Formulas For The Fine-Structure Constant Based On The Number Pi

In this paper I introduce two new formulas for the fine-structure constant based on the number pi. The accuracy of the first formula is 10 decimal places, while the accuracy of the second formula is 11 decimal places. Thus, both formulas improve the accuracy of a previous formula I published in July 2015.

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1. Introduction

In a previous article [1] I introduced the following formula for the fine-structure constant (also known as the electromagnetic coupling constant or atomic constant):

$$\alpha = \frac{2^{10} - 10^3}{(\pi + 3)\,2^{10} - 3 \times 10^3} \tag{1.1}$$

This formula is accurate to 6 decimal places. In this article I present two new formulas:
(a) The first formula (Formula 1) improves the accuracy of formula (1.1) by 4 decimal places. Thus, the accuracy of the new formula is 10 decimal places. The improvement yielded by this formula will be achieved by adding a correction factor to the numerator. Formula 1 was introduced in the first version of this article.
(b) The second formula (Formula 2) improves the accuracy of formula (1.1) by 5 decimal places. Thus, the accuracy of the new formula is 11 decimal places. The improvement yielded by this formula will be achieved by adding a correction factor to both the numerator and the denominator.

2. Formula 1

A correction factor given by

$$\frac{1}{3 \times 360} \tag{2.1}$$

is added to the numerator of formula (1.1). This yields the new formula for the fine-structure constant

Formula 1 (form1)

$$\alpha = \frac{2^{10} - 10^{3} + \dfrac{1}{3 \times 360}}{(\pi + 3)\, 2^{10} - 3 \times 10^{3}} \tag{2.2}$$

The value this formula yields has been calculated with a hand held calculator and is

$$\alpha \approx 0.007\ 297\ 352\ 53 \tag{R2.1}$$

The measured value for the fine-structure constant published by NIST [2] in 2010 is

$$\alpha_{NIST\ 2010} \approx 0.007\ 297\ 352\ 569\ 8(24) \tag{R2.2}$$

The numbers in parenthesis indicate the uncertainty in the last two decimal places of the measured value. Comparing the result, (R2.1), formula (2.2) yields with the corresponding CODATA value, (R2.2), and without rounding off the CODATA value, we find that the accuracy of formula 1 is 10 decimal places.

3. Formula 2 (The Most Accurate Formula)

A correction factor given by

$$\frac{1}{1072} \tag{3.1}$$

is added to both the numerator and the denominator of formula (1.1). This yields the new formula for the fine-structure constant

Formula 2

$$\alpha = \frac{2^{10} - 10^{3} + \dfrac{1}{1072}}{(\pi + 3)\, 2^{10} - 3 \times 10^{3} + \dfrac{1}{1072}} \tag{3.2}$$

The value this formula yields has been calculated with a hand held calculator and is

$$\alpha \approx 0.007\ 297\ 352\ 56 \tag{R3.1}$$

The measured value for the fine-structure constant published by NIST [2] in 2010 is given by R2.2.

Comparing the result, (R3.1), that formula (3.2) yields with the corresponding CODATA value, (R2.2), and without rounding off the CODATA value, we find that the accuracy of formula 2 is 11 decimal places.

4. Other Ways of Writing the Formula 1

Two other forms of expressing formula (2.2) are

Form 2

$$\alpha = \frac{24 + \frac{1}{1080}}{1024 \times \left(\pi + 3 - \frac{375}{128} \right)} \tag{4.1}$$

Form 3

$$\alpha = \frac{1 - \frac{125}{128} + \frac{1}{1080 \times 1024}}{\pi + 3 - \frac{375}{128}} \tag{4.2}$$

5. Conclusions

The formulas presented in this paper were derived from a previous formula by adding a different correction factors. We could have used different correction factors, and in doing so we would have found a different formula. As the reader might have guessed, this proves that the new formula of the fine-structure based on improved versions of formula (1.1) is not unique.

With reference to formula 1, it is worthwhile to remark that form 1 is more "illustrative" that the other two alternative forms (form 2 and form 3) since it shows more clearly that the formula depends not only on the number pi but also on a power of 2 and on a power of 10. It is also important to observe that the number 3 plays three different roles in the formula: firstly, the role of an exponent $\left(10^3\right)$, secondly, the role of a factor $\left(3 \times 10^3\right)$ and $\left(3 \times 360\right)$; and lastly, the role of a term added to the number pi $\left(\pi + 3\right)$.

Finally, we conclude that formula 2 is more accurate than formula 1 by just one decimal place.

REFERENCES

[1] R. A. Frino, *The Role of Powers of 2 in Physics*, viXra.org: viXra 1507.0047, (2015).
[2] NIST, *Fundamental Physical Constants—Extensive Listing*, retrieved from: http://physics.nist.gov/constants, (2010).