The generalized Stokes theorem

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Abstract

When applied to a quaternionic manifold, the generalized Stokes theorem can provide an elucidating space-progression model in which elementary objects float on symmetry centers that act as their living domain.

1 Introduction

This paper uses the fact that separable Hilbert spaces can only cope with number systems that are division rings. We use the most elaborate version of these division rings and that is the quaternionic number system. Quaternionic number systems exist in multiple versions, that differ in the way they are ordered. Ordering influences the arithmetic properties of the number system and it appears that it influences the behavior of quaternionic functions under integration. Another important fact is that every infinite dimensional separable Hilbert system owns a companion Gelfand triple, which is a non-separable Hilbert space. We will use these Hilbert spaces as structured storage media for discrete quaternionic data and for quaternionic manifolds. We use the reverse bra-ket method in order to relate operators and their eigenspaces to pairs of functions and their parameter spaces. Subspaces act as domains in relation to which manifolds are defined.

2 Without discontinuities

Without discontinuities in the manifold ω the generalized Stokes theorem is represented by a simple formula [1].

$$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega; \left(= \oint_{\partial\Omega} \omega \right) \tag{1}$$

The domain Ω is encapsulated by a boundary $\partial\Omega$.

$$\Omega \subset \partial\Omega \tag{2}$$

The manifolds ω and $d\omega$ represent quaternionic fields $\mathfrak{F}$ and d$\mathfrak{F}$, while inside $\partial\Omega$ the manifold ω represents the quaternionic boundary of the quaternionic field $\mathfrak{F}$.

$d\omega$ is the exterior derivative of ω.

2.1 A special boundary between the real part and the imaginary part of the domain

The theorem may construct a rim $\mathfrak{F}(\boldsymbol{x},\tau)$ between the past history of the field $[\mathfrak{F}(\boldsymbol{x},t)]_{t<\tau}$ and the future $[\mathfrak{F}(\boldsymbol{x},t)]_{t>\tau}$ of that field. It means that the boundary $\mathfrak{F}(\boldsymbol{x},\tau)$ of field $[\mathfrak{F}(\boldsymbol{x},t)]_{t<\tau}$ represents a universe wide static status quo of that field.

More specifically:

$$\int_{t=0}^{\tau}\iiint_{V} \mathrm{d}\mathfrak{F}(x) = \int_{t=0}^{\tau}\left(\iiint_{V}\langle\nabla,\mathfrak{F}(x)\rangle\, dx\wedge dy\wedge dz\right)\wedge d\tau = \left[\iiint_{V}\mathfrak{F}(\boldsymbol{x})d\boldsymbol{x}\right]_{t=\tau} \tag{3}$$

$$x = \boldsymbol{x} + \tau \tag{4}$$

Here $[\mathfrak{F}(\boldsymbol{x},t)]_{t=\tau}$ represents the static status quo of a quaternionic field at instance τ. V represents the spatial part of the quaternionic domain of $\mathfrak{F}$, but it may represent only a restricted part of that parameter space. This last situation corresponds to the usual form of the divergence theorem.

2.1.1 Domains and parameter spaces

The quaternionic ***domain*** Ω is supposed to be defined as part of the ***domain*** $\mathfrak{R}$ of a ***reference operator*** $\mathfrak{R}$ that resides in the non-separable Hilbert space $\mathcal{H}$. The bra-ket method relates the eigenspace of reference operator $\mathfrak{R}$ to a flat quaternionic ***function*** $\mathfrak{R}$. The target of function $\mathfrak{R}$ is its ***parameter space***. Here we explicitly use the same symbol $\mathfrak{R}$ for all directly related objects.

The bra-ket method also relates the eigenspace $\mathfrak{R}$ to an equivalent eigenspace $\mathcal{R}$ of a reference operator $\mathcal{R}$, which resides in the separable Hilbert space $\mathfrak{H}$. Both eigenspaces are related to the same version of the quaternionic number system. However, the second eigenspace $\mathcal{R}$ only uses rational quaternions.

Parameter spaces as well as domains correspond to closed subspaces of the Hilbert spaces. The domain subspaces are subspaces of the domains of the corresponding reference operators. The parameter spaces are ordered by a selected coordinate system. The Ω domain is represented by a part of the eigenspace of reference operator $\mathfrak{R}$. The flat quaternionic function $\mathfrak{R}$ defines the parameter space $\mathfrak{R}$. It installs an ordering by selecting a Cartesian coordinate system. Several mutually independent selections are possible. The chosen selection attaches a corresponding symmetry flavor to this parameter space. In the model, this symmetry flavor will become the reference symmetry flavor. Thus, the symmetry flavor of parameter space $\mathfrak{R}^{⓪}$ may be distinguished by its superscript ^{⓪}.

The manifold ω is also defined as the continuum eigenspace of a dedicated normal operator ω which is related to domain Ω and to parameter space $\mathfrak{R}^{⓪}$ via function $\mathfrak{F}$. Within this parameter space $\mathfrak{F}$ may have discontinuities, but these must be excluded from the Ω domain. This exclusion will be treated below.

2.1.2 Interpretation of the selected encapsulation

The boundary $\partial\Omega$ is selected between the real part and the imaginary part of domain $\mathfrak{R}$. But it also excludes part of the real part. That part is the range of the real part from τ to infinity.

The future $\mathfrak{R} - \Omega$ is kept on the outside of the boundary $\partial\Omega$. As a consequence, the mechanisms that generate new data, operate on the rim $\partial\Omega$ between past Ω and future $\mathfrak{R} - \Omega$.

This split of quaternionic space results in a space-progression model that is to a large extent similar to the way that physical theories describe their space time models. However, the physical theories apply a model that has a Minkowski signature. The quaternionic model is strictly Euclidean.

What happens is an ongoing process that embeds the subsequent static status quo's of the separable Hilbert space into the Gelfand triple.

The controlling mechanisms act as a function of progression τ in a stochastic and step-wise fashion in the realm of the separable Hilbert space. The result of their actions are stored in eigenspaces of corresponding operators that reside in the separable Hilbert space. At the same instance this part of the separable Hilbert space is embedded into its companion Gelfand triple.

The controlling mechanisms will provide all generated data with a ***progression stamp*** τ. This progression stamp reflects the state of a model wide clock tick. The whole model, including its physical fields will proceed with these progression steps. However, in the Gelfand triple this progression can be considered to flow.

At the defined rim, any forecasting will be considered as mathematical cheating. Thus, at the rim, the uncertainty principle does not work for the progression part of the parameter spaces. Differential equations that offer advanced as well as retarded solutions must reinterpret the advanced solutions and turn them in retarded solutions, which in that case represent another kind of object. If the original object represents a particle, then the reversed particle is the anti-particle.

As a consequence of the construct, the history, which is stored-free from any uncertainty-in the already processed part of the eigenspaces of the physical operators, is no longer touched. Future is unknown or at least it is inaccessible.

3 Symmetry centers as floating parameter spaces

If we tolerate discontinuities, then these artifacts must be encapsulated by boundaries $\partial \mathrm{H}_n^x$ and in that way they are separated from the main domain Ω.

In that case the model may apply different parameter spaces, which have their own private symmetry flavor [2]. A separable quaternionic Hilbert space can cope with coexisting parameter spaces and these spaces are served by dedicated operators. The bra-ket method relates the parameter space to a corresponding operator. For example [3]:

Let $\{q_i\}$ be the set of ***rational*** quaternions in a selected quaternionic number system and let $\{|q_i\rangle\}$ be the set of corresponding base vectors. They are eigenvectors of a normal operator $\mathcal{R}$.

Here we enumerate the eigenvalues and the base vectors with the same index i.

$$\mathcal{R} \equiv |q_i\rangle q_i \langle q_i| \tag{2}$$

For all bra's $\langle x|$ and ket's $|y\rangle$ hold:

$$\langle x|\mathcal{R}\, y\rangle = \sum_i \langle x|q_i\rangle q_i \langle q_i|y\rangle \tag{3}$$

$\mathcal{R}_0 = \left(\mathcal{R} + \mathcal{R}^\dagger\right)/2$ is a self-adjoint and thus Hermitian operator. Its eigenvalues can be used to arrange the order of the eigenvectors by enumerating them with the eigenvalues. The ordered eigenvalues can be interpreted as ***progression values***.

$\boldsymbol{\mathcal{R}} = \left(\mathcal{R} - \mathcal{R}^\dagger\right)/2$ is the corresponding anti-Hermitian operator.

We will use the same symbol for the operator $\mathcal{R}$, for the eigenspace $\{q_i\}$ and for the defined parameter space. $\boldsymbol{\mathcal{R}}$ is supposed to be ordered by using a selected Cartesian coordinate system. Eight mutually independent selections are possible. The Cartesian ordering determines the symmetry flavor of the eigenspace.

We define a category of anti-Hermitian operators $\{\mathfrak{S}_n^x\}$ that have no Hermitian part and that are distinguished by the way that their eigenspace is ordered by applying a polar coordinate system. We call them symmetry centers $\mathfrak{S}_n^x$. A polar ordering always start with a selected Cartesian ordering. The geometric center of the eigenspace of the symmetry center floats on a background parameter space of the normal reference operator $\mathcal{R}$, whose eigenspace defines a full quaternionic parameter space. The eigenspace of the symmetry center $\mathfrak{S}_n^x$ acts as a three dimensional spatial parameter space. The super script ^{x} refers to the symmetry flavor of $\mathfrak{S}_n^x$. The subscript $_n$enumerates the symmetry centers. Sometimes we omit the subscript.

$$\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x| \tag{4}$$

$$\mathfrak{S}^{x\dagger} = -\mathfrak{S}^x \tag{5}$$

In the companion Gelfand triple of an infinite dimensional separable Hilbert space the reverse bra-ket method can define continuum parameter spaces and relate them to corresponding operators. In this way the countable parameter space $\mathcal{R}$ relates to a continuum parameter space $\mathfrak{R}$.

The quaternionic field $\mathfrak{F}$ can also be represented by a dedicated operator. Here we use a parameter space $\mathfrak{R}$ that is spanned by a full quaternionic number system.

For all bra's $\langle x|$ and ket's $|y\rangle$ hold:

$$\langle x|\mathfrak{R}\, y\rangle = \int_q \langle x|q\rangle q \langle q|y\rangle \; dq \tag{6}$$

$$\langle x|\mathfrak{F}\, y\rangle = \int_q \langle x|q\rangle \mathfrak{F}(q) \langle q|y\rangle \; dq \tag{7}$$

Here, we use the symbol $\mathfrak{F}$ for the field, the function and the operator. However, another parameter space R would deliver another function F for the same field $\mathfrak{F}$. So, what determines the field $\mathfrak{F}$ is stored in the eigenspace $\mathfrak{F}$ of operator $\mathfrak{F}$ and can be coupled to different pairs of functions and parameter spaces.

4 The detailed generalized Stokes theorem

Symmetry centers represent spherically ordered parameter spaces in regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries $\partial\mathrm{H}_n^x$ separate the regions H_n^x from the domain Ω. The regions H_n^x are platforms for local discontinuities in basic fields. These fields are continuous in domain Ω.

The symmetry centers are encapsulated and the encapsulating boundary is part of the disconnected boundary which encapsulates all continuous parts of physical fields that exist in the quaternionic model.

$$\int_{\Omega-\mathrm{H}} d\omega = \int_{\partial\Omega} \omega - \sum_n \int_{\partial\mathrm{H}_n^x} \omega \tag{1}$$

Here domain Ω corresponds to part of the reference parameter space $\mathfrak{R}^{ⓞ}$. As mentioned before the symmetry centers $\{\mathfrak{S}_n^x\}$ represent encapsulated regions $\{\mathrm{H}_n^x\}$ that float on parameter space $\mathfrak{R}^{ⓞ}$.

The geometric center of symmetry center $\mathfrak{S}_n^x$ is represented by a location on parameter space $\mathfrak{R}^{ⓞ}$.

$$H = \bigcup_n \mathrm{H}_n^x \tag{2}$$

The relation between the ***subspaces*** that correspond to the domains and the ***subspaces*** that correspond to the parameter spaces is given by.

$$\underbrace{\Omega} \subset \underbrace{\mathfrak{R}^{ⓞ}} \tag{3}$$

$$\underbrace{\mathrm{H}_n^x} \subset \underbrace{\mathfrak{S}_n^x} \tag{4}$$

Also discontinuities that cover a region of $\mathfrak{R}^{ⓞ}$ can be handled in this way. For example a region that is surrounded by a boundary where the curvature is so high that information contained in ω cannot pass that boundary can be handled by separation from the rest of Ω.

4.1 Symmetry flavor of the symmetry center

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. When the orientation of the coordinate axes is fixed, then eight independent Cartesian orderings are possible [2]. We use the Cartesian ordering of $\mathfrak{R}^{ⓞ}$ as the reference for the orientation of the axes. $\mathfrak{R}^{ⓞ}$ has the same Cartesian ordering as $\mathcal{R}^{ⓞ}$ has.

$$\int_{\Omega-\mathrm{H}} d\omega = \int_{\partial\Omega} \omega - \sum_{n} \int_{\partial \mathrm{H}_n^x} \omega \tag{1}$$

In this formula the boundaries $\partial\Omega$ and $\partial\mathrm{H}_n^x$ are subtracted. This subtraction is affected by the ordering of the domains Ω and H_n^x.

This can best be comprehended when the encapsulation $\partial\mathrm{H}_n^x$ is performed by a cubic space form that is aligned along the Cartesian axes. Now the six sides of the cube contribute different to the effects of the encapsulation when the ordering differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}^{⓪}$. This effect is represented by the symmetry related charge and the color charge of the symmetry center [2]. It is easily related to the algorithm which is introduced for the computation of the symmetry related charge. Also the relation to the color charge will be clear.

The symmetry related charge and the color charge of symmetry center $\mathfrak{S}_n^x$ are located at the geometric center of the symmetry center. A Green's function together with these charges can represent the defining function of the contribution to the symmetry related field $\mathfrak{A}^x$ within and beyond the realm of the floating region H_n^x.

4.2 Path of the symmetry center

The symmetry center $\mathfrak{S}_n^x$ that conforms to encapsulated region H_n^x, keeps its private symmetry flavor. At the passage through the boundary the symmetry flavor of the background parameter space $\mathfrak{R}^{⓪}$ flips. As a consequence the symmetry related charge of the symmetry center will flip.

However, the passage of the symmetry center through the rim may also be interpreted as the annihilation of the historic symmetry center and the creation of a new symmetry center with a reverse symmetry flavor that will extend its live in the future.

The passage of the symmetry centers through the rim goes together with annihilation and creation phenomena for the objects that reside on these platforms. Thus, this passage is related to the annihilation and creation of elementary particles.

In the quaternionic space-progression model the existence of symmetry centers is independent of progression. With other words the number of symmetry centers is a model constant. The passage through the rim does not influence this number. Only the characteristics of the combination of the symmetry center and the background parameter space are affected by the passage.

4.3 The embedding field

Apart from the symmetry related fields $\mathfrak{A}^x$ that are raised by the charges of the symmetry centers at least one other fields exists. That field is the embedding field. The embedding field is not directly affected by the symmetry related charges of the symmetry centers. However, this field is affected by the embedding of artifacts that are picked by controlling mechanisms from the private domain of a symmetry center H_n^x. and then embedded by the controlling mechanism into the embedding continuum, which is represented by the continuum eigenspace of operator $\mathfrak{C}$. The mechanism operates in a cyclic and stochastic fashion. The result is a recurrently regenerated coherent location swarm that also represent a stochastic hopping path. The swarm is generated within the symmetry center $\mathfrak{S}_n^x$ and is encapsulated by $\partial\mathrm{H}_n^x$. The actions of the mechanisms deform the field $\mathfrak{C}$ inside and beyond the floating regions H_n^x.

5 Discussion

The concept of exterior derivative is carefully crafted by skillful mathematicians, such that it becomes independent of the selection of parameter spaces. However, in a situation like this in which one parameter space floats on top of another, the selection of the parameter space does matter. The symmetry flavors of the coupled parameter spaces determine the values of the integrals that account for the contributions of the artifacts. It is represented by the symmetry related charges of these artifacts [4]. These symmetry related charges are supposed to be located at the geometric centers of the symmetry centers.

As happens so often, physical reality reveals facts (the symmetry related charges) that cannot easily be discovered by skilled mathematicians. The standard model contains a short list of electric charges that correspond to the symmetry related charges. Also the standard model does not give an explanation for the existence of this short list. Here it becomes clear that the electric charge is a property of connected spaces and not a property of the objects that use these spaces as parameter spaces. The objects inherit the charge property from the platform on which they reside.

References

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