Transforming Between Massive and Massless States for Bosons by Spontaneous Breakdown of Symmetry in an Alternate Klein-Gordon and Alternate Higgs Fields

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Received:

Abstract A kinetic energy-operated quantum wave equation is used to formulate alternate quantum fields: an alternate Klein-Gordon field, an alternate Dirac field, an alternate Proca field, and an alternate Higgs field.

The alternate Dirac field equations include a vacuum plane wave solution apart from the electron and positron solutions, lending support to the present formulation. The alternate Klein-Gordon field shows scalar bosons transforming between a massive state and a massless, charge state at a particular scalar potential level. The alternate Klein-Gordon Lagrangian directly leads to both the alternate Proca field and the alternate Higgs field by a local U(1) gauge transformation. The result shows vector bosons transforming between a massive state and a massless, charge state by a spontaneous breakdown of symmetry at a minimum potential trough similar to that of a Mexican hat or wine bottle potential in the Brout, Englert, and Higgs (BEH) mechanism, but more generally leaving open a possible presence of entirely different or many alternate Higgs bosons.

Keywords Schrödinger Relativistic Quantum Field Klein-Gordon Dirac Proca Higgs

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1. Motivation

The Lagrangian formalism in quantum field theory describes the massive scalar boson field by the Klein-Gordon equation, the spin half fermion field by the Dirac equation, and the massive vector boson field by the Proca equation, etc. These relativistic quantum wave equations apply the quantum prescriptions to the total energy, *E*, which is the sum of the relativistic external kinetic energy and the internal (rest) energy, and to the relativistic momentum, *P*. In general, however, the external kinetic energy and the internal energy, for instance spin energy, originate from two different motions that may be difficult to describe by a single set of wave equations. In this paper we re-examine the marriage of quantum mechanics and special relativity and present an alternate formulation to resolve this fundamental problem.

2. Relativistic Energy-Momentum Relation

In this section and Section 3.1 that follows, we extract some of the prerequisite from the author's previous paper [1]. We can write the relativistic energy–momentum relation in terms of the total energy, *E*, and momentum, *P*, of a particle [2-10],

$$E^2 = P^2c^2 + M^2c^4 \tag{1}$$

where c is the speed of light and *M* the mass of the particle.

Now the relativistic kinetic energy, *T*, may be written as

$$T = E - Mc^2 . \tag{2}$$

We can then rewrite the energy-momentum relation, Eq. (1), in terms of the kinetic energy and momentum in an alternate form,

$$T^2 + 2Mc^2T = P^2c^2 . \tag{3}$$

If we define $\mathscr{E} \equiv Mc^2$, the internal energy (many authors call this the rest energy) and $\mathscr{P} \equiv M\upsilon$ to be the non-relativistic momentum, we can then call $E \equiv \gamma Mc^2 = \gamma\mathscr{E}$ to be the relativistic total energy and $P = \gamma M\upsilon = \gamma\mathscr{P}$ to be the relativistic momentum where $\gamma = 1\Big/\sqrt{1-\frac{\upsilon^2}{c^2}}$ is the Lorentz factor, v is the velocity of the particle.

The energy-momentum relation, Eq. (1), may then be rewritten,

$$\mathscr{E}^2 = \mathscr{P}^2c^2 + \left(\frac{M}{\gamma}\right)^2 c^4 , \tag{4}$$

in terms of the internal energy, non-relativistic momentum, and mass. We note that the equations **(1)** and **(4)** are of the same form except the mass *M* is replaced with M/γ, a relativistic mass or the mass normalized by the Lorentz factor, with $0 \leqslant 1/\gamma \leqslant 1$. As the velocity of the particle approaches the speed of light, Eq. **(1)** may blow up but Eq. **(4)** behaves well as the relativistic mass term goes to zero. The $1/\gamma$ appears as a normalization factor; for instance each of the electron's orbits in an atom has a particular angular velocity and radius hence a characteristic γ and $1/\gamma$ values. It is crucial information for characterizing particles.

In the same way, Eq. (3) may be rewritten as

$$\mathscr{T}^2 + \frac{2Mc^2}{\gamma}\mathscr{T} = \mathscr{P}^2c^2 , \tag{5}$$

where $\mathscr{T} \equiv T/\gamma$. If a quantum wave equation is built based upon Eq. (4) or (5), the 'observable' counterpart for a scripted quantity, for instance T for $\mathscr{T}$ or E for $\mathscr{E}$, may easily be recovered by multiplying γ appropriately.

3. Alternate Klein-Gordon Equation

The quantum prescriptions are based upon de Broglie's theory [8] that may be expressed by $\mathscr{P} = \hbar \boldsymbol{k}$ and $\mathscr{E} = \hbar\omega$ where $\hbar$ is the reduced Planck constant, $\boldsymbol{k}$ is the wave number and ω is the angular frequency [11]. The bold face indicates a 3-vector.

All we are doing is to separate the external motion from the internal motion of a particle. In Eq. (5), substituting $\mathscr{T}$ by $i\hbar\frac{\partial}{\partial t}$ and $\mathscr{P}$ by $i\hbar\nabla$ and operating on a scalar function Φ, we then obtain,

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\Phi = 2i\frac{M}{\hbar\gamma}\frac{\partial}{\partial t}\Phi . \tag{6}$$

The above may be rewritten as

$$(\Box\,\Phi \equiv)\partial_\mu\partial^\mu\Phi = i\alpha\partial_0\Phi . \tag{7}$$

where $\Box \equiv \partial_\mu\partial^\mu$, the d'Alembertian and

$$\alpha \equiv \frac{2Mc}{\hbar\gamma}. \tag{8}$$

This is the kinetic energy-operated, mass-normalized, relativistic quantum wave equation, an extension of the Schrödinger equation in the free field. Note that if we replace the d'Alembertian with $-\nabla^2$ and take the non-relativistic limit of the relativistic mass $M/\gamma \rightarrow M$ we recover the Schrödinger equation. Conversely, the relativistic extension of the Schrödinger equation may be simply constructed by replacing $-\nabla^2$ in the Schrödinger equation with $\Box \equiv \partial_\mu\partial^\mu$ and the mass M with the relativistic mass M/γ.

We now define a unit four vector,

$$I^\mu = I_0^\mu + I_1^\mu + I_2^\mu + I_3^\mu \tag{9}$$

where

$$I^\mu \equiv \begin{pmatrix}1\\1\\1\\1\end{pmatrix},\ I_0^\mu \equiv \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\ I_1^\mu \equiv \begin{pmatrix}0\\1\\0\\0\end{pmatrix},\ I_2^\mu \equiv \begin{pmatrix}0\\0\\1\\0\end{pmatrix},\ I_3^\mu \equiv \begin{pmatrix}0\\0\\0\\1\end{pmatrix}. \tag{10}$$

When applied to the four derivative, it is understood that

$$\begin{aligned} I^\mu\partial_\mu &= \partial_0 - \partial_1 - \partial_2 - \partial_3 \\ I_0^\mu\partial_\mu &= \partial_0;\ I_1^\mu\partial_\mu = -\partial_1;\ \text{etc.} \end{aligned} \tag{11}$$

This allows Eq. (7) to be rewritten in a more maneuverable form,

$$\partial_\mu\partial^\mu\Phi = i\alpha I_0^\mu\partial_\mu\Phi \tag{12}$$

The above is an alternate Klein-Gordon equation, a new relativistic quantum wave equation for spin zero massive particles that reduces to the Schrödinger equation in the nonrelativistic limit. In the present formulation, it replaces the Klein-Gordon equation

$$\partial_\mu \partial^\mu \Phi + \left(\frac{Mc}{\hbar}\right)^2 \Phi = 0 . \tag{13}$$

Now that we have an alternate Klein-Gordon equation, Eq. (12), it is a daunting task to reformulate the entire quantum field theory based upon the alternate equation. Why do we want to do that? After all, both Eq. (1) and (3) express the same relativistic energy-momentum relation. The difference, however, is where we apply the de Broglie's theory and the author hopes to find if accounting for this difference may help simplifying some of the peculiar treatments in our quantum theories, such as renormalization and Higgs mechanism. This paper focuses on formulating alternate Lagrangian densities for the quantum fields and some preliminary consequence of the new formulation.

4. Spin ½ Fermion

4.1 Alternate Dirac Equation

The alternate Klein-Gordon equation, Eq. (12), may be decoupled into the bi-spinor equations by deploying the Dirac formalism [12]. This was done in the author's previous work [1] and here we only state the result.

We define I (or simply 1) to be a 2x2 unit matrix, and σ^i to be 2 x 2 Pauli matrices, γ^0 to be the first of the 4x4 Dirac matrices (the others are γ^i; i = 1,2,3.) By using the first of the following relationships,

$$\begin{aligned} \gamma^0 - 1 &= 2\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \\ \gamma^0 + 1 &= 2\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{14}$$

the quantum wave equation describing the spin half fermion may be written,

$$i\hbar\gamma^\mu \partial_\mu \Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi = 0 . \tag{15}$$

This is the kinetic energy-operated, alternate Dirac equation compared to the Dirac equation,

$$i\gamma^\mu \partial_\mu \Psi - \frac{Mc}{\hbar}\Psi = 0 . \tag{16}$$

4.2 Lagrangian for the Alternate Dirac Equation

A Dirac Lagrangian may be written as

$$\mathfrak{L} = i\hbar c\overline{\Psi}\gamma^\mu \partial_\mu \Psi - Mc^2\overline{\Psi}\Psi . \tag{17}$$

The Euler-Lagrange equations for the above are

$$\begin{aligned} &\text{EL1}: i\gamma^\mu \partial_\mu \Psi - \frac{Mc}{\hbar}\Psi = 0, \text{ and} \\ &\text{EL2}: i(\partial_\mu \overline{\Psi})\gamma^\mu + \frac{Mc}{\hbar}\overline{\Psi} = 0. \end{aligned} \tag{18}$$

The two Euler-Lagrange equations represent a particle and its anti-particle, respectively. A local U(1) gauge transformation may be performed to Eq. (17) and the result is

$$\mathfrak{L} = i\hbar c\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - Mc^2\overline{\Psi}\Psi \\ -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \left(q\overline{\Psi}\gamma^{\mu}\Psi\right)A_{\mu} \tag{19}$$

which then yields three Euler-Lagrange equations

$$\begin{aligned} &\text{EL1}: i\hbar\gamma^{\mu}\partial_{\mu}\Psi - Mc\Psi - q\gamma^{\mu}\Psi A_{\mu} = 0 \\ &\text{EL2}: i\hbar\partial_{\mu}\overline{\Psi}\gamma^{\mu} + Mc\overline{\Psi} + q\gamma^{\mu}\Psi A_{\mu} = 0 \\ &\text{EL3}: \frac{1}{4\pi}\partial_{\mu}F^{\mu\nu} - q\overline{\Psi}\gamma^{\mu}\Psi = 0. \end{aligned} \tag{20}$$

Similarly, a Lagrangian for the alternate Dirac Equation may be constructed as

$$\mathfrak{L} = i\hbar c\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + \left(\gamma^0 - 1\right)\frac{Mc^2}{\gamma}\overline{\Psi}\Psi \tag{21}$$

of which the Euler-Lagrange equations are

$$\begin{aligned} &\text{EL1}: i\hbar\gamma^{\mu}\partial_{\mu}\Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi = 0, \text{and} \\ &\text{EL2}: -i\hbar(\partial_{\mu}\overline{\Psi})\gamma^{\mu} + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\overline{\Psi} = 0. \end{aligned} \tag{22}$$

The two Euler-Lagrange equations represent a particle and its anti-particle, respectively. The Euler-Lagrange equations, Eq. (22), of the alternate Dirac Lagrangian closely match those of the Dirac Lagrangian, Eq. (18), the only but critical difference being each of Eq. (22) includes both the massive and massless interaction between spinors. As discussed in [1], it can be shown each has plane wave solutions that include a constant solution, which can be set to be zero representing the vacuum state. Eq. (18) lacks this solution and Dirac then had to hypothesize the existence of the so-called Dirac 'sea' [13]. The vacuum solution of Eq. (22) removes this difficulty Dirac tried to resolve and this fact lends support to the present formulation.

The alternate Dirac Lagrangian, Eq. (21), may be gauge-transformed to,

$$\mathfrak{L} = i\hbar c\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + \left(\gamma^0 - 1\right)\frac{Mc^2}{\gamma}\overline{\Psi}\Psi \\ -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \left(q\overline{\Psi}\gamma^{\mu}\Psi\right)A_{\mu}, \tag{23}$$

which then yields three Euler-Lagrange equations

$$\begin{aligned} &\text{EL1}: i\hbar\gamma^{\mu}\partial_{\mu}\Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi - q\gamma^{\mu}\Psi A_{\mu} = 0 \\ &\text{EL2}: i\hbar\partial_{\mu}\overline{\Psi}\gamma^{\mu} - \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\overline{\Psi} + q\overline{\Psi}\gamma^{\mu}A_{\mu} = 0 \\ &\text{EL3}: \frac{1}{4\pi}\partial_{\mu}F^{\mu\nu} - q\overline{\Psi}\gamma^{\mu}\Psi = 0. \end{aligned} \tag{24}$$

The Euler-Lagrange equations, Eq. (24), of the alternate Dirac Lagrangian closely match those of the Dirac Lagrangian, Eq. (20), with the only but critical difference being that the first two of Eq. (24) include both the massive and massless interactions between spinors. The gauge fields (EL3 of each) are exactly the same.

5. Scalar Boson

5.1 Lagrangian for the Alternate Klein-Gordon Equation

Now let $\Phi = \phi_1 + i\phi_2$ and $\Phi^* = \phi_1 - i\phi_2$ where ϕ_1 and ϕ_2 are two real fields. We can then write a Lagrangian for the alternate Klein-Gordon equation, Eq. (12),

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* \left(\partial_\mu \Phi\right). \tag{25}$$

of which the Euler-Lagrange equations are,

$$\begin{aligned} &\text{EL1}: \partial_\mu \partial^\mu \Phi = i\alpha I_0^\mu \partial_\mu \Phi, \text{ and} \\ &\text{EL2}: \partial_\mu \partial^\mu \Phi^* = -i\alpha I_0^\mu \partial_\mu \Phi^*. \end{aligned} \tag{26}$$

EL1 in the above is the same as Eq. (12). EL2 represents its anti-particle. The alternate Klein-Gordon Lagrangian, Eq. (25), describes a massive, scalar, spin-zero boson with mass

$$M = \left(\frac{\gamma\alpha}{2}\right)\frac{\hbar}{c} \tag{27}$$

carried by Φ* times the time derivative of Φ. Note that the 'observable' mass M includes a boost factor, γ, owing to its velocity.

5.2 Massive and Massless Scalar Boson by Gauge Transformation

A local U(1) gauge transformation may be performed for Eq. (25) via

$$\begin{aligned} &\mathcal{D}^\mu \to \partial^\mu + i\beta A^\mu \text{ for } \Phi \\ &\mathcal{D}^\mu \to \partial^\mu - i\beta B^\mu \text{ for } \Phi^* \end{aligned} \tag{28}$$

where

$$\beta \equiv \frac{q}{\hbar c}, \tag{29}$$

q is the charge of the particle, and A^μ and B^μ are some vector fields associated with Φ and Φ^*, respectively. This leads to the gauge transformed, alternate Klein-Gordon Lagrangian density,

$$\begin{aligned} \mathcal{L} = &\left[\partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* \left(\partial_\mu \Phi\right)\right] \\ &- \beta \Phi^* \Phi (\alpha I_0^\mu - \beta B^\mu) A_\mu + i\beta\left[(\partial^\mu \Phi^*)\Phi A_\mu - \Phi^* (\partial^\mu \Phi) B_\mu\right], \end{aligned} \tag{30}$$

where α represents the mass, β represents the charge.

It is interesting to note that when

$$\beta B^\mu = \alpha I_0^\mu \tag{31}$$

i.e., $B^\mu = (\dfrac{2Mc^2}{q\gamma}, 0, 0, 0)$, the second term then vanishes and Eq. (30) reduces to,

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\beta A^\mu (\partial_\mu \Phi^*)\Phi \,. \tag{32}$$

This represents a massless scalar boson with charge q in the vector field, A^μ. The Euler-Lagrange equations of it are

$$\begin{aligned} &\text{EL1}: \partial_\mu \partial^\mu \Phi + i\beta A^\mu \partial_\mu \Phi = 0, \text{ and} \\ &\text{EL2}: \partial_\mu \partial^\mu \Phi^* - i\beta A^\mu \partial_\mu \Phi^* = 0. \end{aligned} \tag{33}$$

Note that we can always select a particular B^μ without losing generality to obtain an A^μ field equation. Hence if we choose $B^\mu = (V, 0, 0, 0)$ with $V = \frac{2Mc^2}{\gamma q}$, where V is a scalar potential, then the massive scalar boson, Eq. (25), transforms into a massless scalar boson, Eq. (32). We will bring γ to the left hand side and rewrite this condition to note that γV is the 'observable' potential,

$$\gamma V = \frac{2Mc^2}{q}. \tag{34}$$

Conversely, a massless scalar boson, Eq. (32) may be shown to transform to a massive scalar boson, Eq. (25), by assuming $A_\mu = B_\mu$, then gauge transforming via

$$\begin{aligned} \mathcal{D}^\mu &\to \partial^\mu - i\alpha I_0^\mu \text{ for } \Phi \\ \mathcal{D}^\mu &\to \partial^\mu + i\alpha I_0^\mu \text{ for } \Phi^*, \end{aligned} \tag{35}$$

and finally taking a similar condition as Eq. (31),

$$\beta A^\mu = \alpha I_0^\mu. \tag{36}$$

If further we choose $A^\mu = (V, 0, 0, 0)$ with V, a scalar potential, we then get

$$M = \left(\frac{\gamma V}{2}\right)\frac{q}{c^2} \tag{37}$$

and we can say the massless boson, Eq. (32), acquired mass M from charge q. In Section 6.3, we will compare this to the acquired mass in the Higgs field. Thus we see that scalar bosons transform between a massive state, Eq. (25), and a massless charge state, Eq. (32), at a particular scalar potential level. We note that a Goldstone boson is not present.

We note that the Lagrangian for the complex-valued scalar field according to the Klein-Gordon equation, Eq. (13), may be written,

$$\mathcal{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi \tag{38}$$

A local U(1) gauge transformation via Eqs. (28) and (29) leads to the gauge transformed, Klein-Gordon Lagrangian,

$$\begin{aligned} \mathcal{L} = &\left[(\partial_\mu \Phi^*)(\partial^\mu \Phi) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi\right] + \beta^2 \Phi^* \Phi B^\mu A_\mu \\ &+ i\beta\left[(\partial^\mu \Phi^*)\Phi A_\mu - \Phi^*(\partial^\mu \Phi) B_\mu\right]. \end{aligned} \tag{39}$$

The above includes the Klein-Gordon Lagrangian, Eq. (38), as expected, and some terms showing the interaction among the scalar fields and the vector fields, but the mass term remains unchanged. This presents a problem in the weak interactions where a transformation between massive and massless scalar field is required [14, 15]. In the following, the alternate formulation is shown to allow this to occur without resorting to a Higgs field.

6. Vector Boson

We define the "Field Strength Tensors" for some vector fields, A^μ and B^μ, respectively,

$$
\begin{aligned}
F^{\mu\nu} &= \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} \\
G^{\mu\nu} &= \partial^{\mu}B^{\nu} - \partial^{\nu}B^{\mu}
\end{aligned}
\tag{40}
$$

and introduce a gauge field, $F^{\mu\nu}G_{\mu\nu}$, into the alternate Klein-Gordon Lagrangian, Eq. (25), to obtain

$$
\mathfrak{L} = \left[\partial_{\mu}\Phi^{*}\partial^{\mu}\Phi + i\alpha I_{0}^{\mu}\Phi^{*}\left(\partial_{\mu}\Phi\right)\right] - \frac{i}{8\pi\hbar c}F^{\mu\nu}G_{\mu\nu}\,. \tag{41}
$$

Gauge transformation of the above according to Eq. (28) results in the following by use of Eq. (30),

$$
\begin{aligned}
\mathfrak{L} = &\left[\partial_{\mu}\Phi^{*}\partial^{\mu}\Phi + i\alpha I_{0}^{\mu}\Phi^{*}\left(\partial_{\mu}\Phi\right)\right] - \frac{i}{8\pi\hbar c}F^{\mu\nu}G_{\mu\nu} \\
&- \beta\Phi^{*}\Phi(\alpha I_{0}^{\mu} - \beta B^{\mu})A_{\mu} + i\beta\left[(\partial^{\mu}\Phi^{*})\Phi A_{\mu} - \Phi^{*}(\partial^{\mu}\Phi)B_{\mu}\right]
\end{aligned}
\tag{42}
$$

If Eq. (31) holds, then Eq. (42) reduces to

$$
\mathfrak{L} = \left[\partial_{\mu}\Phi^{*}\partial^{\mu}\Phi + i\beta A^{\mu}(\partial_{\mu}\Phi^{*})\Phi\right] - \frac{i}{8\pi\hbar c}F^{\mu\nu}G_{\mu\nu} \tag{43}
$$

We see that, in effect, the gauge transformation allows the massive gauge field, Eq. (41) to transform into a massless gauge field, Eq. (43). By the gauge transformation via Eq. (35), the reverse is also true.

Eq. (42) may be rearranged to

$$
\begin{aligned}
\mathfrak{L} = &\left[\partial_{\mu}\Phi^{*}\partial^{\mu}\Phi + i\alpha I_{0}^{\mu}\Phi^{*}\left(\partial_{\mu}\Phi\right)\right] \\
&+ \frac{2i}{\hbar c}\left[-\frac{1}{16\pi}F^{\mu\nu}G_{\mu\nu} + i\frac{q}{2}\Phi^{*}\Phi(\alpha I_{0}^{\mu} - \beta B^{\mu})A_{\mu}\right] \\
&+ i\beta\left[(\partial^{\mu}\Phi^{*})\Phi A_{\mu} - \Phi^{*}(\partial^{\mu}\Phi)B_{\mu}\right].
\end{aligned}
\tag{44}
$$

Eq. (44) combines a massive scalar field in the first square bracket, a massive vector field in the second square bracket, and the interaction of the scalar and the vector fields in the third square bracket. The last is the Noether's conserved current of the fields [14, 15]. In general, the above Lagrangian yields the following four Euler-Lagrange equations carried by the scalar fields, Φ^{*} and Φ, and the vector fields, A_{u} and B_{u}, respectively:

$$
\begin{aligned}
&\text{EL1}: \left(\partial_{\mu}\partial^{\mu}\Phi - i\alpha I_{0}^{\mu}\partial_{\mu}\Phi\right) + \beta\Phi(\alpha I_{0}^{\mu} - \beta B^{\mu})A_{\mu} \\
&\qquad + i\beta\Phi\partial_{\mu}A^{\mu} + i\beta\partial_{\mu}\Phi(A^{\mu} + B^{\mu}) = 0 \\
&\text{EL2}: \left(\partial_{\mu}\partial^{\mu}\Phi^{*} + i\alpha I_{0}^{\mu}\partial_{\mu}\Phi^{*}\right) + \beta\Phi^{*}(\alpha I_{0}^{\mu} - \beta B^{\mu})A_{\mu} \\
&\qquad - i\beta\Phi^{*}\partial_{\mu}B^{\mu} - i\beta\partial_{\mu}\Phi^{*}(A^{\mu} + B^{\mu}) = 0 \\
&\text{EL3}: \left[\frac{1}{4\pi}\partial_{\mu}G^{\mu\nu} + iq\Phi^{*}\Phi(\alpha I_{0}^{\nu} - \beta B^{\nu})\right] + q(\partial^{\nu}\Phi^{*})\Phi = 0 \\
&\text{EL4}: \frac{1}{4\pi}\partial_{\mu}F^{\mu\nu} - q\Phi^{*}(\partial^{\nu}\Phi) = 0.
\end{aligned}
\tag{45}
$$

EL1 and EL2 include a scalar boson with the mass given by Eq. (27) and the scalar boson-vector boson interaction terms. If the Lorentz condition,

$$
\partial_{\mu}A^{\mu} = \partial_{\mu}B^{\mu} = 0\,, \tag{46}
$$

holds and if in addition the special gauge field condition, Eq. (31), is also met, these two transforms into a massless, charged scalar boson, Eq. (33). If the special condition is not met, then the scalar boson interacts with the vector boson in the second term through both charge and mass, and in the fourth term through charge only. EL3 describes a massive vector boson interacting with a scalar boson except for the special case, Eq. (31). EL4 is the Maxwell equation describing a massless vector boson with a current.

6.1 Massless Gauge Vector Field

From the second and the third term of Eq. (44), we can define a vector field,

$$\begin{aligned}\mathfrak{L} = -\frac{1}{16\pi}F^{\mu\nu}G_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu)A_\mu \\ + \frac{q}{2}\left[(\partial^\mu\Phi^*)\Phi A_\mu - \Phi^*(\partial^\mu\Phi)B_\mu\right]\end{aligned}. \tag{47}$$

We can obtain a pure vector field from this under certain conditions: for instance, if the vector fields satisfy $B_\mu = A_\mu$, the above then reads

$$\begin{aligned}\mathfrak{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta A^\mu)A_\mu \\ + \frac{q}{2}\left[(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi)\right]A_\mu\end{aligned}. \tag{48}$$

The square bracket term appears to be a Noether's current. We can take the divergence of it and use Eq, (26) to find

$$\partial_\mu\left[(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi)\right] = -i\alpha I_0^\mu\partial_\mu(\Phi^*\Phi) = 0 \tag{49}$$

since the Noether's current is a conserved quantity. Hence

$$(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi) = -i\alpha I_0^\mu(\Phi^*\Phi) = \text{Constant.} \tag{50}$$

Eq. (48) then reduces to

$$\mathfrak{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}q\beta A^\mu A_\mu \tag{51}$$

after $\Phi^*\Phi = \phi_1^{\,2} + \phi_2^{\,2} = K$ (a constant) is absorbed by A^μ. This is a massless gauge boson field, or an alternate massless Proca Lagrangian, of which the Euler-Lagrange equation is

$$\text{EL1: } \frac{1}{4\pi}\partial_\mu F^{\mu\nu} - iq\beta A^\nu = 0\,. \tag{52}$$

Note that $q\beta = q^2/(\hbar c)$, but we keep β since it appears often separately in the following.

6.2 Massive Gauge Vector Field

We can obtain another pure vector field if the vector fields satisfy $B_\mu = -A_\mu$, Eq. (47) then reads

$$\begin{aligned}\mathfrak{L} = \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu + \beta A^\mu)A_\mu + \frac{q}{2}\partial^\mu(\Phi^*\Phi)A_\mu, \text{ or} \\ \mathfrak{L} = \frac{1}{16\pi}G^{\mu\nu}G_{\mu\nu} - i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu)B_\mu - \frac{q}{2}\partial^\mu(\Phi^*\Phi)B_\mu.\end{aligned} \tag{53}$$

If further the scalar field satisfies $\partial^\mu(\Phi^*\Phi)=0$, then again $\Phi^*\Phi=\phi_1{}^2+\phi_2{}^2=K$(a constant) and K can be absorbed by the vector field. In this case, the above reduces to

$$\begin{aligned}\mathfrak{L}&=\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+i\frac{q}{2}(\alpha I_0^\mu+\beta A^\mu)A_\mu,\text{ or}\\ \mathfrak{L}&=\frac{1}{16\pi}G^{\mu\nu}G_{\mu\nu}-i\frac{q}{2}(\alpha I_0^\mu-\beta B^\mu)B_\mu\end{aligned} \tag{54}$$

which may be called an alternate Proca Lagrangians, special cases of Eq. (44). We can show by use of Eqs. (26) and (50),

$$\begin{aligned}\partial_\mu\partial^\mu(\Phi^*\Phi)&=2\partial_\mu\Phi^*\partial^\mu\Phi+\alpha^2 I_{0\mu}I_0^\mu(\Phi^*\Phi)\\ &=2(\partial_\mu\phi_1\partial^\mu\phi_1+\partial_\mu\phi_2\partial^\mu\phi_2)+\alpha^2 I_{0\mu}I_0^\mu(\phi_1^2+\phi_2^2)\ge 0\end{aligned} \tag{55}$$

so long as both $\partial_\mu\phi_1$ and $\partial_\mu\phi_2$ are time-like, which we assume to be given. It is interesting to note that these scalar field conditions,

$$\begin{aligned}&\partial^\mu(\Phi^*\Phi)=0,\text{ and}\\ &\partial_\mu\partial^\mu(\Phi^*\Phi)\ge 0\end{aligned} \tag{56}$$

define a local minimum potential along the circle, $\phi_1^2+\phi_2^2=0$, under which the above massive vector fields arise. This is remarkably similar to the condition by which the Higgs boson field arises, i.e., the spontaneous symmetry breaking via the Mexican hat or wine bottle potential. Eq. (56) does not exactly define Mexican hat potential but is more general in the sense that there may be many $\Phi^*\Phi$ circles that satisfy this.

The Euler-Lagrange equations for Eq. (54),

$$\begin{aligned}&\text{EL1}:\frac{1}{4\pi}\partial_\mu F^{\mu\nu}-iq(\frac{\alpha}{2}I_0^\nu+\beta A^\nu)=0\\ &\text{EL2}:\frac{1}{4\pi}\partial_\mu G^{\mu\nu}+iq(\frac{\alpha}{2}I_0^\nu-\beta B^\nu)=0,\end{aligned} \tag{57}$$

are the alternate Proca equations with a relativistic mass $M/\gamma=\alpha\hbar/(2c)$.

6.3 Comparison with the Higgs Field

For comparisons, the Proca Lagrangian may be written as

$$\mathfrak{L}=-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+\frac{1}{8\pi}\left(\frac{Mc}{\hbar}\right)^2A^\nu A_\nu \tag{58}$$

with the Euler-Lagrange equation,

$$\partial_\mu F^{\mu\nu}+\left(\frac{Mc}{\hbar}\right)^2A^\nu=0\,. \tag{59}$$

According to the Proca Lagrangian, Eq. (58), mass is carried by the quadratic term of the vector field A^ν and may be created by a mechanism known as Brout-Englert-Higgs (BEH) mechanism along with the Higgs boson. For example, consider a Lagrangian with a self-interaction potential energy terms [14, 15, 16]

$$\mathfrak{L}=\frac{1}{2}\partial_\mu\Phi^*\partial^\mu\Phi+\frac{1}{2}\mu^2\Phi^*\Phi-\frac{1}{4}\lambda^2(\Phi^*\Phi)^2\,. \tag{60}$$

where μ and λ are real constants. By defining $\eta \equiv \phi_1 - \mu/\lambda$, a gage transformed and spontaneously symmetry-broken version of the above is the Lagrangian for the Higgs field, which may be written [14]

$$\begin{aligned}\mathfrak{L} &= \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - (\mu)^2 \eta^2\right] + \left[-\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right] \\ &+ \left\{\frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta A_\mu A^\mu + \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2 \eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \frac{1}{4}\lambda^2\eta^4\right\} + \left(\frac{\mu^2}{2\lambda}\right)^2\end{aligned} \tag{61}$$

where the first square bracket represents the Higgs scalar boson field with mass,

$$M_S = \left(\sqrt{2}\mu\right)\frac{\hbar}{c}, \tag{62}$$

and the second square bracket a gauge boson field with mass,

$$M_A = \left(2\sqrt{\pi}\frac{\mu}{\lambda}\right)\frac{q}{c^2}. \tag{63}$$

Its Euler-Lagrange equations are

$$\begin{aligned}&\text{EL1}: \left[\partial_\mu \partial^\mu \eta - \mu^2 \eta\right] - \beta^2 \left(\frac{\mu}{\lambda} + \eta\right) A_\mu A^\mu + 3\lambda\mu\eta^2 + \lambda^2 \eta^3 = 0 \\ &\text{EL2}: \left[-\frac{1}{4\pi}\partial_\mu F^{\mu\nu} - \left(\beta\frac{\mu}{\lambda}\right)^2 A^\nu\right] - \beta^2\left(\frac{2\mu}{\lambda} + \eta^2\right) A^\nu = 0\end{aligned} \tag{64}$$

The square bracket term of the above EL1 is a Klein-Gordon equation defining the Higgs boson. The square bracket term of the above EL2 is a Proca equation describing a massive gauge boson.

Eq. (44) and Eq. (61) are similar in their structure, combining a massive scalar boson field and a massive gauge boson field. Remarkably, Eq. (44) includes these fields as a result of the local U(1) gauge transformation of the alternate Klein-Gordon equation, Eq. (25), naturally without introducing an arbitrary symmetry breaking process. The mass of the scalar boson given by Eq. (27) will be identical with that of Eq. (62) if

$$\frac{\gamma\alpha}{2} = \sqrt{2}\mu \tag{65}$$

and the mass of the gauge boson given by Eq. (37) will be identical with that of Eq. (63) if

$$\frac{\gamma W}{2} = 2\sqrt{\pi}\frac{\mu}{\lambda}. \tag{66}$$

The Higgs scalar boson has been found experimentally [17-28]. It is interesting to see if the massive scalar boson in Eq. (44), which may be called an alternate Higgs boson, may also be found experimentally. It is possible that the alternate Higgs boson in Eq. (44) is identical to the Higgs boson in Eq. (61), even though we arrive at them in quite different ways and now we see the possibility of multiple alternate Higgs bosons.

7. Maxwell Fields

We can define a four vector potential [14],

$$A^\nu \equiv \left(V, \vec{A}\right) \equiv \left(A^0, A^1, A^2, A^3\right), \tag{67}$$

where V is a scalar potential, $\vec{A}$ is a three-vector potential, and

$$J^{\mu} = (c\rho, \vec{J}) , \tag{68}$$

where ρ is the charge density, $\vec{J}$ is the current density.

The Lagrangian for the Maxwell equation may be written as

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - \frac{1}{c} J^{\mu} A_{\mu} , \tag{69}$$

and we can then write the Maxwell equation,

$$\frac{1}{4\pi} \partial_{\mu} F^{\mu\nu} - \frac{1}{c} J^{\nu} = 0 . \tag{70}$$

This is the electromagnetic field equation with a source current, J^{ν}.

Maxwell field Lagrangians are embedded in the alternate Dirac Lagrangian, Eq. (23), alternate Klein-Gordon Lagrangian, Eq. (44), and alternate Proca Lagrangian, Eq. (51), respectively. EL3 of Eq. (24) is the Maxwell equation under fermion fields. EL4 of Eq. (45) is the Maxwell equation under scalar boson fields. EL1 of Eq.(52) is the Maxwell equation under vector boson fields. Comparing these with Eq. (70), one gets

$$\frac{J^{\nu}}{c} = q_{\Psi} \overline{\Psi} \gamma^{\nu} \Psi = q_{\Phi} \Phi^{*} (\partial^{\nu} \Phi) = i q_{A} \beta A^{\nu} \tag{71}$$

where q_{Ψ}, q_{Φ}, and q_A are the charges in their respective fields.

8. Conclusion

We have critically reviewed the marriage of a quantum wave equation and the special relativity. In our standard physics, the relativistic quantum wave equations are obtained by applying the quantum prescriptions to the total energy, E, and to the relativistic boosted momentum, P. E is the sum of the relativistic boosted kinetic energy and the internal (rest) energy. An elementary particle, however, in general has some internal motion, for example at least that causing spins which contributes to the unique rest mass energy, and an external motion, for example a translational or rotational motion that manifests the external kinetic energy. Since they involve two different mechanisms, it may be difficult to describe both by a single set of wave equations; but this is precisely what we do. This paper presents an alternate approach to resolve this fundamental problem.

External kinetic energy-operated quantum wave equations are used to formulate alternate quantum fields. This leads to an alternate Klein-Gordon field for a massive scalar boson, an alternate Dirac field for a spin half fermion, an alternate Proca field for a massive vector boson, and an alternate Higgs field for a massive scalar boson and massive gauge vector boson. The main results are summarized in the Appendix.

For example, the Klein-Gordon equation is a total energy-operated quantum wave equation while the alternate Klein Gordon equation derived by the present approach is a kinetic energy-operated quantum wave equation. From the alternate Klein-Gordon field, also derived are an alternate Dirac, alternate Proca, and alternate Higgs field.

The alternate Dirac field thus derived closely matches that of the Dirac field, the only but crucial difference being each of the alternates includes both the massive and massless interaction between spinors. The equations of motion then yield plane wave solutions that include a constant solution, which can be set to be zero representing a vacuum state. The original Dirac equation lacks this solution and Dirac then tried hard to resolve this by hypothesizing the existence of the so-called Dirac 'sea'. The vacuum solution of the alternate formulation removes this difficulty. We now see the difficulty arose because the Klein-Gordon equation upon which Dirac equation is based uses the total energy as the basis of quantum prescription. This fact lends a strong support to the present formulation over the original formulation.

The alternate Klein-Gordon field shows scalar bosons transforming at a particular scalar potential level between a massive state and a massless, charge state. The mass comes from the charge, the charge comes from the mass, and i.e., mass and charge are interchangeable under certain conditions.

The alternate Klein-Gordon Lagrangian directly leads to both the alternate Proca field and the alternate Higgs field by a local U(1) gauge transformation. The result shows vector bosons transforming between a massive state and a massless, charge state by a spontaneous breakdown of symmetry at a minimum potential trough similar to, but more generally than that of a Mexican hat or wine bottle potential in the Brout, Englert, and Higgs (BEH) mechanism.

The Higgs and alternate Higgs fields are similar in their structure, combining a massive scalar boson and a massive gauge boson. Remarkably, the alternate Higgs field includes these bosons as a result of the local U(1) gauge transformation of the alternate Klein-Gordon Lagrangian without introducing an arbitrary symmetry breaking process.

It is probable that the scalar bosons in both fields are identical, even though we arrive at them in quite different ways. The Higgs scalar boson has been found experimentally. It will be interesting to see if the alternate scalar boson may also be found experimentally, if not identical to the Higgs boson. Finally, it should be noted that the present theory leaves open a possible presence of entirely different or many "alternate Higgs bosons".

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APPENDIX. Lagrangian Density for Quantum Fields - Standard Formulation vs. Alternate

	Standard Formulation	Alternate
Massive Scalar Boson (Klein-Gordon vs. Alternate)	$\mathfrak{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi$	$\mathfrak{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* (\partial_\mu \Phi); \alpha \equiv \frac{2Mc}{\hbar\gamma}$
Massless Scalar Boson (Goldstone vs. Alternate)	$\mathfrak{L} = \partial_\mu \Phi^* \partial^\mu \Phi$	$\mathfrak{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\beta A^\mu (\partial_\mu \Phi^*)\Phi; \beta \equiv \frac{q}{\hbar c}$
Spin ½ Fermion (Dirac vs. Alternate)	$\mathfrak{L} = i\hbar c \overline{\Psi} \gamma^\mu \partial_\mu \Psi - Mc^2 \overline{\Psi}\Psi$	$\mathfrak{L} = i\hbar c \overline{\Psi} \gamma^\mu \partial_\mu \Psi + (\gamma^0 - 1)\frac{Mc^2}{\gamma} \overline{\Psi}\Psi$
Spin ½ Fermion + Massless Gauge Field (Dirac vs. Alternate)	$\mathfrak{L} = i\hbar c \overline{\Psi} \gamma^\mu \partial_\mu \Psi - Mc^2 \overline{\Psi}\Psi - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - (q\overline{\Psi}\gamma^\mu \Psi) A_\mu$	$\mathfrak{L} = i\hbar c \overline{\Psi} \gamma^\mu \partial_\mu \Psi + (\gamma^0 - 1)\frac{Mc^2}{\gamma} \overline{\Psi}\Psi - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - (q\overline{\Psi}\gamma^\mu \Psi) A_\mu$
Massive Gauge Boson (Proca vs. Alternate)	$\mathfrak{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{Mc}{\hbar}\right)^2 A^\nu A_\nu$	$\mathfrak{L} = \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + i\frac{q}{2}(\alpha I_0^\mu + \beta A^\mu) A_\mu$ $\mathfrak{L} = \frac{1}{16\pi} G^{\mu\nu} G_{\mu\nu} - i\frac{q}{2}(\alpha I_0^\mu - \beta B^\mu) B_\mu$
Massless Gauge Boson (Massless Proca vs. Alternate)	$\mathfrak{L} = \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$	$\mathfrak{L} = \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - i\frac{q}{2}\beta A^\mu A_\mu$
Higgs Field (Φ^4) vs. Alternate	$\mathfrak{L} = \frac{1}{2}\partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\mu^2 \Phi^* \Phi - \frac{1}{4}\lambda^2 (\Phi^*\Phi)^2$	$\mathfrak{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* (\partial_\mu \Phi)$
Massive Scalar Boson + Massive Gauge Boson (Higgs vs. Alternate)	$\mathfrak{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - (\mu)^2 \eta^2\right] + \left[-\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right] + \left\{\frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta A_\mu A^\mu + \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2 \eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \frac{1}{4}\lambda^2\eta^4\right\} + \left(\frac{\mu^2}{2\lambda}\right)^2$	$\mathfrak{L} = [\partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* (\partial_\mu \Phi)] + \frac{2i}{\hbar c}\left[-\frac{1}{16\pi} F^{\mu\nu} G_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu) A_\mu\right] + i\beta[(\partial^\mu \Phi^*)\Phi A_\mu - \Phi^* (\partial^\mu \Phi) B_\mu]$.