$  that vibrate on each tetrahedral site instead of one. These vectors are given in (80), and the doubling of mignon states which they induce gives rise to weak isospin of quarks and leptons, i.e. to partners like  $A(\nu_e)$  and  $A(e)$  in (82).

In explicit terms the mignons from the 2 tetrons fig. 4 on a tetrahedral site are given by isospinors of the form

$$\psi_L = (U_L, D_L) = (1 + \delta U_L, \delta D_L) \quad \chi_R^c = (-D_R^c, U_R^c) = (-\delta D_R^c, -1 + \delta U_R^c) \quad (86)$$

and the corresponding isospin vectors are

$$\vec{Q}_L - (0, 0, \frac{1}{2}) = \begin{pmatrix} \text{Re } \delta D_L \\ \text{Im } \delta D_L \\ \text{Re } \delta U_L \end{pmatrix} \quad \vec{Q}_R - (0, 0, -\frac{1}{2}) = \begin{pmatrix} \text{Re } \delta U_R^c \\ \text{Im } \delta U_R^c \\ \text{Re } \delta D_R^c \end{pmatrix} \quad (87)$$

which makes the weak isospin transformation between the 2 kinds of mignons evident.

Actually, for a more precise analysis, terms of order  $\delta^2$  or, equivalently, fluctuations of  $\text{Im } \delta U$  have to be included in the analysis. For example, they enter in

$$\delta Q_x = \text{Re } \delta D + \frac{1}{2} [\delta U^\dagger \delta D + c.c.] \quad (88)$$



or in  $Q_z$ , because  $Q_z = \psi^\dagger\psi/2 - |\delta D|^2$  with the tetron density given by

$$\psi^\dagger\psi = |1 + \delta U|^2 + |\delta D|^2 \quad (89)$$

This makes the weak isospin transition more complicated than (87).

The complete set of the 24 quark and lepton eigenstates is worked out in [2].

#### 2.4.20 The role of CPT symmetry in the tetron model?

In this section I want to show that CPT symmetry plays an eminent role in the relativistic generalization (7) of the Shubnikov group (5). The point is that in the presence of relativistic particles, CPT supersedes the PT symmetry inherent in ordinary non-relativistic magnetism.

Actually for this argument to work, the parity transformation P is to be interpreted as the product  $P_{in}P_{au}$  of internal and external parity, where  $P_{in}$  reverses the sign of the internal and  $P_{au}$  that of the physical coordinates. In other words, CPT is the CPT transformation not in Minkowski space (this would be  $CP_{au}T$ ) but in the underlying 6+1 dimensional spacetime. Therefore this invariance will sometimes be called CPT6 to distinguish it from ordinary CPT in Minkowski space.

We start the line of reasoning by summarizing what we already know about C, P and T in the tetron model:

-in (2.1.20) the (maximal external) parity violation observed in the SM was related to the maximal violation of internal parity  $P_{in}$  induced by the ground state fig. 1. This relation arises via the appearance of products  $\gamma_5\vec{\tau}$  in the tetron dynamics.

-in (2.4.16) charge conjugation was discussed as a necessary ingredient, if one is working in a relativistic environment and includes antitetrons in the ground state as done in figs. 4 and 5. Due to the antiparticle nature of the isospin vectors pointing inwards, internal parity in fig. 5 is again maximally violated as in fig. 1.

-it is well known that time reversal is an essential ingredient in any kind of magnetic phenomena. One may even go as far to say that magnetism is the physics of the breaking of T symmetry. For example, T appears in the nonrelativistic version (5) of our Shubnikov symmetry. T itself is broken as in any magnetic alignment, but the product  $TP_{in}$  is an implicit part of the symmetry (5).

The action of CPT6 on an isospin vector  $\vec{Q}_L$  pointing up(=outwards) as in figs. 4

and 5 is given by

$$\vec{Q}_L \xrightarrow{C} -\vec{Q}_L \xrightarrow{P_{au}} -\vec{Q}_R \xrightarrow{P_{in}} \vec{Q}_R \xrightarrow{T} -\vec{Q}_R \quad (90)$$

As discussed in section 1, T and  $P_{in}$  reverse the orientation of isospin vectors, while  $P_{au}$  exchanges L and R. C also reverses the orientation of an isospin vector, because antiparticle contributions enter in isospin vectors  $\vec{Q}$  with an opposite sign, cf. (3). Eq. (90) shows that CPT6 invariance is equivalent to demanding that the local ground state must be formed by  $\vec{Q}_L$  and  $\vec{Q}_R$  of opposite orientation like in figs. 4 and 5, more precisely one has  $\langle \vec{Q}_{Li} \rangle = -\langle \vec{Q}_{Ri} \rangle$  on each tetrahedral site  $i$ . In case the local ground state would be  $\langle \vec{Q}_{Li} \rangle = \langle \vec{Q}_{Ri} \rangle$ , i.e. the isospin vectors point in the same direction, one would have PT or CP symmetry, but CPT6 would not be a symmetry.

Note, this discussion of the ground state's discrete symmetries has to be distinguished from the CP violation in the CKM matrix. As discussed in [2], the latter arises from the complex nature of the exchange couplings and the resulting phases in the mixing of mignon states.

#### 2.4.21 Why is top so heavy, why not bottom?

Up to tiny CKM and V+A mixing effects, the top quark corresponds to vibrations of  $\vec{\Sigma}_L = \sum_i \vec{Q}_{Li}$ . This vector plays a special role in the tetron model, which has to do with the chiral nature of the SSB and with the relation between internal and external parity violation[1] induced by (68), and because nature has chosen to break internal parity, i.e. prefers the state fig. 1 with isospin vectors pointing outwards over the one with those pointing inwards. The top-mignon is defined as that internal precession, where all isospin vibrations act against the SSB alignment in the strongest possible way.

#### 2.4.22 Why is $m_b$ much smaller than $m_t$ and from where does it get its predominant contributions?

In contrast to the top quark, the b-quark is mostly an excitation of the  $\sum_i \vec{Q}_{Ri}$ , and therefore does not get a contribution from the  $SU(2)_L$  Dzyaloshinskii-Moriya interaction (15).

I have considered several sources for an  $m_b$  value of order 5 GeV, the most straightforward being the existence of a tiny right handed (=V+A) current. Actually, in the calculation[2] it is more difficult to accommodate the bottom mass than the values of the lighter quark masses. This is because contributions to  $m_b$  are naturally associated to the right handed version of (15) where  $\vec{T}$  instead of  $\vec{S}$  appears, and this V+A contribution must be radically suppressed according to the argument that weak parity violation is due to the internal chirality of the tetrahedrons, as discussed in (2.1.20). Thus from a general point of view the introduction of a small V+A current does not serve as a fully satisfactory explanation for a noteworthy bottom mass.

On the other hand one is talking here about a  $m_b/m_t = 3\%$  effect, and a 3% V+A correction to the leading V-A coupling is compatible with present experimental bounds[6]. Such a correction can then be used to account for the observed value of  $m_b$ [2].

#### **2.4.23 If up type quarks arise mainly from vibrations of $\vec{Q}_L = \vec{S}$ , how can their right-handed version be produced?**

As shown in [2], lepton states originate dominantly from vibrations of the form  $\vec{S} \pm \vec{T}$ , while up and down quark states are related to vibrations of  $\vec{S}$  and  $\vec{T}$ , respectively. Therefore one might suspect that the helicities of quarks and leptons generated in this way are restricted, too. However, it must be noted that an excitation  $\delta$  of a left-handed isospin vector can in principle vibrate into any chiral direction. The same is true for right handed vectors  $\vec{Q}_R$ .

## **2.5 Connections to Gravitation and Cosmology**

This section relies on the interpretation of the world as an elastic system of internal tetrahedrons as described in section 1. As evident from fig. 2, the full universe can be generated by exact repetition of this discrete structure. The following list of questions and answers sheds light on the nature of this repetition and furthermore shows how the tetron particle model may be incorporated within a larger, cosmological framework.

Throughout it will be assumed that after crystallization spacetime is homogeneous and isotropic, i.e. it can be described by a Friedmann / Robertson-Walker (FLRW) metric

$$ds^2 = dt^2 - a^2(t) \left[ \frac{(d\vec{x})^2}{1 - k\vec{x}^2} + \vec{x}^2 d\Omega^2 \right] \quad (91)$$

where  $a(t)$  is the scale factor and  $k=0,+1$  or  $-1$  for a euclidean, spherical or hyperbolic universe. Rewriting this as

$$ds^2 = dt^2 - a^2(t) \gamma_{ij} dy^i dy^j \quad (92)$$

one has

$$\gamma_{ij} = \delta_{ij} + k \frac{y_i y_j}{1 - k y_n y^n} \quad (93)$$

with  $i, j = 1, 2, 3$ . The  $y^i$  are the 'comoving' coordinates, from which the physical coordinates are obtained as  $x^i = a(t)y^i$ . Accordingly, the physical velocity of an object can be decomposed as

$$v^i = \frac{dx^i}{dt} = a(t) \frac{dy^i}{dt} + \frac{da}{dt} y^i = w^i + H x^i \quad (94)$$

where the second term introducing the Hubble parameter  $H$  is called the 'Hubble flow' and the first term  $w^i$  defines the 'peculiar velocity' of an object, i.e. its velocity relative to the Hubble flow.

Note that the FLRW metric fulfills  $g_{00} = 1$  and  $g_{0i} = 0$ , i.e. it corresponds to a metric in a 'synchronous gauge'[93]. Such a metric has a particular simple interpretation in the tetron model, because the only non-trivial elements are the spatial  $g_{ij}$ . These can be identified with variable spatial distances in the 3-dimensional elastic system of tetrahedrons described in section 1. See also fig.6.

### 2.5.1 Is the tetrahedral ground state stable or metastable?

In other words, are there isomagnetic states  $X$  with a lower energy? If yes, this might threaten our world when such a state would be produced in high energy collisions. More precisely, what could happen is that at collider energies of order  $\Lambda_F$  the ordered state fig. 2 is locally destroyed and in the process of re-ordering of isospin vectors a germ of the state  $X$  appears. Energy would then be released, which would destroy the ordering in the neighborhood of the original collision, so that a

chain reaction would start at the end of which the metastable ground state would be completely replaced by the stable one.

In the microscopic model it is possible to compare the energies of all isomagnetic configurations with 4 isovectors to the energy of fig. 1. Essentially, one deals with a system of 4 isospin vectors interacting via an 'anti-ferromagnetic' coupling  $J_A > 0$ . The local energy of any state X is given by

$$E(X) = J_A [\vec{Q}_1\vec{Q}_2 + \vec{Q}_1\vec{Q}_3 + \vec{Q}_1\vec{Q}_4 + \vec{Q}_2\vec{Q}_3 + \vec{Q}_2\vec{Q}_4 + \vec{Q}_3\vec{Q}_4] \quad (95)$$

With this input one may run over all possible ground state configurations of isospin vectors. As a result, one finds 2 minima with exactly the same energy. One minimum (m1) corresponds to fig. 1 while the other (called m2) is characterized by the 4 isospins arranged into 2 pairs of opposite orientation as depicted in fig. 6 of [94].

This conclusion remains unchanged, if one considers separate left- and right-handed isospin vectors on each tetrahedral site as in (14). However, it may get changed, if the *inter*-tetrahedral energies are different. In general, one expects roughly identical inter-tetrahedral energies, because in both cases (m1) and (m2) the same number of isospin pairs are aligned in a ferromagnetic way, with identical exchange couplings  $J_F$ . However, in reality there may be small differences between the  $J_F$  for the (m1) and the (m2) configuration due to the different geometries of the two ground states.

### **2.5.2 Is physical space discrete, i.e. is there a granular structure of physical space in addition to the discrete tetrahedral structure in internal space?**

Most probably yes. Although the discrete structure of physical space is not compelling and the distance  $r$  between two tetrahedrons could be identically zero, the discussions in section 1 suggest that  $r$  has a tiny non-vanishing value of the order of the Planck length. Note, due to the elasticity of the system only an average  $\langle r \rangle$  can be given, in accord with (24).

### 2.5.3 How can such a granular structure be compatible with Lorentz invariance?

In other words: how can the elastic continuum of tetrahedrons be Lorentz invariant? The point to note is that in the tetron model all physical objects that we know are superpositions of excitations which travel as quasi-particles through the hyper-crystal. This holds true even for photons and also for ourselves as well as for all experiments and 'reference frames' we can prepare. In a quantum mechanical framework such excitations always have a wave nature and are to be described by (generalizations of) the d'Alembert wave equation. This fact alone fixes the system of waves, which constitutes our physical environment, to be Lorentzian, because the d'Alembert operator

$$\square = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (96)$$

leaves the squared 4-momentum  $p^2$  invariant. In particular, if one wave packet is emitted from another one, their velocities add up according to the rules of Lorentz transformations.

For massive particles which move at velocities  $< c$ , the operator is modified to  $\square - m^2 c^2 / \hbar^2$  corresponding to a 'dispersion relation'  $p^2 = m^2 c^2 / \hbar^2$ . It is important for this argument (and in general to retain the Lorentz structure) that in all those wave equations there is a universal maximum speed  $c$  that fixes the relation between space and time. This is defined to be the speed of the massless excitations (2.1.3) and according to (26) is given by the ratio of 'lattice constant'  $L_P$  and 'hopping time'  $T_P$ .

The value of  $c$  is universal for all SM particles because all of them arise from the same isomagnetic interactions introduced in section 1. This is discussed in more detail in (2.5.13), and the question of (metrical) velocities larger than  $c$  will be discussed in (2.5.41). The question why gravitational waves propagate at  $c$  will be answered in (2.5.40).

It should be stressed, that in particle physics interactions usually only the excitations move. The tetrahedrons stay fixed at their location in the hyper-crystal. They only move in connection with metrical changes induced by gravity, for example when the universe expands after the crystallization or on a much tinier level in any kind of gravitational interaction, cf. the discussion after (22) and in (2.5.41).

#### 2.5.4 Is time discrete?

Time is an averaged construct induced by the superposition of many elementary thermodynamics processes. In the tetron model it can be interpreted to have a granularity of order  $T_P$  – at least from the standpoint of the quasi-particles which constitute our physical environment, because  $T_P$  is the minimum time it takes a quasi-particle to jump from one tetrahedron to the other. In our world of quasi-particles the duration of any observable physical process can never fall below this value.

#### 2.5.5 Is quantum mechanics just a material property of the hyper crystal?

My claim is yes. The proof will be given in the following question (2.5.6). It relies on the fact that ordinary matter (including the photon) consists of internal excitations traveling as quasi-particles on a discrete structure with Planck length lattice spacing.

#### 2.5.6 Why is the Planck length the natural lattice spacing for the hyper-crystal?

The short answer: the Planck length  $L_P$  arises as a lower limit on  $\Delta x$  in the 'generalized' Heisenberg uncertainty relation (98), which includes the effects of gravity. On the other hand in the tetron model *all* known particles including the photon are interpreted as excitations with an extension of at least one lattice spacing  $r$ , i.e. they are quasi-particle waves with wavelength  $\lambda > L_P$ . Since every physical experiment necessarily makes use solely of these quasi-particles, its resolution cannot be better than  $r$ . This strongly suggests  $r \approx L_P$ , because if  $r$  would be smaller than  $L_P$ ,  $r$  instead of  $L_P$  would determine the uncertainties of quantum mechanics.

Due to the extremely small lattice spacings, spacetime as we perceive it effectively looks like a continuum. This guarantees local rotational symmetry. The item of invariance under Lorentz boosts is discussed in (2.5.3).

Extended answer: in ordinary quantum mechanics there is a fixed dimensionful quantity  $h$  which relates the canonical Fourier variables of frequency to energy and inverse distance to momentum, i.e. it transforms the spacetime quantities  $x$  and  $t$

into physically 'active' quantities

$$\vec{p} = \hbar \vec{k} \quad E = \hbar \omega \quad (97)$$

These relations are the reason why  $\hbar$  appears in the uncertainty principle  $\Delta x \Delta p > \hbar$  which otherwise would just be the Cauchy-Schwarz inequality known from the Fourier analysis of waves, i.e.  $\Delta x \Delta k > 1$  with no dimension on the r.h.s. I consider them as further evidence that  $\hbar$  is a material property of the hyper-crystal not valid outside of it, cf. the discussion at the end of section 1.

The 'generalized' uncertainty relation includes gravitational effects of the photon on a test particle[59]. These occur because the general relativistic effect from the photon with 'mass'  $h\nu/c^2$  adds to the uncertainty about the test particle. The Heisenberg relation is then modified to

$$\Delta x \geq \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar} = \lambda [1 + (\frac{L_P}{\lambda})^2] \quad (98)$$

where  $\lambda$  is the photon wavelength to be identified with the limit of resolution.

Eq. (98) can be derived e.g. by extending the 'Heisenberg microscope' thought experiment (which imagines a photon to measure  $x$  and  $p$  of a probe particle) to include gravitational effects of the photon[59]. It implies that there is a minimum value of

$$\Delta x \sim L_P \quad (99)$$

corresponding to a photon with wavelength  $L_P$ . Usually, this is interpreted in such a way that at distances/wavelengths smaller than  $L_P$  all matter dissolves into quantum fluctuations and the laws of physics do not have a meaning any more.

In the microscopic model the interpretation is a little different. The limitation on distances/wavelengths holds true only inside the hyper-crystal and is restricted to the world of quasi-particles, not of tetrons. For example, since the photon is such a quasi-particle (a correlation among 2 tetrahedrons traveling through space), the minimal wave length of photons, that can be produced and used in experiments, roughly corresponds to the lattice spacing (24), and it is this minimal wavelength which restricts the precision of any experiment.

It is worth mentioning that on the hyper-crystal the value of  $\hbar$  is given by (27) where the Planck energy  $\Lambda_P$  corresponds to the binding energy of the tetrahedrons and  $T_P$  is the hopping time needed to absorb and re-emit the photon.



At first sight the linear momentum dependence of the gravity contribution in (98) does not fit exactly into this picture. However, it can be interpreted in such a way that the gravitational effect of the photon modifies the average spacing  $\langle r \rangle = L_P$  between the tetrahedrons involved in the interaction and thus the resolution  $\Delta p = \lambda$  of the photon. The modification factor can be derived from the metric (22) by inserting the photon's gravitational mass  $M = h\nu/c^2 = h/c\lambda$  and amounts to

$$1 - \frac{\phi}{c^2} = 1 + \frac{Gh}{|\vec{x}|\lambda c^3} \quad (100)$$

Since the relevant regime of discussion is distances  $|\vec{x}| \approx \lambda$ , the factor (100) agrees with the factor in (98).

These considerations may be compared to the Fourier wave analysis which has led to the dispersion relation (33) and which also provides a modified uncertainty relation

$$\Delta x \Delta k = |\langle [x, k] \rangle| = |\langle \cos k L_P \rangle| \quad (101)$$

Eq. (101) may be evaluated in the limit of long wavelengths to give the usual Heisenberg uncertainty relation, i.e. a meaningful result. On the other hand, for wave vectors near the border of the first Brillouin zone, the r.h.s. of (101) vanishes. Thus it would seem that lattice quantum mechanics at Planck scale energies can exhibit classical, non-quantum mechanical behavior. However, in the tetron model, for wavelengths  $\lambda \sim L_P$  the excitation nature of the photon spoils the prerequisite of a normal wave with arbitrary small wave vector on which (33) and (101) rely.

### **2.5.7 How is physical space defined within the 6+1 dimensional world? How is it distinguished from the internal dimensions?**

The aligned tetrahedrons define a 3-dimensional subspace of  $R^6$ . Everything orthogonal gives physical space.

### **2.5.8 What is the exact lattice structure of the hyper-crystal?**

This question is obsolete because of the elastic nature of inter-tetrahedral coordinate bonds. It is an irregular lattice of tetrahedrons in the base 3+1 dimensional spacetime without a discrete symmetry. As discussed in various places - e.g. after

(18) in section 1, in (2.5.9) and (2.5.28) - there are more similarities to a fluid than to a crystal.

### 2.5.9 Then why is this structure called a 'crystal'?

Because there is a rigid tetrahedral structure in the *internal* directions. Concerning physical space it is a disordered system which resembles a plastics or a fluid.

### 2.5.10 \*Why is there no growth of the crystal into the internal directions?\*

I have no complete answer to this question. It is the main reason why internal dimensions need not be compactified, cf. (2.5.31), and it must have to do with the form of the fundamental tetron interaction, cf. (2.2.11) and (2.2.13), and its preference to form the spiky tetrahedral 'star' system fig. 1, which do not allow tetrahedrons to be stacked on top of each other in fig. 2, or, in more scholarly terms, with the fact that there is no space symmetry group corresponding to the point symmetry  $A_4 + S(S_4 - A_4)$ .

### 2.5.11 \*Where do the 6 spatial dimensions come from?\*

I have only a partial answer to this question. A tetron spinor  $\psi = (U, D)$  transforming as 8 under  $SO(6,1)$  can be interpreted as an octonion field living in 6+1 dimensions. Octonions form a rather unique mathematical structure[41, 42, 43]. They are the next thing to use when complex numbers (used for amplitudes in quantum mechanics) and quaternions (used for rotations and spinors in physical space) are not ample enough to describe physical phenomena. The octonion nature of tetrans has been used in (2.1.20) to determine the form of the isomagnetic tetron interactions. The splitting of  $R^6$  into an internal space and physical space corresponds to a splitting of an octonion into two quaternions.

Note that if octonion multiplication is really relevant for the behavior of tetrans, it seems somewhat more natural to have 7 instead of 6 spatial dimensions. However, this is pure speculation. It is the product  $\vec{\tau}\gamma_5$  appearing in (68), which provides the

link between internal and physical space, necessary to explain weak parity violation from the chiral configuration fig. 1.

### **2.5.12 How does the value for $c$ come about? Why is it finite?**

In the tetron model light is interpreted as a (massless) excitation of tetron-antitetron pairs, cf. (2.1.2) and (2.1.3). While the pairs themselves are bound over 2 tetrahedrons with fixed positions within the hyper-crystal, the excitations propagate through physical space as quasi-particles. In contrast to massive excitations, photons cannot cling to one fixed internal tetrahedron, but are constantly moving at the speed  $c$  of light. Nevertheless,  $c$  is not infinite, because even a massless excitation needs a certain 'hopping time'  $T_P$  to jump from one tetrahedron to another. Since the tetrahedrons are distributed over the hyper-crystal with average distance  $L_P$ , this corresponds to  $c = L_P/T_P$  in agreement with (25).

### **2.5.13 Why is $c$ the universal limiting speed for all particles?**

In the tetron model all SM particles are internal excitations whose interactions have the same type of isomagnetism as universal origin. Therefore it is of no surprise that a common maximum speed exists, to be identified with the ratio  $L_P/T_P$ , as explained in (2.5.12).

For the interpretation of (metrical) velocities larger than  $c$  see (2.5.41).

### **2.5.14 Is there a rest system of the hyper-crystal?**

At first sight the existence of such a system seems to contradict Einstein's principle of equivalence and special relativity - well established concepts which I do not want to question. Still I think the answer to the question is affirmative.

The point is that in the tetron model all material objects and all normal matter and gauge bosons including the photons from their very nature are quasi-particle waves, i.e. entities fulfilling Lorentz covariant Klein-Gordon equations, as discussed in (2.5.3). As such they cannot distinguish an absolute rest system, i.e. they naturally fulfill Einstein's principle of equivalence. By contrast, the original tetron matter, which forms the fixed hyper-crystal ground state, is merely the carrier of those

quasi-particles, and mostly invisible for human experiments. This view is close to the Fitzgerald interpretation of special relativity[100] and allows for a rest system of the hyper-crystal. More details are given in the next answers.

### **2.5.15 Why has a rest system never been observed in Michelson Morley type of experiments?**

The essence of the answer to this question has already been given at the end of (2.5.14). An extended version can be found in [100]. In that interpretation of special relativity a Galilean ground state like fig. 2 is not a contradiction to special relativity but rather supplements it. It supplies the 6-dimensional framework which leads to the observed spectrum of physical particles.

### **2.5.16 Can the hyper-crystal's rest system ever be identified?**

It follows from (2.5.14) that it is difficult to experimentally perceive the ground state fig. 2 which forms the rest system of the hyper-crystal - the reason being that our reality (ourselves, our detectors as well as all test particles) consists of quasi particle excitations and not of tetrons themselves.

The puzzle cannot be resolved on the level of excitations. Even gravitational waves propagate at the speed of light, cf. (2.5.40). However, it is well known that the metrical expansion of the universe (and metrical changes in general) can proceed at superluminal velocities, cf. (2.5.24) and (2.5.40).

In section 1 it was shown that metrical changes correspond to displacements of the internal tetrahedrons within the elastic hyper-crystal, cf. the discussion after (19). Applied to the standard model of cosmic expansion, this leads to the idea that *the hyper-crystal's rest system is given by the spatial coordinates of the FLRW metric* (91). Of course one must be aware that due to the expansion factor  $a(t)$  this is an elastic 'rest system' permanently changing with cosmic time, but at least momentarily it can be considered to be at rest. The time dependence of  $a(t)$  defines the expansion of the elastic hyper-crystal, and what comes nearest to the notion of 'matter at rest' are the galaxies on the Hubble flow. The peculiar velocity (94) gives their 'true' nontrivial motion with respect to the elastic rest system.

The underlying idea is that to a good approximation mignon-matter forming the galaxies was originally produced 'at rest' (at least on the average) during and shortly after the big bang / crystallization and has since been moving together with the expanding elastic hyper-crystal. This is in accord with the finding that most large lumps of matter, such as galaxy clusters, are nearly comoving with the Hubble flow.

#### **2.5.17 Is there an internal time different from the ordinary time variable?**

In magnetism, time reversal is a far-spread concept because it allows to reverse the orientation of the magnetization / of a spin vector. This way it enters the definition of magnetic point groups, for example the Shubnikov group (6).

However, as shown in (2.4.16) and (2.4.20), for the case of iso-magnetism one may replace the role of internal time reversal by charge conjugation, so no separate time is needed for the internal dimensions.

#### **2.5.18 Is there an absolute time in the hyper crystal?**

Yes, it is given by the comoving Hubble time coordinate, i.e. the elapsed time since the big bang according to a clock of a comoving observer.

#### **2.5.19 What is the status of the Copernican and of the Cosmological Principle in the tetron model?**

The Copernican Principle states that no place in the universe is 'special' or preferred, while the Cosmological Principle demands that the universe looks the same in all directions (is isotropic) and contains everywhere roughly the same amount and mixture of material (is homogeneous). These principles are not questioned by the microscopic model, assuming a suitable uniformity of the hyper-crystal built from tetrons. However they will *not* be fulfilled at the edges of the DMESC where there may be steadily new accretions of tetrahedrons to the crystal. Those edges are therefore expected to be both 'special' and anisotropic places.

### 2.5.20 Is the original $R^{6,1}$ Lorentzean oder Galilean?

It was argued in section 1, that the speed of light, which is at the heart of Lorentz symmetry, is an intrinsic property of the hyper-crystal, not valid outside of it. Furthermore, the fact that metric expansion of our universe can proceed with velocities larger than  $c$  gives support for Galilean (or, in case of a  $SO(6,1)$  Lorentz type symmetry, with a limiting speed much larger than the speed of light).

### 2.5.21 Do we know anything about the position of our universe in full $R^{6+1}$ ?

To show up in experiments such an information requires *anisotropic* interactions, which would get their anisotropy from 6-dimensional structures which go beyond the hyper-crystal. It cannot be obtained from the dominant isospin interactions (14) and (15), because these are rotationally invariant, i.e. the same mass spectrum is obtained after a global rotation of the isospin axes.

A simple anisotropic Hamiltonian of isospin vectors  $\vec{Q}_i$  would look like

$$H_a = -J_z \sum_{i \neq j}^4 Q_{iz} Q_{jz} - J_{xy} \sum_{i \neq j}^4 (Q_{ix} Q_{jx} + Q_{iy} Q_{jy}) \quad (102)$$

with  $J_z \neq J_{xy}$ .

Since they explicitly break internal rotational symmetry, anisotropic interactions like (102) violate internal angular momentum conservation (17). As discussed in (2.4.12), neutrinos are the Goldstone modes corresponding to that symmetry. More precisely, the 3 neutrino species were identified as the 3 vibrating components of total internal angular momentum. Therefore, the anisotropic Hamiltonian (102) contributes to the neutrino masses, and measuring the neutrino mass matrix has the potential to answer the present question.

### 2.5.22 Is there a contribution from the tetron ground state configuration to the mass/energy density and to the expansion rate of the universe?

According to general relativity the mass/energy density of the universe determines its curvature, its expansion rate and its future. So one might expect that the energy

of the ground state fig. 2 should also contribute. However, as argued in (2.5.29), this energy does not appear in energy balance equations of ordinary matter.

The main effect of the enormous tetron ground state energy is to initialize big bang inflation by releasing a vast amount of crystallization energy.

### 2.5.23 How does all of this fit into inflationary cosmology?

As discussed in (2.1.12) and after (18), inflation in the microscopic model is associated to the crystallization process of tetrahedrons with an accelerated expansion due to the elastic nature of the bindings and the sudden release of crystallization energy.

The major signature of inflation is the exponential increase of the scale parameter  $a(t)$  in (91). According to the Einstein equations  $a(t)$  has to fulfill

$$\ddot{a} = -\frac{4\pi}{3}G(\rho + 3p/c^2)a \quad (103)$$

where  $\rho$  is the mass density of the universe and  $p$  its pressure, and the combination  $\rho + 3p/c^2$  corresponds to the trace of the energy momentum tensor. Eq. (103) makes it clear that the exponential increase needed for inflation can be obtained for constant and negative  $\rho + 3p/c^2$ .

What kind of matter fulfills such a condition? The immediate answer: matter undergoing a phase transitions. Indeed it is normally assumed in models of inflation that a false vacuum decays in the framework of some abstract phase transition being active in the very early universe. However, the physical background of this phase transition is never specified.

In the tetron model the situation is better, because the phase transition relies on the Landau free energy  $\Delta F$  in (18). The order parameter is defined in terms of density fluctuations  $D$  of tetrahedrons and is thus a material quantity. Inflation starts almost immediately after the big bang, at the time when the germ of the hyper-crystal comes into being. This point corresponds to the maximum value of the free energy curve (18) from where the system rolls down to its non-trivial minimum – the moment when the latent heat is released.

A word of warning: although the qualitative features of inflation are well described by (103), one should be aware that the Einstein equations are not valid close to the

crystallization point. According to the discussion at the end of section 1, the least one must expect is a temperature dependence of  $c$  and  $G$  which will modify the details of the description.

#### **2.5.24 How can the release of crystallization energy lead to metric velocities much larger than the speed of light as needed for inflation?**

It is well known that metrical changes proceeding faster than  $c$  do not contradict Einstein's equations, because in GR there are rules about matter moving through space, but there is no rule about space expanding faster than light. They necessarily occur in the standard cosmological model at times shortly after the big bang.

In the microscopic model metric velocities have a special interpretation due to the fact that an expanding space corresponds to elastic motion of tetrahedrons in the hyper-plastics, cf. (2.5.41) and section 1. Since tetron velocities are not bounded by  $c$ , the release of a large amount of crystallization energy at the big bang can push this motion to faster than  $c$ .

#### **2.5.25 What are the tetron model answers to the flatness and the horizon problem?**

They are similar to those in ordinary inflationary models:

–flatness problem (the question why the universe is almost flat everywhere): due to the exponential expansion, triggered by the initial release of crystallization energy, the universe is much larger than anticipated.

–horizon problem (the question why the universe looks almost the same everywhere): all parts of the universe were causally connected at the time when the hyper-crystal was born. Due to the subsequent exponential expansion they have lost their causal contact.

#### **2.5.26 Is there an inflaton field?**

Inflation was explained in section 1 and (2.5.23) as arising from the latent heat released in the crystallization of the hyper-plastics. Therefore in the tetron model,



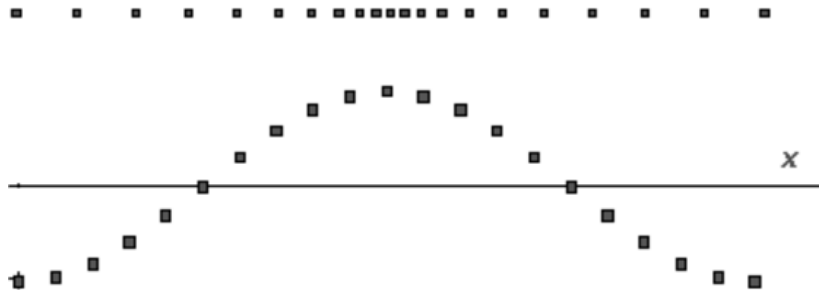


Figure 6: This picture illustrates 'longitudinal'=time vs 'transverse'=spatial curvature for a 1-dimensional chain of tetrahedrons. Physical space is represented by the x-axis, similar to fig. 2. Longitudinal displacements/accelerations are felt as local or global contraction or expansion of physical space and correspond to non-trivial 0-components of the metric. Transverse displacements go into the internal dimension.

the inflaton can be interpreted as the energy/density wave that carries the initial crystallization energy.

### 2.5.27 Nature of the forces among tetrahedrons

The elastic forces among the tetrahedrons are responsible for the curvature of space-time, i.e. for the gravitational interactions. In principle, they should be calculable from the fundamental tetron interaction considered in (2.2.11). Because of the many particles involved, in practice this is not an easy task.

On a heuristic level the forces can be understood as depicted in fig. 6 for a linear chain of tetrahedrons. Longitudinal displacements/accelerations are felt as local or global contraction or expansion of physical space and correspond to non-trivial 0-components of the metric. Transverse displacements go into one of the 3 internal dimensions and induce spatial curvature. According to the discussion in (2.2.11) the force among the tetrahedrons may be of such a form that they are driven towards an equilibrium distance  $r_s$ . In that case the force can be expanded in powers of  $r - r_s$ , and in the leading harmonic approximation it is then of Hooke's form

$$m_T \ddot{r} = -d_T (r - r_s) \quad (104)$$

where  $m_T$  is the tetrahedron mass and  $\omega_T^2 = d_T/m_T$  the frequency characteristic for the interaction.

In an elastic medium of many tetrahedrons this interaction corresponds to

$$\rho_T \ddot{\xi} = \mu \xi'' \quad (105)$$

where  $\rho_T$  is the mass density of tetrahedrons and  $\xi$  the deformation vector. According to (105) one expects elastic waves at speed

$$c^2 = \mu/\rho_T \quad (106)$$

$\mu$  is the Lamé parameter introduced in (29) and can be traced to Hooke's constant via  $m_T = \rho_T L_P^3$  and  $d_T = \mu L_P$ , where the Planck length  $L_P$  corresponds to the average distance among the tetrahedrons. As a consequence,  $c$  in (106) is actually the speed of light.

It must be noted, however, that this kind of elastic wave cannot be perceived by mignon matter, because it corresponds to a pure gauge transformation (113) and thus does not affect the curvature. Furthermore, one obtains

$$\omega_T = \frac{1}{T_P} \quad (107)$$

an extremely high frequency, which does not play any role in the universe's expansion. One point to note is that  $L_P$  - the present day average distance of tetrahedrons in the hyper-crystal - is not equal to the equilibrium distance  $r_s$  of tetrahedrons. In the presence of the many other tetrahedrons, a given tetrahedron is not able to instantaneously attain the distance  $r_s$  to its neighbors. Instead, the characteristic time for this to happen is much larger than  $T_P$ . In other words, there must be another characteristic frequency  $\omega_M \ll \omega_T$  describing the expansion of the hyper-crystal, so that (103) should be replaced by

$$\ddot{a} = -\frac{4\pi}{3}G(\rho + 3p/c^2)a - \omega_M^2(a - a_s) + \Lambda_C c^2 a \quad (108)$$

$\rho$  is the (mignon) matter density, and in a matter dominated universe one has  $\rho \sim 1/a^3$  as usual. For reasons of comparison I have also added a cosmological constant term.

It turns out that it is possible to accommodate the observed dark energy effect by

a harmonic contribution  $\omega_M \neq 0$  instead of a cosmological constant provided –one chooses  $\omega_M = 10^{-18}\text{Hz}$  (very small as compared to  $\omega_T = 10^{43}\text{Hz}$ ). The first 2 terms on the rhs of (108) are then of the same order.

–we live at  $a < a_s$ . This condition corresponds to times where the universe has re-accelerated but not reached an equilibrium. At much later times the Hooke term will dominate and drive it towards  $a_s$ .

Note for a proper treatment of (108) the temperature dependence of  $a_s$  must be taken into account

$$a_s \sim (T_c - T)^{-\frac{1}{6}} \quad (109)$$

where  $T < T_c$  and  $T_c$  was introduced in (18).

### 2.5.28 Is there a similarity to the behavior of superfluids?

There is a certain similarity, but there are also appreciable differences.

The growth and expansion of the elastic hyper-crystal along a lower (in this case 3) dimensional structure is reminiscent of the behavior of superfluids, which can creep along arbitrary surfaces. There are 2 prototypes of superfluids whose representatives are He-3 and He-4. While in He-3 fermion condensates populate the macroscopic quantum state, in He-4 superfluidity is a consequence of a normal Bose-Einstein condensation of the He-4 bosons. There are actually speculations that associate gravity to a He-3 type of superfluid[87]. In the tetron model, however, it is the tetrahedrons and not the tetrans themselves which are involved in the gravitational interactions. The tetrahedrons are bosons, so the ordinary Bose-Einstein approach to superfluidity could be useful to describe the growth and expansion of the hyper-crystal. More precisely, the Bose-Einstein condensate of tetrahedrons within the hyper-crystal would obey a Gross-Pitaevskii equation in 3 dimensions[98]

$$i\hbar \frac{\partial}{\partial t} D(\vec{r}, t) = \left[ -\frac{\hbar^2}{2m} \Delta + \alpha |D(\vec{r}, t)|^2 \right] D(\vec{r}, t) \quad (110)$$

Although it fulfills a (nonlinear) Schrödinger equation and is sometimes called the 'wave function' of the condensate,  $D$  is a classical field and can be identified with the order parameter of the system, i.e. with the density fluctuations of tetrahedrons introduced in (18).

One should, however, be hesitant to accept the idea of a superfluid, for the following reasons:

–quite in general superfluidity is a low temperature effect. There is no indication of an additional cosmic phase transition in the low energy regime.

–in the last 10 billion years the universe has expanded rather slowly. This is not what I would expect from a *superfluid*.

–in a superfluid there is a macroscopic quantum state governed by a Planck constant  $h$ , as appears in (110). This picture is different from the interpretation (2.5.6) of quantum theory on the hyper-crystal and the model with elastic Hooke's inter-tetrahedral forces advocated in sections 1 and (2.5.27).

As a result I prefer to consider the DMESC in its ground state to be a perfect fluid (not a superfluid), and by definition without any stresses. Only if mignons and other excitations are put into the system, this will produce stress and strain, thus modifying metric and curvature in the way discussed in section 1 and giving rise to the effects of gravity.

### 2.5.29 Is energy conserved in the tetron dynamics?

Yes. This is in contrast to general relativity, where energy is only 'covariantly' conserved, i.e. the energy momentum tensor fulfils  $D^\mu T_{\mu\nu} = 0$  where  $D^\mu$  is the covariant derivative involving the Christoffel symbols. The latter point can be better understood by considering the Einstein equation  $G_{\mu\nu} = \kappa T_{\mu\nu}$  which relates the energy-momentum  $T_{\mu\nu}$  of matter(=mignons) to the intrinsic curvature equivalent to displacements of tetrahedrons.

The Einstein equation can be re-interpreted by saying that the total energy momentum of the universe with contributions  $G_{\mu\nu}$  from the tetrahedrons and  $T_{\mu\nu}$  from the mignons is not only constant but actually zero, and that energy can be shifted from one side of the equation to the other, i.e. from mignons to tetrahedrons and vice versa. A famous example is the red shift of photons in the expanding universe where the photons lose energy to the FLRM metric.

In the tetron model, this is not the whole story. As discussed after (31), the vacuum energy density of the universe does not only consist in the (small) contribution from the cosmological constant responsible for dark energy, cf. (2.5.36), and the

(somewhat larger) contribution from the QCD and electroweak symmetry breaking vacua, but there is also a contribution

$$V_T \sim \Lambda_P^4 \quad (111)$$

to the vacuum energy from the tetronic ground state energy fig. 2. This contribution arises as the product of the energy of a bound tetrahedron ( $\sim \Lambda_P$ ) and its inverse volume ( $\sim \Lambda_P^3$ ) and is very large, because the tetrahedrons are closely packed and strongly bound within the hyper-crystal.

In the framework of quantum field theories the vacuum energy density arises as the sum of all zero point oscillations. For example, the vacuum energy for a free field is the sum of the ground state energies  $\omega_k \sim \sqrt{k^2 + m^2}$  of all oscillator modes  $k$ . For a cubic box of side-length  $L_P$ , one has to sum over integers  $n = kL_P/2\pi$ , and in the continuum limit this turns into an integral

$$\int \sqrt{k^2 + m^2} d^3k \sim \Lambda_P^4 \quad (112)$$

Using the Planck scale as a natural cutoff, this is again much larger than the cosmological constant and the energies of the QCD and electroweak symmetry breaking vacua. In field theory[64] it is usually renormalized away, so that it does not appear in any particle or gravitational energy balance consideration. By contrast, in the tetron model it must be included and added to the ground state energy of the hyper-crystal, which according to (111) is of the same order of magnitude.

There is also a third contribution to be included in tetron energy considerations, and that is elastic energy  $\sim (\partial_\mu u_\nu + \partial_\nu u_\mu)^2$  from displacements of tetrahedrons with vanishing curvature. It is true that these contributions do not matter in the energy balance of ordinary matter, because mignons do not react to a 'pure gauge'

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu u_\nu + \partial_\nu u_\mu \quad (113)$$

For the tetron dynamics, however, they are relevant.

### **2.5.30 We are living in a rather cold universe. Why don't we see a transition from the elastic hyper-crystal to a rigid crystalline structure at zero temperature?**

In such a rigid crystal no shifts of the tetrahedrons would be allowed, and therefore one expects gravitation to disappear.

One could speculate that the present temperatures of the universe are not small enough for the solidification to occur, or that pressure is always needed like for the solidification of He-3.

Note that these remarks are only true in the absence of mignons. When a mignon excitation is present, its mass/energy induces a stress in the hyper-crystal which at least locally re-liquidizes the system. In other words, the mignon's mass/energy is automatically accompanied by curvature, i.e. a non-vanishing gravitational potential.

### **2.5.31 Are the internal spaces compact or infinite?**

They are infinite, no assumption about compactification of internal spaces needs to be made. The reason why we cannot step into the internal dimensions is because we are built from quasi-particles and thus cannot leave the hyper-crystal(=our universe).

According to fig. 2, the hyper-crystal is restricted to a 3+1 dimensional 'surface' in  $R^{6+1}$ . Going away from this 'surface', internal space is empty, because according to (2.5.32) and (2.5.10) ordinary matter cannot dissipate into the internal dimensions. Exception: other hyper-crystals may have condensed at big bang times. In general these will lie skewed with respect to the one we live in and form separate 'universes', cf. (2.3.24) and (2.5.37).

### **2.5.32 How can one avoid dissipation of energy into the internal dimensions?**

Since internal space is infinite, matter and energy could in principle disappear into it. This could happen in the form of particles which move in the direction orthogonal to the hyper-crystal. However, in (2.1.2) and (2.1.3) I have taken the viewpoint that all observed particles including the photon are excitations of the crystal and as such cannot exist away from it. Furthermore, the tetrons, from which the crystal is made, are assumed to be so strongly bound, that they can be split off the hyper-crystal only by supply of Planck scale energies.

### **2.5.33 Was there a GUT era in the early universe where electroweak and strong couplings were unified?**

No, because there is no GUT – cf. (2.3.22). In the tetron model the strong force has a different origin than the electroweak one (cf. 2.3.28), and thus GUT unification seems unlikely.

The proper history of the universe starts with the end of the crystallization process (=the inflation era), at which point electromagnetism and weak forces are unified, cf. (2.1.12). In the standard terminology this is the starting point of the radiation dominated era, with photons, effectively massless W/Z and dark matter as the dominant excitations. At the end of the electroweak era at temperatures of order  $\Lambda_F$  there is the alignment of isospin vectors corresponding to the electroweak phase transition which gives masses to q/l and W/Z.

### **2.5.34 Is there a unification of the SM and gravitational forces at the Planck scale?**

Not in the sense of supergravity and related models. Even at big bang temperatures gravity and SM forces have a very different nature. Although it is true that everything observed can eventually be derived from the fundamental forces among tetrons, the SM interactions trace back to interactions between isospin vectors of tetrons, whereas gravity is an elastic force between tetrahedrons which stems from remnant tetron coordinate interactions.

The question, why the forces of gravity appear to be much weaker than particle physics effects, has been treated in (2.3.19) and (2.3.21).

### **2.5.35 Are there dark matter candidates in the model?**

Yes, there are several possibilities:

–further internal excitations of the crystal, like phonons, cf. (2.3.25) and [3]. Their interactions with mignons(=ordinary matter) is tiny, because they are not involved in the isomagnetic correlations giving rise to the SM.

–the pseudoscalar  $\eta$  arising in the 2HDM ansatz (2.1.14). Such a possibility is widely discussed in the literature [68, 69] provided the  $\eta$  is inert, i.e. does not interact with

quarks and leptons. This condition can be fulfilled in the present model essentially because of (49). Note that right after inflation there is the radiation dominated era where the inert scalar is copiously produced together with a soup of many other tetron-antitetron bound state excitations (photon, W/Z,  $\Phi$  and  $\Phi'$ ).

### 2.5.36 Is there an explanation for dark energy?

According to the standard cosmological model the rate of expansion of the universe should decrease in time due to the diminishing matter density of the universe. However, observations indicate that the opposite is true: the universe's expansion rate was decelerating until about 5 billion years ago, after which time the expansion began accelerating. Phenomenologically, this can be explained by including a cosmological constant  $\Lambda_C$  in the theory (this amounts to saying that a volume in space has some intrinsic fundamental vacuum energy creating a pressure which makes the universe expand) or by a weakly fluctuating scalar 'quintessence' field (this does a similar job).

Furthermore, such a type of energy that is not matter or dark matter is also needed to explain the apparent flatness of the universe (absence of any detectable global curvature). According to that argument the contribution of dark energy should be more than twice as large as that of matter and dark matter together.

A third explanation of the dark energy effect is offered within the f(R,T) models (23) discussed in section 1 by suitable accommodation of the 11 phenomenological coupling constants[61].

To understand dark energy within the tetron model one should remember, that the micro-elastic forces that have initially induced cosmic expansion during the big bang crystallization process are still at work today. For example, there may still be accretions to the hyper-crystal at its edges which are setting free large amounts of crystallization energy. The energy is then transferred to the other tetrahedrons of the crystal in the form of weakly fluctuating energy/density waves traveling through the universe. This picture is well along the line of the quintessence idea mentioned above. It would give the accelerations still present in the universe a microscopic meaning.

Another possibility is that the increased acceleration arises, because the average



distance  $L_P$  between tetrahedrons has not yet reached its equilibrium value  $r_s$ . This line of argument is followed in (2.5.27) and gives rise to a self-contained explanation of the dark energy effect.

### 2.5.37 Are there other universes?

Probably yes. One may consider the original  $R^{(6,1)}$  spacetime as a container of universes. When the tetron gas cooled down and temperatures reached the crystallization temperature (Planck energy), germs of 3+1-dimensional hyper-crystals came into being in various places of  $R^{(6,1)}$ , cf. (2.3.24). These crystals then grew in their respective 3+1 dimensional subspaces, each of them making up for a separate  $R^{(3,1)}$  spacetime. Since they are of low dimension as compared to the whole  $R^{(6,1)}$ , they hardly interfere with one another. If at all, they intersect in isolated (1-dimensional) points. At those points a defect in the isomagnetic and/or coordinate crystal structure will show up, because the isospin and/or coordinate vectors of the tetrans do not know how to orient themselves.

### 2.5.38 On the interpretation of black holes in the tetron model

From its very nature the discrete elastic hyper-crystal does not allow for mathematical singularities. It is true that black holes correspond to solutions of the Einstein equations, and the Einstein equations according to section 1 arise from the effective action (23) for the DMESC. However, these equations are not applicable at arbitrary small distances, where the discrete structure becomes perceptible.

It is generally believed, that if enough mass  $M$  is squeezed into a roughly spherical volume of size  $r = GM/c^2$ , it collapses into a black hole. What happens from the standpoint of the tetrahedrons is that inside the black hole's event horizon the crystal becomes extremely compressed, i.e. the distances between the tetrahedrons become smaller and smaller. At the same time the temperature strongly increases as more and more matter (mignons, gauge bosons and other quasi-particle excitations) is accreted. When the temperature exceeds the Fermi scale, the isospin alignment of the hyper-crystal gets lost and the accreted mignons decay to photons and weak gauge bosons. In some sense this scenario is reverse to the appearance of the 'radia-

tion dominated' epoque of the big bang. Finally, if temperatures reach the order of the crystallization energy  $\Lambda_P$ , the DMESC structure completely dissolves and the tetrahedrons vaporize to form a gas which is set free into the full  $R^{(6,1)}$ , i.e. into the internal directions.

It is conceivable that this hot gas becomes the germ of another hyper-crystallization process making up for another universe in the sense of (2.5.37). One could then distinguish 'parent' and 'child' universes in an obvious sense. If our own universe would have been created as a child under such circumstances, the appearance of a big bang with a radially symmetric expansion like described by the FLRW metric (91) would be somewhat easier to understand than in the picture developed in section 1 where accretions of tetrons to the edges of the hyper-crystal could in principle provoke deviations from an FLRW behavior, cf. (2.5.36).

### **2.5.39 Should gravity be quantized?**

As emphasized in (2.5.6), the quantum behavior of nature is closely related to the granularity of physical space. Therefore it seems natural to believe that in circumstances where this discrete structure becomes relevant, gravitational effects should be treated in a quantum theoretical manner.

However, gravity in the tetron model is an effective interaction of (internal) tetrahedrons in an elastic/plastics system. Its description by the Einstein-Hilbert action or its generalization (23) is valid only at distances  $\gg L_P$ , i.e. loses its validity when probed at distances where the discrete structure becomes appearant. Instead of 'quantizing' gravity one should quantize the fundamental interaction among tetrons.

### **2.5.40 Why is the speed of gravitational waves equal to the speed of light?**

In section 1 general relativity has been interpreted as an effective theory for an elastic system of internal tetrahedrons. Gravitational waves exist in this theory on the classical level, as solutions to the Einstein equations. They are transverse metrical waves, whose velocity is forced to be equal to the speed of light by the condition of local Lorentz invariance. In the tetron model they can be associated to

some of the density fluctuations of tetrahedrons<sup>8</sup> discussed in connection with (18). To that end one first has to explain why elastic waves in the DMESC propagate at the same speed as the photon. From the standpoint of the microscopic model this is a rather amazing feature, because it was argued in (2.5.34) that the isomagnetic particle physics interactions of the photon and the elastic gravitational interactions do not have much in common.

In order to answer this question one should first realize that the world according to the microscopic model falls apart into 2 rather disparate pieces:

-the realm of what philosophers would call emergent or appearance phenomena, i.e. isomagnetic quasi-particles like quarks, leptons, Higgs and gauge fields. Since all these excitations fulfill Lorentz invariant wave equations, any phenomenon and signal propagation in this sphere is necessarily limited by the speed of light.

-the realm of what could be called 'true' or tetron matter, consisting of tetrans, of aligned tetrahedrons and of the DMESC with its elastic/metric structure. This may rightfully be called ' $\nu\lambda\eta\ \pi\rho\omega\tau\eta$ '. However, while for Aristotle this was more an idea than a concrete material, here it can be understood in a real, *materialistic* sense. Just for joke one could call it 'tatter' to distinguish it from the ordinary mignon material which remains m-atter.

Since the relevant scales  $\Lambda_P \gg \Lambda_F$  are so vastly different, these two spheres do not have much in common. We ourselves live in the sphere of appearances and can perceive anything coming from the tatter sector only if suitable devices of mignon matter are patched in between. Gravity, for example, which originally corresponds to a shift of tetrahedron locations on the DMESC, becomes visible in our physical world only due to the small and stiff reaction to suitable conglomerations of m-atter. In particular, the physical effects of a gravitational wave can only be seen by plugging appearances in between, i.e. m-atter which 'rides' the gravitational waves. The principle of relativity states, that m-atter must respect local Lorentz invariance. Therefore, although the fundamental interactions among tetrans may proceed at other velocities than  $c$ , the gravitational interactions between m-atter particles always appear to proceed at  $c$ .

---

<sup>8</sup>In distinction the *internal* translational excitations were coined phinons in [3], while the internal rotational excitations are mignons, i.e. quarks and leptons.

### 2.5.41 How can metric velocities larger than $c$ be interpreted in the tetron model?

This question is related to the discussion in (2.5.14),(2.5.15), (2.5.16), (2.5.24) and (2.5.40).  $c$  is the maximum speed for all the isomagnetic quasi-particles that build our known universe. However, this limit does not apply to the bound tetrahedrons which make up the hyper-crystal and are the carriers of the quasi-particles. As evident from (19), in the tetron model metrical changes are associated to displacements of tetrahedrons. The corresponding velocities of the tetrahedrons have been particularly large ( $> c$ ) in the inflationary period shortly after the big bang (crystallization) where a lot of crystallization energy had been released.

According to (94) one can roughly identify the metric velocity in an FLRW universe with the Hubble flow  $Hd$ . In the tetron model this can be interpreted as the relative velocity of 2 tetrahedrons at distance  $d$ .

## 3 Conclusions

The present review is devoted to a model which tries to give a microscopic meaning to physical phenomena usually described by the Standard Model of elementary particles. By introducing an additional level of matter one is able to understand and to calculate known particle properties (like the quark and lepton masses and mixings) from first principles and furthermore to make predictions for future experiments. Most prominent among the latter are:

- the existence of a second Higgs doublet similar as in inert 2HDM models[68, 69].
- the existence of a fourth family of quarks and leptons. This family, however, is distinct from the other three, not only because it has a very massive neutrino but also because its couplings are not given by the SM. The point is that the 8 Dirac particles of this family do not arise from vibrations of iso-'magnetizations'  $\psi^\dagger \vec{\tau} \psi$  but of internal 'densities'  $\psi^\dagger \psi$ , and therefore they do not obtain their masses via the Higgs mechanism and the SSB fig.2.

After discussing particle physics properties, implications of the tetron model on the big bang and on phase transitions in the early universe have been elucidated. This has led to the idea that besides the

–isomagnetic interactions among aligned isospin vectors which are relevant for particle physics

one should consider 2 other forces:

–strong rigid coordinate forces among tetrons fixing the form and extension of the (internal) tetrahedrons.

–weak elastic forces between these tetrahedrons, which are the basis of the gravitational interactions.

All 3 types of forces are assumed to derive from one universal interaction among the tetrons, and it was suggested that one should use octonion multiplication and perhaps supersymmetry as a guideline which eventually will lead to the correct renormalizable theory in 6+1 dimensions.

The various viewpoints on the model presented in the preceding sections have supplied a set of important requirements as to the nature of tetron interaction. We already know that

–tetrons have a tendency to form excited pairs with antitetrons of neighboring tetrahedrons in the crystal with similarity to Cooper pairs or Frenkel excitons of solid state physics.

–tetron bonds are extremely short, of the order of the Planck length.

–they get saturated in tetrahedral configurations.

–these configurations form 'flat' crystal structures, i.e. there is no stacking of tetrahedrons on top of each another, no growth of the hyper-crystal into internal directions.

–isospins within the quartets of tetrons are maximally frustrated (fig. 1).

–once tetrons are in such a saturated hyper-crystal configuration, there is a left-over elastic force among the internal tetrahedrons, which gives rise to the gravitational interactions.

In summary a new picture of the physical world was presented. It was shown that

–quarks and leptons can be interpreted as internal magnons of a discrete iso-magnetic structure and that their spectrum is due to a tetrahedral symmetry group which remains unbroken down to the lowest energies

–the  $SU(2) \times U(1)$  gauge group of the SM is related to an iso-magnetic 'Heisenberg'  $SU(2)$  and the SM SSB can be obtained from a global ordering of internal magnets

–the big bang is due to a crystallization process which has released a large amount of

latent energy and has led to the formation of a rapidly expanding 3+1 dimensional universe

–the physical world including the gauge bosons consists of excitations which travel as quasi-particles through the hyper-crystal. Since they are described by relativistic wave equations, this leads to the appearance of Minkowski space (and the associated Lorentz structure).

–an 'elastic' rest system of the hyper-crystal exists which is identical to the cosmological comoving coordinates. However, due to the quasi-particle nature of our physical world this cannot be observed in Micholsen-Morley type of experiments.

## References

- [1] B. Lampe, arXiv:1311.6058 [hep-ph], Int. J. Mod. Phys. A30 (2015) 1550026.
- [2] B. Lampe, arXiv:1405.6604 [hep-ph], Int. J. Mod. Phys. A30 (2015)1550025.
- [3] B. Lampe, arXiv:1201.2281 [hep-ph], Int. J. Theor. Phys. 51 (2012) 3073.
- [4] B. Lampe, arXiv:1212.0753 [hep-ph], Mod. Phys. Lett. A28 (2013) 135.
- [5] B. Lampe, arXiv:0805.3762 [hep-ph], Found. Phys. 40 (2009) 573.
- [6] J. Beringer et al. (Particle Data Group), Phys. Rev. D86 (2012) 010001.
- [7] J.A. Wheeler, Ann. Phys. (N.Y.) 2 (1057) 604.
- [8] C. Rovelli, arXiv:1102.3660 [gr-qc] (2011).
- [9] A.P. Cracknell, Progr. Theor. Phys. 35 (1966) 196.
- [10] A.P. Cracknell, Progr. Theor. Phys. 33 (1965) 812.
- [11] A.S. Borovik-Romanov and H. Grimmer, International Tables for Crystallography D (2006) 105.
- [12] R.M. White, Quantum Theory of Magnetism, Springer Verlag, ISBN-10 3-540-65116-0.
- [13] D. Baumann, TASI lectures on inflation, arXiv:0907.5424 (hep-th) 2012.

- [14] A. H. Guth, arXiv:astro-ph/0404546 (2004).
- [15] G. Preparata and S.S. Xue, arXiv:hep-th/9503102 (1995).
- [16] P. Jizba, H. Kleinert and F. Scardigli, Phys. Rev. D81 (2010) 084030.
- [17] S.C. Law and K.L. McDonald, arXiv:1303.6384 [hep-ph] (2013).
- [18] R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
- [19] J. Schechter and J.W.F. Valle, Phys. Rev. D 25, 2951 (1982)
- [20] B.C. Allanach, S.F. King, G.K. Leontaris and S.Lola, Phys.Lett. B407 (1997) 275.
- [21] H. Fritzsch and Z. Xing, Phys. Lett. B598 (2004) 237.
- [22] F. Finster, J. Smoller and S. Yau, J. Math. Phys. 41 (2000) 2173.
- [23] A. Ceccucci, Z. Ligeti and Y. Sakai (Particle Data Group), Phys. Rev. D86 (2012) 010001.
- [24] M. Elhadj, B. Canals, R.S. Borrell and C. Lacroix, arXiv:cond-mat/0503009v1 (2005).
- [25] R. Contino, The Higgs as a Composite Nambu-Goldstone Boson, TASI Lectures 2009.
- [26] G. Cvetič, Rev. Mod. Phys. 71 (1999) 513.
- [27] S. Weinberg, Phys. Rev. D19 (1979) 1277.
- [28] S. Dimopoulos and L. Susskind, Nucl. Phys. B 155, 237 (1979).
- [29] F. Sannino, Acta Phys. Polon. B 40, 3533 (2009), arXiv:0911.0931 [hep-ph].
- [30] T.W. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. Lett. 57, 957 (1986).
- [31] E. Eichten et al., Rev. Mod. Phys. 56 (1984) 579.
- [32] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.

- [33] J.O. Andersen and L.T. Kyllingstad, *J. Phys.* G37 (2010) 19.
- [34] D. Ebert and M.K. Volkov, *Z. Phys.* C16 (1983) 205.
- [35] A. Maiezza, M. Nemevsek, F. Nesti and G. Senjanovic, *Phys. Rev. D* 82 (2010) 055022.
- [36] R. Jora, S. Nasrib and J. Schechter, arXiv:1302.6344v3 [hep-ph] (2013).
- [37] S. Oneda and K. Terasaki, *Progr. Theor. Phys. Suppl.* 82 (1985) 1.
- [38] R. Slansky, *Phys. Rep.* 79 (1981) 1.
- [39] J. Schnack, *Dalton Trans.* 39 (2010) 4677, arXiv:0912.0411v1 [cond-mat] (2009).
- [40] G.M. Dixon, *Division Algebras*, Kluwer Books, 2009.
- [41] J. Conway and D. Smith, *On Octonions and Quaternions*, Peters Publishing, Natick, MA (2003).
- [42] I. L. Kantor and A. S. Solodovnikov, *Hypercomplex Numbers*, Berlin, 1989.
- [43] R.D. Schafer, *An Introduction to Nonassociative Algebras*,
- [44] A. Pineda and J. Soto, *Phys. Rev.* D59 (1998) 16.
- [45] P. Labelle, *Phys. Rev.* D58 (1998) 93.
- [46] R. Arnowitt, S. Deser and C. Misner, *Gen. Rel. Grav.* 40 (2008) 1997.
- [47] B. Carter and H. Quintana, *Proc. Roy. Soc.* 331 (1972) 57.
- [48] arXiv:1403.1213v2 [astro-ph.CO] (2014).
- [49] J.M. Cline, *Les Houches Lectures*, arXiv:hep-ph/0609145 (2006).
- [50] K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, *Phys. Rev. Lett.* 77 (1996) 2887.
- [51] J.M. Cline and P.A. Lemieux, arXiv:hep-ph/9609240 (2007).
- [52] C.P. Bean and D.S. Rodbell, *Phys. Rev.* 126 (1962) 104.



- [53] M.J. Duff, *Contemporary Physics* 12 (2014) 56.
- [54] J.W. Moffat, arXiv:0208109 [hep-th] (2012).
- [55] A. Riotto and M. Trodden, *Ann. Rev. Nucl. Part. Sc.* 49 (1999) 46.
- [56] S. J. Huber, arXiv:hep-ph/0606009.
- [57] E. and F. Cosserat, *Thorie des Corps deformables*, Paris 1909, Hermann et Fils.
- [58] D. Röder, J. Ruppert and D.H. Rischke, *Phys. Rev. D* 68 (2003) 016003.
- [59] R.J. Adler, *Am. J. Phys.* 78 (2010) 925.
- [60] T. Hambyea, F.S. Linga, L. Lopez Honoreza and J. Rochera, arXiv:0903.4010v1 [hep-ph] (2009).
- [61] W. Lu, W. Lee and K.W. Ng, arXiv:1401.3155v1 [gr-qc] (2014).
- [62] K. Hayashi and T. Shirafuji, *Prog. Theor. Phys.* 64 (1980) 866, 883, 1435, 2222.
- [63] M.F. Horstemeyer and T.G. Tenev, arXiv:1603.07655 [gr-qc] (2016).
- [64] S.E. Rugh and H. Zinkernagel, *Stud. Hist. Phil. Mod. Phys.* 33 (2002) 663.
- [65] L.N. Mihaila, J. Salomon and M. Steinhauser, *Phys. Rev. D* 86 (2012) 096008.
- [66] V.A. Kosteleck. *Phys. Rev. D* 69 (2004) 105009.
- [67] G.C. Branco et al., *Phys. Rep.* 516 (2012) 1.
- [68] I.F. Ginzburg, K.A. Kanishev, M. Krawczyk and D. Sokolowska, *Phys. Rev. D* 82 (2010) 123533.
- [69] M. Gustafson, arxiv:1106.1719 [hep-ph] (2011).
- [70] A.L. Chernyshev and M.E. Zhitomirsky, *Phys. Rev. B* 79 (2009) 144416.
- [71] R. Yanes, J. Jackson, L. Udvardi, L. Szunyogh and U. Nowak, *Phys. Rev. Lett.* 111 (2013) 217202.
- [72] L. Know and M.S. Turner, *Phys. Rev. Lett.* 73 (1994) 3347.

- [73] D. Spolyar, arXiv:1111.3629v1 [astro-ph.CO] (2011).
- [74] B. Tent, J. Smit and A. Tranberg, arXiv:hep-ph/0404128v2 (2004).
- [75] G. German, G. Ross and S. Sarkar, Nucl. Phys. B608 (2001) 423.
- [76] H. Georgi and C. Jarlskog, Phys. Lett. B86 (1979) 297.
- [77] K. Bora, arXiv:1206.5909v1 [hep-ph] (2012).
- [78] Y. Zhang, H. An, X. Ji and R.N. Mohapatra, Nucl. Phys. B802 (2008) 247.
- [79] G. Altarelli, arXiv:hep-ph/0508053 [hep-ph] (2005).
- [80] G. Altarelli, F. Feruglio and L. Merlo, arXiv:1205.5133 [hep-ph] (2012).
- [81] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B530 (2002) 167.
- [82] C. Jarlskog, Z. Phys. C29 (1985) 491.
- [83] A. D. Sakharov, Doklady Akad. Nauk S. S. R. 177, 70-71 (1987).
- [84] G. Senjanovic, arXiv:1012.4104 [hep-ph] (2011).
- [85] M. Blagojevic, Kopaonik lectures (2002).
- [86] T.P. Sotiriou and V. Faraoni arXiv:0805.1726 [gr-qc] (2008).
- [87] G.E. Volovik, arXiv:gr-qc/0005091 (2000).
- [88] S. Capozziello and M. Laurentis, Phys. Rep. 509 (2011) 167.
- [89] S. L. Adler, Rev. Mod. Phys. 54, 729 (1982).
- [90] F. Gronwald and F.W. Hehl, in *Advances in Modern Continuum Dynamics*, Isola Elba, 1991.
- [91] F.W. Hehl and Y.N. Obukhov, Ann. Fond. L. de Broglie 32 (2007) 157.
- [92] C.G. Bohmer and N. Tamanini, arXiv:1301.5471v2 [gr-qc] (2013).
- [93] C.P. Ma and E. Bertschinger, Astrophys. J. 455 (1995) 7.

- [94] M. Elhajal, B. Canals, R.S. Borrell and C. Lacroix, Phys. Rev. B71 (2005) 094420.
- [95] B. Lampe, Cent. Eur. J. Phys. 8 (2010) 771.
- [96] A. Einstein, Preuss. Akad. d. Wiss., Sitzungsberichte (1928) 217.
- [97] V.C. Andrade, L.C.T. Guillen and J.G. Perreira, arXiv:gr-qc/0011087v1 (2000).
- [98] L.P. Pitaevskii and S. Stringari, BoseEinstein Condensation, Oxford Clarendon Press, 2003.
- [99] G. G. Ross, Grand Unified Theories, Oxford University Press, 1984.
- [100] C. Barcelo and G. Jannes, arXiv:0705.4652v2 [gr-qc] (2007).
- [101] T.M. Davis and C.H. Lineweaver, arXiv:astro-ph/0310808v2 (2003).