On Corda's 'Clarification' of Schwarzschild's Solution

Stephen J. Crothers*

Alpha Institute of Advanced Study
PO Box 1546, Sunshine Plaza, QLD 4558, Australia

Received: XX February 2016, Accepted YY February 2016, Published ZZ May 2016

Abstract: A paper by C. Corda (*A Clarification on the Debate on "the Original Schwarzschild Solution", EJTP 8,* ***No. 25*** *(2011) 65-82*) purports equivalence of Schwarzschild's original solution (1916) and Hilbert's subsequent solution (1917), the latter commonly but incorrectly called 'Schwarzschild's solution'. The derivation of Schwarzschild's actual solution by Corda is in fact a copy of Schwarzschild's original derivation with only changes in notation and equation numbering, and so adds nothing new to the problem. Corda's subsequent arguments on gravitational collapse in terms of Schwarzschild's original solution, follow those advanced by Misner, Thorne and Wheeler for Hilbert's solution, in their book 'Gravitation', and suffer thereby from the very same faults. Consequently, Corda has failed to prove his alleged equivalence of the Schwarzschild and Hilbert solutions. Moreover, it is not difficult to prove that they are not in fact equivalent and that all the methods employed to otherwise 'extend' Droste's solution to Hilbert's solution to produce a black hole constitute a violation of the rules of pure mathematics and are therefore invalid.

Keywords: Black Holes; Schwarzschild Solution; Singularity
PACS (2010): 04.70.-s; 04.70.Bw

1 Introduction

One hundred years ago, on the 13^{th} of January 1916, Karl Schwarzschild communicated his solution to Einstein's gravitational field for a 'mass point'. On the 25^{th} of May 2011, a paper by Christian Corda was published, in which Corda reproduced most of Schwarzschild's paper. In a prelude on page 70 of his paper, Corda stated,

> "*In our approach we will suppose again that a(r,t) = 0, but, differently from the standard analysis, we will assume that the length of the circumference centred in the origin of the coordinate system is not $2\pi r$. We release an apparent different physical assumption, i.e. that arches of circumference are deformed by the presence of mass of the central body M. Note that this different physical hypothesis permits to circumnavigate the Birkhoff Theorem [4] which leads to the 'standard Schwarzschild solution' [3].*"

Then, before launching into his modifications of Schwarzschild's equations, Corda stated, again on page 70,

*steve@plasmaresources.com

"*Then, we proceed assuming $k = -mr^2$, where m is a generic function to be determined in order to obtain the length of circumferences centred in the origin of the coordinate system are not $2\pi r$. In other words, m represents a measure of the deviation from $2\pi r$ of circumferences centred in the origin of the coordinate system.*"

Corda's special generic function $m = m(r)$ plays no special rôle since it already appeared in Schwarzschild's paper as $G = G(r)$, where it deformed nothing.

Schwarzschild's solution [1] for Einstein's equations $R_{\mu\nu} = 0$ is,

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$R = \left(r^3 + \alpha^3\right)^{1/3}, \quad 0 \leq r \tag{1.1}$$

Hilbert's solution [2] is,

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$0 \leq r \tag{1.2}$$

Corda [3] has argued that Schwarzschild's original solution (1916) is equivalent to Hilbert's solution (1917)*. It is evident that Schwarzschild's solution is singular (i.e. undefined) only at $r = 0$. Prima facie, Hilbert's solution seems to be singular at $r = 2m$ and at $r = 0$, by which the black hole originated. In equation (1.1), $r = \sqrt{x^2 + y^2 + z^2}$, where x, y, z are Cartesian coordinates for Euclidean 3-space, and so r is the radius of a Euclidean sphere centred at the origin of coordinates. Note that that neither R nor r is the radius of anything or even a distance in (1.1)†, and similarly r in (1.2) is neither radius nor distance therein. Now let r' be the radius of a Euclidean sphere. It is routinely claimed that in expression (2), $r = r' = \sqrt{x^2 + y^2 + z^2}$ (e.g. Einstein [4]), from which the black hole was constructed. This is incorrect because in (1.2), $r = \sqrt{x_0^2 + y_0^2 + z_0^2} + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} = r_0 + r'$ where $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = 2m$ is the distance of the centre of a Euclidean sphere from the origin of coordinates and r' its radius, which has been explained elsewhere [5, 6, 7]. Only when $x_0 = y_0 = z_0 = 0 = r_0$ does $r = r' = \sqrt{x^2 + y^2 + z^2}$. When a sphere initially centred at the origin of coordinates is moved, it takes its centre with it, and the position of the Euclidean sphere is specified by the coordinates of its centre (x_0, y_0, z_0) so that $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$, whereupon the radius r' of the sphere is no longer given by $r = r' = \sqrt{x^2 + y^2 + z^2}$, but by $r' = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$. The intrinsic geometry of a sphere is not altered by changing its position and so its radius does not change with a change of position. When Hilbert set r^2 as the coefficient of $\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$ in the derivation of his solution, he unwittingly shifted the centre of Schwarzschild's Euclidean sphere from $r = r_0 = 0$ to the coordinates (x_0, y_0, z_0) at the distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = 2m$ from the origin of coordinates, mistakenly thinking the centre still at $r = 0$. Hilbert shifted Schwarzschild's Euclidean sphere but left its centre behind. The result was fantastic. David Hilbert had separated the Euclidean sphere from its centre and even placed the centre outside the sphere. Consequently,

*In Eq.(1.1) $c = 1$, in Eq.(1.2) $c = 1$ and Newton's gravitational constant $G = 1$.

†Although Eq.(1.1) and Eq.(1.2) are spherically symmetric they non-Euclidean.

Hilbert's solution is not equivalent to Schwarzschild's, and black hole theory violates the rules of pure mathematics. Accordingly, Corda has incorrectly argued that in (1.1), $-\alpha \leq r$, in order to produce two singularities in the fashion of Hilbert's solution leading to the black hole: Corda incorrectly argues that $-\alpha \leq \sqrt{x^2+y^2+z^2}$ in (1.1).

2 Notation Manipulation

Schwarzschild used the following notation in his paper: $F, G, H, x_1,\ x_2, x_3, x_4, f_1, f_2, f_3,$ where $F, G, H, f_1, x_1, f_2, f_3$ are functions of $r = \sqrt{x^2+y^2+z^2}$, and $x_1 = r^3/3, x_2 = -\cos\theta, x_3 = \varphi, x_4 = t$.

Corda used the following notation in his paper: $l, m, h, X, Y, Z, t, A, B, C$, where l, m, h, X, A, B, C are functions of $r = \sqrt{x^2+y^2+z^2}$.

That Corda's derivation of Schwarzschild's solution is merely a point by point copy of Schwarzschild's is demonstrated by comparisions:

Schwarzschild:

$$ds^2 = F dt^2 - \left(G + H r^2\right) dr^2 - G r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6}$$

$$x_1 = \frac{r^3}{3}, \quad x_2 = -\cos\theta, \quad x_3 = \varphi \tag{7}$$

$$ds^2 = F dx_4^2 - \left(\frac{G}{r^4} + \frac{H}{r^2}\right) dx_1^2 - G r^2 \left[\frac{dx_2^2}{1-x_2^2} + dx_3^2\left(1-x_2^2\right)\right] \tag{8}$$

Corda:

$$ds^2 = h dr^2 - m r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) + l dt^2 \tag{5}$$

$$X = \frac{r^3}{3}, \quad Y = -\cos\theta, \quad Z = \varphi \tag{6}$$

$$ds^2 = l dt^2 + \frac{h}{r^4} dX^2 - m r^2 \left[\frac{dY^2}{1-Y^2} + dZ^2\left(1-Y^2\right)\right] \tag{7}$$

Inspection reveals that Corda's equations (5), (6) and (7) are precisely Schwarzschild's equations (6), (7) and (8) respectively, with the following change of notation: $l = F, h = -\left(G + H r^2\right)$, $m = G, X = x_1, Y = x_2, Z = x_3, t = x_4$. In his Eq.(5) Corda has also rearranged the order of the components of Schwarzschild's Eq.(6).

At his Eqs.(8) Corda sets $A \equiv -h/r^4, B \equiv m r^2, C \equiv l$, and rewrites his Eq.(7) as,

$$ds^2 = C dt^2 - A dX^2 - B\frac{dY^2}{1-Y^2} - B dZ^2\left(1-Y^2\right) \tag{10}$$

Compare this to Schwarzchild's Eq.(9),

$$ds^2 = f_4 dx_4^2 - f_1 dx_1^2 - f_2\frac{dx_2^2}{1-x_2^2} - f_3 dx_3^2\left(1-x_2^2\right) \tag{9}$$

Corda's Eq.(10) is just Schwarzschild's Eq.(9) relabelled with $C = f_4, t = x_4, A = f_1,\ X = x_1, B = f_2 = f_3$ (since $f_2 = f_3$ in Schwarzschild's analysis), $Y = x_2$ and $Z = x_3$.

Corda then goes on to the Riemann-Christoffel symbols of the second kind at his Eqs.(11). They are the same as Schwarzschild's, except for Corda's relabelling, and the fact that Schwarzschild [1] did not number them. Since there are ten such pairs of symbols, I list only a few by way of example:

Schwarzschild:

$$\Gamma^1_{11} = -\frac{1}{2}\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}, \quad \Gamma^1_{22} = +\frac{1}{2}\frac{1}{f_1}\frac{\partial f_2}{\partial x_1}\frac{1}{1-x_2^2}, \quad \Gamma^3_{31} = -\frac{1}{2}\frac{1}{f_2}\frac{\partial f_2}{\partial x_1} \ \ldots$$

Corda:

$$\Gamma^X_{XX} = -\frac{1}{2A}\frac{\partial A}{\partial X}, \quad \Gamma^X_{YY} = \frac{1}{2A}\frac{\partial B}{\partial X}\frac{1}{1-Y^2}, \quad \Gamma^Z_{Z1} = -\frac{1}{2}\frac{1}{B}\frac{\partial B}{\partial X} \ \ldots \tag{11}$$

wherein Corda set $A = f_1, X = x_1, B = f_2, Y = x_2, Z = x_3$. He also rearranged the order of the ten Riemann-Christoffel symbols appearing in Schwarzschild's paper.

Equations (13), (14), (15), and (16) in Corda's paper are precisely Schwarzschild's equations (b), (a), (c) and (d) respectively, rearranged to equate to zero, and embellished with Corda's relabelling:

Schwarzschild:

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right) = \frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right)^2 + \left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2 + \frac{1}{2}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2 \tag{a}$$

Corda:

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial A}{\partial X}\right) - \frac{1}{2}\left(\frac{1}{A}\frac{\partial A}{\partial X}\right)^2 - \left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2 - \frac{1}{2}\left(\frac{1}{C}\frac{\partial C}{\partial X}\right)^2 = 0 \tag{14}$$

Schwarzschild:

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_2}{\partial x_1}\right) = 2 + \frac{1}{f_1 f_2}\left(\frac{\partial f_2}{\partial x_1}\right)^2 \tag{b}$$

Corda:

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial B}{\partial X}\right) - 2 - \frac{1}{AB}\left(\frac{\partial B}{\partial X}\right)^2 = 0 \tag{13}$$

Schwarzschild:

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_4}{\partial x_1}\right) = \frac{1}{f_1 f_4}\left(\frac{\partial f_4}{\partial x_1}\right)^2 \tag{c}$$

Corda:

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial C}{\partial X}\right) - \frac{1}{AC}\left(\frac{\partial C}{\partial X}\right)^2 = 0 \tag{15}$$

Schwarzschild:

$$\frac{1}{f_1}\frac{\partial f_1}{\partial x_1} + \frac{2}{f_2}\frac{\partial f_2}{\partial x_1} + \frac{1}{f_4}\frac{\partial f_4}{\partial x_1} = 0 \tag{d}$$

Corda:

$$\frac{1}{A}\frac{\partial A}{\partial X} + \frac{2}{B}\frac{\partial B}{\partial X} + \frac{1}{C}\frac{\partial C}{\partial X} = 0 \tag{16}$$

Corda's relabelling is clear, nowithstanding his rearrangment of Schwarzschild's equations to equal zero: $X = x_1, A = f_1, B = f_2, C = f_4$.

Corda's Eqs. (17) and (18) are relabelled copies of Schwarzschild's equations (c′) and (c″),

Schwarzschild:

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right) = \frac{1}{f_1 f_4}\frac{\partial f_1}{\partial x_1}\frac{\partial f_4}{x_1} \tag{c'}$$

$$\frac{1}{f_4}\frac{\partial f_4}{x_1} = \alpha f_1, \;\; (\alpha \textit{ integration constant}) \tag{c''}$$

Corda:

$$\frac{\partial}{\partial X}\left(\frac{1}{C}\frac{\partial C}{\partial X}\right) = \frac{1}{AC}\frac{\partial A}{\partial X}\frac{\partial C}{X} \tag{17}$$

$$\frac{1}{C}\frac{\partial C}{X} = \alpha A, \tag{18}$$

"*where α is an integration constant.*"

and his Eqs. (19) and (20) relabelled copies of Schwarzschild's next two but unnumbered equations, with Schwarzschild's latter equation multiplied by -1:

Schwarzschild:

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1} + \frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right) = \left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2 + \frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1} + \frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2$$

$$-2\frac{\partial}{\partial x_1}\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right) = 3\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2$$

Corda:

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial A}{\partial X} + \frac{1}{C}\frac{\partial C}{\partial X}\right) = \left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2 + \frac{1}{2}\left(\frac{1}{A}\frac{\partial A}{\partial X} + \frac{1}{C}\frac{\partial C}{\partial X}\right)^2 \tag{19}$$

$$2\frac{\partial}{\partial X}\left(\frac{1}{B}\frac{\partial B}{\partial X}\right) = -3\left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2 \tag{20}$$

The relabelling is $X = x_1, A = f_1, C = f_4, B = f_2$. Note that Corda's Eq.(20) is Schwarzschild's second unnumbered equation multiplied through by -1.

Corda's Eqs.(21) and (22) are numbered and relabelled copies of Schwarzschild's very next two unnumbered equations,

Schwarzschild:

$$\frac{1}{f_2}\frac{\partial f_2}{\partial x_1} = \frac{2}{3x_1 + \rho}$$

$$f_2 = \lambda\left(3x_1 + \rho\right)^{2/3} \;\; (\lambda \textit{ integration constant})$$

Corda:

$$\frac{1}{B}\frac{\partial B}{\partial X} = \frac{2}{3X + b} \tag{21}$$

$$B = d\left(3X + b\right)^{2/3} \tag{22}$$

"*where d is an integration constant.*"

Once again Corda's relabelling of Schwarzschild's unnumbered equations is clearly evident: $B = f_2, X = x_1, d = \lambda, b = \rho$. To set $d = 1$ Corda next invokes the same condition that Schwarzschild applied at his Eq.(10) to fix his $\lambda = 1$:

Schwarzschild:

$$f_2 = \left(3x_1 + \rho\right)^{2/3} \tag{10}$$

Corda:

$$B = \left(3X + b\right)^{2/3} \tag{23}$$

The copy precedure continues unabated, with Corda's reproduction of Schwarzschild's derivation in modified raiment. Indeed, immediately after his Eq.(23) Corda presents an unnumbered equation. The corresponding equation in Schwarzschild's paper is also unnumbered. They are as follows:

Schwarzschild:

$$\frac{\partial f_4}{\partial x_1} = \alpha f_1 f_2 = \frac{\alpha}{f_2^2} = \frac{\alpha}{\left(3x_1 + \rho\right)^{4/3}}$$

Corda:

$$\frac{\partial C}{\partial X} = \alpha AC = \frac{\alpha}{B^2} = \alpha\left(3X + b\right)^{-\frac{4}{3}}$$

Here the relabelling is also obvious.

Corda's Eqs.(24) and (25) are again just relabelled copies of Schwarzschild's Eqs.(11) and (12) respectively:

Schwarzschild:

$$f_4 = 1 - \alpha\left(3x_1 + \rho\right)^{-1/3} \tag{11}$$

$$f_1 = \frac{(3x_1+\rho)^{-4/3}}{1-\alpha\,(3x_1+\rho)^{-1/3}} \tag{12}$$

Corda:

$$C = 1-\alpha\,(3X+b)^{-\frac{1}{3}} \tag{24}$$

$$A = \frac{(3X+\rho)^{-\frac{4}{3}}}{1-\alpha\,(3X+b)^{-\frac{1}{3}}} \tag{25}$$

The relabelling is, $C = f_4, A = f_1, X = x_1, b = \rho$.

Corda's Eq.(26) is a relabelled copy of Schwarzschild's Eq.(13) and his three Eqs.(27) are relabelled combinations of Schwarzschild's unnumbered equations immediately following the latter's Eq.(13), as follows:

Schwarzschild:

$$\rho = \alpha^3 \tag{13}$$

$$f_1 = \frac{1}{R^4}\frac{1}{1-\alpha/R},\;\; f_2 = f_3 = R^2,\;\; f_4 = 1-\alpha/R$$

$$R = (3x_1+\rho)^{1/3} = \left(r^3+\alpha^3\right)^{1/3}$$

Corda:

$$b = \alpha^3 \tag{26}$$

$$A = \left(r^3+\alpha^3\right)^{-\frac{4}{3}}\left[1-\alpha\left(r^3+\alpha^3\right)^{-\frac{1}{3}}\right]^{-1}$$

$$B = \left(r^3+\alpha^3\right)^{2/3} \tag{27}$$

$$C = 1-\alpha\left(r^3+\alpha^3\right)^{-\frac{1}{3}}$$

Clearly, from Schwarzschild's Eq.(13) Corda (Eq.(26)) has relabelled $b = \rho$ and from Schwarzschild's unnumbered equations, Corda (Eqs.(27)) relabelled $A = f_1, B = f_2, C = f_4$.

At his Eq.(28) Corda has simply reproduced Schwarzschild's Eq.(14)* wherein he has explicitly substituted Schwarzschild's $R = (r^3+\alpha^3)^{1/3}$, and altered the sequence of terms in the metric:

Schwarzschild:

$$ds^2 = (1-\alpha/R)\,dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\theta^2+\sin^2\theta\; d\varphi^2\right),\quad R = \left(r^3+\alpha^3\right)^{1/3} \tag{14}$$

Corda:

$$ds^2 = \left[1-\frac{\alpha}{(r^3+\alpha^3)^{\frac{1}{3}}}\right]dt^2 - \left(r^3+\alpha^3\right)^{\frac{2}{3}}\left(\sin^2\theta\; d\varphi^2+d\theta^2\right)+ \tag{28}$$

*See Eq.(1) herein.

$$-\frac{d\left(r^3+\alpha^3\right)^{\frac{2}{3}}}{1-\frac{\alpha}{\left(r^3+\alpha^3\right)^{\frac{1}{3}}}}$$

Then, according to the standard practice of cosmology, Corda assigned, at his Eq.(30), by means of Newton's expression for escape speed, $\alpha = r_g = 2GM/c^2$, and also, according to common cosmology practice, set $c = 1$ and $G = 1$ so that $\alpha = r_g = 2M$, where M is alleged to be the mass that is the source of a gravitational field:

Corda:

$$ds^2 = \left[1 - \frac{\alpha}{\left(r^3+r_g^3\right)^{\frac{1}{3}}}\right] dt^2 - \left(r^3+r_g^3\right)^{\frac{2}{3}} \left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \tag{30}$$

$$-\frac{d\left(r^3+r_g^3\right)^{\frac{2}{3}}}{1-\frac{r_g}{\left(r^3+r_g^3\right)^{\frac{1}{3}}}}$$

Corda then remarks,

> "*Historically, the line-element (30) represents 'the original schwarzschild solution' to Einstein field equations as it has been derived for the first time by Karl Schwarzschild in [3] with a slight different analysis.*"

There are two additional problems here: (1) Schwarzschild [1] did not assign $\alpha = r_g = 2M$ in his solution, as even a cursory reading of his original paper attests; (2) Corda's 'analysis' is nothing but a reproduction of Schwarzschild's derivation. Corda's "*slight different analysis*" is different only by his relabelling of the variables, functions and constants of Schwarzschild, with some rearrangement of terms and different numbering of equations, but otherwise precisely the same in every other respects by virtue of it being a reproduction.

At his Eq.(31) Corda relabelled Schwarzschild's 'auxilliary quantity' R,

Schwarzschild:

$$R = \left(r^3+\alpha^3\right)^{1/3}$$

Corda:

$$\widehat{r} = \left(r^3+r_g^3\right)^{1/3} \tag{31}$$

Here Corda has set $\widehat{r} = R$ and $r_g = \alpha$, where $r_g = 2M$.

Finally, at his Eq.(32), Corda presents Schwarzschild's solution (the latter's Eq.(14)), again embellished with $\alpha = r_g$, renumbering, and rearrangement of terms,

Schwarzschild:

$$ds^2 = \left(1-\frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),\ \ R = \left(r^3+\alpha^3\right)^{1/3} \tag{14}$$

Corda:

$$ds^2 = \left(1-\frac{r_g}{\widehat{r}}\right) dt^2 - \widehat{r}^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) - \frac{d\widehat{r}^2}{1-\frac{r_g}{\widehat{r}}} \tag{32}$$

wherein $\widehat{r} = \left(r^3 + r_g^3\right)$, $r_g = \alpha$.

Since Corda's derivation of Schwarzschild's actual solution is nothing but a relabelled point by point copy of Schwarzschild's derivation, he has contributed nothing new whatsoever to the solution of the problem.

3 The spherical surface

The surface embedded in Schwarzschild's metric is,

$$ds^2 = R^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \quad (3.1)$$

This expression is an instance of the First Fundamental Quadratic Form of a surface. The intrinsic geometry of a surface is entirely independent of any space it which it might be embedded. The most important feature of the intrinsic geometry of a surface is its Gaussian curvature*. The Gaussian curvature K of the above surface is easily calculated at $K = 1/R^2$ [7, 8, 9, 10, 11, 14]. Herein is the true meaning of R. Hence, in Hilbert's metric $K = 1/r^2$. Since K is a positive constant the surface (3.1) is a spherical surface [11, 14]. Since they are non-Euclidean, r is neither the radius nor even a distance in the metrics of Schwarzschild or Hilbert. The 'Schwarzschild radius' is not the radius of anything in these metrics. Yet cosmology maintains that the 'Schwarzschild radius' or 'gravitational radius' (i.e. $r = r_g = 2m$ in Hilbert's solution) is the radius of the event horizon of a black hole of mass m. On page 79 of his paper Corda conforms to cosmology:

> "*This assumption enables the origin of the coordinate system to be not a single point, but a spherical surface having radius equal to the gravitational radius, i.e. the surface of the Schwarzschild sphere. . . . In fact, a coordinate transform that transfers the origin of the coordinate system, which is the surface having radius equal to the gravitational radius, in a non-dimensional material point in the core of the black hole, re-obtains the solution re-adapted by Hilbert.*"

4 Corda's deformed circumference

Corda makes much of his function $m = m(r)$:

<u>Corda</u>

$$m = \frac{\left(r^3 + \alpha^3\right)^{\frac{2}{3}}}{r^2} \quad (29)$$

On page 70 of his paper [3] he writes,

> "*. . . we will assume that the length of the circumference centred in the origin of the coordinate system is not $2\pi r$. We release an apparent different physical assumption, i.e. that arches of circumference are deformed by the presence of mass of the central body M.*"

On page 73 he writes,

*The Theorema Egregium of Gauss.

"Hence, we understand that the assumption to locate the mathematical singularity of the function A at X = 0 coincides with the physical condition that the length of the circumference centred in the origin of the coordinate system is $2\pi\left(r^3+\alpha^3\right)^{\frac{1}{3}}$, which is different from the value $2\pi r$. This is the apparent fundamental physical difference between this solution and the 'standard Schwarzschild solution' (1), i.e. the one enabled by Hilbert..."

However, there is neither anything physically meaningful nor mathematically meaningful in Corda's circumference assumption for the simple fact that he has confounded r in Schwarzschild's solution and r in Hilbert's solution as being the radial distance* in both. Moreover, Corda's m is nothing but a relabelling of Schwarzschild's G, i.e. $m(r) = G(r)$, noted in §2 above, and so it already appears in Schwarzschild's Eq.(8), where it does not deform anything, easily seen by the fact that the length of a closed geodesic in the spherical surface embedded in Schwarzschild's metric is given by,

$$C = 2\pi R \tag{4.1}$$

which is indifferent to the functional form that R takes [5, 6, 10, 11]. If $R(r) = r$ then $C = 2\pi r$; if $R(r) = \left(r^3+\alpha^3\right)^{1/3}$ then $C = 2\pi\left(r^3+\alpha^3\right)^{1/3}$; if $R = (\alpha - r)$ then $C = 2\pi(\alpha - r)$. In Eq.(4.1) the quantity R does not care how its value is assigned, and neither does $C^\dagger$.

Corda's function m deforms nothing, because it is simply the ratio of the Gaussian curvatures of two different spherical surfaces. Recall that the Gaussian curvature K_M of the spherical surface in Minkowski's metric is $K_M = 1/r^2$ and the Gaussian curvature K_S of the corresponding spherical surface in Schwarzschild's metric is $K_S = 1/R^2 = 1/\left(r^3+\alpha^3\right)^{2/3}$. Then,

$$m = \frac{\left(r^3+\alpha^3\right)^{\frac{2}{3}}}{r^2} = \frac{R^2}{r^2} = \frac{K_M}{K_S} = G \tag{4.2}$$

which has nothing to do with a deformation of a circumference.

5 Gravitational collapse

In §3 of his paper, Corda mimics the analysis of Misner, Thorne and Wheeler [12], which he admits he reproduces, except for replacing their r with Schwarzschild's $R = \left(r^3+\alpha^3\right)^{1/3}$, wherein Corda sets $\alpha = r_g = 2M$ (the 'Schwarzschild' or 'gravitational' radius). He has therefore not added anything new to the issue. In any event, mathematical and seemingly physical arguments for the extension of Schwarzschild's solution to $-\alpha \le r = \sqrt{x^2+y^2+z^2}$ are easily proven to require violation of the rules of pure mathematics, and so they are invalid.

*It is standard in cosmology to erroneously treat r in Hilbert's solution as radial distance, despite having numerous 'definitions' for it, as is particularly evident in the notion of the 'Schwarzschild radius' (i.e. the radius of the 'event horizon'); but it is in fact neither a radius nor a distance in the these metrics (see for example, [14]).

†Of course, the range on r in all cases must also be correctly specified

6 The infinite equivalence class

The important issues are the invariant curvature scalars and the infinite equivalence class from which all admissible equivalent specific expressions of the solution for $R_{\mu\nu} = 0^*$ are obtained.

The quantity R in Schwarzschild's solution and the quantity r in Hilbert's solution can be replaced by any analytic function $R_c(r)$ without violating $R_{\mu\nu} = 0$ or spherical symmetry. However, not any analytic function of r is permissible for a solution to Einstein's 'gravitational field' $R_{\mu\nu} = 0$. For example, replace Hilbert's r with $R_c(r) = e^r$. The resulting metric is singular only at $r = \ln 2m$. At $r = 0$ nothing special happens; on the unproven assumption that $r < \ln 2m$ is permissible. But $R_c(r) = e^r$ is itself impermissible because the resulting metric is not asymptotically flat. The infinite equivalence class of permissible analytic functions $R_c(r)$ must be ascertained.

The metric for Minkoswki spacetime in Cartesian coordinates is,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{6.1}$$

where c is the speed of light in vacuo†. Changing to spherical coordinates, Eq.(6.1) becomes,

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.2}$$

which is spherically symmetric. The spatial section of Eq.(6.1) is,

$$d\sigma^2 = dx^2 + dy^2 + dz^2 \tag{6.1b}$$

which is Euclidean. The spatial section of Eq.(6.2) is,

$$d\sigma^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.2b}$$

which is spherically symmetric about $r = 0$ for three-dimensional Euclidean space.

It has been known to the geometers since at least 1896 that the most general metrical groundform with spherical symmetry for three-dimensional space is,

$$ds^2 = A^2dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.3}$$

where $R_c = R_c(r)$, $A = A\left(R_c(r)\right)$, and $r = \sqrt{x^2 + y^2 + z^2}$ is the r in Eq.(6.2b). Eq.(6.3) is, in general, non-Euclidean. Note that Eqs. (6.1b), (6.2b) and (6.3) are positive definite, i.e. the signs of the coefficients of the differential elements are all positive‡. Note also that if $R_c(r) = r$ and $A(r) = 1$, then the Euclidean Eq.(6.2b) is recovered.

The required infinite equivalence class must satisfy the following fundamental requirements, in accordance with Einstein's [4] prescription:

1. it must be static,

2. it must be spherically symmetric,

3. it must satisfy $R_{\mu\nu} = 0$,

*Einstein's 'Field Equations of Gravitation in the Absence of Matter' [13, § 14].

†That is, $c = 2.998 \times 10^8$m/s.

‡The signature is fixed at $(+, +, +)$.

4. it must be asymptotically flat.

The infinite equivalence class has been derived elsewhere [5, 6, 10, 11, 14, 15, 16], and it is given by,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$R_c = \left(|r - r_0|^n + \alpha^n\right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+ \tag{6.4}$$

Here α is a positive real constant, r_0 and n entirely arbitrary constants. Note that the spatial section of Eq.(6.4) is precisely of the form of Eq.(6.3), generalised from Eq.(6.2b). Choosing $r_0 = 0$, $n = 3$, $r \geq r_0$ produces Schwarzschild's solution (1). Choosing $r_0 = 0$, $n = 1$, $r \geq r_0$ produces Brillouin's solution [17]. Choosing $r_0 = \alpha$, $n = 1$, $r \geq r_0$ produces Droste's solution [18]. However, Hilbert's solution is not an element of the infinite set (6.4), and so it is not equivalent to Schwarzschid's solution, contrary to the claims of Corda. Note that Hilbert's solution is specifically an alleged 'extension' of Droste's solution, and hence invalid by the very choice of Droste's solution (since selection of Droste's solution places the centre of a Euclidean sphere at the point (x_0, y_0, z_0) at the distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = \alpha$ from the origin of coordinates for Minkowski space). The reader can easily verify for himself that Eq.(6.4) satisfies all four conditions required by Einstein.

Since every element of Eq.(6.4) is equivalent then if any element thereof can be 'extended' to form a black hole, then every element must be extendible in the same fashion. Conversely, if any element of Eq.(6.4) cannot be extended, then none can be extended, on account of equivalence. It is evident that no element of Eq.(6.4) can be extended to form a black hole because the latter requires that in Eq.(6.4), $0 \leq R_c$, which is impossible because $|r - r_0|^n$ is never less than zero. The metric Eq.(6.4) is singular at only one point, $r = r_0$. To amplify this fact, set $r_0 = 0$, $n = 2$, so $R_c = \left(r^2 + \alpha^2\right)^{1/2}$; then the resulting metric is well defined for all real values of r, except $r = 0$. Note that $r^2 \geq 0$ and therefore $R_c \geq \alpha$ necessarily. Since this particular equivalent metric cannot be extended to produce a black hole, no element of Eq.(6.4) can be extended to produce a black hole, due to equivalence. Consequently, neither Droste's solution can be extended to yield Hilbert's 'solution' nor Schwarzschild's solution be extended to $-\alpha \leq r = \sqrt{x^2 + y^2 + z^2}$. Hilbert's solution is invalid and so the black hole is invalid. The theory of black holes requires that $|r - r_0|^n < 0$, which is a violation of the rules of pure mathematics, and so the theory of black holes is false.

The rôle of r is now apparent. Consider a sphere of radius $r' = |r - r_0|$ in Euclidean 3-space (i.e. the spatial section of Minkowski spacetime) such that r and r_0 lie on the same radial line through the origin of coordinates $r = 0$. Then the centre of the sphere is located at a point with Cartesian coordinates (x_0, y_0, z_0) at a distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin of the coordinate system. The equation of this sphere in Cartesian coordinates is,

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = (r - r_0)^2 = r'^2 \tag{6.5}$$

Only when $x_0 = y_0 = z_0 = 0 = r_0$, is the sphere centred at the origin of the coordinate system. In any case, the radius of the sphere is $|r - r_0|$ no matter where in Euclidean 3-space the sphere is centred, as illustrated in Fig.1*.

*The third dimension x has been suppressed for ease of illustration.

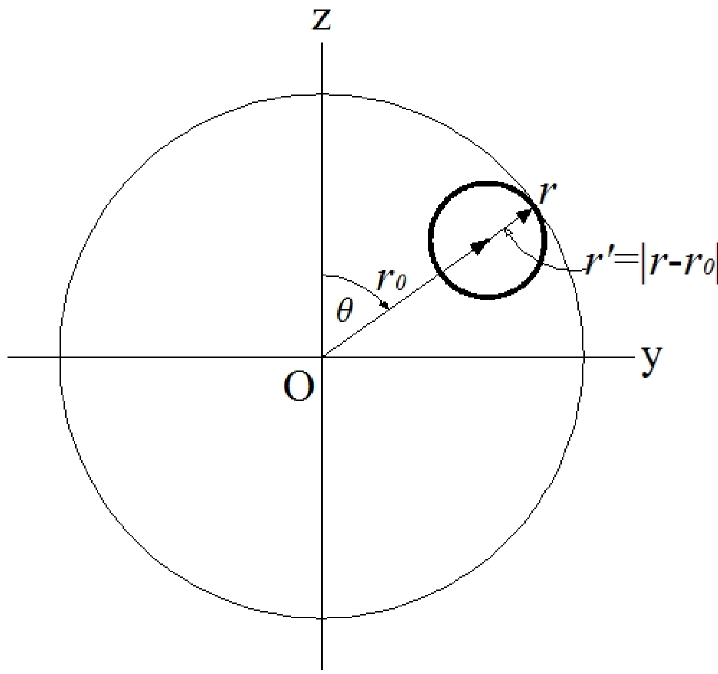

Fig. 1: A Euclidean sphere of radius r' centred at $r = r_0$ (Cartesian coordinates (x_0, y_0, z_0) at a distance $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin O; r and r_0 lying in the same radial line through O). Schwarzschild centred at the origin O but Hilbert, by setting the coefficient of $\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$ at r^2, unwittingly centred at $r_0 = \alpha = 2m$ (see Eq.(6.4)), mistakenly thinking he had centred at $r = 0$, thereby separating the sphere from its centre, misplacing its centre at $r = 0$. In other words, Hilbert unwittingly moved the Euclidean sphere of Minkowski space to $r = r_0$ leaving its centre behind at $r = 0$. Thus, when $r' = 0$, $r = r_0$, the latter misinterpreted as the radius of a black hole event horizon, with $r = 0$ misinterpreted as an infinitely dense 'physical' singularity of a black hole, where spacetime curvature is 'infinite'. Note that as r' grows its sphere engulfs the origin O and beyond, so none of Euclidean 3-space is left out. Cosmology however, seeks to stop at $r = 0$, all the while attempting to drive the centre of the sphere r' down to $r = 0$ when in fact it is at $r_0 = \alpha = 2m$. When $r = 0$ in Hilbert's solution, the radius $r' = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ and the centre of the sphere at (x_0, y_0, z_0), as Eq.(6.5) attests. Hilbert confounded the origin of the coordinate system for the centre of the Euclidean sphere, and so his solution is invalid.

The radius $|r - r_0|$ in Euclidean 3-space (i.e. the spatial section of Minkowski spacetime) is mapped by Eq.(6.4) into the corresponding radius R_p in Schwarzschild non-Euclidean 3-space (i.e. the spatial section of Schwarzschild spacetime) [5, 6, 10, 11, 14, 15, 16],

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right)$$

$$R_c = \left(|r - r_0|^n + \alpha^n\right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+ \tag{6.6}$$

Note that $R_p\left(r_0\right) = 0\ \forall r_0\ \forall n$ (the centre of the non-Euclidean sphere is located at $R_p\left(r_0\right)$), although the metric itself is undefined there (and only there). Also note that $R_c(r_0) = \alpha\ \forall r_0\ \forall n$.

Following §3 above, $R_c(r)$ maps the the Gaussian curvature $K = 1/\left|r - r_0\right|^2$ of the spherical surface of a sphere centred anywhere in Minkowski spacetime, into the corresponding Gaussian curvature $K = 1/R_c^2$ of the spherical surface embedded in Schwarschild spacetime.

Hence, in general, by Eq.(6.4) and Eq.(6.6), the radius $|r - r_0|$ of a Euclidean sphere centred at the point (x_0, y_0, z_0) in Minkowski spacetime is mapped into the corresponding radius R_p of a non-Euclidean sphere centred at $R_p(r_0) = 0$ in Schwarzschild spacetime, and the Gaussian curvature $1/|r - r_0|^2$ of the spherical surface embedded in Minkowski spacetime is mapped to the corresponding Gaussian curvature $1/R_c^2$ of the spherical surface embedded in Schwarzschild spacetime. From this it follows that choosing $R_c(r) = r$ (i.e. $r_0 = \alpha = 2m$ and $n = 1$ in Eq.(6.4)) shifts the centre of the Euclidean sphere associated with Hilbert's solution, to a point (x_0, y_0, z_0) at a distance $\sqrt{x_0^2 + y_0^2 + z_0^2} = 2m$ from the origin of coordinates of the associated Euclidean space and so the radius of that Euclidean sphere is zero when $r = r_0 = 2m$, then making $R_p(r_0) = 0$ in Schwarzschild spacetime. Note that r and r_0 are just distances (scalars) from the origin of coordinates and so vector notation is not required. Indeed, considering vectors $\vec{r}$ and $\vec{r}_0$, if they are collinear then the radius r' of the Euclidean sphere is $r' = |\vec{r} - \vec{r}_0| = |r - r_0|$ where $r = |\vec{r}|$ and $r_0 = |\vec{r}_0|$. Consequently, when Hilbert set the coefficient of $\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$ to r^2, he unwittingly moved the centre of Schwarzschild's Euclidean sphere from $r = 0$ to $r = 2m$ so that instead of $r = r' = \sqrt{x^2 + y^2 + z^2}$ he should have obtained $r = \sqrt{x_0^2 + y_0^2 + z_0^2} + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} = r_0 + r'$ so that when the radius r' of the Euclidean sphere, centred at the point (x_0, y_0, z_0), is zero, i.e. when $x = x_0, y = y_0, z = z_0$, $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$. Hilbert confounded the radius r' of the Euclidean sphere with the radius r from the origin of the coordinate system of the associated Euclidean space. This is amplied by the fact that since r and r_0 lie on the very same radial line through the origin of coordinates,

$$\sqrt{x^2 + y^2 + z^2} = \sqrt{x_0^2 + y_0^2 + z_0^2} + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \tag{6.6}$$

so that when $x = x_0, y = y_0, z = z_0$, Eq.(6.6) reduces to the *identity*,

$$\sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

in which case,

$$r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

because the radius r' of the Euclidean sphere has now converged to $r' = 0$ at the centre of the sphere, located at the point (x_0, y_0, z_0) at the distance $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin of the coordinate system. The origin of the coordinate system of the spatial section of Minkowski spacetime is always at $r = 0$ but the centre of the Euclidean sphere need not be there - it can be placed anywhere, as Eq.(6.4) and Eq.(6.5) permit, illustrated in Fig.1.

If $\vec{r}$ and $\vec{r}_0$ are not collinear, then the radius vector $\vec{r}'$ of the Euclidean sphere is $\vec{r}' = (\vec{r} - \vec{r}_0)$ the radius of that sphere is $r' = |\vec{r}'| = |\vec{r} - \vec{r}_0|$ (see Fig.2), and all other related expressions become, trivially, scalar functions of a vector variable:

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c - \alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right)$$

$$R_c = \left(|\vec{r} - \vec{r}_o|^n + \alpha^n\right)^{1/n}, \quad \vec{r}, \vec{r}_o \in \mathbf{R}^3, \quad n \in \Re^+$$

Then as $\vec{r} \to \vec{r}_0$, $\vec{r}' \to \vec{O}$ and $r' \to 0$.

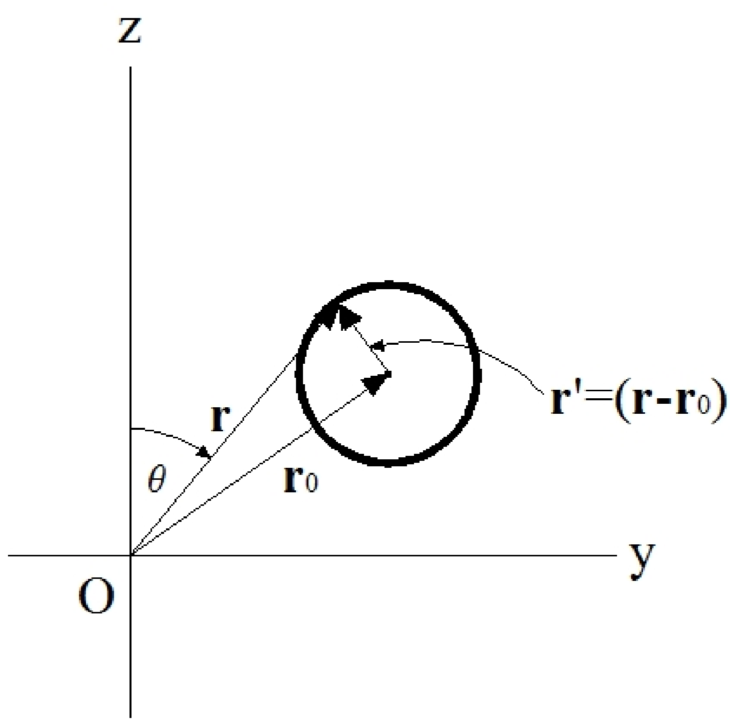

Fig. 2: The radius vector $\vec{r}'$ of the Euclidean sphere centred at the pointed end of the position vector $\vec{r}_0$ is $\vec{r}' = (\vec{r} - \vec{r}_0)$ and the radius of that sphere is $r' = |\vec{r} - \vec{r}_0|$. As the vector $\vec{r} \to \vec{r}_0$ the radius vector of the Euclidean sphere $\vec{r}' \to \vec{O}$ and so the radius $r' \to 0$.

Note that $|r - r_0|$ means that $(r - r_0)$ may be ≥ 0 or ≤ 0, or both. In fact, when $r = 0$ the radius of the Euclidean sphere is $r' = |-r_0| = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$. By elementary analytic geometry, r can take values less than zero because $-r$ is the radius pointing in the opposite direction to r.

Consider Schwarzschild's unnumbered equation, $3x_1 = \alpha^3 - \rho$. To eliminate the integration constant ρ he had two obvious choices, (a) $\rho = \alpha^3$ or (b) $\rho = 0$. Schwarzschild chose (a) "*In order that this discontinuity coincides with the origin*"[1]. If he chose (b) he would have obtained Droste's solution [18], in which case the centre of the Euclidean sphere is moved along any radial line to the point (x_0, y_0, z_0), at the distance $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin of coordinates (as depicted in Fig.1).

It follows that all the scalar curvature invariants are finite - contrary to cosmology, there are no curvature singularities anywhere [5, 6, 7, 8, 9, 10, 11, 14, 15, 21].

7 Isotropic coordinates

The infinite equivalence class for Schwarzchild spacetime can be expressed in isotropic coordinates [14, 15, 19]*:

$$ds^2 = \frac{\left(1 - \frac{\alpha}{4R_c}\right)^2}{\left(1 + \frac{\alpha}{4R_c}\right)^2} dt^2 - \left(1 + \frac{\alpha}{4R_c}\right)^4 dR_c^2 - R_c^2 \left(1 + \frac{\alpha}{4R_c}\right)^4 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$R_c = \left[|r - r_0|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

The radius R_p is given by,

$$R_p = \int \left(1 + \frac{\alpha}{4R_c}\right)^2 dR_c = R_c + \frac{\alpha}{2} \ln\left(\frac{4R_c}{\alpha}\right) - \frac{\alpha^2}{8R_c} + \frac{\alpha}{4}$$

Note that $R_p(r_0) = 0 \ \forall r_0 \ \forall n$ and $R_c(r_0) = \alpha/4 \ \forall r_0 \ \forall n$. Clearly no element of this infinite equivalence class can be 'extended' to produce a black hole, as amplified by the case $r_0 = 0$, $n = 2$.

A similar result is obtained for the Reissner-Nordström solution in isotropic coordinates [14, 15, 19]. In no case does the black hole obtain, for the reason that it constitutes a violation of the rules of pure mathematics in the same fashion as for Hilbert's solution.

8 Other black holes

The Kerr-Newman solution subsumes the Kerr, Reissner-Nordström, and Schwarzschild solutions. In similar fashion the Kerr-Newman configuration requires an infinite equivalence class for its solution. The infinite equivalence class is [14, 15, 20]:

$$ds^2 = -\frac{\Delta - a^2\sin^2\theta}{\rho^2} dt^2 - \frac{2a\sin^2\theta\,(R_c^2 + a^2 - \Delta)}{\rho^2} dt d\varphi + \frac{\left(R_c^2 + a^2\right)^2 - a^2\Delta\sin^2\theta}{\rho^2} \sin^2\theta \, d\varphi^2$$

$$+ \frac{\rho^2}{\Delta} dR_c^2 + \rho^2 \, d\theta^2$$

$$\Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad \rho^2 = R_c^2 + a^2\cos^2\theta, \quad R_c = \left(|r - r_0|^n + \xi^n\right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2\cos^2\theta}, \quad a^2 + q^2 < \frac{\alpha^2}{4}$$

Note that no element of this infinite equivalence class can be extended to produce a black hole, once again amplified by the case $r_0 = 0$, $n = 2$. Consequently there is no black hole. If there is no 'angular momentum', $a = 0$ and the Reissner-Nordström solution is recovered. If charge $q = 0$ the Kerr solution is recovered. If $a = 0$ and $q = 0$ the Schwarzschild solution is recovered. In all cases, contrary to cosmology, there are curvature singularities nowhere [14].

*Here $c = 1$.

9 Conclusions

Corda's derivation of Schwarzschild's solution is merely a point by point reproduction of Schwarzschild's derivation, embellished with relabelling of Schwarzschild's variables and functions, renumbering of his equations, rearrangment of elements of his equations, and numbering of his unnumbered equations. Corda has thereby not advanced anything new to the solution of the problem.

Corda's claim that Schwarzschild's solution and Hilbert's solution are equivalent is demonstrably false. Droste's solution and Brillouin's solution are equivalent to Schwarzschild's solution. The solution to $R_{\mu\nu} = 0$ requires an infinite equivalence class in order to provide for all permissible 'transformations of coordinates'. Hilbert's solution is not an element of the infinite equivalence class and is therefore invalid, amplified by the fact that Hilbert's solution is an alleged extension of Droste's solution, which cannot be extended by its very selection. The 'extension' of Droste's solution to Hilbert's solution to produce a black hole constitutes a violation of the rules of pure mathematics and so it is invalid. It is from the extension of Droste's solution that Hilbert enabled the black hole. Hence, the theory of black holes is invalid.

Corda's claim that "*the length the circumference centred in the origin of the coordinate system is not $2\pi r$*" is false because the length of a closed geodesic in a spherical surface *always* has the form $2\pi R$, being indifferent to how R is assigned a value. Corda's assertion that r in Hilbert's solution and Schwarzschild's solution is radial distance therein, although standard cosmology, is demonstrably false. The 'Schwarzschild radius' is not the radius of anything, or even a distance, in the Schwarzschild metric or Hilbert's metric. It is therefore not the radius of a black hole event horizon.

Corda's [3] conclusion that "*. . . Hilbert was not wrong but they are definitely wrong the authors who claim that 'the original Schwarzschild solution' implies the non existence of BHs*" is incorrect. The black hole is a product of violations of the rules of pure mathematics and therefore false. Schwarzschild's solution does not permit a black hole. Since the Michell-Laplace dark body does not share the properties of the black hole, it is not a black hole [14, 22]. Hence, there is no legitimate mathematical theory of black holes.

References

[1] K. Schwarzschild, On the gravitational field of a mass point according to Einstein's theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 189, (1916), http://arxiv.org/pdf/physics/9905030v1

[2] D. Hilbert, *Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl.*, 53 (1917)

[3] C. Corda, A Clarification of the Debate on "the Original Schwarzschild Solution", *EJTP* **8. No. 25** (2011) 65-82, http://arxiv.org/pdf/1010.6031v5

[4] A. Einstein, The Meaning of Relativity, Princeton University Press, (1988)

[5] S.J. Crothers, On the General Solution to Einstein's Vacuum Field and Its Implications for Relativistic Degeneracy, *Progress in Physics*, v.1, pp.68-73, (2005), http://vixra.org/pdf/1012.0018.pdf

[6] S.J. Crothers, A Short Discussion of Relativistic Geometry, *Bulletin of Pure and Applied Sciences*, Vol.24E (No.2) 2005: P.267-273, http://vixra.org/pdf/1208.0229v1.pdf

[7] S.J. Crothers, The Black Hole Catastrophe: A Short Reply to J. J. Sharples, *The Hadronic Journal*, **34**, 197-224 (2011), http://vixra.org/pdf/1111.0032v1.pdf

[8] S.J. Crothers, 'Flaws in Black Hole Theory and General Relativity', for the Proceedings of the XXIX$^{\text{th}}$ International Workshop on High Energy Physics, Protvino, Russia, (26-28 June 2013), http://vixra.org/pdf/1308.0073v1.pdf

[9] S.J. Crothers, The Schwarzschild solution and its implications for gravitational waves, for 'Mathematics, Physics and Philosophy in the Interpretations of Relativity Theory', Budapest, (4-6 September, 2009), www.sjcrothers.plasmaresources.com/Budapest09-b.pdf

[10] S.J. Crothers, On the Geometry of the General Solution for the Vacuum Field of the Point-Mass, *Progress in Physics*, v.2, pp.3-14, 2005, http://vixra.org/pdf/1101.0003v1.pdf

[11] S.J. Crothers, Gravitation on a Spherically Symmetric Metric Manifold, *Progress in Physics*, v2. pp.68-74, (2007), http://vixra.org/pdf/1101.0005v1.pdf

[12] C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation, W.H. Freeman and Company, New York, (1973)

[13] A. Einstein, The Foundations of the General Theory of Relativity, in The Principle of Relativity, Dover Publications, Inc. (1952)

[14] S.J. Crothers, General Relativity: In Acknowledgement Of Professor Gerardus 't Hooft, Nobel Laureate, 4 August, 2014, http://vixra.org/pdf/1409.0072v7.pdf

[15] S.J. Crothers, On the Generation of Equivalent 'Black Hole' Metrics: A Review, *American Journal of Space Science*, July 6, 2015, DOI: 10.3844/ajssp.2015, http://vixra.org/pdf/1507.0098v1.pdf

[16] S.J. Crothers, The Painlevé-Gullstrand 'Extension' - A Black Hole Fallacy, *American Journal of Modern Physics*, Special Issue: Physics Without Higgs and Without Supersymmetry. Vol.5, Issue 1-1, February 2016, Pages:33-39, doi: 10.11648/j.ajmp.s.2016050101.15, http://vixra.org/pdf/1512.0089v1.pdf

[17] M. Brillouin, The singular points of Einstein's Universe, *Journ. Phys. Radium*, v.23, 43, (1923), www.sjcrothers.plasmaresources.com/brillouin.pdf.

[18] J. Droste, The field of a single centre in Einstein's theory of gravitation, and the motion of a particle in that field, *Ned. Acad. Wet.*, S.A., v.19, 197, (1917), http://www.dwc.knaw.nl/DL/publications/PU00012346.pdf.

[19] S.J. Crothers, On Isotropic Coordinates and Einstein's Gravitational Field, *Progress in Physics*, v.3, pp.7-12, (2006), http://vixra.org/pdf/1101.0008v1.pdf

[20] S.J. Crothers, On the Ramifications of the Schwarzschild Space-Time Metric, *Progress in Physics*, v.1, pp.74-80, (2005), http://vixra.org/pdf/1101.0002v1.pdf

[21] L.S. Abrams, Black holes: the legacy of Hilbert's error. *Can. J. Phys.*, v.67, 919, (1989), http://arxiv.org/pdf/gr-qc/0102055v1

[22] G.C. McVittie, Laplace's Alleged 'Black Hole', *Observatory* 98: 272-274 (Dec 1978), www.sjcrothers.plasmaresources.com/McVittie.pdf