Right-Handed Four-Fermion Condensation, LHC 750 GeV Diphoton Resonance, and Potential Dark Matter Candidate

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Abstract

We propose a Clifford algebra based model, which includes local gauge symmetries $SO(1,3) \otimes SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. There are two sectors of bosonic fields as electroweak and Majorana bosons. The electroweak boson sector is composed of scalar Higgs, pseudoscalar Higgs, and antisymmetric tensor components. The Majorana boson sector is responsible for flavor mixing and neutrino Majorana masses. The LHC 750 GeV diphoton resonance is identified as a Majorana sector quadruon, which is the pseudo-Nambu-Goldstone boson of $\bar{u}_R s_R \bar{c}_R d_R$ four-quark condensation. The quadruon results from spontaneous symmetry breaking of a flavor-related global $U(1)$ symmetry involving right-handed up, down, charm, and strange quarks. In addition to $\bar{u}_R s_R \bar{c}_R d_R$, four-fermion condensations can also involve three other right-handed configurations $\bar{u}_R \tau_R \bar{\nu}_{\tau R} d_R$, $\bar{t}_R e_R \bar{\nu}_{eR} b_R$, and $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$. Free from gauge interactions, these four-fermion condensations are potential dark matter candidates.

Keywords. Higgs bosons, diphoton resonance, quadruon, dark matter.

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1 Introduction

The experiments at LHC recently indicated a diphoton resonance at about 750 Gev[1, 2], in addition to the earlier finding of Higgs boson with $m_h = 125$ Gev[3, 4]. Scenarios with either an isospin singlet state or an isospin doublet state can not accommodate the observed signal and an extended particle content is necessary[5, 6, 7, 8, 9, 10, 11, 12, 13].

We propose a Clifford algebra based model which encompasses Yang-Mills interactions as well as gravity. The 750 GeV diphoton resonance corresponds to a pseudo-Nambu-Goldston boson of underlying all right-handed $\bar{u}_R s_R \bar{c}_R d_R$ four-quark condensation. No further extended particle content is needed.

In addition to $\bar{u}_R s_R \bar{c}_R d_R$, four-fermion condensations can also involve three other right-handed configurations $\bar{u}_R \tau_R \bar{\nu}_{\tau R} d_R, \bar{t}_R e_R \bar{\nu}_{eR} b_R$, and $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$. Free from gauge interactions, these four-fermion condensations are potential dark matter candidates.

With the purpose of studying 3 generations of standard model fermions, a ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The flavor projection operators facilitate flavor mixing via Majorana bosons.

The current paper is a continuation of our previous work[14], which is based on three premises. Firstly, both gravity and Yang-Mills interactions should be treated as gauge theories and integrated in a single overarching framework. The key is to take a page from effective field theory, where an infinite number of terms allowed by symmetry requirements should be included in a generalized action. Only the first few order terms of the action are relevant in low-energy limit.

The second premise is that all idempotent projections of the original algebraic spinor should be realized as fermions of physical world. In other words, no spinor projection should be casually discarded. Hence, finding the right Clifford algebra turns out to be a simple process of counting numbers of fermion species. There are 16 Weyl fermions (including right-handed neutrino) with $16 \times 4 = 64$ real components in one generation. Clifford algebra $C\ell_{0,6}$, with $2^6 = 64$ degrees of freedom, seems to be a natural choice.

The third premise is that rotations should be generalized. As well known in Clifford algebra approaches, a rotation is realized by a rotor, which is an exponential of bivectors. It rotates a vector into another vector. However, a rotor could be defined to be an exponential of any multivectors. It could rotate a vector into a multivector, generalizing definition of rotations. Hence, one can entertain large symmetry groups with lower dimensional Clifford algebras, whereas the same symmetry groups would otherwise require higher Clifford dimensions within the conventional framework. While the conventional Dirac matrix operators $\gamma_1, \gamma_2, \gamma_3$ correspond to vectors in $C\ell_{0,6}$, the conventional matrix operator γ_0 corresponds to a trivector $\gamma_0 = \Gamma_1\Gamma_2\Gamma_3$ in $C\ell_{0,6}$. Lorentz boost rotations are represented as exponentials of Clifford 4-vectors $\Gamma_1\Gamma_2\Gamma_3\gamma_1$, $\Gamma_1\Gamma_2\Gamma_3\gamma_2$, $\Gamma_1\Gamma_2\Gamma_3\gamma_3$.

This paper is structured as follows: Section 2 introduces binary Clifford algebra, gauge symmetries, and the action of the world. In section 3, an additional ternary Clifford algebra is defined. The Majorana boson sector, flavor mixing, dark matter candidates, and 750 Gev diphoton resonance are discussed. In section 4, we study electroweak boson sector.

In section 5, we touch upon the topic of grand unification. In the last section we draw our conclusions.

2 Gauge- and Diffeomorphism-Invariant Action

2.1 Clifford Algebra $C\ell_{0,6}$

We begin with a review of orthogonal Clifford algebra $C\ell_{0,6}$. It is defined by anticommutators of orthonormal vector basis $\{\gamma_j, \Gamma_j; j = 1, 2, 3\}$

$$[\gamma_j, \gamma_k] = \frac{1}{2}(\gamma_j\gamma_k + \gamma_k\gamma_j) = -\delta_{jk}, \tag{1}$$

$$[\Gamma_j, \Gamma_k] = -\delta_{jk}, \tag{2}$$

$$[\gamma_j, \Gamma_k] = 0, \tag{3}$$

where $j, k = 1, 2, 3$. All basis vectors are space-like. There are $\binom{6}{k}$ independent k-vectors. The complete basis for $C\ell_{0,6}$ is given by the set of all k-vectors. Any multivector can be expressed as a linear combination of $2^6 = 64$ basis elements.

Two trivectors

$$\gamma_0 = \Gamma_1\Gamma_2\Gamma_3, \tag{4}$$

$$\Gamma_0 = \gamma_1\gamma_2\gamma_3 \tag{5}$$

square to 1, so they are time-like. The orthonormal vector-trivector basis $\{\gamma_a, a = 0, 1, 2, 3\}$ defines space-time Clifford algebra $C\ell_{1,3}$, with

$$\eta_{ab} = \langle\gamma_a\gamma_b\rangle = \begin{pmatrix} +1, 0, 0, 0 \\ 0, -1, 0, 0 \\ 0, 0, -1, 0 \\ 0, 0, 0, -1 \end{pmatrix}, \tag{6}$$

where $\langle\cdots\rangle$ means scalar part of enclosed expression. The reciprocal vectors $\{\gamma^a\}$ are defined by

$$\gamma^a\eta_{ab} = \gamma_b, \tag{7}$$

thus

$$\langle\gamma^a\gamma_b\rangle = \delta^a_b. \tag{8}$$

Here we adopt the summation convention for repeated indices. Notice that γ_0 is a trivector, rather than a vector.

The unit pseudoscalar

$$i = \Gamma_1\Gamma_2\Gamma_3\gamma_1\gamma_2\gamma_3 = \gamma_0\gamma_1\gamma_2\gamma_3 = \gamma_0\Gamma_0 \tag{9}$$

squares to -1, anticommutes with odd-grade elements, and commutes with even-grade elements.

Reversion of a multivector $M \in C\ell_{0,6}$, denoted $\tilde{M}$, reverses the order in any product of vectors. For any multivectors M and N, there are algebraic properties

$$(MN)\tilde{} = \tilde{N}\tilde{M}, \tag{10}$$

$$\langle MN \rangle = \langle NM \rangle \, . \tag{11}$$

The magnitude of a multivector M is defined as

$$|M| = \sqrt{\langle M^\dagger M \rangle}, \tag{12}$$

where

$$M^\dagger = -i\tilde{M}i, \tag{13}$$

is the Hermitian conjugate.

2.2 Algeraic Spinor

Algebraic spinor $\psi \in C\ell_{0,6}$ is a multivector , which is expressed as a linear combination (with Grassmann-odd coefficients) of all $2^6 = 64$ basis elements.

Spinors with left/right chirality correspond to odd/even multivectors

$$\psi = \psi_L + \psi_R, \tag{14}$$

$$\psi_L = \frac{1}{2}(\psi + i\psi i), \tag{15}$$

$$\psi_R = \frac{1}{2}(\psi - i\psi i). \tag{16}$$

A projection operator squares to itself. Idempotents are a set of projection operators

$$P_l = \frac{1}{4}(1 + iJ_1 + iJ_2 + iJ_3) = \frac{1}{4}(1 + 3iJ), \tag{17}$$

$$P_{q1} = \frac{1}{4}(1 + iJ_1 - iJ_2 - iJ_3), \tag{18}$$

$$P_{q2} = \frac{1}{4}(1 - iJ_1 + iJ_2 - iJ_3), \tag{19}$$

$$P_{q3} = \frac{1}{4}(1 - iJ_1 - iJ_2 + iJ_3), \tag{20}$$

$$P_q = P_{q1} + P_{q2} + P_{q3} = \frac{3}{4}(1 - iJ), \tag{21}$$

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_0\Gamma_3), \tag{22}$$

where

$$J_1 = \gamma_1\Gamma_1, J_2 = \gamma_2\Gamma_2, J_3 = \gamma_3\Gamma_3, \tag{23}$$

$$J = \frac{1}{3}(J_1 + J_2 + J_3), \tag{24}$$

$$P_l + P_{q1} + P_{q2} + P_{q3} = P_l + P_q = 1, \tag{25}$$

$$P_a P_b = \delta_{ab}, \quad (a, b = l, q1, q2, q3), \tag{26}$$

$$P_+ + P_- = 1. \tag{27}$$

Here P_l is lepton projection operator, P_q is quark projection operator, and P_{qj} are color projection operators. The bivectors J_j appearing in the color projectors P_{qj} suggest an interesting duality between 3 space dimensions and 3 colors of quarks.

Now we are ready to identify idempotent projections of spinor

$$\psi = (P_+ + P_-)(\psi_L + \psi_R)(P_l + P_{q1} + P_{q2} + P_{q3}) \tag{28}$$

with left-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_L = P_+\psi_L P_l, \\ e_L = P_-\psi_L P_l, \\ u_L = P_+\psi_L P_{q1} + P_+\psi_L P_{q2} + P_+\psi_L P_{q3} = P_+\psi_L P_q, \\ d_L = P_-\psi_L P_{q1} + P_-\psi_L P_{q2} + P_-\psi_L P_{q3} = P_-\psi_L P_q, \end{cases} \tag{29}$$

and right-handed leptons, red, green, and blue quarks

$$\begin{cases} \nu_R = P_-\psi_R P_l, \\ e_R = P_+\psi_R P_l, \\ u_R = P_-\psi_R P_{q1} + P_-\psi_R P_{q2} + P_-\psi_R P_{q3} = P_-\psi_R P_q, \\ d_R = P_+\psi_R P_{q1} + P_+\psi_R P_{q2} + P_+\psi_R P_{q3} = P_+\psi_R P_q. \end{cases} \tag{30}$$

2.3 Gauge Symmetries

Spinors transform as

$$\psi_L \to e^{\Theta_{LOR} + \Theta_{WL}} \psi_L e^{\Theta_J - \Theta_{STR}}, \tag{31}$$

$$\psi_R \to e^{\Theta_{LOR} + \Theta_{WR}} \psi_R e^{\Theta_J - \Theta_{STR}}. \tag{32}$$

It is worth noting that all gauge transformations are with Grassmann-even rotation angles, so that the transformed spinors remain to be Grassmann-odd.

There are Lorentz $SO(1,3)$ gauge transformations

$$\{\gamma_a\gamma_b\} \in \Theta_{LOR}, (a, b = 0, 1, 2, 3, a \neq b), \tag{33}$$

weak isospin $SU(2)_L$ gauge transformations acting on left-handed fermions

$$\{\frac{1}{2}\Gamma_2\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_3, \frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WL}, \tag{34}$$

weak $U(1)_R$ gauge transformation acting on right-handed fermions

$$\{\frac{1}{2}\Gamma_1\Gamma_2\} \in \Theta_{WR}, \tag{35}$$

J $U(1)$ gauge transformation

$$\{\frac{1}{2}J\} \in \Theta_J, \tag{36}$$

and color $SU(3)$ gauge transformations

$$\left\{\begin{array}{l} T_1, T_2, T_3, \\ T_4, T_5, \\ T_6, T_7, \\ T_8 \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{4}(\gamma_1\Gamma_2 + \gamma_2\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_2 + \gamma_1\gamma_2), \frac{1}{4}(\Gamma_1\gamma_1 - \Gamma_2\gamma_2), \\ \frac{1}{4}(\gamma_1\Gamma_3 + \gamma_3\Gamma_1), \frac{1}{4}(\Gamma_1\Gamma_3 + \gamma_1\gamma_3), \\ \frac{1}{4}(\gamma_2\Gamma_3 + \gamma_3\Gamma_2), \frac{1}{4}(\Gamma_2\Gamma_3 + \gamma_2\gamma_3), \\ \frac{1}{4\sqrt{3}}(\Gamma_1\gamma_1 + \Gamma_2\gamma_2 - 2\Gamma_3\gamma_3) \end{array}\right\} \in \Theta_{STR}. \tag{37}$$

It is remarkable that the gauge groups contain both gravitational (Θ_{LOR}) and internal gauge transformations.

Because the product of lepton projector P_l with any generator in color algebra (37) is zero $P_l T_k = 0$, leptons are invariant under color gauge transformations.

After symmetry breaking of Θ_{WR}, Θ_{WL}, and Θ_J via Majorana and electroweak Higgs bosons, which will be detailed in later sections, the remaining electromagnetic $U(1)$ symmetry is a synchronized double-sided transformation

$$\psi \to e^{\frac{1}{2}\epsilon_E\Gamma_1\Gamma_2}\psi e^{\frac{1}{2}\epsilon_E J}, \tag{38}$$

where a shared rotation angle ϵ_E synchronizes the double-sided gauge transformation.

Thanks to the properties

$$JP_l = -iP_l, \tag{39}$$

$$JP_{qj} = \frac{1}{3}iP_{qj}, \tag{40}$$

$$\Gamma_1\Gamma_2 P_\pm = \mp iP_\pm, \tag{41}$$

electric charges q_k as in

$$e^{\frac{1}{2}\Gamma_1\Gamma_2}\psi_k e^{\frac{1}{2}J} = \psi_k e^{q_k i} \tag{42}$$

are calculated as $q_k = 0, -1, \frac{2}{3}$, and $-\frac{1}{3}$ for neutrino, electron, up quarks, and down quarks, respectively.

2.4 Gauge Field 1-Forms, Gauge-Covariant Derivatives, and Curvature 2-Forms

Gauge fields are Clifford-valued 1-forms (Clifforms with Grassmann-even coefficients) on 4-dimensional space-time manifold (x_μ, $\mu = 0, 1, 2, 3$)

$$e = e_\mu dx^\mu = e^a_\mu \gamma_a dx^\mu, \tag{43}$$

$$\omega = \omega_\mu dx^\mu = \frac{1}{4}\omega^{ab}_\mu \gamma_a\gamma_b dx^\mu \quad \in \quad \Theta_{LOR}, \tag{44}$$

$$W_L = W_{L\mu} dx^\mu = \frac{1}{2}(W^1_{L\mu}\Gamma_2\Gamma_3 + W^2_{L\mu}\Gamma_1\Gamma_3 + W^3_{L\mu}\Gamma_1\Gamma_2)dx^\mu \quad \in \quad \Theta_{WL}, \tag{45}$$

$$W_R = W_{R\mu} dx^\mu = \frac{1}{2}W^3_{R\mu}\Gamma_1\Gamma_2 dx^\mu \quad \in \quad \Theta_{WR}, \tag{46}$$

$$C = C_\mu dx^\mu = \frac{1}{2}C^J_\mu J dx^\mu \quad \in \quad \Theta_J, \tag{47}$$

$$G = G_\mu dx^\mu = G^k_\mu T_k dx^\mu \quad \in \quad \Theta_{STR}, \tag{48}$$

where e is vierbein, ω is gravity spin connection, G is strong interaction, and the rest are electroweak related interactions. Notice that we adopt the same notation for vierbein e , mathematical number e, and electron e. One should be able to differentiate them based on contexts.

The vierbein field e acts like space-time frame field, which is essential in building all actions as diffeomorphism-invariant integration of 4-forms on 4-dimensional space-time manifold. The space-time manifold is initially without metric. It's the vierbein field which gives notion to metric

$$g_{\mu\nu} = \langle e_\mu e_\nu \rangle = e^a_\mu e^b_\nu \eta_{ab}. \tag{49}$$

Local gauge transformations are coordinate-dependent gauge transformations. Gauge fields obey local gauge transformation laws

$$e(x) \quad \to \quad e^{\Theta_{LOR}(x)} e(x) e^{-\Theta_{LOR}(x)}, \tag{50}$$

$$\omega(x) \quad \to \quad e^{\Theta_{LOR}(x)} \omega(x) e^{-\Theta_{LOR}(x)} - (de^{\Theta_{LOR}(x)})e^{-\Theta_{LOR}(x)}, \tag{51}$$

$$W_L(x) \quad \to \quad e^{\Theta_{WL}(x)} W_L(x) e^{-\Theta_{WL}(x)} - (de^{\Theta_{WL}(x)})e^{-\Theta_{WL}(x)}, \tag{52}$$

$$W_R(x) \quad \to \quad W_R(x) - (de^{\Theta_{WR}(x)})e^{-\Theta_{WR}(x)}, \tag{53}$$

$$C(x) \quad \to \quad C(x) - e^{-\Theta_J(x)}(de^{\Theta_J(x)}), \tag{54}$$

$$G(x) \quad \to \quad e^{\Theta_{STR}(x)} G(x) e^{-\Theta_{STR}(x)} + e^{\Theta_{STR}(x)}(de^{-\Theta_{STR}(x)}), \tag{55}$$

where $d = dx^\mu \partial_\mu$.

It's worth emphasizing that gravity related fields $e(x)$ and $\omega(x)$ are treated as gauge fields with local gauge transformation properties, as the rest Yang-Mills gauge fields.

Gauge-covariant derivatives of spinor fields $\psi_{L/R}(x)$ are defined by

$$D\psi_L = (d + \omega + W_L)\psi_L + \psi_L(C - G), \tag{56}$$

$$D\psi_R = (d + \omega + W_R)\psi_R + \psi_R(C - G). \tag{57}$$

The gravitational spin connection ω is essential in maintaining *local* Lorentz covariance of $D\psi_{L/R}$.

We introduce gauge curvature 2-forms by applying the covariant derivative to the 0-form spinor ψ and then to the 1-form spinor $D\psi$

$$D(D\psi_{L/R}) = (d + \omega + W_{L/R})D\psi_{L/R} - D\psi_{L/R}(C - G) \tag{58}$$
$$= (R + F_{WL/WR})\psi_{L/R}(F_J - F_{STR}), \tag{59}$$

where gravity, left/right weak, J, and strong force curvature 2-forms are

$$R = d\omega + \omega^2 = \frac{1}{2}R_{\mu\nu}dx^\mu dx^\nu, \tag{60}$$
$$F_{WL} = dW_L + W_L^2 = \frac{1}{2}F_{WL\mu\nu}dx^\mu dx^\nu, \tag{61}$$
$$F_{WR} = dW_R = \frac{1}{2}F_{WR\mu\nu}dx^\mu dx^\nu, \tag{62}$$
$$F_J = dC = \frac{1}{2}F_{J\mu\nu}dx^\mu dx^\nu, \tag{63}$$
$$F_{STR} = dG + G^2 = \frac{1}{2}F_{STR\mu\nu}dx^\mu dx^\nu. \tag{64}$$

$F^{\mu\nu k}$ is defined by

$$F^{\mu\nu k}\eta_{\mu\alpha}\eta_{\nu\beta} = F^k_{\alpha\beta}, \tag{65}$$

where k enumerates the Clifford components of each gauge field.

Notice that the connection fields are defined to absorb gauge coupling constants. As a result, gauge coupling constants neither appear in the definition of gauge-covariant derivatives of fermions $D\psi_{L/R}$, nor appear in the gauge curvature 2-forms such as $F_{WL} = dW_L + W_L^2$. They will show up in the gauge field actions instead.

2.5 Gauge- and Diffeomorphism-Invariant Action of the World

The local gauge- and diffeomorphism-invariant action of the world is

$$S_{World} = S_{Spinor-Kinetic} \tag{66}$$
$$+ S_{Gravity} + S_{Yang-Mills} \tag{67}$$
$$+ S_{Majorana-Yukawa} + S_{Majorana-Bosons} \tag{68}$$
$$+ S_{Electroweak-Yukawa} + S_{Electroweak-Bosons}. \tag{69}$$

The spinor kinetic action is now written down as

$$S_{Spinor-Kinetic} \sim \int \langle \bar{\psi}_L ie^3 D\psi_L + \bar{\psi}_R ie^3 D\psi_R \rangle, \tag{70}$$

where e^3 is vierbein 3-form, and $\bar{\psi}_{L/R}$ are defined as

$$\bar{\psi}_{L/R} = \psi^{\dagger}_{L/R}\gamma_0 = -i\tilde{\psi}_{L/R}i\gamma_0 = \mp\tilde{\psi}_{L/R}\gamma_0. \tag{71}$$

Here outer products between differential forms are implicitly assumed.

One can write down the action for gravity as

$$S_{Gravity} \sim \int \left\langle ie^2(R + \frac{\Lambda}{24}e^2) \right\rangle, \tag{72}$$

where e^2 is vierbein 2-form, $R = d\omega + \omega^2$ is spin connection curvature 2-form, and Λ is cosmological constant.

The Yang-Mills action is written as

$$S_{Yang-Mills} = S_{WL} + S_{WR} + S_J + S_{STR}, \tag{73}$$

$$S_{WL} \sim \int \left\langle (e^2 F_{WL})^2 \right\rangle / \left\langle ie^4 \right\rangle, \tag{74}$$

$$S_{WR} \sim \int \left\langle (e^2 F_{WR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \tag{75}$$

$$S_J \sim \int \left\langle (e^2 F_J)^2 \right\rangle / \left\langle ie^4 \right\rangle, \tag{76}$$

$$S_{STR} \sim \int \left\langle (e^2 F_{STR})^2 \right\rangle / \left\langle ie^4 \right\rangle, \tag{77}$$

where e^4 is vierbein 4-form.

The Clifford algebra elements, which are related to left-(e, ω, W_L, W_R) and right-(C, G)sided gauge fields, are formally assigned to two sets of Clifford algebras in Yang-Mills action (and other actions without spinor fields). Elements from different sets formally commute with each other. Here $\langle \cdots \rangle$ means scalar part of both sets.

It's understood that 4-form factor d^4x in one of e^2F in each Yang-Mills term should be canceled out by 4-form factor d^4x in the denominator before any further outer multiplication of differential forms as

$$\int \left\langle \frac{e^2 F}{\langle ie^4 \rangle} e^2 F \right\rangle. \tag{78}$$

In this way, the Yang-Mills action is a diffeomorphism-invariant integration of 4-form on 4-dimensional space-time manifold.

There is no explicit Hodge dual in Yang-Mills action. Vierbein plays the role of Hodge dual, when it acquires nonzero vacuum expectation value (VEV) in the case of flat space-time, which will be discussed in next section.

Yukawa and boson portions of the action will be subjects of later chapters.

2.6 Local Lorentz Symmetry Breaking and Minkowskian Space-time

Up to this point, the action of the world is constructed in *curved* space-time, with space-time dependent vierbein and spin connection. In a vacuum with zero cosmological constant $\Lambda = 0$, vierbein field e acquires a nonzero Minkowskian flat space-time VEV

$$< 0|e|0 >= \delta_\mu^a \gamma_a dx^\mu = \gamma_\mu dx^\mu, \tag{79}$$

while VEV of spin connection is zero

$$< 0|\omega|0 >= 0. \tag{80}$$

The space-time metric reduces to

$$g_{\mu\nu} = \langle e_\mu e_\nu \rangle = \eta_{\mu\nu}. \tag{81}$$

The soldering form $\gamma_\mu dx^\mu$ breaks the independent *local* Lorentz gauge invariance and diffeomorphism invariance. The action of the world is left with a residual *global* Lorentz symmetry, with synchronized Clifford space and x coordinate space global Lorentz rotations. Actually the specific VEV form $\gamma_\mu dx^\mu$ is a result of coordinating the above two kinds of global rotations.

With the substitution of vierbein and spin connection with their VEVs, the spinor kinetic action(70) in flat Minkowskian space-time can be rewritten as

$$S_{Spinor-Kinetic} = \int \left\langle \bar{\psi}_L \gamma^\mu D_\mu \psi_L + \bar{\psi}_R \gamma^\mu D_\mu \psi_R \right\rangle d^4x, \tag{82}$$

where

$$D_\mu \psi_{L/R} = (\partial_\mu + W_{L/R\mu})\psi_{L/R} + \psi_{L/R}(C_\mu - G_\mu). \tag{83}$$

Similarly, the Yang-Mills action(73) can be rewritten as

$$\begin{aligned} S_{Yang-Mills} = & -\frac{1}{4g_{WL}^2}\int F_{WL\mu\nu}^k F_{WL}^{\mu\nu k} d^4x && (84)\\ & -\frac{1}{4g_{WR}^2}\int F_{WR\mu\nu} F_{WR}^{\mu\nu} d^4x && (85)\\ & -\frac{1}{4g_J^2}\int F_{J\mu\nu} F_J^{\mu\nu} d^4x && (86)\\ & -\frac{1}{4g_{STR}^2}\int F_{STR\mu\nu}^k F_{STR}^{\mu\nu k} d^4x, && (87) \end{aligned}$$

where g_{WL}, g_{WR}, g_J, and g_{STR} are dimensionless gauge coupling constants.

In the following chapters, however, we will stay with local Lorentz gauge invariant curved space-time formulation.

2.7 Relation to Conventional Matrix Formulation

A map [14] can be constructed by placing the Dirac column spinor $\hat{\psi}$ in one-to-one correspondence with the algebraic spinor ψ. And the mappings for the operators are

$$\hat{\gamma}^{\mu}\hat{\psi} \leftrightarrow \gamma^{\mu}\psi, (\mu = 0, 1, 2, 3) \tag{88}$$

$$\hat{i}\hat{\psi} \leftrightarrow \psi i, \tag{89}$$

$$\hat{\gamma}^{5}\hat{\psi} \leftrightarrow -i\psi i \tag{90}$$

where $\hat{i}$ is the conventional unit imaginary number, and $\hat{\gamma}^{\mu}$ and $\hat{\gamma}^{5}$ are the Dirac matrix operators.

We will not go into the details of further mappings in this paper.

3 Majorana Bosons, Flavor Structure, and 750 Gev Diphoton Resonance

3.1 Ternary Clifford Algebra and Flavor Projection Operators

With the purpose of studying 3 generations of fermions, we turn to another kind of Clifford algebra involving ternary communication relationships rather than the usual binary ones. Let's consider ternary $C\ell_{T1}$, which is defined by

$$[\zeta, \zeta, \zeta] = \zeta^3 = 1, \tag{91}$$

with ζ commuting with $C\ell_{0,6}$

$$\zeta\gamma_j - \gamma_j\zeta = 0, \tag{92}$$

$$\zeta\Gamma_j - \Gamma_j\zeta = 0. \tag{93}$$

Flavor projection operators are define by

$$P_1 = \frac{1}{3}(1 + e^{\sigma'+\sigma}\zeta + e^{-\sigma'-\sigma}\zeta^2) \tag{94}$$

$$= \frac{1}{3}P_l(1 + \zeta + \zeta^2) + \frac{1}{3}P_q(1 + e^{-\sigma}\zeta + e^{\sigma}\zeta^2), \tag{95}$$

$$P_2 = \frac{1}{3}(1 + e^{\sigma'}\zeta + e^{-\sigma'}\zeta^2) \tag{96}$$

$$= \frac{1}{3}P_l(1 + e^{-\sigma}\zeta + e^{\sigma}\zeta^2) + \frac{1}{3}P_q(1 + e^{\sigma}\zeta + e^{-\sigma}\zeta^2), \tag{97}$$

$$P_3 = \frac{1}{3}(1 + e^{\sigma'-\sigma}\zeta + e^{-\sigma'+\sigma}\zeta^2) \tag{98}$$

$$= \frac{1}{3}P_l(1 + e^{\sigma}\zeta + e^{-\sigma}\zeta^2) + \frac{1}{3}P_q(1 + \zeta + \zeta^2), \tag{99}$$

where

$$P_1 + P_2 + P_3 = 1, \tag{100}$$

$$P_j P_k = \delta_{jk}, \quad (j, k = 1, 2, 3), \tag{101}$$

$$\sigma = \frac{2\pi}{3} i, \sigma' = \frac{2\pi}{3} i', \tag{102}$$

$$i' = \frac{1}{2}(i + 3J), i'^2 = -1, \tag{103}$$

and P_l and P_q are lepton and quark projection operators, respectively.

We label 3 generations of spinors as $\psi_{L/Rj}$. They are valued in $C\ell_{0,6}$. The spinor kinetic action involves 3 families of fermions as

$$S_{Spinor-Kinetic} \sim \int \left\langle \bar{\psi}_{Lj} i e^3 D \psi_{Lj} P_j + \bar{\psi}_{Rj} i e^3 D \psi_{Rj} P_j \right\rangle. \tag{104}$$

Here $\langle \cdots \rangle$ means scalar part of both $C\ell_{0,6}$ and $C\ell_{T1}$. There is flavor-mixing cross term in kinetic action. Flavor mixing is the subject of next section. It is induced via Majorana Boson fields.

3.2 Yukawa Action and Flavor Mixing

Fields in Majorana boson section interact with right-handed fermions only. The Lorentz, isospin, and color singlet Majorana boson section contains two fields

$$\phi_{MAJ} = \phi^\nu + \Phi. \tag{105}$$

The neutrino Higgs field $\phi^\nu = \phi^{\nu\dagger}$ is valued in Clifford space spanned by 2 trivectors

$$\{\Gamma_0 P_l, i\Gamma_0 P_l\}. \tag{106}$$

It obeys gauge transformation rules

$$\phi^\nu \quad \rightarrow \quad e^{-\check{\Theta}_{WR} - \Theta_J} \phi^\nu e^{\check{\Theta}_{WR} + \Theta_J}, \tag{107}$$

where

$$\check{\Theta}_{WR} = \frac{1}{2} \epsilon_{WR} i \tag{108}$$

shares rotation angle ϵ_{WR} with

$$\Theta_{WR} = \frac{1}{2} \epsilon_{WR} \Gamma_1 \Gamma_2. \tag{109}$$

Boson field

$$\Phi = \Phi_{12} + \Phi_{13} + \Phi_{13B} + \Phi_{23} \tag{110}$$

is valued in Clifford space spanned by scalar and pseudoscalar

$$\{1, i\}. \tag{111}$$

It is invariant under all gauge interaction transformations. It plays an essential role in LHC 750 Gev diphoton resonance, and is a potential dark matter candidate.

We can write Majorana Yukawa action of right-handed fermions as

$$
\begin{aligned}
S_{Majorana-Yukawa} \quad \sim \quad & y_{11} \int \left\langle \phi^{\nu} P_1 \bar{\nu}_{R1} e^4 e^{\varepsilon_{11}\Gamma_1\Gamma_2} \Gamma_2 \Gamma_3 \nu_{R1} P_1 \right\rangle & (112)\\
& + y_{23} \int \left\langle \phi^{\nu} P_2 \bar{\nu}_{R2} e^4 e^{\varepsilon_{23}\Gamma_1\Gamma_2} \Gamma_2 \Gamma_3 \nu_{R3} P_3 \right\rangle + h.c. & (113)\\
& + Y_{12} \int \left\langle \Phi_{12} P_1 \bar{u}_{R1} e^4 d_{R2} P_2 \bar{u}_{R2} e^4 d_{R1} \right\rangle / \left\langle i e^4 \right\rangle + h.c. & (114)\\
& + Y_{13A} \int \left\langle \Phi_{13A} P_1 \bar{u}_{R1} e^4 e_{R3} P_3 \bar{\nu}_{R3} e^4 d_{R1} \right\rangle / \left\langle i e^4 \right\rangle + h.c. & (115)\\
& + Y_{13B} \int \left\langle \Phi_{13B} P_3 \bar{u}_{R3} e^4 e_{R1} P_1 \bar{\nu}_{R1} e^4 d_{R3} \right\rangle / \left\langle i e^4 \right\rangle + h.c. & (116)\\
& + Y_{23} \int \left\langle \Phi_{23} P_2 \bar{\nu}_{R2} e^4 e_{R3} P_3 \bar{\nu}_{R3} e^4 e_{R2} \right\rangle / \left\langle i e^4 \right\rangle + h.c., & (117)
\end{aligned}
$$

where y_{jk} and Y_{jk} are Majorana Yukawa coupling constants, and $e^{\varepsilon_{jk}\Gamma_1\Gamma_2}$ are phase factors.

There are four fermions in the Yukawa terms of Φ_{jk}, different from Higgs boson ϕ^{ν}, which interacts with two fermions. The four-fermion Yukawa coupling constants Y_{jk} are of mass dimension -3. Thus it is nonrenormalizable. A later section will discuss the effective theory point of view and the issue of nonrenormalizability.

Since $e^{\frac{2\pi}{3}i}$ phases in flavor projection operators anticommute with Clifford odd fields, there are properties

$$
\begin{aligned}
P_1 \phi^{\nu} &= \phi^{\nu} P_1, & (118)\\
P_2 \phi^{\nu} &= \phi^{\nu} P_3, & (119)\\
P_3 \phi^{\nu} &= \phi^{\nu} P_2, & (120)
\end{aligned}
$$

according to the definition of flavor projection operators (95, 97, 99). Therefore, there are flavor-mixing Yukawa terms between 2nd and 3rd generation neutrinos.

Likewise, allowable flavor-changing four-fermion Yukawa terms are also dictated by the properties of flavor projection operators. Additionally, we require that the four-fermion combination should accommodate a global $U(1)$ symmetry, which will be studies in next section. The four $\Phi_{12}, \Phi_{13A}, \Phi_{13B}$, and Φ_{23} Yukawa terms, with fermion configurations $\bar{u}_R s_R \bar{c}_R d_R$, $\bar{u}_R \tau_R \bar{\nu}_{\tau R} d_R$, $\bar{t}_R e_R \bar{\nu}_{eR} b_R$, and $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$, are the ones satisfying both conditions.

After ϕ^{ν} and Φ acquire nonzero VEVs, which will be investigated in later section, the flavor mixing between right-handed fermions is represented by neutrino Majorana mass terms and four-fermion interaction terms. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

3.3 Flavor-Related Global $U(1)$ Symmetry, Quadruon, and Dark Matter

As mentioned earlier, Φ boson is invariant under all gauge transformations related to gauge interactions. Nevertheless, for Φ_{12} there is a flavor-related *global* $U(1)$ symmetry under the following transformations

$$\Phi_{12} \quad \rightarrow \quad \Phi_{12}e^{\theta_{12}i}, \tag{121}$$

$$u_1 = u \quad \rightarrow \quad ue^{\theta_u i}, \tag{122}$$

$$d_1 = d \quad \rightarrow \quad de^{\theta_d i}, \tag{123}$$

$$u_2 = c \quad \rightarrow \quad ce^{\theta_c i}, \tag{124}$$

$$d_2 = s \quad \rightarrow \quad se^{\theta_s i}, \tag{125}$$

where

$$\theta_{12} = (\theta_u - \theta_d) - (\theta_c - \theta_s), \tag{126}$$

and u, d, c, and s are up, down, charm, and strange quarks. The phase θ_{12} measures rotation angle difference between first and second generation quarks in $\bar{u}_R s_R \bar{c}_R d_R$.

Because of

$$\bar{u}_R = u_R^\dagger \gamma_0, \tag{127}$$

$$\bar{c}_R = u_R^\dagger \gamma_0, \tag{128}$$

moving a phase factor θi around Clifford-odd γ_0 changes its sign as

$$\theta i u_R^\dagger \gamma_0 = u_R^\dagger \gamma_0 (-\theta i), \tag{129}$$

$$\theta i c_R^\dagger \gamma_0 = c_R^\dagger \gamma_0 (-\theta i). \tag{130}$$

$$\tag{131}$$

This is the reason of the specific sign for each θ in θ_{12}.

By the same token, $\bar{u}_R \tau_R \bar{\nu}_{\tau R} d_R$, $\bar{t}_R e_R \bar{\nu}_{eR} b_R$, and $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$ correspond to phases

$$\theta_{13A} = (\theta_u - \theta_d) - (\theta_{\nu_\tau} - \theta_\tau), \tag{132}$$

$$\theta_{13B} = (\theta_t - \theta_b) - (\theta_{\nu_e} - \theta_e), \tag{133}$$

$$\theta_{23} = (\theta_{\nu_\mu} - \theta_\mu) - (\theta_{\nu_\tau} - \theta_\tau). \tag{134}$$

In the event of spontaneous symmetry breaking (SSB), there will be a massive sigma mode and a massless Nambu-Goldstone mode for each Φ_{ij}. As opposed to the Higgs mechanism, the Nambu-Goldstone mode is not 'eaten' by gauge field.

Notice that above global symmetry is an approximate symmetry. The electroweak section spoils the symmetries $\theta_{12}, \theta_{13A}, \theta_{13B}$, and θ_{23} explicitly. And the neutrino Higgs field ϕ^ν Yukawa terms do not respect symmetries $\theta_{13A}, \theta_{13B}$, and θ_{23}. The Nambu-Goldstone modes are not exactly massless. The size of the masses grow with the strength of the explicit symmetry breaking. A not-quite-massless would-be Nambu-Goldstone particle for an approximate symmetry is often called a pseudo-Nambu-Goldstone (pNG) boson. We call these flavor-related pNG bosons quadruons, as they represent phase differences between four different fermions from two generations.

Electroweak Yukawa coupling constants of quarks are usually larger than leptons. If Majorana Yukawa coupling constants follow the same pattern, Y_{12} might be the largest and Y_{23} might be the smallest.

Since boson fields Φ_{ij} are free from gauge interactions, they are potential dark matter candidates. Φ_{23} might be the prime candidate due to its small Yukawa coupling constant, which means suppressed decaying rate.

Given that the Yukawa term with Y_{12} is the dominant one, in the following analysis we will concentrate on Φ_{12}, and treat the model as if $\Phi = \Phi_{12}$. The treatment of other fields Φ_{13A}, Φ_{13B}, and Φ_{23} should follow the same logic as Φ_{12}. Going forward, we will not explicitly write down formulas for Φ_{13A}, Φ_{13B}, and Φ_{23}.

3.4 Symmetry Breaking and Majorana Masses

Majorana Boson action reads

$$S_{Majorana-Bosons} = S_{Majorana-Kenetic} - V_{Majorana}, \tag{135}$$

with

$$S_{Majorana-Kenetic}(\phi^\nu) \sim \int \left\langle (e^3 D\phi^{\nu\dagger})(e^3 D\phi^\nu) \right\rangle / \left\langle ie^4 \right\rangle, \tag{136}$$

$$V_{Majorana-Bosons}(\phi^\nu, -\mu_\nu^2, \lambda_\nu) \sim \int \left(-\mu_\nu^2 |\phi^\nu|^2 + \lambda_\nu |\phi^\nu|^4\right) \left\langle ie^4 \right\rangle, \tag{137}$$

and

$$S_{Majorana-Kenetic}(\Phi) \sim \int \left\langle (e^3 D\Phi^\dagger)(e^3 D\Phi) \right\rangle / \left\langle ie^4 \right\rangle, \tag{138}$$

$$V_{Majorana-Bosons}(\Phi, -\mu_\Phi^2, \lambda_\Phi) \sim \int \left(-\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4\right) \left\langle ie^4 \right\rangle, \tag{139}$$

where

$$D\phi^{\nu} = (d - \check{W}_R - C)\phi^{\nu} + \phi^{\nu}(\check{W}_R + C), \tag{140}$$

$$D\Phi = d\Phi, \tag{141}$$

$$\check{W}_R = \check{W}_{R\mu}dx^{\mu} = \frac{1}{2}W^3_{R\mu}idx^{\mu}. \tag{142}$$

Notice that ϕ^{ν} and Φ have negative $-\mu_{\nu}^2$ and $-\mu_{\Phi}^2$. It means that ϕ^{ν} and Φ acquire nonzero VEVs as

$$< 0|\phi^{\nu}|0 >= \frac{1}{\sqrt{2}}\upsilon_{\nu}\Gamma_0 P_l e^{\alpha i} = \frac{1}{\sqrt{2}}\frac{\mu_{\nu}}{\sqrt{\lambda_{\nu}}}\Gamma_0 P_l e^{\alpha i}, \tag{143}$$

$$< 0|\Phi|0 >= \frac{1}{\sqrt{2}}\upsilon_{\Phi}e^{\alpha_{12}i} = \frac{1}{\sqrt{2}}\frac{\mu_{\Phi}}{\sqrt{\lambda_{\Phi}}}e^{\alpha_{12}i}, \tag{144}$$

As a result, the gauge symmetry related to gauge field

$$Z'_{\mu} = W^3_{R\mu} - C^J_{\mu}, \tag{145}$$

and the global symmetry of Φ are spontaneously broken. Notice that the minus sign in above equation stems from the fact that $JP_l = -iP_l$.

After replacing ϕ^{ν} and Φ with their VEVs, the Majorana Yukawa action reduces to

$$S_{Majorana-Yukawa} \sim \int \left\langle m_{11}P_1\bar{\nu}_{R1}e^4\Gamma_2\Gamma_3\nu_{R1}P_1\Gamma_0 P_l\right\rangle, \tag{146}$$

$$+\int \left\langle m_{23}P_2\bar{\nu}_{R2}e^4\Gamma_2\Gamma_3\nu_{R3}P_3\Gamma_0 P_l\right\rangle + h.c., \tag{147}$$

$$+\frac{1}{\sqrt{2}}Y_{12}\upsilon_{\Phi}\int \left\langle e^{\alpha_{12}i}P_1\bar{u}_{R1}e^4 d_{R2}P_2\bar{u}_{R2}e^4 d_{R1}\right\rangle / \left\langle ie^4\right\rangle + h.c., \tag{148}$$

with Majorana masses

$$m_{11} = \frac{1}{\sqrt{2}}y_{11}\upsilon_{\nu}e^{(\alpha+\varepsilon_{11})i}, \tag{149}$$

$$m_{23} = \frac{1}{\sqrt{2}}y_{23}\upsilon_{\nu}e^{(\alpha+\varepsilon_{23})i}. \tag{150}$$

Neutrino Majorana masses are much heavier than neutrino Dirac masses, if we assume

$$y_{jk}\upsilon_{\nu} >> y^{\nu}\upsilon, \tag{151}$$

where constants y^{ν} and υ are electroweak Higgs counterparts, which will be defined in later section. Because of the hierarchy, very small effective masses are generated for neutrinos, known as seesaw mechanism.

Now we express gauge fields W_R^3 and C^J in terms of B and Z'

$$W_{R\mu}^3 = B_\mu + (cos\theta'_W)^2 Z'_\mu, \tag{152}$$
$$C_\mu^J = B_\mu - (sin\theta'_W)^2 Z'_\mu, \tag{153}$$

where

$$cos\theta'_W = \frac{g_{WR}}{g_{Z'}}, \tag{154}$$
$$sin\theta'_W = \frac{g_J}{g_{Z'}}, \tag{155}$$
$$g_{Z'} = \sqrt{g_{WR}^2 + g_J^2}. \tag{156}$$

Gauge field B remains massless with an effective coupling of

$$g_B = \frac{g_{WR} g_J}{g_{Z'}}, \tag{157}$$

while gauge field Z' acquires a mass from neutrino part of the Majorana Kinetic action

$$M_{Z'} = \frac{1}{2} \upsilon_\nu g_{Z'}. \tag{158}$$

Higgs boson ϕ^ν and the sigma mode of Φ acquire masses

$$m_{h\nu} = \sqrt{2}\mu_\nu, \tag{159}$$
$$m_\Phi = \sqrt{2}\mu_\Phi. \tag{160}$$

3.5 LHC 750 Gev Diphoton Resonance from Quadruon

If we assume that $\upsilon_\nu >> \upsilon$ and $\upsilon_\Phi >> \upsilon$, gauge boson Z', Higgs boson ϕ^ν, and sigma mode of Φ would be too heavy to be detected at electroweak energy scale. On the other hand, the pNG quadruon of Φ_{12} is not exactly massless, since the electroweak sector explicitly breaks the global symmetry and can generate mass for it. The size of the Φ_{12} quadruon mass is proportional to electroweak scale. Hence it is detectable at LHC.

The LHC 750 GeV diphoton resonance is identified as the quadruon, which is the pNG boson of four-quark condensation $\bar{u}_R s_R \bar{c}_R d_R$. The quadruon results from SSB of the flavor-related global $U(1)$ symmetry involving first and second generation quarks.

A resonance starts with four quarks (two quark-antiquark pairs) produced by two gluons. Two of the quarks interact with VEV $< 0|\Phi_{12}|0 >$ and turn into other two quarks. And then the four quarks turn into a quadruon via the four-fermion Yukawa term. The quadruon propagates and finally decays in a reverted generation process. The reverted process has two external photon lines, instead of two external gluon lines.

The width of the resonance might be due to involvement of other two bosons Φ_{13A} and Φ_{13B}. The weaker resonance of Φ_{13A} (Φ_{13B}) also starts with two quark-antiquark pairs produce by two gluons. Two of the quarks interact with VEV $< 0|\Phi_{13A}|0 >$ ($< 0|\Phi_{13B}|0 >$) and turn into two leptons. And then the four fermions turn into Φ_{13A} (Φ_{13B}) quadruon mode. The width of the 750 Gev resonance might be due to the addition of weaker resonance signals coming from Φ_{13A} and Φ_{13B} quadruons with different masses from Φ_{12}.

3.6 Four-Fermion Condensation

Boson sectors might be just an effective Ginzbrug-Landay-type description of the low energy physics represented by composite boson fields. One approach is to assume effective four-quark interactions strong enough to induce top quark-antiquark condensation into composite electroweak Higgs fields[15, 16, 17], via dynamical symmetry breaking mechanism in Nambu-Jona-Lasinio[18] (NJL) like models.

The four-quark contact term in top condensation model is

$$\int \left\langle \bar{q}_L e q_L \bar{t}_R e^3 t_R \right\rangle, \tag{161}$$

where $q_L = t_L + b_L$, e and e^3 are vierbein 1-form and 3-form.

Likewise, the Majorana boson fields are also collective excitations of underlying composite spinors. For example, ϕ^ν and Φ are effective representation of two-neutrino and four-quark condensations

$$\check{\phi}^\nu = y_{11} P_1 \bar{\nu}_{R1} e^4 \Gamma_2 \Gamma_3 \nu_{R1} P_1, \tag{162}$$

$$+ y_{23} P_2 \bar{\nu}_{R2} e^4 \Gamma_2 \Gamma_3 \nu_{R3} P_3 + h.c., \tag{163}$$

$$\check{\Phi} = Y_{12} P_1 \bar{u}_R e^4 s_R P_2 \bar{c}_R e^4 d_R / \left\langle ie^4 \right\rangle + h.c.. \tag{164}$$

The four-neutrino and eight-quark interactions are

$$\int \left\langle \check{\phi}^{\nu\dagger} \check{\phi}^\nu \right\rangle / \left\langle ie^4 \right\rangle + \int \left\langle \check{\Phi}^\dagger \check{\Phi} \right\rangle / \left\langle ie^4 \right\rangle. \tag{165}$$

A collective mode is determined as the pole of bosonic channel of the four-fermion interaction by summing to infinite order chains of bubble perturbation diagrams. The leading order calculation goes by different names such as random-phase approximation, Bethe-Salpeter T-matrix equation, and 1/N expansion.

If the Majorana bosonic field Φ is indeed a collective excitation of the underlying four fermions, the first order approximation would involve summing to infinite order chains of 'bubble' diagrams, linked together via eight-fermion contact interactions. Each 'bubble' contains four lines of fermion propagators, rather than two lines.

3.7 The Issue of Nonrenormalizability

The four-fermion Yukawa terms of Φ and four/eight-quark contact interactions are nonrenormalizable in the conventional sense.

For nonrenormalizable models, the renomalization procedure can be made only at the cost of adding increasing numbers of term to the original Lagrangian. In principle, there is no problem with a theory having an infinite number of coupling constants as an effective field theory[19]. However, the NJL model is often regarded as regularization-dependent and its predictability is called into question.

A novel strategy for handling divergences is called implicit regularization [20]. It avoids the critical step of explicit evaluation of divergent integrals. The finite parts are separated from the divergent ones and integrated free from effects of regulation. The application to NJL model reveals that it can be ambiguity-free and symmetry-preserving can be obtained, making the NJL model predictive.

Likewise, we expect that models with four-fermion Yukawa interactions or eight-fermion contact terms are as predictive as renormalizable theories.

4 Electroweak Bosons

4.1 Electroweak Bosons and Yukawa Action

Electroweak boson field ϕ_{EW} interacts with both left-handed and right-handed fermions, while Majorana boson field ϕ_{MAJ} interacts with right-handed fermions only. Electroweak Boson field ϕ_{EW} spans the whole 32 component $C\ell_{0,6}$ even space. It obeys gauge transformation rules

$$\phi_{EW} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}}\phi_{EW}e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{166}$$

It can be broken down into three fields as

$$\phi_{EW} = \phi_S + \phi_P + \phi_{AT}, \tag{167}$$

with scalar ϕ_S valued in Clifford space spanned by 4 multivectors

$$\{1, \Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{168}$$

pseudoscalar ϕ_P valued in Clifford space spanned by 4 multivectors

$$\{i, i\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k\}, \tag{169}$$

and antisymmetric tensor ϕ_{AT} valued in Clifford space spanned by $4*6 = 24$ multivectors

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k = 1,2,3, j \neq k, a,b = 0,1,2,3, a \neq b\}. \tag{170}$$

The scalar and pseudoscalar electroweak Higgs fields ϕ_S and ϕ_P transform as

$$\phi_{S/P} \quad \rightarrow \quad e^{\Theta_{WL}} \phi_{S/P} e^{-\Theta_{WR}}, \tag{171}$$

while the antisymmetric tensor electroweak boson field ϕ_{AT} transforms as

$$\phi_{AT} \quad \rightarrow \quad e^{\Theta_{LOR}+\Theta_{WL}} \phi_{AT} e^{-\Theta_{LOR}-\Theta_{WR}}. \tag{172}$$

Notice that ϕ_{AT} is *not a Lorentz singlet*, since it's not invariant under local Lorentz gauge transformations.

We can write electroweak Yukawa action of fermions as

$$S_{Electroweak-Yukawa} \quad \sim \tag{173}$$

$$\int \langle \bar{\psi}_{Lj} i e^4 \phi_{EW} (y_j^\nu \nu_{Rj} + y_j^e e_{Rj} + y_j^u u_{Rj} + y_j^d d_{Rj}) i P_j \rangle \tag{174}$$

$$+ \int \langle (y_j^\nu \bar{\nu}_{Rj} + y_j^e \bar{e}_{Rj} + y_j^u \bar{u}_{Rj} + y_j^d \bar{d}_{Rj}) i e^4 \bar{\phi}_{EW} \psi_{Lj} i P_j \rangle, \tag{175}$$

where

$$\bar{\phi}_{EW} = \gamma_0 \phi_{EW}^\dagger \gamma_0 = \gamma_0 \tilde{\phi}_{EW} \gamma_0, \tag{176}$$

and y_j^ν, y_j^e, y_j^u, and y_j^d are electroweak Yukawa coupling constants.

4.2 Electroweak Boson Action, Symmetry Breaking, and Dirac Mass

Electroweak boson action reads

$$S_{Electroweak-Bosons} = S_{Electroweak-Kenetic} - V_{Electroweak}, \tag{177}$$

with

$$S_{Electroweak-Kenetic}(\phi_S) \sim \int \langle (e^3 (\bar{D\phi_S}))(e^3 D\phi_S) \rangle / \langle ie^4 \rangle, \tag{178}$$

$$V_{Electroweak-Bosons}(\phi_S, -\mu_S^2, \lambda_S) \sim \int (-\mu_S^2 |\phi_S|^2 + \lambda_S |\phi_S|^4) \langle ie^4 \rangle, \tag{179}$$

and

$$S_{Electroweak-Kenetic}(\phi_P), V_{Electroweak}(\phi_P, -\mu_P^2, \lambda_P), \tag{180}$$

$$S_{Electroweak-Kenetic}(\phi_{AT}), \tag{181}$$

$$V_{Electroweak}(\phi_{AT}, +\mu_{AT}^2, \lambda_{AT}) \sim \int (\mu_{AT}^2 \langle \bar{\phi}_{AT} \phi_{AT} \rangle + \lambda_{AT} (\langle \bar{\phi}_{AT} \phi_{AT} \rangle)^2) \langle ie^4 \rangle, \tag{182}$$

where

$$D\phi_{P/S} = (d + W_L)\phi_{P/S} - \phi_{P/S}(W_R), \tag{183}$$
$$D\phi_{AT} = (d + \omega + W_L)\phi_{AT} - \phi_{AT}(\omega + W_R), \tag{184}$$

Notice that ϕ_S and ϕ_P have negative $-\mu_S^2$ and $-\mu_P^2$. It means that ϕ_S and ϕ_P acquire nonzero VEVs via SSB

$$< 0|\phi_S|0 >= \frac{1}{\sqrt{2}}\upsilon_S = \frac{1}{\sqrt{2}}\frac{\mu_S}{\sqrt{\lambda_S}}, \tag{185}$$
$$< 0|\phi_P|0 >= \frac{1}{\sqrt{2}}\upsilon_P i = \frac{1}{\sqrt{2}}\frac{\mu_P}{\sqrt{\lambda_P}}i. \tag{186}$$

The situation of ϕ_{AT} is a bit complicated, and will be discussed in later section. Let's for the moment assume that its VEV is zero.

After replacing ϕ_S, ϕ_P, and ϕ_{AT} with their VEVs, the electroweak Yukawa action reduces to

$$\int \big\langle (\bar{\nu}_j ie^4 m_j^\nu \nu_j i + \bar{e}_j ie^4 m_j^e e_j i + \bar{u}_j ie^4 m_j^u u_j i + \bar{d}_j ie^4 m_j^d d_j i) P_j \big\rangle, \tag{187}$$

where 'complex' (scalar plus pseudoscalar) Dirac masses are

$$m_j^{\nu/e/u/d} = \frac{1}{\sqrt{2}} y_j^{\nu/e/u/d}(\upsilon_S + \upsilon_P i) = \frac{1}{\sqrt{2}} y_j^{\nu/e/u/d} \upsilon e^{\beta i}, \tag{188}$$

with

$$\upsilon = \sqrt{\upsilon_S^2 + \upsilon_P^2}, \tag{189}$$
$$tan(\beta) = \frac{\upsilon_P}{\upsilon_S}. \tag{190}$$

However the $e^{\beta i}$ phase factor can be canceled out via a global rotation of spinor

$$\psi \quad \rightarrow \quad e^{-\frac{1}{2}\beta i}\psi, \tag{191}$$

so that the fermion Dirac masses are 'real' (scalar) valued. If scalar and pseudoscalar Higgs fields have different configurations of Yukawa coupling constants, the rotation angles are spinor ($\nu/e/u/d$) specific.

Since the experiments at LHC indicated only one Higgs boson with $m_h = 125$ Gev[3, 4], there could be two scenarios. Case one is that both scalar and pseudoscalar Higgs fields contribute to the electroweak symmetry breaking and their masses are degenerate

$$m_h = m_S = m_P. \tag{192}$$

Case two is that only one of them acquires a nonzero VEV (with negative $-\mu^2$), which is the $m_h = 125$ Gev Higgs. The other maintains a zero VEV (with positive μ^2), which is still waiting to be detected at LHC.

Now we express gauge fields W_L^3, B, and W_R^3 in terms of A, Z, and Z'

$$W_{L\mu}^3 = A_\mu + (cos\theta_W)^2 Z_\mu, \tag{193}$$
$$B_\mu = A_\mu - (sin\theta_W)^2 Z_\mu, \tag{194}$$
$$W_{R\mu}^3 = B_\mu + (cos\theta_W')^2 Z_\mu' = A_\mu - (sin\theta_W)^2 Z_\mu + (cos\theta_W')^2 Z_\mu', \tag{195}$$

where

$$cos\theta_W = \frac{g_{WL}}{g_Z}, \tag{196}$$
$$sin\theta_W = \frac{g_B}{g_Z}, \tag{197}$$
$$g_Z = \sqrt{g_{WL}^2 + g_B^2}. \tag{198}$$

Electromagnetic field A remains massless with an effective coupling of

$$g = \frac{g_{WL} g_B}{g_Z} = \frac{g_{WL} g_{WR} g_J}{\sqrt{g_{WL} g_{WR} + g_{WL} g_J + g_{WR} g_J}}, \tag{199}$$

while gauge field Z acquires a mass

$$M_Z = \frac{1}{2} \upsilon g_Z. \tag{200}$$

4.3 Antisymmetric Tensor Boson and Dark Spin Current

The antisymmetric tensor boson is a bridge between gravity and electroweak sector. The strong connection between gravity field and electroweak Higgs field has also been studied in a geometrical 5D unification approach[21], which deduces all the known interactions from an induced symmetry breaking of the non-unitary GL(4)-group of diffeomorphisms.

As stated earlier, the antisymmetric tensor field ϕ_{AT} is not invariant under Lorentz gauge transformations. Hence, its boson potential should involve Lorentz invariant

$$\left\langle \bar{\phi}_{AT} \phi_{AT} \right\rangle = \left\langle \gamma_0 \phi_{AT}^\dagger \gamma_0 \phi_{AT} \right\rangle, \tag{201}$$

as opposed to

$$|\phi_{AT}|^2 = \left\langle \phi_{AT}^\dagger \phi_{AT} \right\rangle, \tag{202}$$

which is not Lorentz invariant.

It's easy to see that $\langle \bar{\phi}_{AT}\phi_{AT}\rangle$ is not a positive definite quantity. Components of

$$\{\gamma_a\gamma_b, \gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k=1,2,3, j\neq k, a,b=1,2,3, a\neq b\}, \tag{203}$$

have positive 'metric' and components of

$$\{i\gamma_a\gamma_b, i\gamma_a\gamma_b\Gamma_j\Gamma_k; \quad j,k=1,2,3, j\neq k, a,b=1,2,3, a\neq b\}, \tag{204}$$

have negative 'metric'.

We can divide ϕ_{AT} as

$$\phi_{AT} = \phi_{ATs} + \phi_{ATp}, \tag{205}$$

where ϕ_{ATs} and ϕ_{ATp} are valued in positive and negative 'metric' components, respectively. Thus we have

$$\langle \bar{\phi}_{AT}\phi_{AT}\rangle = |\phi_{ATs}|^2 - |\phi_{ATp}|^2. \tag{206}$$

A zero VEV $< 0|\phi_{AT}|0 >$ is allowed only if $\mu_{AT}^2 = 0$. On the other hand, nonzero VEV can be acquired for any value of μ_{AT}^2, including $\mu_{AT}^2 = 0$. Nonzero VEV simultaneously breaks electroweak and Lorentz symmetries.

In the case of $\mu_{AT}^2 = 0$, the four-boson-field term in the boson potential enforces

$$(\langle \bar{\phi}_{AT}\phi_{AT}\rangle)^2 = (|\phi_{ATs}|^2 - |\phi_{ATp}|^2)^2 = 0. \tag{207}$$

Therefore, the VEV should be on the 'light cone', which means

$$|\phi_{ATs}|^2 = |\phi_{ATp}|^2. \tag{208}$$

Replacing ϕ_{AT} with nonzero $< 0|\phi_{AT}|0 >$ in the boson kinetic action, we have a Lorentz symmetry breaking term

$$\int \langle (e^3(\omega < 0|\phi_{AT}|0 > - < 0|\phi_{AT}|0 > \omega))(e^3(\omega < 0|\phi_{AT}|0 > - < 0|\phi_{AT}|0 > \omega))\rangle \,/\, \langle ie^4\rangle. \tag{209}$$

This spin connection ω related term can contribute to space-time torsion equation. We call it 'dark spin current'. It is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature.

Since we know that modifications to torsion could have gravitational and cosmological consequences[22, 23], it's worth further research on the above antisymmetric-tensor-induced scenario.

5 Possible Grand Unification Symmetries

Embolden by the power of Clifford algebra, we now explore more symmetries allowed by an algebraic spinor. Let's begin with general gauge transformations

$$\psi \quad \rightarrow \quad e^{\Theta}\psi e^{\Theta'}, \tag{210}$$

where e^{Θ} and $e^{\Theta'} \in C\ell_{0,6}$ are independent gauge transformations. Spinor bilinear

$$\left\langle \tilde{\psi}\gamma_0\psi \right\rangle \tag{211}$$

is invariant if

$$e^{\tilde{\Theta}}\gamma_0 e^{\Theta} = \gamma_0, \tag{212}$$

$$e^{\Theta'}e^{\tilde{\Theta}'} = 1, \tag{213}$$

where we restrict our discussion to gauge transformations continuously connected to identity. General solution of these equations includes $\Theta \sim so(4,4)$, which is a linear combination of 28 gauge transformation generators

$$\{\gamma_a, \gamma_a\gamma_b, \Gamma_a\Gamma_b, i\Gamma_j, \Gamma_0\gamma_j\Gamma_k; j,k = 1,2,3, a,b = 0,1,2,3, a > b\} \in \Theta, \tag{214}$$

and $\Theta' \sim sp(8)$, which is a linear combination of 36 gauge transformation generators of pseudoscalar, all bivectors, and all trivectors

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l, \gamma_0, \Gamma_0, \gamma_0\gamma_j\Gamma_k, \Gamma_0\gamma_j\Gamma_k; j,k,l = 1,2,3, k > l\} \in \Theta'. \tag{215}$$

The de Sitter algebra $\Theta_{DS} \sim so(1,4)$

$$\{\gamma_a, \gamma_a\gamma_b\} \in \Theta_{DS} \tag{216}$$

is a subalgebra of Θ.

The Clifford odd parts of Θ and Θ' mix odd (left-handed ψ_L) and even (right-handed ψ_R) spinors. Since we know that left- and right-handed spinors transform differentially, only Clifford even subalgebras of Θ and Θ' are permitted, namely

$$\{\gamma_a\gamma_b, \Gamma_a\Gamma_b\} \in \Theta_{Even} \sim so(1,3) \oplus so(1,3), \tag{217}$$

$$\{i, \gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \in \Theta'_{Even} \sim u(1) \oplus so(6) \sim u(1) \oplus su(4). \tag{218}$$

The gauge transformations $\{\Gamma_a\Gamma_b\}$ can be further decomposed into weak transformations $\{\Gamma_k\Gamma_l\}$ and weak boost transformations $\{\Gamma_0\Gamma_j\}$, which are counterparts of spacial rotation $\{\gamma_k\gamma_l\}$ and Lorentz boost transformations $\{\gamma_0\gamma_j\}$.

Unitary algebra $u(3)$ is embedded in $\{\gamma_j\Gamma_k, \gamma_k\gamma_l, \Gamma_k\Gamma_l\} \sim su(4)$. Removing $u(1)$ $\{J\}$ from $u(3)$ defines the color algebra $su(3)$.

Since there are left-handed weak $su(2)_L$ and right-handed weak $u(1)_R$, one might expect left-right symmetric $su(2)_R$ as well. We can even go further and entertain the possibility of two exact copies of left-handed Θ_{EvenL} and right-handed Θ_{EvenR}. It means left- and right-handed fermions have separate local Lorentz gauge symmetries $\{\gamma_a\gamma_b\}_L$ and $\{\gamma_a\gamma_b\}_R$. Thus there are left- and right-handed gravities.

Of course, the grand unification symmetries studied in this section are speculative in nature. If there is indeed grand unification scale physics involving Θ_{EvenL}, Θ_{EvenR} and Θ'_{Even}, either symmetry breaking or other mechanism is needed to prevent detection of gauge interactions related to pseudoscalar $\{i\}$, quark/lepton mixing part of $su(4)$, weak boost $\{\Gamma_0\Gamma_j\}$, $W_R^{\pm}$ part of $su(2)_R$, and differences between left-handed $\{\gamma_a\gamma_b\}_L$ and right-handed $\{\gamma_a\gamma_b\}_R$ Lorentz transformations. It's an interesting topic. Nevertheless, we leave grand unification to future research.

6 Conclusion

We propose a Clifford algebra based model. A ternary Clifford vector is introduced alongside 6 binary Clifford vectors. The model includes local gauge symmetries $SO(1,3) \otimes SU_L(2) \otimes U_R(1) \otimes U(1) \otimes SU(3)$. Both gravitational and Yang-Mills interactions are treated as gauge fields.

There are two sectors of bosonic fields as Majorana and electroweak bosons. Majorana boson field interacts with right-handed fermions only. Electroweak boson field interacts with both left-handed and right-handed fermions.

The neutrino Higgs field part of Majorana boson sector acquires a nonzero VEV via spontaneous symmetry breaking, inducing Majorana masses of right-handed neutrinos via Yukawa-like couplings. The Majorana boson sector causes flavor mixing between generations. Higher order processes can introduce further effective mixing between generations. One may potentially couple above effects with appropriate choices of Majorana and electroweak Yukawa coupling constants to explain the quite different patterns of CKM and PMNS matrices.

The LHC 750 GeV diphoton resonance is identified as a Majorana sector quadruon, which is the pseudo-Nambu-Goldstone boson of $\bar{u}_R s_R \bar{c}_R d_R$ four-quark condensation. The quadruon results from spontaneous symmetry breaking of a flavor-related global $U(1)$ symmetry involving right-handed up, down, charm, and strange quarks. The quadruon is not exactly massless, since the electroweak sector explicitly breaks the global symmetry and can generate mass for it. The size of quadruon mass is proportional to electroweak scale.

In addition to $\bar{u}_R s_R \bar{c}_R d_R$, four-fermion condensations can also involve three other right-handed configurations $\bar{u}_R \tau_R \bar{\nu}_{\tau R} d_R, \bar{t}_R e_R \bar{\nu}_{eR} b_R,$ and $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$. Free from gauge interactions, these four-fermion condensations are potential dark matter candidates. The prime candidate might be four lepton condensation $\bar{\nu}_{\mu R} \tau_R \bar{\nu}_{\tau R} \mu_R$, due to its small effective Yukawa coupling constant and suppressed decaying rate.

The electroweak boson sector is composed of scalar, pseudoscalar, and antisymmetric tensor components. Scalar and/or pseudoscalar Higgs fields break the electroweak symmetry, contributing masses to fermions.

The antisymmetric tensor boson is not a Lorentz singlet. Its nonzero VEV would simultaneously break electroweak and Lorentz symmetries, giving rise to 'dark spin current'. 'Dark spin current' is a counterpart of dark energy, with the former affecting space-time torsion and the later affecting space-time curvature. Since we know that modifications to torsion could have gravitational and cosmological consequences[22, 23], it's worth further research on the antisymmetric-tensor-induced scenario.

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