

# Dark Matter and weak field limit of General Relativity

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(Dated: January 23, 2016)

## Abstract

Followed the Dr. Cooperstock (the emeritus professor) idea to solve Dark Matter problem by means of Einstein's General Relativity. Then is discovered, what the problem of galaxies (even with help of the proposed novel numerical algorithm) do not converge into needed solution. The reason is obvious: the small factors as the non-azimuthal motion of stars have been neglected. But the exact, the non-approximative, equations are simple enough for the stationary rotating dust cylinder of huge height. Turns out, what weak fields limit of the General Relativity fully coincides with the Newton Gravity

The Fred Cooperstock was the brave man, first one tried to solve the seeming lack of matter inside the Galaxies. Because he was missed some technical points, his life work was rejected by the majority of scientists. They have rejected all: the idea and the way of this idea. It is too soon. The idea remains actual, however one shall find other ways for it.

Let me show actuality of idea first. Take an object with density  $\rho_0$ . Then move along it with velocity  $v/c \ll 1$ . You will experience the density  $\rho \approx \rho_0$ . Thus, by comparing density profiles of observation and the Cooperstock's theory one sees practically no difference. Thus, Cooperstock's idea remains in tact.

## I. SEEKING THE TRUTH

Let there be following order: Greek indexes run  $\kappa = t, r, z, \phi$ ; the Latin ones run  $i = r, z, \phi$  in cylindric coordinates and they run  $n = x, y, z$  in Cartesian coordinates.

The calculations show, what the rotating disk solution is not converging. The reason is the simplifications in the original equations. Indeed, strictly speaking, the dust velocity of disk is not  $U^\kappa = (U^t, 0, 0, \omega U^t)$ , but has non-zero  $U^r(r, z, t)$  and  $U^z(r, z, t)$ . Hereby the  $U^t = U^t(r, z, t)$ . The metric is time-dependent. In the limit  $t \rightarrow \infty$  it turns to singularity at equatorial plane  $z = 0$ . I think so, because of my physical intuition. So, let us study first the infinite ( $-\infty < z < \infty$ ) cylinder of dust, if angular velocity is allowed to be not rigid one:  $\omega_r \neq 0$ . Here and after for any function  $f(r)$  the  $f_r$  means first order  $r$ -derivation, the  $f_{rr}$  is the second order and the  $f_{rrr}$  is the third order.

### A. What says the Newton's Gravity?

Holds balance of forces  $m v^2/r = -m g$ . So, let us find the acceleration  $g$ . The point of mass  $m_0$  has Cartesian coordinates  $x^n = (r_0, 0, 0)$ , the point of mass  $dm = \rho(r) r d\phi dz$  has position  $u^n = (r \cos \phi, r \sin \phi, z)$ . Thus, the distance between masses is

$$\Delta = \sqrt{(x^1 - u^1)^2 + (x^2 - u^2)^2 + (x^3 - u^3)^2}. \quad (1)$$

Then the norm of acceleration is the triple integral

$$g = - \int \int \int (x^1 - u^1) \frac{dm}{\Delta^3}. \quad (2)$$

We recommend to take first integral over the range of  $z$ , using the

$$\int_{-\infty}^{\infty} \frac{dz}{(a^2 + z^2)^{3/2}} = 2/a^2. \quad (3)$$

then MapleV gives integral over  $0 < \phi \leq 2\pi$  and then the integration over  $0 < r < R$  has

$$g = 4\pi \frac{1}{r_0} \int_0^{r_0} \rho(r) r dr, \quad (4)$$

where  $g = \omega^2(r_0) r_0$ . Then by taken the derivative  $d(g(r_0) - \omega^2(r_0) r_0)/dr_0 = 0$ , we arrive at differential law (let now the  $r_0$  be denoted as simply  $r$ )

$$\rho(r) = \frac{1}{2\pi} (\omega r \omega_r + \omega^2). \quad (5)$$

The solution to  $\rho = 0$  is the flat velocity profile  $v = \omega(r) r = C_1 = const$ . The superposition principle (do you remember the integral in Eq.(4)?) demands the free constant  $C_1 = 0$ , but there is no superposition principle in General Relativity. But the Galaxies' velocity anomaly can not be explained within the Newton's formulas. Indeed, from integration of the Eq.(5) follows the superposition (the Eq.(4)). So, must be the  $C_1 = 0$ . But let us now watch the formulas of General Relativity.

## B. Wonders in the General Relativity

The metric in cylindric curvature coordinates reads

$$ds^2 = -e^{k(r)} dt^2 - 2 H(r) dt d\Phi + e^{\nu(r)} (f(r) dr^2 + dz^2) + e^{p(r)} r^2 d\Phi^2. \quad (6)$$

The function  $f(r)$  by coordinate transformation is put to unity. In the co-moving coordinates holds  $\Phi = \phi + \omega(r) t$ . Therefore the new metric has non-diagonal elements like  $g_{rt}$  and others. In co-moving coordinates the  $k(r)$  is chosen so, what  $g_{tt} = -\exp(j(r))$ . The zero of geodesic motion  $dU^\kappa/Ds = U^\kappa_{;\alpha} U^\alpha = 0$ , where is put  $U^\kappa = (\exp(-j/2), 0, 0, 0)$ , has the ordinary acceleration  $dU^i/ds \sim \Gamma_{tt}^i = 0$ , where  $i = r, z, \phi$ , and allows the first metric function to be extracted:

$$H = e^p r^2 \omega + e^j j_r / (2 \omega_r). \quad (7)$$

NB! the  $p$  and  $j$  do look here as indexes, but they are not! They are functions of  $r$ .

And calculate with this the Einstein equations. By inserting into  $8\pi U^t U^t \rho = G^{tt}$ , where  $U^t = 1/\exp(j/2) \neq 1$ , the derivatives  $\nu_{rr}$ ,  $\nu_r$  and  $p_{rr}$  from remaining Einstein equations

(they are not solved, but a needed derivative is being "written out" to the left hand of an equation), we arrive at the result:

$$\rho = \frac{1}{32 \pi r^2 (\omega_r)^4} ((j_r \omega_{rr} - j_{rr} \omega_r)^2 - 4 r^4 (\omega_r)^6), \quad (8)$$

where  $j$  can be found from

$$\begin{aligned} 6 r^2 (\omega_r)^4 - (\omega_r)^2 j_{rr} + \omega_{rr} j_r (\omega_r) + 2 (\omega_r)^3 r^3 \omega_{rr} + 2 (\omega_{rr})^2 j_r r - \omega_{rrr} (\omega_r) j_r r + \\ + (\omega_r)^2 j_{rrr} r - 2 \omega_{rr} (\omega_r) j_{rr} r = 0. \end{aligned} \quad (9)$$

Note, what in weak approach (constant  $\epsilon \ll 1$ ) we have

$$p(r) = \epsilon^2 P(r), \quad \nu(r) = \epsilon^2 n(r), \quad j(r) = \epsilon^2 J(r), \quad \omega(r) = \epsilon \Omega(r). \quad (10)$$

Note, what  $\omega(r)$  is larger than other functions, because we are in co-moving coordinates. Then the above solution is found by MapleV productions of the very first term in Taylor series.

Then for example take the non-singular function

$$\omega = V/(r + 1), \quad (11)$$

where  $V$  is constant. Then the (it is non-singular!) solution has

$$\rho = \frac{1}{8 V^2 \pi (r + 1)^3} (4 V^4 + (6 r + 4 + 2 r^2) C_3 V^2 + (3 r + r^3 + 3 r^2 + 1) (C_3)^2). \quad (12)$$

The integration constant  $C_3$ , despite the looking, is not free one: it must be inside the metric tensor, thus, it can be inside the curvature tensor. And the relation  $F = (\omega r)^2/r$  must give it for  $\omega$ . But where is no such constant in Eq.(11). The Newton's theory above (Eq.(5)) for this  $\omega$  gives  $C_3 = 0$ .

In Russian Wikipedia for the Andromeda nebula the local orbital star-speed decreases as  $v \sim 1/\sqrt{r}$ , if to calculate within the Newton's Gravity (hereby you can not use Eq.(5), because we have the galaxy disk, not the our simple cylinder). To this corresponds the gravity force  $F \sim 1/r^2$ . Latter is the point-mass gravity field. Thus, in the Andromeda the density (of baryon matter, the Dark Matter is not added) rapidly decreases to the edge. Therefore, in the flat profile region the  $\rho \approx 0$ .

## II. MODELS, WHERE THE SUPERPOSITION COULD FAIL

In Newton's Gravity the superposition principle takes the most important place. However the Newton's Gravity fails describing the flat Universe model with small homogeneous density  $\rho$ . Therefore by believing, what superposition is true, one says: the universe is "closed" and finite in space and time (past and future). Can one describe the rotating planet made entirely of water? Indeed, the form of water surface is conditioned by the gravity, but the gravity is found by the form of water surface. The circular argument.

## III. NUMERICAL ALGORITHM

Perhaps following way the complexity of Einstein's General Relativity can be managed. Start with initial metric functions:  $f_i(r, z)$  (you should pre-suppose the initial functions, like perhaps  $f_1 = 1/(1+r)$ ). Let us divide the object into grid:  $r_i = R i/1000$ , where  $R$  is galaxy radius. The  $z_j = -\delta/2 + \delta j/1000$ . The  $i, j = 0, 1, 2, \dots, 1000$ . Then use this to manage the numerically all of the derivatives: Wikipedia 2015, "Finite difference". As example:

$$\frac{\partial^2 f(r, z)}{\partial z^2} \approx (f(r_i, z_i) + f(r_i, z_i + 2 \Delta z) - 2 f(r_i, z_i + \Delta z))/(\Delta z)^2, \quad (13)$$

where  $\Delta z = \delta/1000$ . Then compose zero expressions:  $\psi^{\alpha\beta} := G^{\alpha\beta} - 8\pi T^{\alpha\beta}$ . Then make the functional

$$S = (\psi^{tt})^2 + (\psi^{tr})^2 + (\psi^{rr})^2 + (\psi^{zz})^2 + \dots \quad (14)$$

Then take  $f_1$  at point  $(r_1, z_1)$  and add (or take away) to it small  $\beta = f_1/100$ . Then if the  $S$  decreased, then write instead of  $f_1$  its new value. Then make the same procedure with the next function  $f_2$  and so on. After completion take the next point:  $(r_1, z_2)$  and find the  $f_1$ . Then increase/decrease latter by  $\beta$ . If the  $S$  has decreased, then save this new  $f_1$  value at this point. And the cycle starts over. Builds the attractor, which gives you the required solution. The super-PC is required.

But never the less, the Numerical calculations are approximate. But not the calculations in above sections. The simple calculations above are analytic. They show the weak field limit as density turns to zero. The approximate Numerical methods might get support to the main calculations above, but nevertheless can not debunk them (in case of disagreements with algorithm's output). Therefore, I suggest to rely on the above calculations, and save the time by not doing the complex calculations on super-PC.

#### IV. ROTATING DISC IN NEWTON GRAVITY

Because of non-stationaries in dust, one shall think very hard to solve the problem. In equatorial plane  $z = 0$  because of symmetries holds:  $\vec{g} = (-g, 0, 0)$ . Then the balance of forces is  $\omega^2 r = g$ . Let us check, is the  $\rho(r, 0) = 0$ , if  $\omega = V/r$ ?

The Newton's superposition, where  $\omega^2 r^2 = r g$ , turns to local theory (which would be part of General Relativity, latter is local too) by derivative  $d(\omega^2 r^2 - r g)/dr = 0$ , so  $2\omega \omega_r r^2 + 2\omega^2 r - g - r g_r = 0$ . Latter has solution  $\omega = V/r$ ,  $\rho(r, z) = 0$ . Thus, there can be flat velocity profile in weak fields limit of General Relativity. Hereby the rapidly decreasing density  $\rho \sim 1/(r + 1)^3$ , in case of disc, do not produce the flat velocity profile, if the superposition principle holds.

#### V. CAN PIONEER ANOMALY BE FULLY EXPLAINED BY DR. GROOTE TEAM?

The Solar system with all it's planets, dust, asteroids, meteoroids is kind of galaxy, do you agree? So, applying the Newton's theory but without the superposition Principle, one gets, what the real velocity of Pioneers is noticeably lower, than the one was predicted. Is it correct?

Wikipedia 2016: "The apparent anomaly was a matter of tremendous interest for many years, but has been subsequently explained by an anisotropic radiation pressure caused by the spacecraft's heat loss." But in Discovery TV in 2016 is transmitted following information: the 1/4 of the anomaly is still unexplained.