Using formula for searching a prime number in the interval[p_m, p_{m+1}^2]

Nguyen Van Quang

Abstract.There is now a method for searching a prime number in the interval [p_m, p_{m+1}^2] by using formula, and there is no known useful formula that sets apart all prime numbers from composites. In this paper, we try to use a formula for searching a prime number in the interval [p_m, p_{m+1}^2], if a complete list of prime numbers up to p_m is known.

1.Introduce

.

We have known the famous Euclid's proof by formula $N = 2.3.5...p_{m-1}.p_m + 1$, the primality test by using formula $N = n! \pm 1$. Assume $p_m \leqslant n < p_{m+1}$, then N is not divisible by all prime numbers $\leqslant p_m$, but they require more test, since $\sqrt{N} > p_{m+1}$ if $p_m \geq 7$ in these formulas, and they do not give a value which is belong to the interval [p_m, p_{m+1}^2] .

The most basic method of checking the primality of a given integer N is call *trial division*, This routine consists of dividing n by each integer m that is greater than 1 and less than or equal to the square root of n. This routine can be implemented more efficiently if a complete list of prime numbers up to $\sqrt{N}$ is known- then trial divisions need to be checked only for those m that are prime. An algorithm yielding all prime numbers up to a given limit, such as required in the prime numbers - only trial method, is called a prime number sieve. The oldest example, the sieve of Eratosthenes is still the most commonly used.

2.Using formula for searching a prime number in the interval [p_m, p_{m+1}^2]

.

Let be given prime numbers from 2 to p_m, we divide these prime numbers into two groups, the first group contains $p_1, p_2, ..., p_k$ prime numbers, we make the first product: $p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k}$, the second group contains remaining prime numbers $q_1, q_2, ..., q_h$, and we make the second product: $q_1^{\beta_1}.q_2^{\beta_2}...q_h^{\beta_h}$.

Here: α_i, β_j are powers, max $\{p_k, q_h\} = p_m$

Then make absolute value of difference of two products:

$$N = |p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k} - q_1^{\beta_1}.q_2^{\beta_2}...q_h^{\beta_h}|$$

N is not divisible by any prime numbers from 2 to p_m, and N can be following values:
a. $N = 1$.
b. $p_m < N < p_{m+1}^2$, then N is a prime number, since N is not divisible by any prime number $\leq \sqrt{N}$.
c. $N \geq p_{m+1}^2$, then N is a prime number or a composite of two or more prime factors, each of them is equal or lager than P_{m+1}.

If p_{m+1} is unknown, since $p_{m+1} \geq p_m + 2$, so if $N < (p_m + 2)^2$, then N is certain a prime number.

Example: given prime numbers 2,3,5,7,11.

Apply above formula, we obtain some prime numbers : $7 < N < 11^2$ as follows:

$$N_1 = |3.5.7 - 2^7| = 23$$

$$N_2 = |3.5.7 - 2^6| = 41$$

$$N_3 = |3.5.7 - 2^5| = 73$$

$$N_4 = |3.5.7 - 2^4| = 89$$

$$N_5 = |3.5.7 - 2^3| = 97$$

$$N_6 = |3.5.7 - 2^2| = 101$$

$$N_7 = |3.5.7 - 2| = 103$$

$$N_8 = |3.7 - 2.5| = 11$$

$$N_9 = |3^2.7 - 2.5^2| = 13$$

$$N_{10} = |3^2.7 - 2.5| = 53$$

$$N_{11} = |3^2.7 - 2^2.5^2| = 37$$

And can this formula give all prime numbers in the interval[p_m, p_{m+1}^2].

Reference

- Prime number- Wikipedia

Emai: quangnhu67@yahoo.com.vn