LHC 750 GeV Diphoton Resonance and Flavor Mixing from Four-Quark Condensation

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Abstract

We propose a Clifford algebra based model, which includes local gauge symmetries $SO(1,3) \otimes SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. There are two sectors of bosonic fields as Majorana and electroweak bosons. The Majorana boson sector is responsible for flavor mixing and neutrino Majorana masses. The electroweak boson sector is composed of scalar Higgs, pseudoscalar Higgs, and antisymmetric tensor components. The LHC 750 GeV diphoton resonance is explained by flavon, which is the pseudo-Nambu-Goldstone boson of four-quark condensation. The flavon is resulted from spontaneous symmetry breaking of a global phase symmetry involving first and second generation quarks. Being a standard model singlet, flavon is a potential dark matter candidate.

Keywords. Clifford algebra, diphoton resonance, Higgs bosons, flavor structure, gravity.

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1 Introduction

The experiments at LHC recently indicated a diphoton resonance at about 750 Gev[1, 2], in addition to the earlier finding of Higgs boson with $m_h = 125$ Gev[3, 4]. Scenarios with either an isospin singlet state or an isospin doublet state can not accommodate the observed signal and an extended particle content is necessary[5, 6, 7, 8, 9, 10, 11, 12, 13].

We propose a Clifford algebra based model which encompasses Yang-Mills interactions as well as gravity. The 750 GeV diphoton resonance corresponds to a pseudo-Nambu-Goldston boson of underlying four-quark condensation. No further extended particle content is needed.

With the purpose of studying 3 generations of Standard Model fermions, a ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The flavor projection operators facilitate flavor mixing via Majorana bosons.

The current paper is a continuation of our previous work[14, 15], which is based on three premises. Firstly, both gravity and Yang-Mills interactions should be treated as gauge theories and integrated in a single overarching framework. The key is to take a page from effective field theory, where an infinite number of terms allowed by symmetry requirements should be included in a generalized action. Only the first order terms of the action are relevant in low-energy limit.

The second premise is that all idempotent projections of the original algebraic spinor should be realized as fermions of physical world. In other words, no spinor projection should be casually discarded. Hence, finding the right Clifford algebra turns out to be a simple process of counting numbers of fermion species. There are 16 Weyl fermions (including right-handed neutrino) with $16 \times 4 = 64$ real components in one generation. Clifford algebra $C\ell_{0,6}$, with $2^6 = 64$ degrees of freedom, seems to be a natural choice.

The third premise is that rotations should be generalized. As well known in Clifford algebra approaches, a rotation is realized by a rotor, which is an exponential of bivectors. It rotates a vector into another vector. However, a rotor could be defined to be an exponential of any multivectors. It could rotate a vector into a multivector, generalizing definition of rotations. Hence, one can entertain large symmetry groups with lower dimensional Clifford algebras, whereas the same symmetry groups would otherwise require higher Clifford dimensions within the conventional framework. While the conventional Dirac matrix operators $\gamma_1, \gamma_2, \gamma_3$ correspond to vectors in $C\ell_{0,6}$, the matrix operator γ_0 corresponds to a trivector $\gamma_0 = \Gamma_1\Gamma_2\Gamma_3$. Lorentz boost rotations are represented as exponentials of Clifford 4-vectors $\Gamma_1\Gamma_2\Gamma_3\gamma_1$, $\Gamma_1\Gamma_2\Gamma_3\gamma_2$, $\Gamma_1\Gamma_2\Gamma_3\gamma_3$.

This paper is structured as follows: Section 2 introduces binary Clifford algebra, gauge symmetries, and the action of the world. In section 3, an additional ternary Clifford algebra is defined. The Majorana boson sector, flavor mixing, and 750 Gev diphoton resonance are discussed. In section 4, we study electroweak boson sector. In section 5, we touch upon the topic of grand unification. In the last section we draw our conclusions.

2 Gauge- and Diffeomorphism-Invariant Action

2.1 6D Clifford Algebra

We begin with a review of orthogonal Clifford algebra $C\ell_{0,6}$. It is defined by anticommutators of orthonormal vector basis $(\gamma_j, \Gamma_j; j = 1, 2, 3)$

$$[\gamma_j, \gamma_k] = \frac{1}{2}(\gamma_j\gamma_k + \gamma_k\gamma_j) = -\delta_{jk}, \tag{1}$$

$$[\Gamma_j, \Gamma_k] = -\delta_{jk}, \tag{2}$$

$$[\gamma_j, \Gamma_k] = 0, \tag{3}$$

where $j, k = 1, 2, 3$. All basis vectors are space-like. There are $\binom{6}{k}$ independent k-vectors. The complete basis for $C\ell_{0,6}$ is given by the set of all k-vectors. Any multivector can be expressed as a linear combination of $2^6 = 64$ basis elements.

Two trivectors

$$\gamma_0 = \Gamma_1\Gamma_2\Gamma_3, \tag{4}$$

$$\Gamma_0 = \gamma_1\gamma_2\gamma_3 \tag{5}$$

square to 1, so they are time-like. The orthonormal vector-trivector basis $\{\gamma_a, a = 0, 1, 2, 3\}$ defines space-time Clifford algebra $C\ell_{1,3}$, with

$$\eta_{ab} = \langle\gamma_a\gamma_b\rangle = \begin{pmatrix} +1, 0, 0, 0 \\ 0, -1, 0, 0 \\ 0, 0, -1, 0 \\ 0, 0, 0, -1 \end{pmatrix}, \tag{6}$$

where $\langle\cdots\rangle$ means scalar part of enclosed expression. The reciprocal vectors $\{\gamma^a\}$ are defined by

$$\gamma^a\eta_{ab} = \gamma_b, \tag{7}$$

thus

$$\langle\gamma^a\gamma_b\rangle = \delta^a_b. \tag{8}$$

Here we adopt the summation convention for repeated indices. Notice that γ_0 is a trivector, rather than a vector.

The unit pseudoscalar

$$i = \Gamma_1\Gamma_2\Gamma_3\gamma_1\gamma_2\gamma_3 = \gamma_0\gamma_1\gamma_2\gamma_3 = \gamma_0\Gamma_0 \tag{9}$$

squares to -1, anticommutes with odd-grade elements, and commutes with even-grade elements.

Reversion of a multivector $M \in C\ell_{0,6}$, denoted $\tilde{M}$, reverses the order in any product of vectors. For any multivectors M and N, there are algebraic properties

$$(MN)\tilde{} = \tilde{N}\tilde{M}, \tag{10}$$

$$\langle MN \rangle = \langle NM \rangle \,. \tag{11}$$

The magnitude of a multivector M is defined as

$$|M| = \sqrt{\langle M^\dagger M \rangle}, \tag{12}$$

where

$$M^\dagger = -i\tilde{M}i, \tag{13}$$

is the Hermitian conjugate.

2.2 Algeraic Spinor

Algebraic spinor $\psi \in C\ell_{0,6}$ is a multivector , which is expressed as a linear combination (with Grassmann odd coefficients) of all $2^6 = 64$ basis elements.

Spinors with left/right chirality correspond to odd/even multivectors

$$\psi = \psi_L + \psi_R, \tag{14}$$

$$\psi_L = \frac{1}{2}(\psi + i\psi i) \tag{15}$$

$$\psi_R = \frac{1}{2}(\psi - i\psi i). \tag{16}$$

A projection operator squares to itself. Idempotents are a set of projection operators

$$P_0 = \frac{1}{4}(1 + iJ_1 + iJ_2 + iJ_3) = \frac{1}{4}(1 + 3iJ), \tag{17}$$

$$P_1 = \frac{1}{4}(1 + iJ_1 - iJ_2 - iJ_3), \tag{18}$$

$$P_2 = \frac{1}{4}(1 - iJ_1 + iJ_2 - iJ_3), \tag{19}$$

$$P_3 = \frac{1}{4}(1 - iJ_1 - iJ_2 + iJ_3), \tag{20}$$

$$P_q = P_1 + P_2 + P_3 = \frac{3}{4}(1 - iJ), \tag{21}$$

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_0\Gamma_3), \tag{22}$$

where

$$J_1 = \gamma_1\Gamma_1, J_2 = \gamma_2\Gamma_2, J_3 = \gamma_3\Gamma_3, \tag{23}$$

$$J = \frac{1}{3}(J_1 + J_2 + J_3), \tag{24}$$

$$P_0 + P_1 + P_2 + P_3 = P_0 + P_q = 1, \tag{25}$$

$$P_a P_b = \delta_{ab}, \quad (a, b = 0, 1, 2, 3), \tag{26}$$

$$P_+ + P_- = 1. \tag{27}$$

Here P_0 is lepton projection operator, P_q is quark projection operator, and P_j are color projection operators. The bivectors J_j appearing in the color projectors P_j suggest an interesting duality between 3 space dimensions and 3 colors of quarks.

Now we are ready to identify idempotent projections of spinor

$$\psi = (P_+ + P_-)(\psi_L + \psi_R)(P_0 + P_1 + P_2 + P_3) \tag{28}$$

with left-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_L = P_+\psi_L P_0, \\ e_L = P_-\psi_L P_0, \\ u_L = P_+\psi_L P_1 + P_+\psi_L P_2 + P_+\psi_L P_3 = P_+\psi_L P_q, \\ d_L = P_-\psi_L P_1 + P_-\psi_L P_2 + P_-\psi_L P_3 = P_-\psi_L P_q, \end{cases} \tag{29}$$

and right-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_R = P_-\psi_R P_0, \\ e_R = P_+\psi_R P_0, \\ u_R = P_-\psi_R P_1 + P_-\psi_R P_2 + P_-\psi_R P_3 = P_-\psi_R P_q, \\ d_R = P_+\psi_R P_1 + P_+\psi_R P_2 + P_+\psi_R P_3 = P_+\psi_R P_q. \end{cases} \tag{30}$$

2.3 Symmetries

Spinors transformation as

$$\begin{aligned} \psi_L &\to e^{\Theta_{LOR}+\Theta_{WL}}\psi_L e^{\Theta_J - \Theta_{STR}}, \\ \psi_R &\to e^{\Theta_{LOR}+\Theta_{WR}}\psi_R e^{\Theta_J - \Theta_{STR}}. \end{aligned} \tag{31}$$

It worth noting that all gauge transformations are with Grassmann even rotation angles, so that the transformed spinors remains to be Grassmann odd.

There are Lorentz $SO(1,3)$ gauge transformations

$$\{\gamma_a\gamma_b\} \in \Theta_{LOR}, (a, b = 0, 1, 2, 3, a \neq b), \tag{32}$$

weak isospin $SU(2)_L$ gauge transformations acting on left-handed fermions

$$\{\frac{1}{2}\Gamma_2\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WL}, \tag{33}$$

weak $U(1)_R$ gauge transformation acting on right-handed fermions

$$\{\frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WR}, \tag{34}$$

$J\, U(1)$ gauge transformation

$$\{\frac{1}{2}J\} \in \Theta_J, \tag{35}$$

and color $SU(3)$ gauge transformations

$$\left\{\begin{array}{l} T_1, T_2, T_3, \\ T_4, T_5, \\ T_6, T_7, \\ T_8 \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{4}(\gamma_1\Gamma_2 + \gamma_2\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_2 + \gamma_1\gamma_2), \frac{1}{4}(\Gamma_1\gamma_1 - \Gamma_2\gamma_2), \\ \frac{1}{4}(\gamma_1\Gamma_3 + \gamma_3\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_3 + \gamma_1\gamma_3), \\ \frac{1}{4}(\gamma_2\Gamma_3 + \gamma_3\Gamma_2), \frac{1}{4}(\Gamma_2\Gamma_3 + \gamma_2\gamma_3), \\ \frac{1}{4\sqrt{3}}(\Gamma_1\gamma_1 + \Gamma_2\gamma_2 - 2\Gamma_3\gamma_3) \end{array}\right\} \in \Theta_{STR}. \tag{36}$$

It is remarkable that the gauge groups contain both gravitational (Θ_{LOR}) and internal gauge transformations.

Because the product of lepton projector P_0 with any generator in color algebra (36) is zero $P_0T_k = 0$, leptons are invariant under color gauge transformation.

After symmetry breaking of Θ_{WR}, Θ_{WL}, and Θ_J via Majorana and electroweak Higgs bosons, which will be detailed in later sections, the remaining electromagnetic $U(1)$ symmetry is a synchronized double-sided gauge transformation

$$\psi \to e^{\frac{1}{2}\epsilon_E\Gamma_1\Gamma_2}\psi e^{\frac{1}{2}\epsilon_E J}. \tag{37}$$

where a shared rotation angle ϵ_E synchronizes the double-sided gauge transformation.

Thanks to the properties

$$\begin{aligned} JP_0 &= -iP_0, \\ JP_j &= \frac{1}{3}iP_j, \\ \Gamma_1\Gamma_2P_\pm &= \mp iP_\pm, \end{aligned} \tag{38}$$

electric charges q_k as in

$$e^{\frac{1}{2}\Gamma_1\Gamma_2}\psi_k e^{\frac{1}{2}J} = \psi_k e^{q_k i} \tag{39}$$

are calculated as $q_k = 0, -1, \frac{2}{3}$, and $-\frac{1}{3}$ for neutrino, electron, up quarks, and down quarks, respectively.

2.4 Gauge Field 1-Forms, Gauge-Covariant Derivatives, and Curvature 2-Forms

Gauge fields are Clifford-valued 1-forms (Cliffooms with Grassmann even coefficients) on 4-dimensional space-time manifold (x_μ, $\mu = 0, 1, 2, 3$)

$$
\begin{aligned}
e &= e_\mu dx^\mu = e_\mu^a \gamma_a dx^\mu, && (40)\\
\omega &= \omega_\mu dx^\mu = \frac{1}{4}\omega_\mu^{ab}\gamma_a\gamma_b dx^\mu \quad \in \quad \Theta_{LOR}, && (41)\\
W_L &= W_{L\mu} dx^\mu = \frac{1}{2}(W_{L\mu}^1 \Gamma_2\Gamma_3 + W_{L\mu}^2 \Gamma_1\Gamma_3 + W_{L\mu}^3 \Gamma_1\Gamma_2) dx^\mu \quad \in \quad \Theta_{WL}, && (42)\\
W_R &= W_{R\mu} dx^\mu = \frac{1}{2} W_{R\mu}^3 \Gamma_1\Gamma_2 dx^\mu \quad \in \quad \Theta_{WR}, && (43)\\
C &= C_\mu dx^\mu = \frac{1}{2} C_\mu^J J dx^\mu \quad \in \quad \Theta_J, && (44)\\
G &= G_\mu dx^\mu = G_\mu^k T_k dx^\mu \quad \in \quad \Theta_{STR}, && (45)
\end{aligned}
$$

where e is vierbein, ω is gravity spin connection, G is strong interaction, and the rest are electroweak related interactions.

The vierbein field e acts like space-time frame field, which is essential in building all actions as diffeomorphism-invariant integration of 4-forms on 4-dimensional space-time manifold. The space-time manifold is initially without metric. It's the vierbein field which gives notion to metric

$$
g_{\mu\nu} = \langle e_\mu e_\nu \rangle = e_\mu^a e_\nu^b \eta_{ab}. \tag{46}
$$

Local gauge transformations are coordinate-dependent gauge transformations. Gauge fields obey local gauge transformation laws

$$
\begin{aligned}
e(x) &\rightarrow e^{\Theta_{LOR}(x)} e(x) e^{-\Theta_{LOR}(x)}, && (47)\\
\omega(x) &\rightarrow e^{\Theta_{LOR}(x)} \omega(x) e^{-\Theta_{LOR}(x)} - (de^{\Theta_{LOR}(x)}) e^{-\Theta_{LOR}(x)}, && (48)\\
W_L(x) &\rightarrow e^{\Theta_{WL}(x)} W_L(x) e^{-\Theta_{WL}(x)} - (de^{\Theta_{WL}(x)}) e^{-\Theta_{WL}(x)}, && (49)\\
W_R(x) &\rightarrow W_R(x) - (de^{\Theta_{WR}(x)}) e^{-\Theta_{WR}(x)}, && (50)\\
C(x) &\rightarrow C(x) - e^{-\Theta_J(x)} (de^{\Theta_J(x)}), && (51)\\
G(x) &\rightarrow e^{\Theta_{STR}(x)} G(x) e^{-\Theta_{STR}(x)} + e^{\Theta_{STR}(x)} (de^{-\Theta_{STR}(x)}) && (52)
\end{aligned}
$$

where $d = dx^\mu \partial_\mu$.

It's worth emphasizing that gravity related fields $e(x)$ and $\omega(x)$ are treated as gauge fields with local gauge transformation properties, as the rest Yang-Mills gauge fields.

Gauge-covariant derivatives of spinor fields $\psi_{L/R}(x)$ are defined by

$$
\begin{aligned}
D\psi_L &= (d + \omega + W_L)\psi_L + \psi_L(C - G), && (53)\\
D\psi_R &= (d + \omega + W_R)\psi_R + \psi_R(C - G). && (54)
\end{aligned}
$$

The gravitational spin connection ω is essential in maintaining *local* Lorentz covariance of $D\psi_{L/R}$.

We introduce gauge curvature 2-forms by applying the covariant derivative to the 0-form spinor ψ and then to the 1-form spinor $D\psi$

$$\begin{aligned} D(D\psi_{L/R}) &= (d+\omega+W_{L/R})D\psi_{L/R} - D\psi_{L/R}(C-G) \\ &= (R+F_{WL/WR})\psi_{L/R}(F_J - F_{STR}), \end{aligned} \tag{55}$$

where gravity, left/right weak, J, and Strong force curvature 2-forms are

$$R = d\omega + \omega^2 = \frac{1}{2}R_{\mu\nu}dx^\mu dx^\nu, \tag{56}$$

$$F_{WL} = dW_L + W_L^2 = \frac{1}{2}F_{WL\mu\nu}dx^\mu dx^\nu, \tag{57}$$

$$F_{WR} = dW_R = \frac{1}{2}F_{WR\mu\nu}dx^\mu dx^\nu, \tag{58}$$

$$F_J = dC = \frac{1}{2}F_{J\mu\nu}dx^\mu dx^\nu, \tag{59}$$

$$F_{STR} = dG + G^2 = \frac{1}{2}F_{STR\mu\nu}dx^\mu dx^\nu. \tag{60}$$

$F^{\mu\nu k}$ is defined by

$$F^{\mu\nu k}\eta_{\mu\alpha}\eta_{\nu\beta} = F^k_{\alpha\beta}, \tag{61}$$

where k enumerates the Clifford components of each gauge field.

Notice that the connection fields are defined to absorb gauge coupling constants, which neither appear in the definition of gauge-covariant derivatives of fermions $D\psi_{L/R}$, nor in the gauge curvature 2-forms such as $F_{WL} = dW_L + W_L^2$. Gauge coupling constants will show up in the gauge field actions in stead.

2.5 Gauge- and Diffeomorphism-Invariant Action of the World

The local gauge- and diffeomorphism-invariant action of the world is

$$\begin{aligned} S_{World} = & S_{Spinor-Kinetic} \\ & + S_{Gravity} + S_{Yang-Mills} \\ & + S_{Majorana-Yukawa} + S_{Majorana-Bosons} \\ & + S_{Electroweak-Yukawa} + S_{Electroweak-Bosons}. \end{aligned} \tag{62}$$

The spinor kinetic action is now written down as

$$S_{Spinor-Kinetic} \sim \int \langle \bar{\psi}_L ie^3 D\psi_L + \bar{\psi}_R ie^3 D\psi_R \rangle, \tag{63}$$

where e^3 is vierbein 3-form, and $\bar{\psi}_{L/R}$ is defined as

$$\bar{\psi}_{L/R} = \psi^{\dagger}_{L/R}\gamma_0 = -i\tilde{\psi}_{L/R}i\gamma_0 = \mp\tilde{\psi}_{L/R}\gamma_0. \tag{64}$$

Here outer products between differential forms are implicitly assumed.

One can write down the action for gravity as

$$S_{Gravity} \sim \int \left\langle ie^2(R + \frac{\Lambda}{24}e^2) \right\rangle, \tag{65}$$

where e^2 is vierbein 2-form, $R = d\omega + \omega^2$ is spin connection curvature 2-form, and Λ is cosmological constant.

The Yang-Mills action is written as

$$\begin{aligned} &S_{Yang-Mills} = S_{WL} + S_{WR} + S_J + S_{STR}, \\ &S_{WL} \sim \int \left\langle (e^2 F_{WL})^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_{WR} \sim \int \left\langle (e^2 F_{WR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_J \sim \int \left\langle (e^2 F_J)^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &S_{STR} \sim \int \left\langle (e^2 F_{STR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \end{aligned} \tag{66}$$

where e^4 is vierbein 4-form.

The Clifford algebra elements, which are related to left-(e, ω, W_L, W_R) and right-(C, G)sided gauge fields, are formally assigned to two sets of Clifford algebras in Yang-Mills action (and other actions without spinor fields). Elements from different sets formally commute with each other. Here $\langle \cdots \rangle$ means scalar part of both sets.

It's understood that 4-form factor d^4x in one of e^2F in each Yang-Mills term should be canceled out by 4-form factor d^4x in the denominator before any further outer multiplication of differential forms as

$$\int \left\langle \frac{e^2F}{\langle ie^4 \rangle} e^2F \right\rangle, \tag{67}$$

In this way, the Yang-Mills action is a diffeomorphism-invariant integration of 4-form on 4-dimensional space-time manifold.

There is no explicit Hodge dual in Yang-Mills action. Vierbein plays the role of Hodge dual, when it acquires nonzero vacuum expectation value (VEV) in the case of flat space-time, which will be discussed in next section.

Yukawa and Boson portion of the action will be subjects of later chapters.

2.6 Local Lorentz Symmetry Breaking and Minkowskian space-time

Up to this point, the action of the world is constructed in *curved* space-time, with space-time dependent vierbein and spin connection. In a vacuum with zero cosmological constant $\Lambda = 0$, vierbein field e acquires a nonzero Minkowskian flat space-time VEV

$$< 0|e|0 >= \delta^a_\mu \gamma_a dx^\mu, \tag{68}$$

while VEV of spin connection is zero

$$< 0|\omega|0 >= 0. \tag{69}$$

The space-time metric reduces to

$$g_{\mu\nu} = \langle e_\mu e_\nu \rangle = \eta_{\mu\nu}. \tag{70}$$

The soldering form $\delta^a_\mu \gamma_a dx^\mu$ breaks the independent *local* Lorentz gauge invariance and diffeomorphism invariance. The action of the world is left with a residual *global* Lorentz symmetry, with synchronized Clifford space and x coordinate space global Lorentz rotations. Actually the specific form in VEV $\delta^a_\mu \gamma_a dx^\mu$ is a result of coordinating the above two kinds of global rotations.

With the substitution of vierbein and spin connection with their VEVs, the spinor kinetic action(63) in flat Minkowskian space-time can be rewritten as

$$S_{Spinor-Kinetic} = \int \left\langle \bar{\psi}_L \gamma^\mu D_\mu \psi_L + \bar{\psi}_R \gamma^\mu D_\mu \psi_R \right\rangle d^4x, \tag{71}$$

where

$$D_\mu \psi_{L/R} = (\partial_\mu + W_{L/R\mu})\psi_{L/R} + \psi_{L/R}(C_\mu - G_\mu). \tag{72}$$

Similarly, the Yang-Mills action(66) can be rewritten as

$$\begin{aligned} S_{Yang-Mills} = & -\frac{1}{4g^2_{WL}} \int F^k_{WL\mu\nu} F^{\mu\nu k}_{WL} d^4x \\ & -\frac{1}{4g^2_{WR}} \int F_{WR\mu\nu} F^{\mu\nu}_{WR} d^4x \\ & -\frac{1}{4g^2_J} \int F_{J\mu\nu} F^{\mu\nu}_J d^4x \\ & -\frac{1}{4g^2_{STR}} \int F^k_{STR\mu\nu} F^{\mu\nu k}_{STR} d^4x, \end{aligned} \tag{73}$$

where g_{WL}, g_{WR}, g_J, and g_{STR} are dimensionless gauge coupling constants.

In the following chapters, however, we will stay with local Lorentz gauge invariant curved space-time formulation.

2.7 Relation to Conventional Matrix Formulation

A map [14] can be constructed by placing the Dirac column spinor $\hat{\psi}$ in one-to-one correspondence with the algebraic spinor. And the mappings for the operators are

$$\hat{\gamma}^{\mu}\hat{\psi} \leftrightarrow \gamma^{\mu}\psi, (\mu = 0, 1, 2, 3) \tag{74}$$

$$\hat{i}\hat{\psi} \leftrightarrow \psi i, \tag{75}$$

$$\hat{\gamma}^{5}\hat{\psi} \leftrightarrow -i\psi i \tag{76}$$

where $\hat{i}$ is the conventional unit imaginary number, and $\hat{\gamma}^{\mu}$ and $\hat{\gamma}^{5}$ are the Dirac matrix operators.

We will not go into the details of further mappings in this paper.

3 Majorana Bosons, Flavor Structure, and 750 Gev Diphoton Resonance

3.1 Ternary Clifford Algebra and Flavor Projection Operators

With the purpose of studying 3 generations of fermions, we turn to another kind of Clifford algebra involving ternary communication relationships rather than the usual binary ones. Let's consider ternary $C\ell_{T1}$, which is defined by

$$[\zeta, \zeta, \zeta] = \zeta^3 = 1, \tag{77}$$

with ζ commuting with $C\ell_{0,6}$

$$\zeta\gamma_j - \gamma_j\zeta = 0, \tag{78}$$

$$\zeta\Gamma_j - \Gamma_j\zeta = 0. \tag{79}$$

Flavor projection operators are define by

$$P_{G1} = \frac{1}{3}(1 + e^{\theta'+\theta}\zeta + e^{-\theta'-\theta}\zeta^2) \tag{80}$$

$$= \frac{1}{3}P_0(1 + \zeta + \zeta^2) + \frac{1}{3}P_q(1 + e^{-\theta}\zeta + e^{\theta}\zeta^2), \tag{81}$$

$$P_{G2} = \frac{1}{3}(1 + e^{\theta'}\zeta + e^{-\theta'}\zeta^2) \tag{82}$$

$$= \frac{1}{3}P_0(1 + e^{-\theta}\zeta + e^{\theta}\zeta^2) + \frac{1}{3}P_q(1 + e^{\theta}\zeta + e^{-\theta}\zeta^2), \tag{83}$$

$$P_{G3} = \frac{1}{3}(1 + e^{\theta'-\theta}\zeta + e^{-\theta'+\theta}\zeta^2) \tag{84}$$

$$= \frac{1}{3}P_0(1 + e^{\theta}\zeta + e^{-\theta}\zeta^2) + \frac{1}{3}P_q(1 + \zeta + \zeta^2), \tag{85}$$

where

$$P_{G1} + P_{G2} + P_{G3} = 1, \tag{86}$$

$$P_{Gj} P_{Gk} = \delta_{jk}, \quad (j, k = 1, 2, 3), \tag{87}$$

$$\theta = \frac{2\pi}{3} i, \theta' = \frac{2\pi}{3} I, \tag{88}$$

$$I = \frac{1}{2}(i + 3J), I^2 = -1, \tag{89}$$

and P_0 and P_q are lepton and quark projection operators, respectively.

We label 3 generations of spinors as $\psi_{L/Rj}$, valued in $C\ell_{0,6}$. The spinor kinetic action involves 3 families of fermions as

$$S_{Spinor-Kinetic} \sim \int \langle \bar{\psi}_{Lj} i e^3 D \psi_{Lj} P_{Gj} + \bar{\psi}_{Rj} i e^3 D \psi_{Rj} P_{Gj} \rangle, \tag{90}$$

without flavor-mixing cross terms. Here $\langle \cdots \rangle$ means scalar part of both $C\ell_{0,6}$ and $C\ell_{T1}$.

Flavor-mixing is induced via Majorana Boson fields, which is the subject of next section.

3.2 Yukawa Action and Flavor Mixing

Fields in Majorana boson section interact with right-handed fermions only. The Lorentz, isospin, and color singlet Majorana boson section contains two fields

$$\phi_{MAJ} = \phi^{\dagger}_{MAJ} = \phi^{\nu} + \Phi. \tag{91}$$

The neutrino Higgs field $\phi^{\nu} = \phi^{\nu\dagger}$ is valued in Clifford space spanned by 2 trivectors

$$\{\Gamma_0 P_0, i\Gamma_0 P_0\}. \tag{92}$$

It obeys gauge transformation rules

$$\phi^{\nu} \quad \to \quad e^{-\check{\Theta}_{WR} - \Theta_J} \phi^{\nu} e^{\check{\Theta}_{WR} + \Theta_J}, \tag{93}$$

where

$$\check{\Theta}_{WR} = \frac{1}{2} \epsilon_{WR} i \tag{94}$$

shares rotation angle ϵ_{WR} with

$$\Theta_{WR} = \frac{1}{2} \epsilon_{WR} \Gamma_1 \Gamma_2. \tag{95}$$

Boson field

$$\Phi = \Phi_{12} + \Phi_{13} + \Phi_{31} + \Phi_{23} \tag{96}$$

is valued in Clifford space spanned by scalar and pseudoscalar

$$\{1, i\}. \tag{97}$$

It is invariant under all gauge interaction transformations. It plays an essential role in LHC 750 Gev diphoton resonance.

We can write Majorana Yukawa action of right-handed fermions as

$$\begin{aligned} S_{Majorana-Yukawa} \quad \sim \quad & y_{jk} \int \left\langle \phi^\nu P_{Gj} \bar{\nu}_{Rj} e^4 e^{\varepsilon_{jk}\Gamma_1\Gamma_2} \Gamma_2 \Gamma_3 \nu_{Rk} P_{Gk} \right\rangle \\ & + Y_{12} \int \left\langle \Phi_{12} P_{G1} \bar{u}_{R1} e^4 d_{R2} P_{G2} \bar{u}_{R2} e^4 d_{R1} \right\rangle / \left\langle ie^4 \right\rangle + h.c. \\ & + Y_{13} \int \left\langle \Phi_{13} P_{G1} \bar{u}_{R1} e^4 e_{R3} P_{G2} \bar{\nu}_{R3} e^4 d_{R1} \right\rangle / \left\langle ie^4 \right\rangle + h.c. \\ & + Y_{31} \int \left\langle \Phi_{31} P_{G1} \bar{u}_{R3} e^4 e_{R1} P_{G2} \bar{\nu}_{R1} e^4 d_{R3} \right\rangle / \left\langle ie^4 \right\rangle + h.c. \\ & + Y_{23} \int \left\langle \Phi_{23} P_{G1} \bar{\nu}_{R2} e^4 e_{R3} P_{G2} \bar{\nu}_{R3} e^4 e_{R2} \right\rangle / \left\langle ie^4 \right\rangle + h.c., \end{aligned} \tag{98}$$

where y_{jk} and Y_{jk} are Majorana Yukawa coupling constants.

There are four fermions in the Yukawa terms of Φ_{jk}, different from Higgs boson ϕ^ν, which interacts with two fermions. The four-fermion Yukawa coupling constants Y_{jk} are of mass dimension -3. Thus it is nonrenormalizable. A later section will discuss the effective theory point of view and issue of nonrenormalizability.

Since $e^{\frac{2\pi}{3}i}$ phases in flavor projections operators anticommute with Clifford odd fields, there are properties

$$\begin{aligned} P_{G1}\phi^\nu &= \phi^\nu P_{G1}, \\ P_{G2}\phi^\nu &= \phi^\nu P_{G3}, \\ P_{G3}\phi^\nu &= \phi^\nu P_{G2}, \end{aligned} \tag{99}$$

according to the definition of flavor projection operators (81, 83, 85). Therefore, there are flavor-mixing two-neutrino Yukawa terms between 2nd and 3rd generations, in addition to flavor-mixing four-fermion Yukawa terms between all generations.

After ϕ^ν and Φ acquire nonzero VEVs, which will be investigated in later section, the flavor mixing between right-handed fermions is represented by neutrino Majorana mass terms and four-fermion interaction terms. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

Electroweak Yukawa coupling constants of quarks are usually larger than leptons. If Majorana Yukawa coupling constants follow the same pattern, Y_{12} should be the largest. Hence, the Yukawa term with Y_{12} is the dominant one. In the following analysis we will concentrate on this term, and treat the model as if $\Phi = \Phi_{12}$. The properties of the other terms are similar to Φ_{12}.

3.3 Global Phase Symmetry and Flavon

As mentioned earlier, Φ boson is invariant under all gauge transformations related to gauge interactions. Nevertheless, there is a *global* phase symmetry under the following transformations

$$\begin{aligned} \Phi &\rightarrow \Phi e^{\theta i}, && (100)\\ u_1 = u &\rightarrow u e^{\theta_u i}, && (101)\\ d_1 = d &\rightarrow d e^{\theta_d i}, && (102)\\ u_2 = c &\rightarrow c e^{\theta_c i}, && (103)\\ d_2 = s &\rightarrow s e^{\theta_s i}, && (104) \end{aligned}$$

where

$$\theta = (\theta_u - \theta_d) - (\theta_c - \theta_s), \tag{105}$$

and $u, d, c,$ and s are up, down, charm, and strange quarks. The phase $e^{\theta i}$ measures differences in rotation angles between first and second generation quarks.

In the event of spontaneous symmetry breaking (SSB), there will be a massive sigma mode and a massless Nambu-Goldstone mode. As opposed to the Higgs mechanism, the Nambu-Goldstone mode is not 'eaten' by gauge field.

Notice that above global symmetry is an approximate symmetry, in the sense that the electroweak section spoils the symmetry explicitly. The Nambu-Goldstone mode is not exactly massless. The size of the mass grows with the strength of the explicit symmetry breaking. A not-quite-massless would be Nambu-Goldstone particle for an approximate symmetry is often called a pseudo-Nambu-Goldstone (PNG) boson. We call this PNG boson flavon, as it represents phase differences between flavors.

3.4 Symmetry Breaking and Majorana Masses

Majorana Boson action reads

$$S_{Majorana-Bosons} = S_{Majorana-Kenetic} - V_{Majorana}, \tag{106}$$

with

$$\begin{aligned} &S_{Majorana-Kenetic}(\phi^\nu) \sim \int \left\langle (e^3 D\phi^\nu)^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &V_{Majorana-Bosons}(\phi^\nu, -\mu_\nu^2, \lambda_\nu) \sim \int (-\mu_\nu^2 |\phi^\nu|^2 + \lambda_\nu |\phi^\nu|^4) \left\langle ie^4 \right\rangle, \end{aligned} \tag{107}$$

and

$$\begin{aligned} &S_{Majorana-Kenetic}(\Phi) \sim \int \left\langle (e^3 D\Phi)^2 \right\rangle / \left\langle ie^4 \right\rangle, \\ &V_{Majorana-Bosons}(\Phi, -\mu_\Phi^2, \lambda_\Phi) \sim \int (-\mu_\Phi^2 (\Phi)^2 + \lambda_\Phi (\Phi)^4) \left\langle ie^4 \right\rangle, \end{aligned} \tag{108}$$

where

$$
\begin{aligned}
D\phi^{\nu} &= (d-\check{W}_R-C)\phi^{\nu}+\phi^{\nu}(\check{W}_R+C), && (109)\\
D\Phi &= d\Phi, && (110)\\
\check{W}_R &= \check{W}_{R\mu}dx^{\mu}=\frac{1}{2}W^3_{R\mu}idx^{\mu}. && (111)
\end{aligned}
$$

Notice that ϕ^{ν} and Φ have negative $-\mu_{\nu}^2$ and $-\mu_{\Phi}^2$. It means that ϕ^{ν} and Φ acquire nonzero VEVs as

$$
\begin{aligned}
<0|\phi^{\nu}|0> &= \frac{1}{\sqrt{2}}\upsilon_{\nu}e^{\alpha i}\Gamma_0P_0=\frac{1}{\sqrt{2}}e^{\alpha i}\frac{\mu_{\nu}}{\sqrt{\lambda_{\nu}}}\Gamma_0P_0, && (112)\\
<0|\Phi|0> &= \frac{1}{\sqrt{2}}\upsilon_{\Phi}e^{\alpha_{12}i}=\frac{1}{\sqrt{2}}e^{\alpha_{12}i}\frac{\mu_{\Phi}}{\sqrt{\lambda_{\Phi}}}, && (113)
\end{aligned}
$$

As a result, the gauge symmetry related to gauge field

$$Z'_{\mu}=W^3_{R\mu}-C^J_{\mu}, \tag{114}$$

and the global phase symmetry of Φ are spontaneously broken. Notice that the minus sign in above equation stems from the fact that $JP_0=-iP_0$.

After replacing ϕ^{ν} and Φ with their VEVs, the Majorana Yukawa action reduces to

$$
\begin{aligned}
S_{Majorana-Yukawa} \sim\ & m_{jk}\int\left\langle e^{\alpha i}P_{Gj}\bar{\nu}_{Rj}e^4e^{\varepsilon_{jk}}\Gamma_2\Gamma_3\nu_{Rk}P_{Gk}\Gamma_0P_0\right\rangle,\\
&+\frac{1}{\sqrt{2}}Y_{12}\upsilon_{\Phi}\int\left\langle e^{\alpha_{12}i}P_{G1}\bar{u}_{R1}e^4d_{R2}P_{G2}\bar{u}_{R2}e^4d_{R1}\right\rangle/\left\langle ie^4\right\rangle+h.c.
\end{aligned}
\tag{115}
$$

with Majorana masses

$$m_{jk}=\frac{1}{\sqrt{2}}y_{jk}\upsilon_{\nu}. \tag{116}$$

Neutrino Majorana masses are much heavier than neutrino Dirac masses, if we assume

$$y_{jk}\upsilon_{\nu}>>y^{\nu}\upsilon \tag{117}$$

where constants y^{ν} and υ are electroweak Higgs ounterparts, which will be defined in later section. Because of the hierarchy, very small effective masses are generated for neutrinos, known as seesaw mechanism.

Now we express gauge fields W^3_R and C^J in terms of B and Z'

$$
\begin{aligned}
W^3_{R\mu} &= B_{\mu}+(cos\theta'_W)^2Z'_{\mu},\\
C^J_{\mu} &= B_{\mu}-(sin\theta'_W)^2Z'_{\mu},
\end{aligned}
\tag{118}
$$

where

$$cos\theta'_W = \frac{g_{WR}}{g_{Z'}},$$
$$sin\theta'_W = \frac{g_J}{g_{Z'}}, \tag{119}$$
$$g_{Z'} = \sqrt{g_{WR}^2 + g_J^2}.$$

Gauge field B remains massless with an effective coupling of

$$g_B = \frac{g_{WR} g_J}{g_{Z'}}, \tag{120}$$

while gauge field Z' acquires a mass from neutrino part of the Majorana Kinetic action

$$M_{Z'} = \frac{1}{2} \upsilon_\nu g_{Z'}. \tag{121}$$

Higgs boson ϕ^ν and the sigma mode of Φ acquire masses

$$m_{h\nu} = \sqrt{2}\mu_\nu,$$
$$m_\Phi = \sqrt{2}\mu_\Phi. \tag{122}$$

3.5 LHC 750 Gev Diphoton Resonance from Flavon

If we assume that $\upsilon_\nu >> \upsilon$ and $\upsilon_\Phi >> \upsilon$, gauge boson Z', Higgs boson ϕ^ν, and sigma mode of Φ would be too heavy to be detected at electroweak energy scale. On the other hand, the PNG flavon is not exactly massless, since the electroweak sector explicitly breaks the global phase symmetry and can generate mass for it. The size of the flavon mass is proportional to electroweak scale. Hence it is detectable at LHC.

The LHC 750 GeV diphoton resonance is explained by flavon, which is the PNG boson of four-quark condensation. Flavon is resulted from spontaneous symmetry breaking of the global phase symmetry involving first and second generation quarks.

A resonance starts with four quarks produced by two gluons. Two of the quarks turn into other two quarks via the four-fermion Yukawa interaction with VEV υ_Φ. After one further internal gluon line, the four quarks turn into a flavon. The flavon propagates and finally decays like the generation process in reverse order. The only difference of the reverse process is that there are two external B boson lines in stead of two external gluon lines.

Gauge field B contains massive gauge field Z (upon electroweak symmetry breaking) and massless electromagnetic gauge field A. Thus we are expecting the detection of resonance decaying into Z bosons as well, in addition into decaying to photons.

3.6 Four-Fermion Condensation and Pseudo-Nambu-Goldstone-Boson

Boson sectors might be just an effective Ginzbrug-Landay-type description of the low energy physics represented by composite boson fields. One approach is to assume effective four-quark interactions strong enough to induce top quark-antiquark condensation into composite electroweak Higgs fields[16, 17, 18], via dynamical symmetry breaking mechanism in Nambu-Jona-Lasinio[19] like models.

Likewise, the Majorana boson fields might also be collective excitations of underlying composite spinors. For example, ϕ^ν and Φ could be effective representation of two-neutrino and four-quark condensations

$$\begin{aligned} \check{\phi}^\nu &= y_{jk} P_{Gj} \bar{\nu}_{Rj} e^4 \Gamma_2 \Gamma_3 \nu_{Rk} P_{Gk}, \\ \check{\Phi} &= Y_{12} P_{G1} \bar{u}_R e^4 s_R P_{G2} \bar{c}_R e^4 d_R / \left\langle ie^4 \right\rangle . \end{aligned} \tag{123}$$

The four-neutrino and eight-quark interactions are

$$\int \left\langle (\check{\phi}^\nu)^2 \right\rangle / \left\langle ie^4 \right\rangle + \int \left\langle (\check{\Phi})^2 \right\rangle / \left\langle ie^4 \right\rangle. \tag{124}$$

A collective mode is determined as the pole of bosonic channel of the four-fermion interaction by summing to infinite order chains of 'bubble' perturbation diagrams. The leading order calculation goes by different names such as random-phase approximation (RPA), Bethe-Salpeter T-matrix equation, and 1/N expansion.

If the Majorana bosonic field Φ is indeed a collective excitation of the underlying four fermions, the first order approximation would involve RPA summing to infinite order chains of 'bubble' diagrams, linked together via eight-fermion contact interactions. Each 'bubble' contains four lines of fermion propagators.

3.7 The Issue of Nonrenormalizability

The four-fermion Yukawa terms of Φ and four/eight-quark contact interactions are nonrenormalizable in the conventional sense.

For nonrenormalizable models, the renomalization procedure can be made only at the cost of adding an increasing numbers of term to the original Lagrangian. In principle, there is no problem with a theory having an infinite number of coupling constants as an effective field theory[20]. However, the NJL model is often regarded as regularization dependent and its predictability is called into question.

A novel strategy for handling divergences is called implicit regularization [21]. It avoids the critical step of explicit evaluation of divergent integrals. The finite parts are separated from the divergent ones and integrated free from effects of regulation. The application to NJL model reveals that calculations can be ambiguity-free and symmetry-preserving can be obtained, making the NJL model predictive.

Likewise, we expect that models with four-fermion Yukawa interactions are as predictable as renormalizable theories.

4 Electroweak Bosons

4.1 Electroweak Bosons and Yukawa Action

Electroweak boson field ϕ_{EW} interacts with both left-handed and right-handed fermions, while Majorana boson field ϕ_{MAJ} interacts with right-handed fermions only. Electroweak Boson field ϕ_{EW} spans the whole 32 component $C\ell_{0,6}$ even space. It obeys gauge transformation rules

$$\phi_{EW} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}}\phi_{EW}e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{125}$$

It can be broken down into three sectors as

$$\phi_{EW} = \phi_S + \phi_P + \phi_{AT}, \tag{126}$$

with scalar ϕ_S valued in Clifford space spanned by 4 multivectors

$$\{1, \Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{127}$$

pseudoscalar ϕ_P valued in Clifford space spanned by 4 multivectors

$$\{i, i\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{128}$$

and antisymmetric tensor ϕ_{AT} valued in Clifford space spanned by $4*6 = 24$ multivectors

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k, a,b = 0,1,2,3, a \neq b\}. \tag{129}$$

The scalar and pseudoscalar electroweak Higgs fields ϕ_S and ϕ_P transform as

$$\phi_{S/P} \quad \rightarrow \quad e^{\Theta_{WL}}\phi_{S/P}e^{-\Theta_{WR}}, \tag{130}$$

while up antisymmetric tensor electroweak boson field ϕ_{AT} transforms as

$$\phi_{AT} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}}\phi_{AT}e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{131}$$

Notice that ϕ_{AT} is *not a Lorentz singlet*, since it's not invariant under local Lorentz gauge transformations.

We can write electroweak Yukawa action of fermions as

$$\begin{aligned} S_{Electroweak-Yukawa} \quad &\sim \\ \int \big\langle \bar{\psi}_{Lj} i e^4 \phi_{EW} (y_j^\nu \nu_{Rj} + y_j^e e_{Rj} + y_j^u u_{Rj} + y_j^d d_{Rj}) i P_{Gj} \big\rangle \\ + \int \big\langle (y_j^\nu \bar{\nu}_{Rj} + y_j^e \bar{e}_{Rj} + y_j^u \bar{u}_{Rj} + y_j^d \bar{d}_{Rj}) i e^4 \bar{\phi}_{EW} \psi_{Lj} i P_{Gj} \big\rangle, \end{aligned} \tag{132}$$

where

$$\bar{\phi}_{EW} = \gamma_0 \phi_{EW}^\dagger \gamma_0 = \gamma_0 \tilde{\phi}_{EW} \gamma_0 \tag{133}$$

and y_j^ν, y_j^e, y_j^u, and y_j^d are electroweak Yukawa coupling constants.

4.2 Electroweak Boson Action, Symmetry breaking, and Dirac Mass

Electroweak boson action reads

$$S_{Electroweak-Bosons} = S_{Electroweak-Kenetic} - V_{Electroweak}, \tag{134}$$

with

$$\begin{aligned} &S_{Electroweak-Kenetic}(\phi_S) \sim \int \left\langle (e^3(\bar{D\phi_S}))(e^3 D\phi_S) \right\rangle / \left\langle ie^4 \right\rangle \\ &V_{Electroweak-Bosons}(\phi_S, -\mu_S^2, \lambda_S) \sim \int (-\mu_S^2|\phi_S|^2 + \lambda_S|\phi_S|^4) \left\langle ie^4 \right\rangle, \end{aligned} \tag{135}$$

and

$$\begin{aligned} &S_{Electroweak-Kenetic}(\phi_P), V_{Electroweak}(\phi_P, -\mu_P^2, \lambda_P), \\ &S_{Electroweak-Kenetic}(\phi_{AT}), \\ &V_{Electroweak}(\phi_{AT}, +\mu_{AT}^2, \lambda_{AT}) \sim \int (\mu_{AT}^2 \left\langle \bar{\phi}_{AT}\phi_{AT} \right\rangle + \lambda_{AT} \left\langle \bar{\phi}_{AT}\phi_{AT} \right\rangle^2) \left\langle ie^4 \right\rangle, \end{aligned} \tag{136}$$

where

$$D\phi_{P/S} = (d + W_L)\phi_{P/S} - \phi_{P/S}(W_R), \tag{137}$$

$$D\phi_{AT} = (d + \omega + W_L)\phi_{AT} - \phi_{AT}(\omega + W_R), \tag{138}$$

Notice that ϕ_S and ϕ_P have negative $-\mu_S^2$ and $-\mu_P^2$. It means that ϕ_S and ϕ_P acquire nonzero VEVs via SSB

$$< 0|\phi_S|0 >= \frac{1}{\sqrt{2}}\upsilon_S = \frac{1}{\sqrt{2}}\frac{\mu_S}{\sqrt{\lambda_S}}, \tag{139}$$

$$< 0|\phi_P|0 >= \frac{1}{\sqrt{2}}\upsilon_P i = \frac{1}{\sqrt{2}}\frac{\mu_P}{\sqrt{\lambda_P}}i. \tag{140}$$

The situation of ϕ_{AT} is a bit complicated, and will be discussed in later section. Let's for the moment assume that its VEV is zero.

After replacing ϕ_S, ϕ_P, and ϕ_{AT} with their VEVs, the electroweak Yukawa action reduces to

$$\int \left\langle (\bar{\nu}_j ie^4 m_j^\nu \nu_j i + \bar{e}_j ie^4 m_j^e e_j i + \bar{u}_j ie^4 m_j^u u_j i + \bar{d}_j ie^4 m_j^d d_j i) P_{Gj} \right\rangle, \tag{141}$$

where 'complex' (scalar plus pseudoscalar) Dirac masses are

$$m_j^{\nu/e/u/d} = \frac{1}{\sqrt{2}} y_j^{\nu/e/u/d}(\upsilon_S + \upsilon_P i) = \frac{1}{\sqrt{2}} y_j^{\nu/e/u/d} \upsilon e^{\beta i}, \tag{142}$$

with

$$\begin{aligned} &\upsilon = \sqrt{\upsilon_S^2 + \upsilon_P^2}, \\ &tan(\beta) = \frac{\upsilon_P}{\upsilon_S}. \end{aligned} \tag{143}$$

However the $e^{\beta i}$ phase factor can be canceled out via a global rotation of spinor

$$\psi \quad \rightarrow \quad e^{-\frac{1}{2}\beta i}\psi, \tag{144}$$

so that the fermion Dirac masses are 'real' (scalar) valued. Notice that ie^4 is a Clifford-scalar-valued 4-form.

Since the experiments at LHC indicated only one Higgs boson with $m_h = 125$ Gev[3, 4], there could be two scenarios. Case one is that both scalar and pseudoscalar Higgs fields contribute to the electroweak symmetry breaking and their masses are degenerate

$$m_h = m_S = m_P. \tag{145}$$

Case two is that only one of them acquires a nonzero VEV (with negative $-\mu^2$), which is the $m_h = 125$ Gev Higgs. The other maintains a zero VEV (with positive μ^2), which is still waiting to be detected at LHC.

Now we express gauge fields W_L^3, B, and W_R^3 in terms of A, Z, and Z'

$$\begin{aligned}
&W_{L\mu}^3 = A_\mu + (cos\theta_W)^2 Z_\mu,\\
&B_\mu = A_\mu - (sin\theta_W)^2 Z_\mu,\\
&W_{R\mu}^3 = B_\mu + (cos\theta'_W)^2 Z'_\mu = A_\mu - (sin\theta_W)^2 Z_\mu + (cos\theta'_W)^2 Z'_\mu,
\end{aligned} \tag{146}$$

where

$$\begin{aligned}
cos\theta_W &= \frac{g_{WL}}{g_Z},\\
sin\theta_W &= \frac{g_B}{g_Z},\\
g_Z &= \sqrt{g_{WL}^2 + g_B^2}.
\end{aligned} \tag{147}$$

Electromatic field A remains massless with an effective coupling of

$$g_A = \frac{g_{WL} g_B}{g_Z} = \frac{g_{WL} g_{WR} g_J}{\sqrt{g_{WL} g_{WR} + g_{WL} g_J + g_{WR} g_J}}, \tag{148}$$

while gauge field Z acquires a mass

$$M_Z = \frac{1}{2} \upsilon g_Z. \tag{149}$$

4.3 Antisymmetric Tensor Boson and Dark Spin Current

As stated earlier, the antisymmetric tensor electroweak boson field ϕ_{AT} is not invariant under local Lorentz gauge transformations. Hence, its boson potential should involve Lorentz invariant

$$\left\langle \bar{\phi}_{AT} \phi_{AT} \right\rangle = \left\langle \gamma_0 \phi_{AT}^\dagger \gamma_0 \phi_{AT} \right\rangle, \tag{150}$$

as opposed to

$$|\phi_{AT}|^2 = \left\langle \phi^\dagger_{AT}\phi_{AT} \right\rangle, \tag{151}$$

which is not Lorentz invariant.

It's easy to see that $\langle \bar{\phi}_{AT}\phi_{AT} \rangle$ is not a positive definite quantity. Components of

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k, a,b = 1,2,3, a \neq b\}, \tag{152}$$

have positive 'metric' and components of

$$\{i\gamma_a\gamma_b, i\gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k, a,b = 1,2,3, a \neq b\}, \tag{153}$$

have negative 'metric'.

A zero VEV $< 0|\phi_{AT}|0 >$ is allowed only if $\mu^2_{AT} = 0$. On the other hand, nonzero VEV can be acquired for any value of μ^2_{AT}, including $\mu^2_{AT} = 0$. Nonzero VEV breaks both Lorentz and electroweak symmetries. Replacing ϕ_{AT} with nonzero $< 0|\phi_{AT}|0 >$ in the boson kinetic action, we have a Lorentz symmetry breaking term

$$\int \left\langle (e^3(\omega < 0|\phi_{AT}|0 > - < 0|\phi_{AT}|0 > \omega)\tilde{)}(e^3(\omega < 0|\phi_{AT}|0 > - < 0|\phi_{AT}|0 > \omega)) \right\rangle / \left\langle ie^4 \right\rangle \tag{154}$$

This spin connection ω related term can contribute to space-time torsion equation. We call it 'dark spin current'. It is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature.

Since we know that modifications to torsion could have gravitational and cosmological consequences[22, 23], it's worth further research on the above antisymmetric-tensor-induced scenario.

5 Possible Grand Unification Symmetries

Embolden by the power of Clifford algebra, we now explore more symmetries allowed by an algebraic spinor. Let's begin with general gauge transformations

$$\psi \quad \rightarrow \quad e^{\Theta}\psi e^{\Theta'}, \tag{155}$$

where e^{Θ} and $e^{\Theta'} \in C\ell_{0,6}$ are independent gauge transformations. Spinor bilinear

$$\left\langle \tilde{\psi}\gamma_0\psi \right\rangle \tag{156}$$

is invariant if

$$e^{\tilde{\Theta}}\gamma_0 e^{\Theta} = \gamma_0, \tag{157}$$

$$e^{\Theta'}e^{\tilde{\Theta}'} = 1, \tag{158}$$

where we restrict our discussion to gauge transformations continuously connected to identity. General solution of these equations includes $\Theta \sim so(4,4)$, which is a linear combination of 28 gauge transformation generators

$$\{\gamma_a, \gamma_a\gamma_b, \Gamma_a\Gamma_b, i\Gamma_j, \Gamma_0\gamma_j\Gamma_k; j,k=1,2,3, a,b=0,1,2,3, a>b\} \in \Theta, \tag{159}$$

and $\Theta' \sim sp(8)$, which is a linear combination of 36 gauge transformation generators of pseudoscalar, all bivectors, and all trivectors

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l, \gamma_0, \Gamma_0, \gamma_0\gamma_j\Gamma_k, \Gamma_0\gamma_j\Gamma_k; j,k,l=1,2,3, k>l\} \in \Theta'. \tag{160}$$

The de Sitter algebra $\Theta_{DS} \sim so(1,4)$

$$\{\gamma_a, \gamma_a\gamma_b\} \in \Theta_{DS} \tag{161}$$

is a subalgebra of Θ.

The Clifford odd parts of Θ and Θ' mix odd (left-handed ψ_L) and even (right-handed ψ_R) spinors. Since we know that left- and right-handed spinors transform differentially, only Clifford even subalgebras of Θ and Θ' are permitted, namely

$$\{\gamma_a\gamma_b, \Gamma_a\Gamma_b\} \in \Theta_{Even} \sim so(1,3) \oplus so(1,3), \tag{162}$$

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \in \Theta'_{Even} \sim u(1) \oplus so(6) \sim u(1) \oplus su(4). \tag{163}$$

The gauge transformations $\{\Gamma_a\Gamma_b\}$ can be further decomposed into weak transformations $\{\Gamma_k\Gamma_l\}$ and weak boost transformations $\{\Gamma_0\Gamma_j\}$, which are counterparts of spacial rotation $\{\gamma_k\gamma_l\}$ and Lorentz boost transformations $\{\gamma_0\gamma_j\}$.

Unitary algebra $u(3)$ is embedded in $\{\gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \sim su(4)$. Removing $u(1)$ $\{J\}$ from $u(3)$ defines the color algebra $su(3)$.

Since there are left-handed weak $su(2)_L$ and right-handed weak $u(1)_R$, one might expect left-right symmetric $su(2)_R$ as well. We can even go further and entertain the possibility of two exact copies of left-handed Θ_{EvenL} and right-handed Θ_{EvenR}.

Of course, the grand unification symmetries studied in this section are speculative in nature. If there is indeed grand unification scale physics involving Θ_{EvenL}, Θ_{EvenR} and Θ'_{Even}, either symmetry breaking or other mechanism is needed to prevent detection of gauge interactions related to pseudoscalar $\{i\}$, quark/lepton mixing $su(4) \ominus u(3)$, weak boost $\{\Gamma_0\Gamma_j\}$, right-handed $su(2)_R$, and differences between left-handed $\{\gamma_a\gamma_b\}_L$ and right-handed $\{\gamma_a\gamma_b\}_R$ Lorentz transformations. It's an interesting topic. Nevertheless, we leave grand unification to future research.

6 Conclusion

We propose a Clifford algebra based model. A ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The model includes local gauge symmetries $SO(1,3) \otimes$

$SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. Both gravitational and Yang-Mills interactions are treated as gauge fields.

There are two sectors of boson fields as Majorana and electroweak bosons. Majorana boson field interacts with right-handed fermions only. Electroweak boson field interacts with both left-handed and right-handed fermions.

The Majorana boson sector causes flavor mixing between all generations. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

The LHC 750 GeV diphoton resonance is explained by flavon, which is the pseudo-Nambu-Goldstone boson of four-quark condensation. Flavon is resulted from spontaneous symmetry breaking of a global phase symmetry involving first and second generation quarks. Flavon is not exactly massless, since the electroweak sector explicitly breaks the global phase symmetry and can generate mass for it. The size of the flavon mass is proportional to electroweak scale. We expect the detection of resonance decaying into Z bosons as well, in addition to decaying into photons. Being a standard model singlet, flavon is a potential dark matter candidate.

The neutrino Higgs field part of Majorana boson sector acquires a nonzero VEV via SSB, inducing Majorana masses of right-handed neutrinos via Yukawa-like couplings.

The electroweak boson sector is composed of scalar, pseudoscalar, and antisymmetric tensor components. Scalar and/or pseudoscalar Higgs fields break the electroweak symmetry, contributing masses to fermions.

The antisymmetric tensor boson is not a Lorentz singlet. Its nonzero VEV would break Lorentz and electroweak symmetries, giving rise to 'dark spin current'. 'Dark spin current' is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature. Since we know that modifications to torsion could have gravitational and cosmological consequences[22, 23], it's worth further research on the antisymmetric-tensor-induced scenario.

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