Thermogravity and a Statistical Model of Spacetime and Matter

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Abstract

I propose first a simple model for quantum black holes and gravity based on a harmonic oscillator representing the black hole horizon covered by Planck length sized squares carrying soft hair. Next the heat density of matter and gravity is defined based on the assumption that gravitational equations are invariant with respect to adding a constant to the Lagrangian. Extremizing the heat density with respect to normals to null surfaces leads to the equation for the background metric of the spacetime with the cosmological constant as an integration constant. Issues in consistent quantization of both spacetime and matter fields are discussed. Secondly, I redefine the partition function sum over horizon squares by a sum over black hole stretched horizon constituents which are black holes themselves. From this partition function Bekenstein-Hawking entropy law, Hawking radiation and Davies-Unruh effects are predicted. Based on this model I propose that the structures of black holes and spacetime atoms are the same. Requiring consistent quantization for both spacetime and matter, the black hole structure is suggested to matter particles using an old composite model for quarks and gluons. This note is a brief survey of research of current personal interest in gravitation with a number of points by this author.

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1 Introduction

Thermodynamical properties, like entropy and temperature, of black holes have been established about four decades ago [1, 2]. More recently thermodynamics has been considered as the major agent behind general relativity by Padmanabhan [3]. Thermodynamical concepts have been applied to black holes as well as to local Rindler, or acceleration, frames [4]. In [5] acceleration frame considerations have been applied to a model of stretched horizon black holes by calculating the partition function of the system.

I describe first a warmup model for the structure of quantum black holes. The black hole horizon is a spherical membrane covered with $l_{\rm Pl}^2$ size squares each of which can be in k states. The membrane dynamics is represented by a two dimensional harmonic oscillator.

Heat density of matter and gravity is considered for the basis of gravity assuming that the gravitational equations of motion are invariant with respect of adding a constant to the Lagrangian. Gravitational equations are obtained using the normal to null surface extremization method. Spacetime and quantum matter fields themselves remain as different as before.

Secondly, I redefine the membrane partition function as a sum over black hole stretched horizon constituents based on [5]. This method gives as predictions Bekenstein-Hawking entropy, Hawking radiation and Davies-Unruh effects. The author regards the stretched horizon black holes as atoms of spacetime. I propose here that the structure of black holes is the same as the structure of spacetime atoms. A unified quantum picture for both spacetime and matter is proposed.

Large amounts of the results of research by the various schools of thought towards quantum gravity are widely scattered around in various journals in the literature. The motivation of this note is to compile together some of them as I see appropriate from the point of view of physics, with a some thoughts of mine.

This note is organized as follows. The presentation is very concise in all sections. After the Introduction the simple oscillator model for black holes is described in section 2. In section 3 heat density is defined and considered as the origin of gravity. This section relies on the work of done in [3, 6, 7]. In section 5 I present the main points of the stretched horizon black hole model [5]. Consistent quantization of spacetime and matter is discussed in section 6 in connection with a model for composite quarks and leptons. Finally in section 7 I give a brief discussion of results and conclusions.

2 Membrane Model of Horizon

As the first model for black holes of any size I assume the standard picture of a hole as a spherical horizon covered with $l_{\rm Pl}^2$ size squares. The minimal horizon radius is of the order of $l_{\rm Pl}$. All physics takes place on the surface of the sphere, and tentatively, none inside. Suppose there are n squares on the horizon and each square can be in k soft hair states [8]. Then the total number of states is k^n. This gives for entropy S of the sphere the well known result

$$S = k_B \; log \; k^n = k_B \; n \; ln \; k \propto \frac{A}{l_{\rm Pl}^2} \tag{1}$$

whre k_B is the Boltzmann constant and A is the area of the horizon.

The vibrations of an oscillator can be calculated in normal way. The geometry is a two dimensional sphere

$$H = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2\vec{x}^2 \quad (2)$$

The energy eigenvalues for a square are given $E_n = \hbar\omega(n+1)$ where $n = 0, 1, 2, \ldots$.

The partition function Q is (to be used in section 5)

$$Z = \Sigma_n g_n exp(E_n/kT) \quad (3)$$

where g_n (= k in (1)) is the degeneracy of the n'th state and E_n its energy and T the temperature.

3 Heat Density and Gravity

In quantum field theories of matter, like the standard model, the equations of motion are invariant under the addition of a constant to the Lagrangian. Following [9], I assume that the gravitational field equations have this invariance too. I call it Padmanabhan invariance. Here it is supposed to hold near the Planck scale only. This leads to two important results: (i) one cannot vary the metric g_{ab} in a suitable action to obtain the field equations, and (ii) the form of energy-momentum tensor T_{ab} which obeys this invariance under $T_{ab} \to T_{ab} + c \times \delta_{ab}$, is given by

$$\mathcal{H}_m[n_a] = T_{ab}n^a n^b \quad (4)$$

where n_a is a null vector.

For ideal fluid the energy-momentum tensor $T_{ab} = (\rho + p)u_a u_b + p\delta_{ab}$ the quantity $T^{ab}n_a n_b$ is the heat density $\rho + p = Ts$ where T is the temperature and s is the entropy density of the fluid. The invariance principle of the previous paragraph suggests that it is the heat density rather than the energy density which is the source of gravity.

It has been shown in [9] that around any event in spacetime there exists a class of local observers who will interpret $T_{ab}n^a n^b$ as the heat density contributed by matter to a null surface which they perceive as a horizon. We introduce the concept of local Rindler frame (LRF) [10] with coordinates $(t, \mathbf{x})$ and local Rindler observers which will allow us to provide a thermodynamic interpretation of $T_{ab}n^a n^b$ for any T_{ab}. We also introduce a freely falling frame (FFF) with coordinates $(T, \mathbf{X})$. LRF and FFF are connected by a boost having an acceleration a

$$X = \sqrt{2ax}\cosh(at), \; T = \sqrt{2ax}\sinh(at) \quad (5)$$

when $X > |T|$ and similarly for other wedges. One of the null surfaces passing though $\mathcal{P}$, will get mapped to the $X = T$ surface in the FFF and will act as a patch of horizon to the $x =$ constant Rindler observers.

The gravitational term $\mathcal{H}_g$, balancing the matter term, is estimated in terms of number of spacetime atoms at an event x^i together with a null vector n_a. It has the dimension of energy density, so it can be written as $L_P^{-4}f(x^i, n_a)$ where f denotes the number of atoms of spacetime. The function $f(x^i, n_a)$ must be proportional to the simplest objects of spacetime, either the area or volume at x^i. The discrete structure in spacetime defined in terms of L_P. After some analysis in [9] the gravitational contribution to the heat density on a null surface is obtained by integrating $L_P^{-4}f(x^i, n_j)$ over the volume:

$$Q_g = \int \frac{\sqrt{\gamma}d^2xd\lambda}{L_P^4} f(x^i, n_j) = \int \frac{\sqrt{\gamma}d^2xd\lambda}{L_P^4}\left[1 - \frac{1}{6}L_0^2(R_{ab}n^a n^b)\right] \quad (6)$$

One gets the correct equations with the choice $L_0^2 = (3/4\pi)L_P^2$ for the numerical factor of the second term.

The variational method is thus applied to the heat density on a null surface functional [9]

$$Q_{\mathrm{tot}} = \int \sqrt{\gamma}\, d^2x\, d\lambda \left[\frac{1}{L_P^4} + \left\{ T_{ab} - \frac{1}{8\pi L_P^2} R_{ab} \right\} n^a n^b \right] \tag{7}$$

where R_{ab} is the Ricci tensor. Extremizing this with respect to n_a and introducing a Lagrange multiplier $\lambda(x)$ to keep $n^2 = 0$, and demanding the extremum holds for all n_a at an event leads to the result $R_{ab} - (8\pi L_P^2)T_{ab} = \lambda(x)\delta_{ab}$. Using the Bianchi identity and $\nabla^a T_{ab} = 0$, the gravitational field equations are recovered except for a cosmological constant.

In passing, I mention that one may be able to relate $f(x^i, n_a)$ in to the entanglement entropy which could provide an alternative interpretation to the above results [9].

4 Matter-Spacetime Coupling

The equation $R_{ab} = 8\pi T_{ab}$ connects objects of different categories [9]. The left hand side is purely geometrical while the right hand side describes matter which obeys the equations of quantum field theory. The application of the quantum field theory methods to gravity have failed distinctly.

An equation like $R_{ab}n^a n^b = 8\pi T_{ab} n^a n^b$ does not do much better in this regard. The hope is to interpret both sides independently as heat densities and think of this equation as a balancing procedure. But one can require one step further that both spacetime and matter themselves are treated on equal footing. This will be done in the next two sections.

5 Model of Stretched Horizon

I consider a micro black hole dressed by a (virtual reality [11]) stretched horizon, which is a membrane hovering about a Planck length outside the event horizon and which is both physical and hot. A treatment of the stretced horizon has been done by Mäkelä in [5] which paper I cite below. He assumes that the stretched horizon consists of finite number of discrete constituents each contributing to the stretched horizon an area of a non-negative integer times a constant

$$A = \alpha l_{\mathrm{Pl}}^2 (n_1 + n_2 + ... + n_N) \tag{8}$$

where N is the number of constituents, the n_i define their area quantum states and α is a number of the order unity, to be determined later. For the constituents themselves one assumes simply black holes of size l_{Pl}. It is supposed that each stationary quantum state of a black hole is determined by the quantum numbers $n_1, n_2, ..., n_N$ of its stretched horizon.

To calculate the partition function of a Schwarzschild black hole one needs to know the energy levels of the system. The energy of the hole from the point of view of an observer on its stretched horizon is called Brown-York energy [12]

$$E = \frac{ac^2}{8\pi G} A \tag{9}$$

where a is the constant proper acceleration of an observer on the stretched horizon and A is the area of the horizon. The possible energy values of a black hole are, from the point of view of an observer located on its stretched horizon, in terms of the acceleration

$$E_n = n\alpha \frac{\hbar a}{8\pi c} \tag{10}$$

where $n = n_1 + n_2 + ... + n_N$. The number of microscopic states associated with energy E_n is the number of ways of writing a given positive integer n as a sum of exactly N positive integers, whith $N \leq n$, which is given by the binomial coefficient

$$\Omega_N(n) = \left(\begin{array}{c} n-1 \\ N-1 \end{array} \right). \tag{11}$$

It depends on n and N only, and it gives the degeneracy function $g(E_n)$ needed to calculate the partition function

$$Z(\beta) = \Sigma_n g(E_n) e^{-\beta E_n} \tag{12}$$

The resulting partition function $Z(\beta)$ of the Schwarzschild black hole may be calculated explicitly yielding a simple expression [5]:

$$Z(\beta) = \frac{1}{2^{\beta T_C} - 2} \left[1 - \left(\frac{1}{2^{\beta T_C} - 1} \right)^{N+1} \right] \tag{13}$$

where the temperature

$$T_C = \frac{\alpha \hbar a}{4(\ln 2)\pi k_B c} \tag{14}$$

is called the characteristic, or critical, temperature of the hole.

From the partition function one can calculate the average energy

$$E(\beta) = -\frac{\partial}{\partial \beta} \ln Z(\beta) \tag{15}$$

of the hole at temperature $T = 1/\beta$ which yields

$$E(\beta) = \left[\frac{2^{\beta T_C}}{2^{\beta T_C} - 2} - \frac{(N+1)2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1} \right] T_C \ln 2 \tag{16}$$

The average energy per constituent is

$$\bar{E}(\beta) = \frac{E(\beta)}{N} \tag{17}$$

and one gets for large N

$$\bar{E}(\beta) = \bar{E}_1(\beta) + \bar{E}_2(\beta) \tag{18}$$

where

$$\bar{E}_1(\beta) = \frac{1}{N} \frac{2^{\beta T_C}}{2^{\beta T_C} - 2} T_C \ln 2, \tag{19a}$$

$$\bar{E}_2(\beta) = -\frac{2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1} T_C \ln 2. \tag{19b}$$

where $(N+1)/N \approx 1$ has been used.

It is shown in [5] that when $T = T_C$ the average energy per a constituent of the stretched horizon is, in SI units,

$$\bar{E} = k_B T_C \ln 2 \tag{20}$$

and that

$$\frac{d\bar{E}}{dT}|_{T=T_C} = \frac{1}{6}k_B(\ln 2)^2 N + \mathcal{O}(1) \tag{21}$$

where $\mathcal{O}(1)$ denotes the terms, which are of the order N^0, or less. When the number N of constituents becomes large increase of energy does not change the temperature of the hole at $T = T_C$. So the hole undergoes a phase transition at $T = T_C$. When $T < T_C$, $\bar{E}$ is nearly zero. When $T = T_C$, the curve $\bar{E} = \bar{E}(T)$ becomes practically vertical. When T is slightly greater than T_C, $\bar{E}(T)$ is approximately $1.4k_BT_C$, which is about the same as $2\ln 2$. Finally, the dependence of $\bar{E}(T)$ on T becomes approximately linear when $T \gg T_C$.

The most important implication of the observed phase transition at the characteristic temperature T_C is that it predicts the Hawking effect: the result that $\bar{E}(T)$ is practically zero, when $T < T_C$, and then suddenly jumps to $\bar{L} = 2k_BT_C \ln 2$, when $T = T_C$, indicates that the characteristic temperature T_C is the lowest possible temperature a black hole may have. If the temperature T of the black hole were less than its characteristic temperature T_C, all of the constituents of its stretched horizon, except one, would be in vacuum, and there would be no black hole. The characteristic temperature T_C may be written in terms of the Schwarzschild mass M and the Schwarzschild radial coordinate r of an observer on the stretched horizon as:

$$T_C = \frac{\alpha}{8\pi \ln 2}\left(1 - \frac{2M}{r}\right)^{-1/2}\frac{M}{r^2} \tag{22}$$

With some more effort one can obtain the Bekenstein-Hawking entropy law for the Schwarzschild black hole from its partition function which, in turn, followed from the specific microscopic model of its stretched horizon [5]

$$S(A) = \frac{1}{4}\frac{k_B c^3}{\hbar G}A \tag{23}$$

When $T = T_C$, the energy of the hole from the point of view of an observer on its stretched horizon is exactly

$$E = (N+2)k_BT_C \ln 2 \tag{24}$$

It is interesting that, up to an unimportant numerical factor $2\ln 2$, this expression for energy is the same as the one used as a starting point in the scenario for an entropic theory of gravity in [13].

How do we interprete the stretched horizon model? It applies both to black holes and atoms of spacetime. The next question is are matter particles pointlike in this spacetime or do particles have some kind of substructure? In a unified model both spacetime and matter particles have the same type of substructure as we see in the next section.

6 Unified Quantization of Spacetime and Matter

Statistical methods of section 5 for spacetime offer a possibility to study the matter sector from a different point of view to build a consistent statistical picture of both spacetime and matter. This goal would imply some internal structure at scale of the order of l_{Pl} for standard model particles. Such a model has been proposed in [14] (though there never was experimental support for it). The Padmanabhan invariance postulate of section 3 may be of help in determining the masses of the heavy constituent bound states. Can a single extra mass counter term in the Lagrangian shift the quark or lepton masses to the observed range of values? I assume tentatively it can but the details may be subtle.

The basic idea in [14] is that the quarks and lepton are made of color triplet maxons with charge 0 or $\frac{1}{3}$ as follows

$$u_k = \epsilon_{ijk} m_i^+ m_j^+ m^0 \tag{25}$$

$$d_k = m_k^- m^0 m^0 \tag{26}$$

$$e^- = \epsilon_{ijk} m_i^- m_j^- m_k^- \tag{27}$$

$$\nu = \epsilon_{ijk} m_i^0 m_j^0 m_k^0 \tag{28}$$

where i, j and k are color indices. This construction should be considered as an example rather than a fundamental statement.

The maxons are black holes ($T = 0$), now with a stretched horizon. The three generations would be due to a gravitational mechanism or new symmetry. The huge mass difference of Planck mass and hadron masses is accomplished by the invariance of the Lagrangian under adding a constant term introduced in section 3. The gauge bosons and the Higgs would be elementary (but their compositeness is not ruled out). This looks logically possible, or even satisfactory. But missing at the moment are calculational methods for bound states. The heavy constituents are non-relativistic and numerical methods can be developed at some level of accuracy.

7 Discussion and Conclusions

Any non-inertial observer who perceives a horizon will attribute to it the Davies-Unruh temperature (14)

$$T = \frac{\hbar}{k_B c} \frac{\kappa}{2\pi} \tag{29}$$

where κ is the acceleration of the observer, which is predicted in [5]. This result makes the notion of temperature and all of thermodynamics observer dependent phenomena. It was taken into account in the extremization procedure in the earlier sections.

It has turned out that horizons have profound importance in gravity both on thermodynamical and statistical levels. There are interesting questions of heat as inertial effect in a quantum equivalence principle and static observer's virtual reality in [11].

In the UV black holes cannot be probed deeper than $l_{\rm Pl}$. With increasing energy the hole begins to grow approaching the classical regime. This model is therefore consistent with the concept of self-completeness [15].

The regime of real quantum gravity is limited to the vicinity of mini black holes and very early universe. Otherwise classical theory is accurate, see however for Lanczos-Lovelock theories in [6].

Having the heat density, rather than the metric, as central concept in the model of section 3 has lead to an interesting role for the cosmological constant: a constant of integration. Its numerical value has been determined in [7].

A model of decay and radiation of black holes has been proposed in [16, 17]. The lightest black hole state $E_{n=0}$, the gravon, is expected to decay via a grand unified theory phase finally into standard model particles. Otherwise black holes radiate by the Hawking mechanism and by a classical no-hair theorem based mechanism producing non-thermal particles, dominantly light leptons.

There are at present a number competing theoretical schemes for quantum gravity like string theory, loop quantum gravity, causal dynamical triangulation, and others.

The model of section 5 goes deep into the structure of the physical universe and can be considered a promising candidate. In that scenario the horizon properties of black holes and Rindler frames are the origin of gravity and general relativity is its IR limit. These ideas have been applied also to the matter sector in connection of a preon model in section 6.

In this microscopic model the role of gravitons is open at the moment and the role of geometry is minimal. Finally, this note is a program definition for future work with a number of model results as guidelines.

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