Time dependent analysis of the 10S2 Quintet

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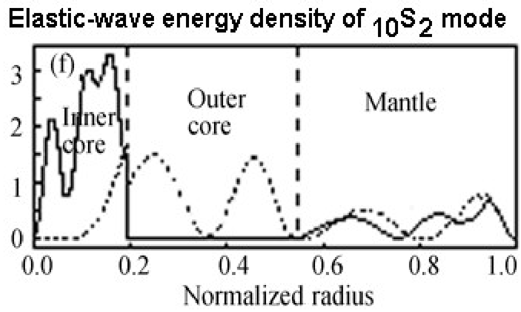
Abstract: The time dependent analysis of the normal mode ${}_{10}S_2$ near 4040 µHz allows a precise determination of the frequencies of the five singlets and reveals a frequency modulation by 23 µHz, probably generated by the moon. The results of the data from 18 stations distributed worldwide deliver very good matching results and may help to figure out some properties of the isotropic layer at the top of the inner core.

Introduction

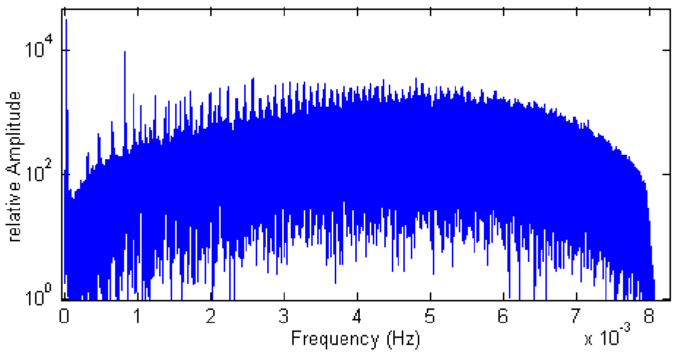


After earthquakes, the Earth vibrates like a bell and a set of different natural frequencies is recorded by various instruments. The theory provides information, which parts of the globe influence these normal modes. The figure shows that the mode ${}_{10}S_2$ is well suited as a probe to study certain properties of the Earth's inner core. The solid curve indicates the shear energy density and the dotted line the density of the compression energy. Both values are supposed to change abruptly the near the surface of of the inner core.

The spectral line near 4040 µHz is difficult to observe because it is weak and can be detected only during the first hundred hours after a strong earthquake. In addition, the very strong tides with frequencies around 20 µHz conceal the low amplitude of ${}_{10}S_2$. In order to discover these weak oscillations at all, one has to filter out a narrow frequency band around 4040 µHz. With careless use of standard filtering software, the huge dynamic range of at least 100 dB generates intermodulation and numeric noise in standard math co-processors, whereby the SNR deteriorates noticeably. The comparison of two spectra below shows clear differences.

The Preparation of the CORMIN data

In a first step, a two-week cluster with the start time December 26, 2004 was taken from the CORMIN records of all available SG stations. The influence of atmosphere pressure variation on the gravity data may be omitted because the air mass above the instruments changes much slower than the oscillation time of ${}_{10}S_2$ (about 4 minutes).



The extremely strong tides can be removed very easily with a comb filter. The application is very simple: all samples are shifted by two positions (postponed for two minutes) and subtracted from the original record. The ratio of this time difference to the period(s) of the oscillations produces the desired effect: The very intense amplitudes of the slow tides (T ≈ severals hours) are largely compensated, while the amplitude of the much faster ${}_{10}S_2$ oscillations (T ≈ four minutes) is doubled.
Repeating this "*shift and subtract*" procedure generates the final data string. The spectrum shows how effective this method attenuates the low frequencies although it is not a high-pass filter. It is a

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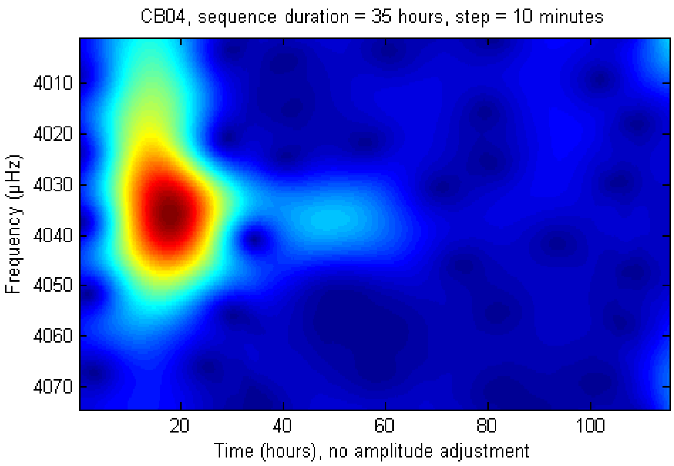
feedforward comb filter with a notch at $f = 0$ and a very broad peak near 4.2 mHz. It has all the positive characteristics of an FIR filter and and is extremly fast. Because it uses no feedback like an IIR filter, glitches are eliminated without affecting the subsequent data. Unfortunately, this filter can not improve the SNR.

After the comb filter has reduced the dynamic range of the data below 40 dB, the mean frequency of ${}_{10}S_2$ is lowered to 360 ± 150 µHz, without modifying the properties of the spectral frequencies[1]. Using this mixing method, the absolute frequency difference Δf between adjacent spectral lines remains constant while the ratio $\Delta f/f$ increases by a factor of eleven. This simplifies the data analysis and allows to alter the sampling rate to 600 seconds. In order to avoid unwanted impulse responses, IIR filters are strictly forbidden, only Sinc filters were used, having a rectangular pass-band. A detailed discussion follows below.

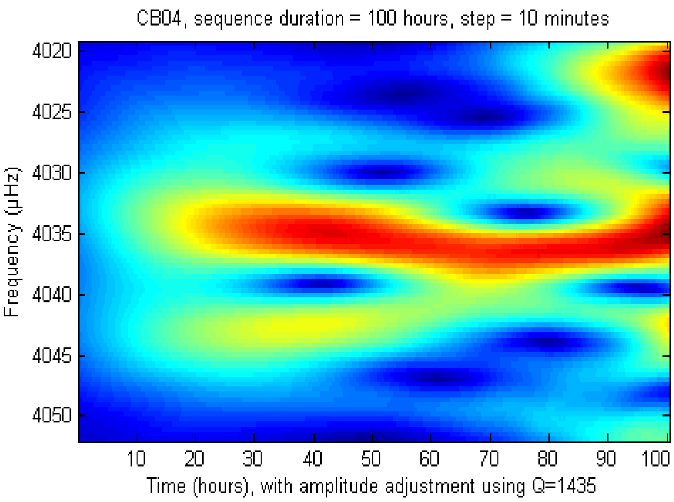
Despite all efforts, only the CORMIN-data of one single station (CB) are good enough to achieve an acceptable time and frequency resolution. Regrettably, the 2004/05 CORMIN data of the stations H1, H2, M1, M2, S1, S2, W1 and W2 were unusable because the ran through a low pass filter deleting all frequency components above 3 mHz.

First attempts with FFT



The target of this study was to analyze the temporal evolution of ${}_{10}S_2$. Using Fourier transformation, the data stream must be divided into short overlapping sequences. Then fast amplitude and frequency changes may be detected, but the frequency resolution is poor. The image consists of 692 individual spectra, each based on only 210 samples, corresponding to a time frame of 35 hours. Despite this short sequence, it is very difficult to discover details that are significantly shorter than about 10 hours.

The analysis window starts shortly after the earthquake, 8655 hours after 2004-01-01 and includes the following 110 hours. The next sequence starts ten minutes later. Since the data stream begins abruptly a few hours after the earthquake, the step response results in an initial broadening of the spectrum.



The ${}_{10}S_2$ oscillations soon get weaker and after a few hours, they disappear in the noise. Because the amplitude decreases exponentially, the visibility of the spectra near the right side of the image can be improved by multiplying with the formula $e^{\frac{\omega t}{2Q}}$. With proper choice of the Q-factor, the spectrogram shows a constant color gradient. The increased sequence duration (100 hours) in the picture allows a high frequency resolution and delivers more details, but the temporal resolution is very poor. The indicated split into three spectral lines can be real or just an artefact. This detail can not be verified for lack of comparable CORMIN data sets with sufficient quality. Below, a much higher resolution is achieved with another data base. And an improved time resolution reveals that the peculiar frequency change at t = 60 hours is generated by a phase shift, which can not be detected by FFT.

The resolution of every FFT is basically limited by the formula $\Delta f \cdot \Delta t \geqslant 0.5$, which can not be circumvented by a change in the sampling time. A time resolution of two hours requires a minimum bandwidth of 70 µHz. However, a so wide surrounding area of ${}_{10}S_2$ contains many other, interfering spectral lines, which distort the results. The Küpfmüller rules apply also for upstream band filters, because limiting the bandwidth necessarily causes ringing artifacts in the time domain. Sticking to FFT, there is no way out of this dilemma.

A second try

Because the planned studies can not be performed using FFT, a homodyne receiver was programmed and used. Using this *zero-beat* principle, one can measure both frequency and modulation with extreme precision. Of course, the above-mentioned uncertainty principle applies here also. After a strong earthquake, the ${}_{10}S_2$ spectral line can be detected for about 100 hours. Consequently, one can expect the frequency tolerance 2.8 µHz. The principle of coherent detection (also called direct-conversion) was already described[2] and used to examine ${}_{0}S_0$.

The second problem was to find a better data base, because the restriction on the records of a single station (CB) excludes any cross-checking of the results. In fact, there are numerous raw files of the 2004 event with samples every second or so. These data were made machine readable and proved to be extremely valuable.

The necessary data reduction on a format similar to CORMIN is time-consuming. But it has the advantage that all program steps can be optimized for the desired frequency range and the impact of every operation on the data is immediately visible.

There are many different ways to filter noisy data and each method has desirable main effects and side effects that are sometimes overlooked. Here is a summary of the essential characteristics.

- Stay away from IIR filters ("*quick and dirty*"). Long time ago, they were indispensable for fast data processing on slow computers. The computational speed of modern processors is much higher and at least with offline calculations, IIR filters are no longer necessary. Main disadvantages: Malignant impulse response after discontinuities with very strong data corruption and long lasting "ringing". I spent a lot of time chasing ghost signals generated by narrow IIR filters.
  IIR is the abbreviation for *Infinite-duration Impulse Response*. The frequency-dependent phase shift may produce additional distortion. Analyzing the internal machinery of IIR reveals that the math inside often produces surprisingly high values of intermediate results, thus reducing the allowable dynamic range of the data. This causes unwanted cross modulation due to a loss of numeric precision, producing noise and additional spectral lines. Each earthquake generates signals that begin abruptly and each IIR filter responds with wild oscillations which can not be suppressed. Reversing the direction of time before filtering reduces the problem, but does not eliminate it.
  Be suspicious: Many commercial procedures hide IIR filters without an explicit declaration. If, nevertheless, IIR filters are to be used, always restict to the *Second Order Section* (SOS) versions, avoid a sharp transition from passband to stopband and never use an "elliptic" filter characteristic.

- In modern computers, even high order FIR filters with many coefficients are processed sufficiently fast and produce significantly less errors than IIR filters. The filter startup transients have finite duration and glitches affect only a limited number of subsequent samples. Caution: In order to shorten the processing time, many commercial programs replace a FIR filter by three faster steps: FFT / multiplication with the transfer function / FFT. This saves computation time and reaches almost the same results as the FIR filter. However, an accurate analysis of noisy signals shows a noticeable loss of quality.

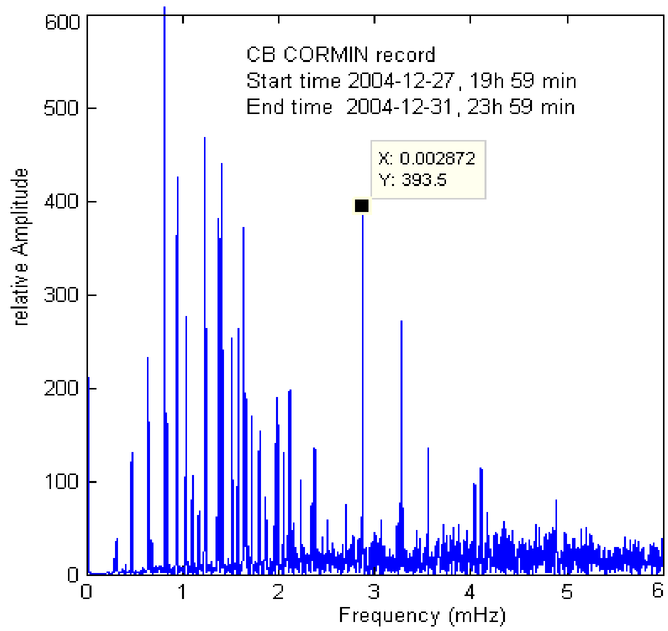
- A comb filter is an extremely fast FIR filter with a single coefficient and a very special filter shape. It adds or subtracts only ever two samples and does not generate distortions. But the comb filter doubles extremely short peaks, so they must be removed beforehand. First, manually during a careful repair of the data, followed by a high quality low pass filter.
- In the early years of digital data processing, the “*moving average*“ was a very popular filter method. However, the rectangular window function changes the sign in certain frequency ranges and therefore generates errors. An FIR or WSF lowpass is considerably better.
- Windowed Sinc Filters (WSF) use a lot of coefficients and work relatively slowly, but they treat the data with particular care. A unique feature of this convolution is the almost perfect rectangular filter shape. That is the only way to prevent that the measurable frequency of a noisy signal is pulled towards the middle of a “*tapered window*”.
  WSF are no causal filters. This means that glitches in the data stream change data samples which were measured chronologically *before* the occurrence of glitches. For all that follows, WSF is the prefered filter type. Quality is more important than time saving.

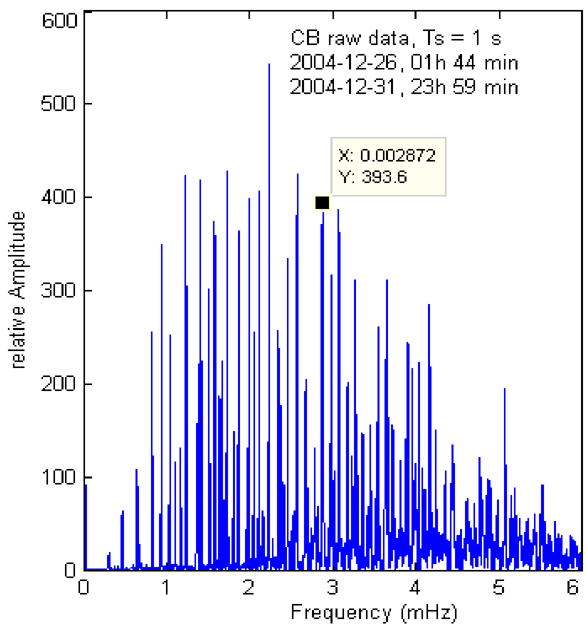
The Preparation of the raw data

First, the samples during the period 2004-12-26 till 2005-01-01 were extracted from the raw data of 18 SG stations. The influence of the atmosphere pressure variation on the gravity data was omitted because the air mass above the instruments changes much slower than the oscillation time of ${}_{10}S_2$. After the big faults have been removed manually, the data were smoothed by a WSF lowpass (cutoff frequency 8 mHz). Then, a comb filter (notch frequency 8 mHz) eliminated the tides and in a last step, the sampling time was changed to 60 seconds.

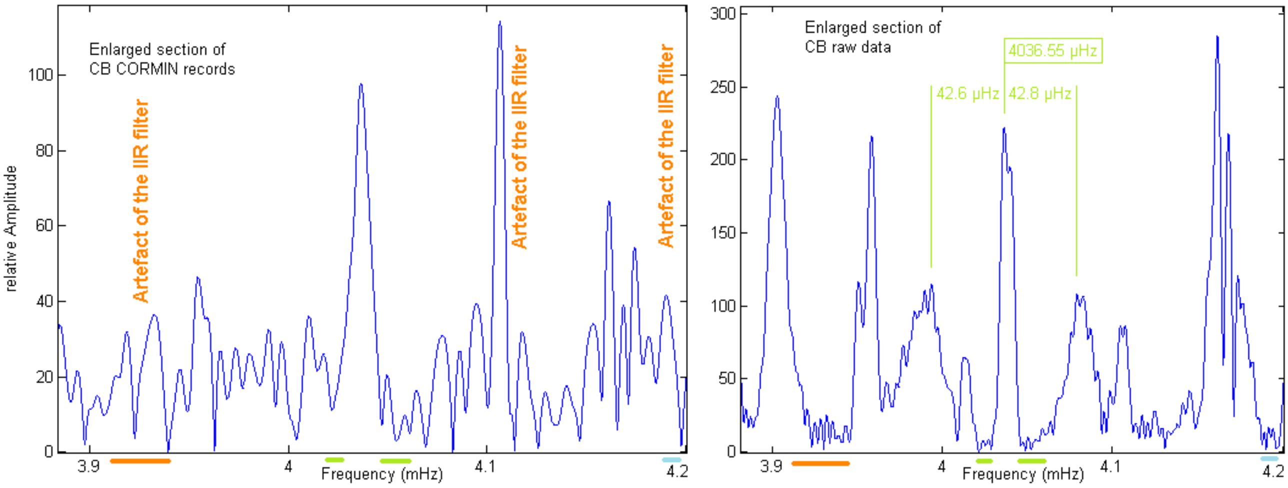
Type of data processing	Raw data	CORMIN data from CB
Start of data collection (UTC)	2004-12-26, 01 h, 44 min	2004-12-27, 19 h, 59 min
Lowpass filter	WSF, 0<f<8 mHz	Unknown, probably IIR
Second stage	Comb+comb, notch = 8 mHz	IIR-Decimation → Ts = 60 sec
Third stage	FIR-Decimation → Ts = 60 sec	Comb+comb, notch = 8 mHz

The colored processes are immutable characteristics of the data. Now, it's time to compare the spectra of the CORMIN data and the decimated raw data.





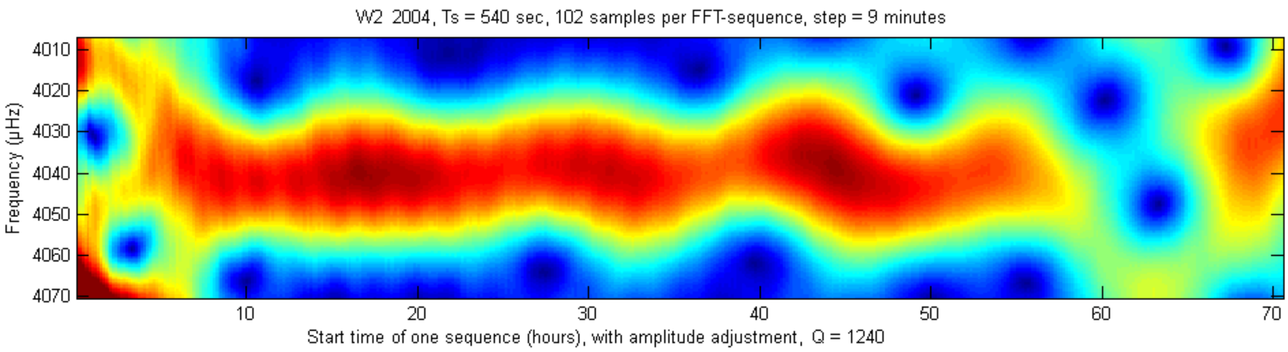
The amplitudes were scaled that a randomly chosen spectral line appears with the same amplitude in both spectra. Since the data recording of CORMIN data starts about 63 hours later than in the raw data, many spectral lines are no longer recognizable. An enlarged representation of a narrow range around 4040 μHz shows much more significant differences.



In the color-coded areas, the CORMIN spectrum shows far too high values (relative to ${}_{10}S_2$), most likely typical errors, generated by IIR filters. The spectrum of the raw data shows a symmetric arrangement of three frequencies around 4036 μHz, which is not found in the CORMIN data. It must be emphasized that in the frequency range shown, the CORMIN records of the station CB are much less noisy than the data of all other stations. The resolution of both spectra is still too low to confirm the indicated split into three spectral lines ($\Delta f \approx 8$ μHz) in the spectrogram above.

The Temporal Evolution of ${}_{10}S_2$

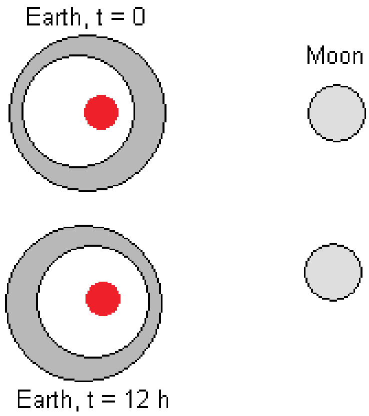
The Fourier analysis of an unmodulated oscillation (constant amplitude) provides a monotonic result, which hardly depends on the length of the sequence. With short sequences, one gets large half-widths (FWHM), whereas long sequences produce very narrow spectra. If there is an exponential decay with a known *Q*-factor, this decay should be compensated for by a suitable function in order to obtain a crisp spectral line. This ensures an approximately constant color in the spectrogram and a low FWHM. In the figure below, the exponential decay of the ${}_{10}S_2$ amplitude was compensated in accordance with the *Q*-factor of 1240.



The spectrogram is composed of 349 individual spectra, each one generated from a sequence of 102 samples and whose start times advance in steps of nine minutes. The rapid fluctuations are caused by interference with another spectral line 127 μHz higher.

The change of the red color corresponds to a weak amplitude modulation. Drawing a horizontal line in the spectrogram at 4040 μHz yields the average amplitude of ${}_{10}S_2$ as a function of time. Impressive is the slow frequency modulation with T ≈ 12 hours (see below) which is generated by the gravitational effect of the moon. Our companion causes a periodic asymmetry of the earth and changes the natural frequency of ${}_{10}S_2$.

Possibly, the deformation of the earth's crust is not sufficient to explain this periodic change of the resonance frequency. Another explanation would be that the gravitational constant *G* depends on the material. What if the value which applies the dense inner core of the earth, is slightly different from the value which applies to the rest of the (asymmetric) globe with its much lower density? Even a tiny difference would ensure that the inner core of the Earth is dislocated by the moon and not resident in the gravitational center of the surrounding outer core and mantel. Then the moon would be an eternal driving force for the inner core, and never let him rest.



The volume between the asymmetric crust (gray) and the inner core (red) is filled with the liquid outer core (white). Because there is no rigid coupling therebetween, the inner core does not need to follow the movement (in particular the rotation) of the earth's crust.

If even a part of this slow frequency modulation is generated by the movement of the Earth's core, the weak equivalence principle is questioned. This could perhaps replace the space science experiment STEP.

Coherent Detection

The reasons for the zero beat procedure were already mentioned above. After the conversion of the raw data into the same format as CORMIN, the decline of the ${}_{10}S_2$ amplitude is first compensated by multiplication with an appropriate factor $\exp\left(\frac{\omega t}{2Q}\right)$. Then, the intermediate result is mixed with the frequency which is to be investigated. This mixing operation is the standard procedure of the entire communications technology, because it allows the most accurate measurements.

Now follows the search for the signal frequency. FFT provides a clue where to search. Desirable and pleasant is an isolated, single spectral line. A dense collection of equally strong spectral lines, however, is far more problematic and can affect the convergence of the method. Frequency and phase of the local oscillator are varied until the long-term mean value at the output of the mixer is zero. Now, we have lock-in. With a clean signal, this is called *zero beat*. But normally, this signal (called *in-phase* component) fluctuates - either by noise or by frequency modulation.

Like in a Lock-in amplifier, a second detector with an additional 90 degree phase shift produces the *quadrature* component, needed to create the analytical signal below. Here, a periodic admixture indicates amplitude modulation. In both cases, FFT is used to analyze if there are any periodic components and to estimate the modulation frequencies.

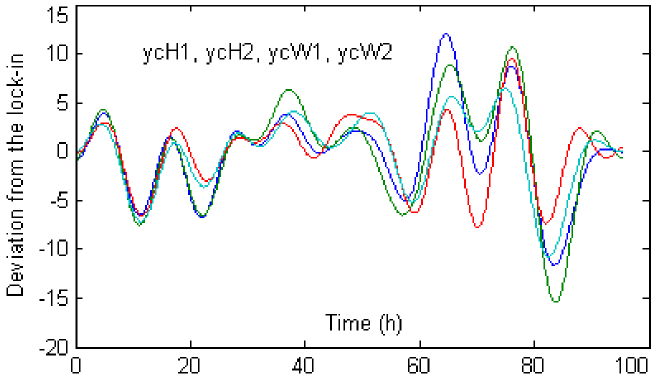
A subsequent lowpass specifies which highest modulation frequency is taken into account. In principle, all adjacent frequencies can be sidebands of the studied oscillation. They may contain important information and must not be simply cut off. At first you do not know yet what is important, therefore, this neighborhood has to be large. But not too large, so not an unnecessary amount of noise is further processed. The examination of the data of the station NY (below) is a good example to show that for ${}_{10}S_2$, the cutoff frequency of the low pass filter should be 30 µHz.

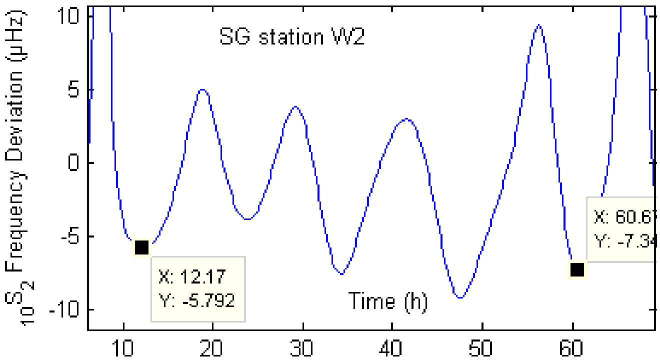
The slow Frequency Modulation

The *in-phase* component should be a horizontal straight line. In fact, the records of all stations show periodic deviations, which can be traced for many hours.

The superposition of the *in-phase* components of the stations H1, H2, W1 and W2 indicates that there must be a common cause. But the vertical axis of the image (the *in-phase* component) shows only one part of the solution. Linking with the *quadrature* component generates an analytical signal. Differentiating the phase of this signal yields the desired frequency deviation.

$$f = \frac{1}{2\pi}\frac{d\varphi}{dt}$$





The characteristic data of the FM can be identified in the picture. Obviously, the (maximal) frequency deviation is 5 µHz and the modulation frequency is 22.9 µHz.

It would be wrong to assume that the spectrum becomes a broad band, twice as wide as the frequency deviation. If, like in the present case, the modulation signal is a single sine wave, the spectrum consists of three separate lines at intervals of the modulation frequency. The amplitudes of each spectral line can be calculated with the Bessel functions of the first kind: The central frequency (amplitude lowered to 98% compared with the unmodulated case) and two symmetrical side frequencies, each in 22.9 µHz distance and with 11% amplitude (compared with the central frequency). In between is nothing but noise. That is the reason for the above-mentioned minimum cutoff frequency of the low pass filter (30 µHz). Any reduction in the bandwidth cuts off the sidebands and distorts the signal.

Below we will see that the frequencies of two outer singlets (m = -2 and m = 2) can also be found at a distance of about ± 23.16 µHz symmetrical to the central line (m = 0). Usually, both singlets can be measured with a rather high amplitude, covering the weaker FM-Lines. The very low frequency spacing (zero?) is likely to lead to a strong coupling. Perhaps the frequencies of the two outer singlets are even defined by the remarkable frequency modulation, which is imposed by the moon.

Measuring Spectral Lines

The normal mode $_{10}S_2$ may split into five singlets. Depending on the geographical position of the SG station, they can be received with different strengths. In the "normal case", the amplitude of one spectral line exceeds all others. In a few special cases, all five singlets can be measured. The table below shows the individual frequencies and relative amplitudes of the singlets and the *Q*-factor of the center line (m = 0).

Some values deserve special attention: The central frequencies for CB and WU are split and show a sharp minimum at this frequency. Below and based on the data of the station CB, it is shown in detail, that the phase changes during the measurement period by π (it jumps to anti-phase).

The stations CB, S1 and S2 receive the central frequency with a much higher amplitude than average. On the other hand, ME and NY record remarkably small amplitudes. Possibly, this may help to determine the directions of the axes of symmetry of this normal mode. The central frequencies of the stations NY, S1 and S2 are significantly lower than the average. This difference could not be eliminated even after repeated inspection.

	m = -2	m = -1	m = 0	m = 1	m =2	*Q*-factor
CB (µHz)	4013,41	4029,84	4038,18	4049,75	---	830
Rel. Ampl.	2000	1280	9000	712	---	
ES	4018,07	4031,29	4038,54	4048,61	4061,86	1692
	1459,74	1384,55	2060,06	2795,55	1370,98	
H1	4017	4030,83	4040,37	4048,94	4058,78	1120
	728,96	479,8	2536,7	456,34	354,98	
H2	4018,07	4031,29	4038,54	4048,61	4061,86	1120
	1459,74	1384,55	2060,06	2795,55	1370,98	
M1	4016,59	4029,87	4039,28	4049,14	4061,12	1154
	528,41	618,9	2722,97	713,8	371,77	
MA	4012,19	4026,2	4038,26	4054,25	4066,39	1088
	761,83	565,64	3776,87	790,39	703,03	
MB	4013,97	4027,58	4040,6	4048,53	4065,39	802
	1296,67	1584,54	6211,77	1703,46	1562,1	
MC	4016,72	4029,15	4040,5	4047,06	4056,21	1360
	586,26	523,38	2277,11	917,88	376,72	
ME	4014,05	4022,49	4038,41	---	4058,35	1532
	104,21	107,82	661,95	---	106,99	
NY	4014,01	4027,83	4036,76	4046,98	4061,11	2232
	650	948,71	791,96	1131,72	596,34	
S1	---	4020,76	4035,77	---	---	692
	---	805,01	11374,52	---	---	
S2	4014,15	---	4035,87	4043,19	4061,35	762
	970,71	---	11157,94	2436,69	797,02	
ST	4016,13	---	4040,47	4047,72	4060,69	1104
	544,5	---	2476,76	842,26	312,02	
TC	4014,1	4021,02	4040,45	4047,52	4056,52	1190
	837,64	735,93	2787,65	556,2	351,93	
VI	4016,49	4026,1	4038,77	4048,84	---	1248
	425,53	184,46	2180,5	511,94	---	
W1	4013,64	4029,2	4039,5	---	4058,78	1248
	700	302,36	2181,17	---	359,1	
W2	4016,41	---	4039,4	4047,49	4061,53	1248
	516,19	---	2182,86	403,97	323,38	
WU	---	4021,12	4038,26	---	4068,32	1209
	---	1694,66	4500	---	513,35	

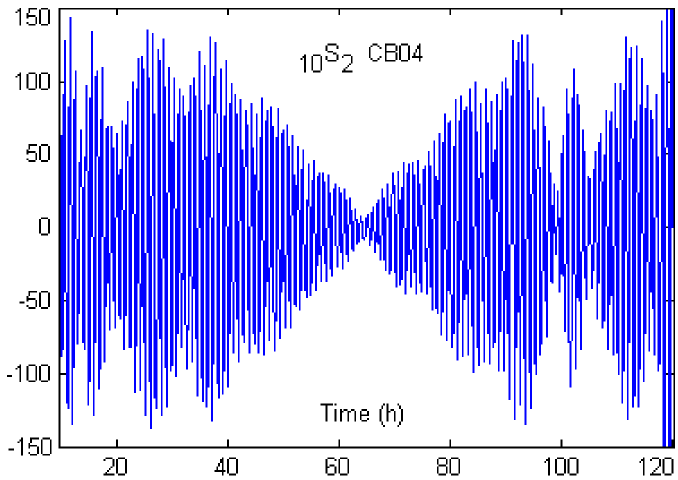
Since the data are sometimes very noisy, different methods were used to calculate the average of each table column.

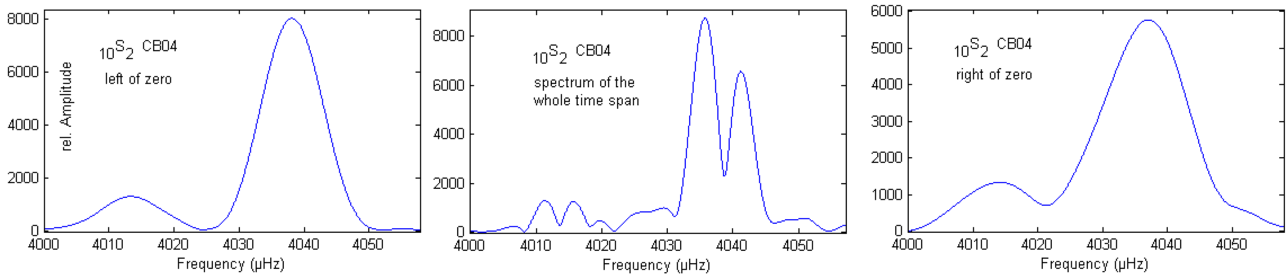
	m = -2	m = -1	m = 0	m = 1	m = 2
Unweighted average (μHz) (jackknife)	4015,31 ± 0.45	4026,97 ± 1.00	4038,77 ± 0.35	4048,33 ± 0.62	4061,22 ± 0.88
Unweighted without NY, S1 and S2	4015,49	4027,38	4039,3	4048,87	4061,22
Weighted with Amplitudes	4015,32	4027,17	4038,12	4047,86	4062,26
Weighted with Amplitudes, without S1 and S2	4015,64	4027,43	4039,3	4048,62	4062,35

The normal mode $_{10}S_2$ is splitted into a quintett. The frequency spacing between the center frequency and the two outer satellites (m = -2 and m = 2) is obviously equal and can be determined fairly accurately to be 23.16 ± 0.27 Hz. The frequencies of the two inner satellites can not be measured accurately and differ greatly (-11.6 μHz for m = -1 and +9.5μHz for m = +1). Whether this is caused by a frequency modulation like in $_0S_2$[3], has yet to be tested.

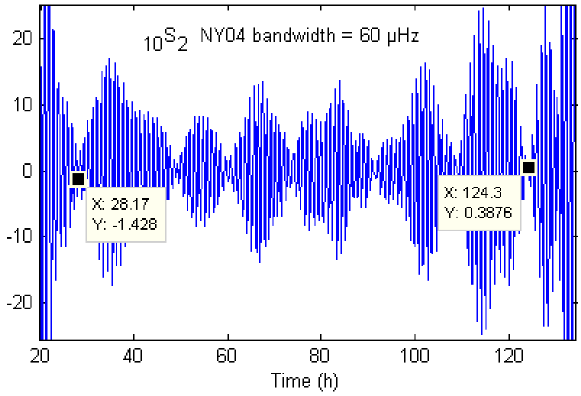
Special case CB

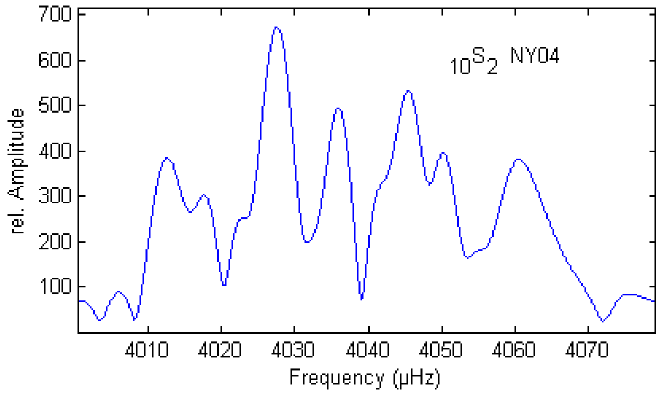
Choosing a *Q*-factor of 830, the chronological representation of the filtered data of station CB looks like a butterfly. The spectra left and right of the striking zero point are almost identical with the two peak frequencies 4013.5 μHz and 4038.2 μHz. But exactly at the zero point, the oscillations of both spectral lines undergo a phase shift of π, generating a minimum in the overall spectrum, where a maximum should be. Ignoring this phase jump within a data sequence, the Fourier analysis shows a frequency split, which does not exist in reality. The middle image in the figure below is misleading. Nevertheless, the FWHM decreases because of the extended sequence length.





Special case NY





The data stream of station NY was prepared as usual and the decline of the ${}_{10}S_2$ amplitude was compensated by multiplication with the time-dependent factor $\exp\left(\frac{\omega t}{2Q}\right)$. Choosing Q-factors between 2100 and 2500, the maximum amplitudes are approximately balanced. The narrowband filtered data stream of station NY differs from all others because of the regularly arranged series of minimums looks like a string of pearls. This amplitude modulation can arise only through the superposition of several frequencies and actually the spectrum shows the splitting of ${}_{10}S_2$ into a nearly symmetrical quintet.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[4].

[1] H. Weidner, A New method for High-resolution Frequency Measurements, http://viXra.org/abs/1506.0005
[2] H. Weidner, High-resolution Frequency Measurements of 0S0, http://vixra.org/abs/1504.0115
[3] H. Weidner, Frequency Modulation of 0S2-B, http://viXra.org/abs/1506.0129
[4] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html