Relating spontaneous and explicit symmetry breaking in the presence of the Higgs mechanism

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Abstract

One common way to define spontaneous symmetry breaking involves necessarily explicit symmetry breaking. Thus, how could we have spontaneous without explicit symmetry breaking? We study the concept of sectorial symmetries, which generalizes and relates different types of symmetry: gauge, spontaneously broken and spurion symmetries. For instance, since the Higgs potential is gauge invariant, the gauge group acts on a different sector than the parameters of the Higgs potential, so that we do not need to make assertions on whether the gauge symmetry is spontaneously broken or not when studying the Higgs potential.

Consider the most general Higgs potential V which is invariant under a compact group G_l. Consider a group G_L which includes G_l as a normal subgroup. We assume that the observables are invariant under G_l, then:
1) if V is necessarily invariant under G_L, then the observables do not show spontaneous symmetry breaking of G_L/G_l;
2) if V can break explicitly G_L, then there are observables showing spontaneous symmetry breaking of G_L/G_l.

Using the above proposition:
1) we show that it is possible to study the Higgs potential without making assertions on the spontaneous breaking of the local gauge $SU(2)_L$ symmetry, which (non-perturbatively) may not be possible to break spontaneously without gauge fixing;
2) we relate the proposition with the accidental custodial symmetry of the Higgs potential of the Standard Model and the spontaneous breaking of $U(1)_{em}$ in multi-Higgs-doublet models;
3) we explain a recent and related conjecture related with the charge-parity symmetry in multi-Higgs-doublet models.

1 Introduction

There are several definitions of spontaneous breaking of global symmetries [1, 2]. In the following common definition[1], spontaneous symmetry breaking is defined as a particular case of explicit symmetry breaking via the external source J.

Let $\mathcal{A}$ be an algebra of operators, the global symmetry β is a bijective map $\beta : \mathcal{A} \to \mathcal{A}$.

The system's expectation value ω_J is a positive linear functional $\omega_J : \mathcal{A} \to \mathbb{R}$, $J \geq 0$ is the intensity of an external source breaking the symmetry β. The system has infinite size $\omega_J = \lim_{N\to\infty} \omega_{J,N}$.

For finite size N, the system is well behaved with continuous expectation values as a function of J: $J > 0$: $\omega_{J,N}(A - \beta(A)) = a_{J,N} \neq 0$ $J = 0$: $\omega_{0,N}(A) = \omega_{0,N}(\beta(A))$ $\lim_{J\to 0} \omega_{J,N}(A-\beta(A)) = 0$ for some operator A.

1) The spontaneous symmetry breaking happens when $\lim_{J\to 0}\{\lim_{N\to\infty} \omega_{J,N}(A-\beta(A))\} \to a_0 \neq 0$.

The value $a_0 \neq 0$ is possible since the (pointwise) limit of a convergent sequence of continuous functions is not necessarily continuous.

Other definitions do not consider an external source [1], at least not explicitly.

In statistical mechanics, it is widely accepted that these definitions should be all equivalent (e.g. in the Ising model [1]), although it is not easy to prove it as the systems with or without external source are physically different [3].

When it comes to quantum non-abelian gauge field theories, the theories themselves lack a non-perturbative mathematical definition [4], so it is even more difficult to relate these different definitions. By analogy with statistical mechanics, we expect that they are related. In the presence of the Higgs mechanism, there is yet another definition of spontaneous symmetry breaking, most common in the context of perturbation theory:

2) After a suitable perturbative non-abelian gauge fixing, the vacuum expectation value of the Higgs field is determined (up to quantum corrections) by one of the possible minima of the Higgs potential. The symmetries broken by the vacuum expectation value of the Higgs field are spontaneously broken symmetries.

It is not at all obvious that the definition (2) is physically (not to mention mathematically) equivalent to the first definition (1) in the context of the quantum Electroweak theory, since spontaneous symmetry breaking is a non-perturbative phenomenon. Note that in definition (2), the determination of the spontaneously broken symmetries is a classical problem of minimization of a polynomial [5].

However, the fact is that the perturbative predictions from the Electroweak theory seem to be a very good approximation to the existing experimental data in high-energy physics[6], and the lattice simulations so far agree with this picture [7–9] (also for two-Higgs-doublet models [10]). Therefore, for consistency these definitions should be related. While we cannot give a solid proof that this is so, we can check in concrete models that the perturbative definition is consistent with the non-perturbative ones.

There is a further ingredient to take into account [11]: a spontaneous breaking of local gauge symmetry without gauge fixing may be impossible in a gauge theory such as the Electroweak theory. The argument is based on the fact that local gauge transformations affect only a small sized system near each space-time point and so the non-commutativity of the limits seen above does not applies (under some assumptions on the analiticity of $\omega_{J,N}$). It can be argued that the Higgs mechanism avoids the presence of Nambu-Goldstone bosons precisely because the local gauge symmetry is not spontaneously broken [12, 13]. Moreover, there is a group-theory correspondence between gauge-invariant composite operators and the gauge-dependent elementary fields in the Electroweak theory [9, 13, 14].

The above discussion implies that there must exist specific relations between the gauge-dependent minima of the Higgs potential and the gauge-invariant operators appearing in the Lagrangian, for consistency reasons. That is, relations between explicit and spontaneous symmetry breaking. Some of these relations were noted recently in the context of the study of the CP symmetry in multi-Higgs-doublet models [15] and were summarized in the form of a conjecture. In this paper we will study the concept of sectorial symmetries and how they apply to the Higgs mechanism. We will then address three problems which as we will see are related: how to study the Higgs potential without assuming spontaneous symmetry breaking of the gauge symmetry $SU(2)_L$, why the custodial symmetry is accidentally conserved in the Higgs potential of the Standard Model, the recent conjecture on CP symmetry mentioned above [15].

2 Sectorial symmetries and observables

What is a measurement[1]? It is a physical process which we use to extract information about an open system. However in a theory of the Universe, say the Standard Model, the system (universe) is closed and so in some sense there can be no measurements of the system. There are however different sectors of the universe, which interact in precise ways. For instance, inside a pion there are QCD $SU(3)$ gauge interactions but it only interacts with an electron via electroweak interactions. These electroweak interactions are in turn (due to the Higgs mechanism) a remnant of the $SU(2)_L$ gauge interactions, to which the quiral part e_R of the electron is insensible. Thus the observables are the operators which we can use to relate different sectors of a theory of the Universe. In the end, the stable matter and so the devices we can build are mostly sensitive to electromagnetic and termodynamic phenomena, so it is frequent to consider observables the electromagnetic and termodynamic properties of the subsystems, but this only for practical reasons and there are no fundamental reasons to consider some properties of a system an observable or not.

Suppose that we have a group of transformations G_I and G_{II} which act in the elementary fields of Sectors I and II of our theory. With the fields of each sector we can build composite operators which are representations of a common group G, there are then interactions involving the composite operators of both sectors.

We thus have an homormophism $G_I \to G$ (a similar one for the sector II as well), we call sectorial symmetry to G_n, with $G = G_I/G_n$ (it is the the kernel of the homomorphism and so it is a normal subgroup of G_I).

[1]this is not a discussion about quantum foundations, we are considering only the classical Lagrangian

The sector II is insensible to any assertion we can make about the sectorial symmetry of the sector I and vice-versa.

3 Higgs potential and minima

A spurion or (non-dynamical) background field enters in the definition of the Lagrangian but it is not a variable of the Lagrangian. When calculating the observables, the background fields are replaced by numerical values. It is a representation of a group of background symmetries of the Lagrangian, but there are no Noether's currents associated with such background symmetries if the numerical values are non-trivial. The observables are invariant under the action of the group of the background symmetries. See [16] for details and related studies.

The Higgs field $\sqrt{2}\phi$ can be written as the sum of a background field $v\phi_0$ and a dynamical field φ with null vacuum expectation value like all other fields. When calculating the observables, the background field $v\phi_0$ is replaced by the numerical value of the Higgs vacuum expectation value.

Thus spontaneously broken symmetries are symmetries of the Lagrangian which get broken with the replacement $\sqrt{2}\phi \to v\phi_0 + \varphi$. After that replacement we can deal with the parameters of the Higgs potential and the Higgs vacuum expectation value (which is a function of them) in the same way, as background fields.

Note that there is an important difference with respect to the background fields, the vacuum expectation value is not unique. Consider a group G_L which includes G_l as a normal subgroup and the observables are invariant under G_l. For each allowed vacuum expectation value we can define a correspondent G_l-orbit, such orbit breaks G_L/G_l if all points in the orbit break G_L (since we can break G_l instead). The criteria to exist spontaneous symmetry breaking is that there are observables breaking G_L/G_l for all the allowed G_l-orbits; this is important in case the Higgs field is a direct sum of non-equivalent representations of a group.

To study the Higgs potential and in particular the global symmetries which are spontaneously broken or not, we assume that it suffices to consider only correlation functions in one point of the space-time. We assume that interactions with other (non-Higgs) fields as well as the quantum dynamics of the Higgs field introduce only small deviations to the our results. This is a non-trivial assumption, but since non-perturbative studies of Quantum Field Theory are hard in general we do not have much better alternatives. We are thus left with only the Higgs potential as the Lagrangian and the Higgs field is not really a field since we consider only a space-time point. We neglect quantum effects and so we are left with a non-perturbative purely classical problem of minimization of a polynomial [5]. The Higgs field is simply replaced by its vacuum expectation value $\sqrt{2}\phi \to v\phi_0$ just like a background field.

4 The proposition

Consider the most general Higgs potential V which is invariant under a compact group G_l. Consider a group G_L which includes G_l as a normal subgroup. We assume that the observables are invariant under G_l.

4.1 Part 1

If V is necessarily invariant under G_L, then for any ϕ we have that $X_2 = \int_{G_l} dg R_g \phi\phi^\dagger R_g^\dagger$ and $X_4 = \int_{G_l} dg R_g \phi\phi^\dagger R_g^\dagger \otimes R_g \phi\phi^\dagger R_g^\dagger$ must also be invariant under G_L (where dg is the Haar measure of the group G_l such that $\int_{G_l} dg = 1$), since X_2 and X_4 can appear in a G_l-invariant Higgs potential. Therefore, for such class of Higgs potentials there is no spontaneous symmetry breaking of G_L/G_l, since the observables (defined by the homomorphism $G_I \to G$) are invariant under G_L .

4.2 Part 2

On the other hand, if V breaks explicitly G_L, let V_2 be the quadratic part and V_4 the quartic part of V.

There must exist φ (with $\varphi^\dagger\varphi = 1$) such that $V_2(R_g\varphi) \neq V_2(\varphi)$ or $V_4(R_g\varphi) \neq V_4(\varphi)$ for some $g \in G_L$. Note that $V_{2,4}$ are not linear, but we have that $V_4(a\phi) = a^4 V_4(\phi)$, so the conditions are independent of the normalization of ϕ. But since V_2 and V_4 are G_l-invariant then we can replace in the inequalities $\varphi\varphi^\dagger$ by $X_2 = \int_{G_l} dg R_g \varphi\varphi^\dagger R_g^\dagger$ $\varphi\varphi^\dagger \otimes \varphi\varphi^\dagger$ by $X_4 = \int_{G_l} dg R_g \varphi\varphi^\dagger R_g^\dagger \otimes R_g \varphi\varphi^\dagger R_g^\dagger$. Both X_2 and X_4 are G_l-invariant but at least one of them is not G_L-invariant, otherwise the potential V would not break G_L. Note that if X_2 breaks G_L, then $R_g X_2 R_g^\dagger$ also breaks G_L for any $g \in G_L$, and the analogous result applies to X_4.

So, let $Y = \int_{G_L} dg R_g \varphi\varphi^\dagger R_g^\dagger$.

We now consider the Higgs potential: $U = -\phi^\dagger Y \phi + \frac{1}{2}(\phi^\dagger\phi)^2$, the minimization implies that $Y\phi_0 = \frac{v^2}{2}\phi_0$. Thus $\phi_0 = y R_g \varphi$ for any $g \in G_L$ are minima of the potential.

Therefore, all the symmetries explicitly broken by V are spontaneously broken in U since V_2 and V_4 are observables. Thus U is an example of a potential G_L-symmetric where G_L/G_l can be spontaneously broken.

Note that we can always play with the particular values of the parameters so to avoid spontaneous symmetry breaking, for instance setting $y = 0$ would avoid any spontaneous symmetry breaking. Also, we make no assumptions on whether or not G_l is spontaneously broken, this is important.

5 $SU(2)_L$ gauge symmetry

The $SU(2)_L$ gauge symmetry is one example of a sectorial symmetry. The Higgs-$SU(2)_L$ gauge bosons and the left handed quiral fermions form a sector, due to the Higgs mechanism only operators which are singlets of $SU(2)_L$ interact with the right handed quiral fermions and the $U(1)_Y$ gauge boson.

The parameters of the Higgs potential and the Yukawa couplings are singlets of $SU(2)_L$ and form another sector, when studying these parameters any assertion about the spontaneous breaking of $SU(2)_L$ is meaningless.

The proposition implies that we can study the Higgs potential without making assumptions on whether or not $SU(2)_L$ is spontaneously broken, since $SU(2)_L$ is always a normal subgroup.

6 Custodial symmetry

The Higgs potential of the Standard Model involves only one Higgs doublet. It is symmetric under the compact symmetry$SO(4)$. If we impose a symmetry $SU(2)_L$ to the potential, since the representation is already irreducible there are no Hermitian operators commuting with $SU(2)_L$ up to a sign (since the group is continuous, this is to account for the quartic terms of the potential). Thus, no Higgs potential exists breaking $SO(4)$ and therefore $SO(4)/SU(2)_L \simeq SO(3)$ (we need to consider the real fundamental representation of $SU(2)_L$, which is 4-dimensional).

In the context of multi-Higgs-doublet models, the vacuum may break $U(1)_{em}$ [17]. With more than one-Higgs doublet, the $SU(2)_L$ representation is reducible, we can have the hermitian Pauli matrices breaking $SO(4)/SU(2)_L$ completely, which includes $U(1)_{em}$.

7 CP-violation

The proposition can be applied to CP-violation. We have a group G and a normal subgroup G_n, a CP-group is any group G_c which is a subgroup of G but not of G_n.

Suppose that we have a family group $G_f \subset G_n$. Let G_L be the minimal group which includes G_f and G_c. and $G_l = G_L \cap G_n$

G_l does not contain G_c since G_l is a subgroup of G_n which does not contain G_c. G_l is a normal subgroup of G_L, since for all $g \in G_L$ and $h \in G_l$ we have that $g^{-1}hg \in G_L$ since $g, h \in G_L$ and G_L is a group, also $g^{-1}hg \in G_n$ since $h \in G_n$ and G_n is a normal subgroup.

After applying the symmetry G_f to a general potential, if there is a CP-group G_L which is conserved explicitly then G_L/G_l is not spontaneously broken and thus G_L/G_n is also not spontaneously broken.

On the other hand, if a potential V conserving G_f breaks all CP-groups, then for each CP-group G_L which includes G_f as a subgroup there is a particular Higgs field ϕ_L (with arbitrary normalization) which breaks G_L. We can construct a potential which is G_L-symmetric, explicitly breaks all CP-groups that do not conserve the space generated by $G_L\phi_L$ and it spontaneously breaks all CP-groups that do conserve the space generated by $G_L\phi_L$. Thus, all CP-groups are broken with G_L spontaneously broken.

8 Avoiding the charged vacuum

In the context of CP-violation we usually want the vacuum to conserve $U(1)_{em}$.

So, if CP is conserved regardless of the charge of the vacuum, then for a neutral vacuum it is also conserved.

However, if it is possible to have terms in the potential breaking $U(1)_{em}$ and CP, then it is possible to have spontaneous breaking of both CP and/or $U(1)_Y$. If we additionally assume that we will choose parameters of the Higgs potential such that $U(1)_{em}$ is conserved, then we have to look for the terms which verify 3 conditions: break CP, conserve $U(1)_{em}$ and finally are non-null at the neutral minima. If no such term exists then there is no spontaneous breaking of CP. If such term exists then spontaneous breaking of CP is allowed.

Note that the terms in the Higgs potential which will be null for a neutral vacuum are irrelevant when evaluating spontaneous symmetry breaking at neutral minima, since changing such terms leaves the potential invariant at the minimum (they correspond to Lagrange multipliers in the bilinear formalism to minimize the potential). Dropping such terms is similar to consider a system where all the Higgs fields are neutral (the neutral Higgs sector of Ref. [15]).

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