Defining a modified adjacency value product following unique prime labeling of graph vertices and undertaking a small step toward possible application for testing Graph isomorphism

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Abstract: In a previous paper we described a method to represent graph information as a single numerical value by distinctly labeling each of its vertices with unique primes. In this paper, we modify the previous approach to again represent a graph as a single numeric value, we log transform this value and approximate it with an optimum value which if minimized by appropriate prime labeling of the graph should allow us to compare it with another graph on which an identical algorithm is implemented. Identical optimum value minima is a necessary but not sufficient condition for graph isomorphism.

Results:

Consider a graph with "n" vertices and edges. Label each vertex of this graph using distinct primes from the prime series 2,3,5,7,………p_{x},……p_n. Consider a vertex that has been labeled by the prime p_x and let it be connected to "k" vertices labeled p_a, p_b, p_c, ………p_k. We define the modified adjacent value m_x for this vertex as
$m_x= \{(p_x)^((p_a.p_b.p_c……p_k)(p_x^4))\}$

We calculate the modified adjacency value for all "n" vertices of the graph as
m_1,m_2, m_3,………..,m_n

The modified adjacency value product is represented as
$M= m_1.m_2.m_3…………m_n$

Theoretically, given this value it would be possible to reconstruct the graph completely.

Ideally we would like to label a graph such that its M value is minimum.

Therefore $\log_e M$ would also be minimum for the particular prime labeling.

$\log_e M= \log_e m_1+ \log_e m_2+ \log_e m_3+……….+ \log_e m_n$

For any vertex labeled with the prime p_x, and if p_a,p_b,p_c,……,p_k represents the prime labels of all its connected vertices, then

$\log_e m_x= (p_a.p_b.p_c……p_k)(p_x^4)(\log_e(p_x))$
or
$\log_e m_x= (p_a.p_b.p_c……p_k)(p_x^3)(p_x)(\log_e(p_x))$

Since $(p_x)(\log_e(p_x)$ is always less than the p_x^{th} prime number say q_x (Ref 1, Rosser, 1939), we replace the former by the latter to derive an optimized log modified adjacency product and get rid of decimal values (Table 1).

# n=	**n^{th} prime = p_n**	**p_n^{th} prime = q_n**
1	First prime =2	Second prime=3
2	Second prime =3	Third prime= 5
3	Third prime= 5	Fifth prime=11
4	Fourth prime= 7	Seventh prime=17
………	………	…………
………	………	………….
x	x^{th} prime $=p_x$	p_x^{th} prime$=q_x$
………..	………….	……………
n	n^{th} prime= p_n	p_n^{th} prime$=q_n$

Table 1: For a graph with "n" vertices, we need the first "n" primes from 2 to p_n and the corresponding "n" values of the p_n^{th} prime from 3 to q_n to calculate the optimized Log_eM value for a distinct labeling of the prime vertices with primes 2,3,5,……….,p_n.

Therefore for each vertex labeled with prime p_x and connected to vertices $p_a,p_b,p_c,\ldots\ldots,p_k,$

$\log_e m_x= (p_a.p_b.p_c\ldots\ldots.p_k)\ (p_x^3)\{p_x\log_e(p_x)\}$

Applying optimization the

Optimized $\log_e m_x= (p_a.p_b.p_c\ldots\ldots.p_k)(p_x^3)(p_x^{th}$ prime)

Optimized $\log_e m_x= (p_a.p_b.p_c\ldots\ldots.p_k)(p_x^3)(q_x)$

(where q_x represents the p_x^{th} prime)

For non-empty simple graphs, we can expect for each vertex that is labeled p_x, optimized $\log_e m_x$ value is a positive integer and is unique representation for that vertex.

Optimized $\log_e M$= optimized $\log_e m_1$+ optimized $\log_e m_2$+ optimized $\log_e m_3$+………+ optimized $\log_e m_n$

Choosing the prime labeling such that this optimized $\log_e M$ is minimum for each graph to be tested for isomorphism is a necessary next step toward using this method for graph isomorphism. Two isomorphic graphs may be expected to yield identical optimized minimum values. However we need to ascertain if alternate labelings of the same graph may lead to same optimized $\log_e M$ values even in case of a highly non-symmetric graph. We have not ruled out this possibility since we recently determined that integer solutions $a_1,a_2,\ldots,a_n$ of the equation $(\mathbf{a_1p_1^m + a_2p_2^m + \ldots + a_np_n^m = 0})$ exists where m is a positive integer and $p_1,p_2,p_3,\ldots,p_n$ represent the sequence of n primes (Rao, 2016).
This would imply that for m=3 and two alternate optimized labelings of the graph one could obtain values, **optimized $\log_e$M1** and **optimized $\log_e$M2** such that

opt $\log_e$M1-**opt** $\log_e$M2=0, which is of the form $a_1p_1^3 + a_2p_2^3 + \ldots + a_np_n^3 = 0$

In which case the product of the optimized $\log_e m$ terms for the two equations may also need to be minimized to decide optimal labeling.

It is also possible that distinct graphs may yield identical minimum optimized $\log_e M$ values, therefore once the minimum values are obtained we must use this optimal labeling for each graph and calculate/represent the optimized Modified adjacency value product
opt M= $\mathrm{opt}(m_1).\mathrm{opt}(m_2).\mathrm{opt}(m_3)\ldots\mathrm{opt}(m_n)$ for the optimized labeling which must be identical for the graphs to be isomorphic.

Reference:

1. **B.Rosser** The n-th Prime is Greater than nlogn
   Proceedings of the London Mathematical Society January 1939

2. **P.R.Rao** Proof of existence of integral solutions $(a_1, a_2,\ldots,a_n)$ of the equation $a_1p_1^m + a_2p_2^m + \ldots + a_np_n^m = 0$ for any integer "m" greater than or equal to one, for sequence of prime $p_1,p_2,\ldots,p_n$ http://vixra.org/abs/1601.0214 (2016)