On the relation between the spontaneous and explicit breaking of global symmetries in the presence of the Higgs mechanism

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Abstract

One common way to define spontaneous symmetry breaking involves necessarily explicit symmetry breaking. Thus, how could we have spontaneous without explicit symmetry breaking? We study the concept of Hidden symmetries, which are not necessarily spontaneously broken, and its real representations on Higgs fields. Given G_p is a normal subgroup of G_a (compact group), we proved for a multi-Higgs-doublet model that:

1) if we impose a symmetry G_p to the Higgs potential and no term exists violating G_a, then there is no spontaneous breaking of G_a.

2) if explicit symmetry breaking of G_a is allowed while G_p is conserved, then G_a is allowed to break spontaneously;

Using this theorem we explain a recent and related conjecture related with the CP symmetry.

1 Introduction

There are several definitions of spontaneous breaking of global symmetries [2, 3]. In the following common definition[2], spontaneous symmetry breaking is defined as a particular case of explicit symmetry breaking via the external source J.

Let $\mathcal{A}$ be an algebra of operators, the global symmetry β is a bijective map $\beta : \mathcal{A} \to \mathcal{A}$.

The system's expectation value ω_J is a positive linear functional $\omega_J : \mathcal{A} \to \mathbb{R}$,
$J \geq 0$ is the intensity of an external source breaking the symmetry β. The system has infinite size $\omega_J = \lim_{N\to\infty} \omega_{J,N}$.

For finite size N, the system is well behaved with continuous expectation values $J > 0$: $\omega_{J,N}(A - \beta(A)) = a_{J,N} \neq 0$ $J = 0$: $\omega_{0,N}(A) = \omega_{0,N}(\beta(A))$ $\lim_{J\to 0} \omega_{J,N}(A - \beta(A)) = 0$ for some A.

1) The spontaneous symmetry breaking happens when: $\lim_{J\to 0}\{\lim_{N\to\infty} \omega_{J,N}(A-\beta(A))\} \to a_0 \neq 0$ and it is possible due to the fact that the limits are non-commutative if $\omega_{J,N}$ is continuous but not uniformly continuous.

Other definitions do not consider an external source [2], at least not explicitly.

In statistical mechanics, it is more or less clear that these definitions should be all equivalent (e.g. in the Ising model [2]), although it is not easy to mathematically prove it as the systems with or without external source are physically different [4].

When it comes to quantum non-abelian gauge field theories, the theories themselves lack a non-perturbative mathematical definition [5], so it is even more difficult to relate these different definitions. By analogy with statistical mechanics, we expect that they are related. In the presence of the Higgs mechanism, there is yet another definition of spontaneous symmetry breaking, most common in the context of perturbation theory:

2) After a suitable perturbative non-abelian gauge fixing, the vacuum expectation value of the Higgs field is determined (up to quantum corrections) by one of the possible minima of the Higgs potential. The symmetries broken by the vacuum expectation value of the Higgs field are spontaneously broken symmetries.

It is not at all obvious that the last definition (2) is physically (not to mention mathematically) equivalent to the first definition (1) in the context of the Electroweak theory, since spontaneous symmetry breaking is a non-perturbative phenomenon.

However, the fact is that the perturbative predictions from the Electroweak theory seem to be a very good approximation to the existing experimental data in high-energy physics[6], and the lattice simulations so far agree with this picture [7–9] (also for two-Higgs-doublet models [10]). Therefore, for consistency these definitions should be related. While we cannot give a solid proof that this is so, we can check in concrete models that the perturbative definition is consistent with the non-perturbative ones.

There is a further ingredient to take into account [11]: a spontaneous breaking of local gauge symmetry without gauge fixing may be impossible in a gauge theory such as the Electroweak theory. The argument is based on the fact that local gauge transformations affect only a small sized system near each space-time point and so the non-commutativity of the limits seen above does not applies (under some assumptions on the analiticity of $\omega_{J,N}(A-\beta(A))$). It can be argued that the Higgs mechanism avoids the presence of Nambu-Goldstone bosons precisely because the local gauge symmetry is not spontaneously broken [12, 13]. Moreover, there is a group-theory correspondence between gauge-invariant composite operators and the gauge-dependent elementary fields in the Electroweak theory [9, 13, 14].

The above discussion implies that there must exist specific relations between the gauge-dependent minima of the Higgs potential and the gauge-invariant operators appearing in the Lagrangian, for consistency reasons. That is, relations between explicit and spontaneous symmetry breaking. Some of these relations were noted recently in the context of the study of the CP symmetry in multi-Higgs-doublet models [1] and were summarized in the form of a conjecture. We will study the concept of Hidden symmetries, which are not necessarily spontaneously broken, and its real representations in Higgs fields. Then we will generalize and try to understand the recent conjecture mentioned above [1].

2 Hidden symmetries and minima

An important remark is that the minimization of the potential is done classically. Then quantum corrections may be added, but evaluating if spontaneous symmetry breaking is allowed or not is always done purely classically and more importantly, only Higgs fields and the potential are considered. Another remark is that the Higgs potential is a real function, so we have to work with irreducible representations over the real numbers (which admit more operators than the corresponding irreducible complex representations, in this sense they are more general than the complex representations) [15].

Suppose we have a compact group of symmetries G. We consider a real irreducible representation of G, ϕ. Also we call to the sub-groups G_p the physical group and to G_a the absent group. G_p is a normal subgroup of $G_a \subset G$.

We now assume that we can decompose the representation space ϕ into a direct sum of irreducible equivalent representations of G_a. Now we assume that every irreducible subspace of ϕ conserved by G_p is also conserved by G_a. This implies that every hermitian H operator acting on ϕ that is invariant under G_p is also invariant under G_a. (the converse is also true: every hermitian operator that is conserved by G_p is proportional to the identity in each irreducible representation of G_p— by the Schur's lemma over the real numbers— if all of them are conserved by G_a that implies that G_a leaves invariant the projection operators defining the subspace of each irreducible representation of G_p and that we can decompose the representation space ϕ into a direct sum of irreducible equivalent representations of G_a).

We write the most general quartic potential symmetric under G_p, $V(\phi)$ and therefore verify that it is also invariant under G_a. Then the group of background symmetries G_b is the subgroup of G such that G_a, not G_p is a normal sub-group. That is because we cannot tell the difference, whether we wanted or not for the symmetry G_a to be conserved, as the potential is exactly the same. The parameters of the potential will be an irreducible representation of G_b, with G_a acting trivially.

Then the Higgs basis will necessarily conserve G_a. These symmetries are hidden in the problem of minimization. They are not broken by the bilinears in the Higgs basis. This is what lies behind the fact that the Higgs mechanism in the SM involves a minimization problem, that does not lead necessarily to the breaking of the $SU(2)_R \times SU(2)_L \to SO(3)$ [12–14].

The coordinates of a vector space covariant under a group G are not meaningful mathematically (neither physically). The only relevant information we can extract from a group representation are its invariants, in the case of the Higgs potential which only depends on Higgs fields in one space-time point, these are the bilinears which always absorb the G_a group, thus these symmetries are hidden at least in the Higgs potential which determines what are the spontaneously broken symmetries.

3 The case when explicit symmetry breaking is allowed

When the normal subgroup $G_p \subset G_a$ is a hidden symmetry, suppose that explicit symmetry breaking of G_a is possible. Then, in the Higgs basis we can have one such term breaking G_a. That implies spontaneous symmetry breaking if in the Lagrangian basis the symmetry G_a is conserved. Note that the terms of the Lagrangian are an irreducible representation of the group of background symmetries G_b, therefore there are no invariant subspaces under G_b. Of course that we can play with the parameters so to avoid spontaneous symmetry breaking, an extreme case would be for instance setting the quadratic terms to zero.

4 Avoiding the charged vacuum

In the context of multi-Higgs-doublet models, the vacuum may break $U(1)_{em}$. That is because we are not considering all the possible terms which are invariant under $SU(2)_L$ for real irreducible representations (note that the Higgs potential is a real function), we are setting some of the terms in the potential manually to zero, such that $U(1)_{em}$ is conserved explicitly [16]. So, we have to work with the most general potential invariant under $SU(2)_L$, or modify the representation space.

The terms in the Higgs potential which will be null for a neutral vacuum are irrelevant when evaluating spontaneous symmetry breaking at neutral minima, since changing such terms leaves the potential invariant at the minimum (they correspond to Lagrange multipliers in the bilinear formalism to minimize the potential). Dropping such terms is equivalent to consider a smaller system where all the Higgs fields are aligned.

Hence, our irreducible representation space is instead two-dimensional, i.e. a real representation of $U(1)_Y$. Then we can apply the theorem.

In order to apply the theorem to CP violation, we need to define the group that includes CP. The theorem is valid for any subgroup of $SU(n) \rtimes Z_2$ where n is the number of Higgs doublets, so it is valid for any subgroup including CP [17, 18].

5 Conclusion

Consider the groups G_p the physical group and to G_a the absent group. G_p is a normal subgroup of G_a. The Higgs field representation should be real since the potential is a real function. Under the non-perturbative assumption that the vacuum expectation value of the Higgs field is determined (up to quantum corrections) by one of the possible minima of the Higgs potential, we proved for a multi-Higgs-doublet model that:
1) if we impose a symmetry G_p to the Higgs potential and no term exists violating G_a, then there is no spontaneous breaking of G_a. 2) if explicit symmetry breaking of G_a is allowed while G_p is conserved, then G_a is allowed to break spontaneously;

Using this theorem we explained a recent and related conjecture [1], related with the CP symmetry. Note that the theorem of this paper is not just valid for the breaking of the CP symmetry, it also considers restrictions from compact groups (not necessarily finite or abelian), unlike the mentioned conjecture [1].

Already for consistency reasons, the above theorem (modulo technical details) should be valid: probably the most popular way to define spontaneous symmetry breaking involves necessarily explicit symmetry breaking (def. 1 of the introduction). Thus, how could we have spontaneous without explicit symmetry breaking? (1st part of theorem). Also, since we are assuming that the spontaneous symmetry breaking is determined by the minima of the Higgs potential, then if we allow explicit symmetry breaking of the Higgs potential we necessarily allow spontaneous symmetry breaking, since we can redefine the field coordinates with respect to a particular minimum that breaks explicitly the symmetry (2nd part of theorem).

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