A CRITICAL ANALYSIS OF LIGO'S RECENT DETECTION OF GRAVITATIONAL WAVES CAUSED BY MERGING BLACK HOLES

Stephen J. Crothers

4 March 2016

e-mail: steve@plasmaresources.com

Abstract

The LIGO Scientific Collaboration and the Virgo Collaboration have announced that on 24 September 2015, LIGO detected an Einstein gravitational wave directly for the first time, with the first observation of a binary black hole merger. The announcement was made with much media attention. Not so long ago similar media excitement surrounded the announcement by the BICEP2 Team of detection of primordial gravitational waves imprinted in B-mode polarisations of a Cosmic Microwave Background, which proved to be naught. According to the LIGO and Virgo Collaborations, the gravitational wave LIGO allegedly detected was generated by two merging black holes, one of ~29 solar masses, the other of ~36 solar masses, at a distance of some 1.3 billion light years. The insurmountable problem for the credibility of LIGO's claims is the questionable character of the theoretical assumptions upon which they are based. In this paper various arguments are presented according to which the basic theoretical assumptions, and the consequential claims of detecting gravitational waves, are proven false. The apparent detection by the LIGO-Virgo Collaborations is not related to gravitational waves or to the collision and merger of black holes.

Keywords: Gravitational waves; Black holes; Big Bang Cosmology; General Relativity.

1 Introduction

The LIGO Scientific Collaboration and Virgo Collaboration have announced [1] that on the 14th of September 2015, at 09:50:45 UTC, they detected a transient Einstein gravitational wave, designated GW150914, produced by two merging black holes forming a single black hole. The two black holes that merged are reported to have been of ~29 solar masses and ~36 solar masses respectively, the newly formed black hole at ~62 solar masses, radiating away ~3 solar masses as Einstein gravitational waves.

The reported detection was obtained, not during an observation run of LIGO, but during an engineering test run, prior to the first scheduled observation run on 18 September 2015.

> *"In the early morning hours of September 14, 2015 — during an engineering run just days before official data-taking started — a strong signal, consistent with merging black holes, appeared simultaneously in LIGO's two observatories, located in Hanford, Washington and Livingston, Louisiana."* Conover [2]

> *"The eighth engineering run (ER8) began on 2015 August 17 at 15:00 and critical software was frozen by August 30. The rest of ER8 was to be used to calibrate the detectors, to carry out diagnostic studies, to practice maintaining a high coincident duty cycle, and to train and tune the data analysis pipelines. Calibration was complete by September 12 and O1 was scheduled to begin on September 18. On 2015 September 14, cWB reported a burst candidate to have occurred at 09:50:45 with a network signal-to-noise ratio (S/N) of 23.45 ..."*
>
> Abbott *et al*. [3]

Magnitudes with error margins have been presented by the LIGO-Virgo Collaborations for the masses of the black holes, along with other related source quantities, in their TABLE I [1].

TABLE I. Source parameters for GW150914. We report median values with 90% credible intervals that include statistical errors, and systematic errors from averaging the results of different waveform models. Masses are given in the source frame; to convert to the detector frame multiply by $(1+z)$ [90]. The source redshift assumes standard cosmology [91].

Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

Reproduced from Abbott, B.P. *et al*., Observation of Gravitational Waves from a Binary Black Hole Merger, *PRL* 116, 061102 (2016), DOI: 10.1103/PhysRevLett.116.061102

There are two ways by which the LIGO report can be analysed: (a) examination of the LIGO interferometer operation and data acquisition, and (b) consideration of the theories of black holes and gravitational waves upon which all else relies. Only theoretical considerations are considered herein, as there is no need to analyse the LIGO apparatus and its signal data to understand that the claim for detection of Einstein gravitational waves and black holes is built upon theoretical fallacies and conformational bias.

2 Gravitational waves, black holes and Big Bangs combined

Presumably the gravitational waves reported by LIGO-Virgo are present inside some Big Bang expanding universe as there has been no report that Big Bang cosmology has been abandoned. Of the gravitational wave detection the LIGO-Virgo Collaborations have stated,

> "*It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole.*" Abbott *et al*. [1]

All purported black hole equations are solutions to corresponding specific forms of Einstein's nonlinear field equations. Gravitational waves on the other hand are obtained from a linearisation of Einstein's nonlinear field equations, combined with a deliberate selection of coordinates that produce the assumed afore propagation at the speed of light *in vacuo*. Because General Relativity is a nonlinear theory, the Principle of Superposition does not hold. Let ***X*** be some black hole universe and ***Y*** be some Big Bang universe. Then the linear combination (i.e. superposition) ***X*** + ***Y*** is not a universe. Indeed, ***X*** and ***Y*** pertain to completely different sets of Einstein field equations and so they have absolutely nothing to do with one another. The same argument holds if ***X*** and ***Y*** are both black hole universes, be they the same or not, and if ***X*** and ***Y*** are Big Bang universes, be they the same or not. Consequently, a black hole universe cannot co-exist with any other black hole universe or with any Big Bang universe.

All black hole universes:

(1) are spatially infinite
(2) are eternal
(3) contain only one mass
(4) are not expanding (i.e. are not non-static)
(5) are either asymptotically flat or asymptotically curved.

All Big Bang universes:

(1) are either spatially finite (1 case; k = 1) or spatially infinite (2 different cases; k = -1, k = 0)
(2) are of finite age (~13.8 billion years)
(3) contain radiation and many masses
(4) are expanding (i.e. are non-static)
(5) are not asymptotically anything.

Note also that no black hole universe even possesses a Big Bang universe k-curvature.

It is clearly evident that black holes and Big Bang universes cannot co-exist by their very definitions.

All the different black hole 'solutions' are also applicable to stars, planets and the like. Thus, these equations do not permit the presence of more than one star or planet in the universe. In the case of a body such as a star, the only significant difference is that its spacetime does not bend to infinite curvature at the star because there is no singularity and no event horizon in this case. Newton's theory on the other hand, places no restriction on the number of masses that can be present.

Since a black hole universe is a solution to a specific set of Einstein's nonlinear field equations it is not possible to extract from it any gravitational waves that are produced from linearised field equations. No gravitational waves can in fact be extracted from Einstein's nonlinear field equations [4]. Superposing solutions obtained from the nonlinear system with those from the linearised system violates the mathematical structure of General Relativity. Accordingly, contrary to the report of the LIGO-Virgo Collaborations, gravitational waves cannot exist in any black hole universe. Neither can they exist in any Big Bang universe because all Big Bang models are in fact single mass universes by mathematical construction [4,5]. Nonetheless the LIGO-Virgo Collaborations superpose everything [1]:

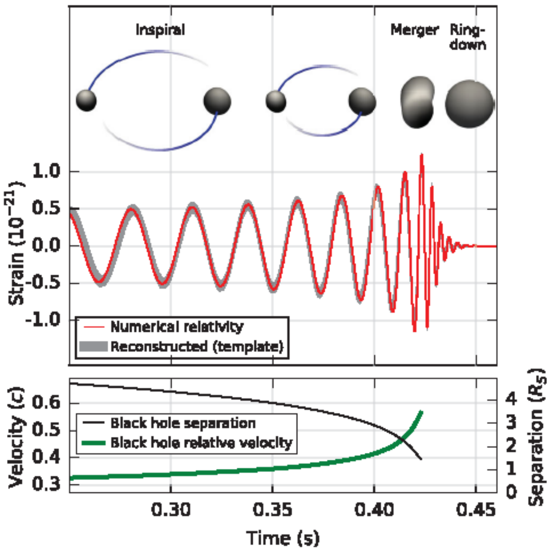

FIG.2."Top: Estimated gravitational-wave strain amplitude from GW150914 projected onto H1. This shows the full bandwidth of the waveforms, without the filtering used for Fig. 1. The inset images show numerical relativity models of the black hole horizons as the black holes coalesce. Bottom: The Keplerian effective black hole separation in units of Schwarzschild radii ($R_S = 2GM/c^2$) and the effective relative velocity given by the post-Newtonian parameter $v/c = (GM\pi f/c^3)^{1/3}$, where f is the gravitational-wave frequency calculated with numerical relativity and M is the total mass (value from Table I)."

Figure and text reproduced from Abbott, B.P. *et al.*, Observation of Gravitational Waves from a Binary Black Hole Merger, *PRL* **116**, 061102 (2016), DOI: 10.1103/PhysRevLett.116.061102

3 Gravitational wave propagation speed

The LIGO-Virgo Collaborations opened the *Introduction* to their paper with the following:

> "*In 1916, the year after the final formulation of the field equations of general relativity, Albert Einstein predicted the existence of gravitational waves. He found that the linearized weak-field equations had wave solutions: transverse waves of spatial strain that travel at the speed of light, generated by time variations of the mass quadrupole moment of the source*".
>
> Abbott *et al*. [1]

The impression given here that the speed of propagation of Einstein's gravitational waves is uniquely that of light *in vacuo* is false. Propagation speed of Einstein's gravitational waves is arbitrary, because it is coordinate dependent. That Einstein's waves seem to travel uniquely at the speed of light *in vacuo* is simply because this speed was deliberately selected in order to conform to the presupposition. The method employed to determine the wave equation for Einstein's gravitational waves is the weak-field 'linearisation' of his field equations and concomitant selection of a specific set of coordinates.

Maxwell's electromagnetic theory predicts sinusoidal electromagnetic-wave propagation *in vacuo* at the unique speed *v,* given by,

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

The speed of light changes according to the permittivity and permeability of the dielectric medium in which it travels,

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\kappa\kappa_m}}$$

wherein κ and κ_m are the dielectric constant and relative permeability respectively of the medium. Note that the speed of electromagnetic wave propagation *in vacuo* is not arbitrary. Since the speed of light 'in vacuo' plays a central role in Einstein's Relativity Theory, the motive for choosing coordinates in order to make gravitational waves, emerging from the linearisation game[1], travel at the speed of light 'in vacuo', is abundantly clear.

Appendix A presents the mathematical means of linearisation of Einstein's field equations, revealing the arbitrary nature of the speed of his gravitational waves.

[1]"*The rules of the 'linearization game' are as follows: (a) $h_{\mu\nu}$ together with its first derivatives $h_{\mu\nu,\rho}$ and higher derivatives are small, and all products of these are ignored; (b) suffixes are raised and lowered using $\eta^{\mu\nu}$ and $\eta_{\mu\nu}$, rather than $g^{\mu\nu}$ and $g_{\mu\nu}$.*" Foster & Nightingale [6]

4 A black hole is a universe

Each and every black hole is an independent universe by its very definition, no less than the Big Bang universes are independent universes, because the black hole universe is not contained within its event horizon. Its spacetime extends indefinitely far from its singularity. All types of black hole universes are spatially infinite and eternal, and are either asymptotically flat or, in more esoteric cases, asymptotically curved. There is no bound on asymptotic, for otherwise it would not be asymptotic. Thus every type of black hole constitutes an independent infinite and eternal universe; bearing in mind also that each different type of black hole universe pertains to a different set of Einstein field equations and therefore have nothing to do with one another. Without the asymptotic condition the black hole equations do not result, and one can then write as many non-asymptotic solutions to the corresponding Einstein field equations for the supposed different types of black holes as one pleases [4,7], none of which produces a black hole.

> *"Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time."*
>
> Dictionary of Geophysics, Astrophysics and Astronomy [9]

> *"The Kerr-Newman solutions … are explicit asymptotically flat stationary solutions of the Einstein-Maxwell equation (λ = 0) involving just three free parameters **m**, **a** and **e**. … the mass, as measured asymptotically, is the parameter **m** (in gravitational units). The solution also possesses angular momentum, of magnitude **am**. Finally, the total charge is given by **e**. When **a** = **e** = 0 we get the Schwarzschild solution."* Penrose [10]

> "*The charged Kerr metrics are all stationary and axisymmetric … They are asymptotically flat…*" Wald [11]

5 Black hole gravity

Cosmology maintains that the finite mass of a black hole is concentrated at its 'singularity', where volume is zero, density is infinite, and spacetime curvature is infinite. There are two types of black hole singularity proposed by cosmologists, according to whether or not the black hole is rotating. In the case of no rotation the singularity is a point; in the case of rotation the singularity is the circumference of a circle. Cosmologists call them 'physical singularities'. These and other mathematical singularities of black hole equations are reified so as to contain the masses of black holes and to locate their event horizons. Singularities are actually only places in an equation where the equation is undefined, owing for example, to a division by zero. Although they have been construed as such by cosmology, singularities are not in fact physical entities. A singularity also occurs in Newton's theory; it is called the *centre of gravity*. The centre of gravity of a body is not a physical object, rather a mathematical artifice.

There are forces in General Relativity but gravity is not one of them, because it is spacetime curvature. It immediately follows that according to cosmology gravity is infinite at a black hole singularity. Infinities of density, spacetime curvature, and gravity are claimed to be physically real.

> "*The work that Roger Penrose and I did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole.*" Hawking [12]

> *"Once a body of matter, of any mass m, lies inside its Schwarzschild radius 2m it undergoes gravitational collapse . . . and the singularity becomes physical, not a limiting fiction."*
> Dodson and Poston [13]

> *"A nonrotating black hole has a particularly simple structure. At the center is the singularity, a point of zero volume and infinite density where all of the black hole's mass is located. Spacetime is infinitely curved at the singularity. . . . The black hole's singularity is a real physical entity. It is not a mathematical artifact . . ."*
> Carroll and Ostlie [14]

> *"As r decreases, the space-time curvature mounts (in proportion to r^{-3}), becoming theoretically infinite at r = 0."* Penrose [10]

> *"One says that 'r=0 is a physical singularity of spacetime.'"*
> Misner, Thorne & Wheeler [15]

However, no finite mass possesses zero volume, infinite density, or infinite gravity, anywhere.

6 The mathematical theory of black holes

The LIGO-Virgo Collaborations have invoked a binary black hole system, merging to cause emission of their reported gravitational waves.

> "*The basic features of GW150914 point to it being produced by the coalescence of two black holes—i.e., their orbital inspiral and merger, and subsequent final black hole ringdown. Over 0.2s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum. The most plausible explanation for this evolution is the inspiral of two orbiting masses, m_1 and m_2, due to gravitational-wave emission*."
> Abbott *et al*. [1]

In the *Introduction* of their paper the LIGO-Virgo Collaborations incorrectly attribute the black hole to K. Schwarzschild.

> "*Also in 1916, Schwarzschild published a solution for the field equations [4] that was later understood to describe a black hole [5,6], and in 1963 Kerr generalized the solution to rotating black holes [7].*"
>
> Abbott *et al*. [1]

The resultant black hole type is identified in [1].

> "*A pair of neutron stars, while compact, would not have the required mass, while a black hole neutron star binary with the deduced chirp mass would have a very large total mass, and would thus merge at much lower frequency. This leaves black holes as the only known objects compact enough to reach an orbital frequency of 75 Hz without contact. Furthermore, the decay of the waveform after it peaks is consistent with the damped oscillations of a black hole relaxing to a final stationary Kerr configuration*." Abbott *et al*. [1]

All the black holes are identified in [8].

> *"Here we perform several studies of GW150914, aimed at detecting deviations from the predictions of GR. Within the limits set by LIGO's sensitivity and by the nature of GW150914, we find no statistically significant evidence against the hypothesis that, indeed, GW150914 was emitted by a binary system composed of two black holes (i.e., by the Schwarzschild [17] or Kerr [18] GR solutions), that the binary evolved dynamically toward merger, and that it formed a merged rotating black hole consistent with the GR solution."* Abbott *et al.* [8]

Note the invalid superposition of the two 'Schwarzschild' or 'Kerr' black holes, due to violation of their asymptotic flatness (each encounters infinite spacetime curvature i.e. infinite gravity, at the singularity of the other). The Kerr configuration subsumes the Schwarzschild configuration and so depends upon the existence of the latter. The Schwarzschild solution has no physical meaning because it is the solution to a set of physically meaningless equations (see §7 below). Furthermore, all black hole equations are obtained by violations of the rules of pure mathematics, as will now be proven.

Einstein's field equations 'in the absence of matter' are,

$$R_{\mu\nu} = 0 \tag{6.1}$$

Consider Schwarzschild's actual solution [16] to Eq.(6.1), which is not the solution that has been assigned to him by cosmologists.

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (6.2)$$

$$R = \left(r^3 + \alpha^3\right)^{1/3} \quad 0 \leq r$$

Here α is a positive real-valued constant and $r = \sqrt{x^2 + y^2 + z^2}$. The speed of light *in vacuo* is set to unity, i.e. $c = 1$. Eq.(6.2) is singular (i.e. undefined) only at $r = 0$ (i.e. when $x = y = z = 0$). Contrast this with the so-called 'Schwarzschild solution' attributed to Schwarzschild but actually due to the German mathematician D. Hilbert [17],

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (6.3)$$

$$0 \leq r$$

Here $c = 1$, Newton's gravitational constant $G = 1$, and M is claimed to be the mass that produces the gravitational field described by Eq.(6.3) for a universe according to Einstein's field equations $R_{\mu\nu} = 0$ [18]. Note that *prima facie* Eq.(6.3) is singular (i.e. undefined) at $r = 2M$ and at $r = 0$. Eq.(6.2) and Eq.(6.3) are not equivalent owing to $0 \leq r$ in Eq.(6.3). If they are equivalent then in Eq.(6.2) it must be that $-\alpha \leq r = \sqrt{x^2 + y^2 + z^2}$. Eq.(6.3) is somewhat deceptive. Rewriting it explicitly with c and G is much more informative,

$$ds^2 = c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (6.3b)$$

$$0 \leq r$$

In Eq.(6.3) the so-called 'Schwarzschild radius' is $r_S = 2M$. From Eq.(6.3b) the Schwarzschild radius r_S is easily identified in full,

> *"This value is known as the Schwarzschild radius. In terms of the mass of the object that is the source of the gravitational field, it is given by*
>
> $$r_s = \frac{2GM}{c^2}$$
>
> *"For ordinary stars, the Schwarzschild radius lies buried deep in the stellar interior*." McMahon [19]

> *"Remarkably, as we shall see this is exactly the modern formula for the radius of a black hole in general relativity …"*
>
> Schutz [20]

According to cosmologists, the Schwarzschild radius (or 'gravitational radius') is the radius of the event horizon of a black hole. That *r* is incorrectly treated as the radius by cosmologists is most clearly evident by the very 'Schwarzschild radius', which for stars, *"lies buried deep in the stellar interior"* [19].

> *"The Schwarzschild radius for the Earth is about 1.0 cm and that of the Sun is 3.0 km."* d'Inverno [21]
>
> "*For example, a Schwarzschild black hole of mass equal to that of the Earth, $M_E = 6 \times 10^{27}$g, has $r_s = 2GM_E/c^2 \sim 1$ cm. … A black hole of one solar mass has a Schwarzschild radius of only 3km*." Wald [11]
>
> "*'ordinary' stars and planets contain matter ($T_{\mu\nu} \neq 0$) within a certain radius $r > 2M$, so that for them the validity of the Schwarzschild solution stops there*." 't Hooft [22]

In Hilbert's solution cosmologists call $r = r_s$ a 'coordinate' or 'apparent' singularity, and $r = 0$ a 'physical' singularity, because it is endowed with fantastic physical properties (see §5 above).

Let *r'* be the radius of a Euclidean sphere. It is routinely claimed for Eq.(6.3) and Eq.(6.3b) that $r = r' = \sqrt{x^2 + y^2 + z^2}$ (Einstein [18]), from which the black hole was constructed. This is incorrect [4,7,23-35] because here,

$$r = \sqrt{x_0^2 + y_0^2 + z_0^2} + \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} = r_0 + r' \qquad (6.4)$$

where $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = \dfrac{2GM}{c^2}$

Notwithstanding, *r* is neither the radius nor even a distance in any black hole equation [4,7,23-35]; a mathematical fact which subverts the entire theory of black holes. The reader is referred to [4,7,23-35] for all the mathematical details, which I only summarise herein.

Geometrically speaking Eq.(6.4) means that the black hole is the result of unwittingly moving a sphere, initially centred at the origin of coordinates, to some other place, but leaving its centre behind. Analytically speaking this is revealed by,

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (6.5)$$

$$R_c = \left(|r - r_0|^n + \alpha^n\right)^{1/n} \quad r, r_0 \in \Re,\, n \in \Re^+$$

The radius R_p is,

$$R_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c(R_c - \alpha)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c - \alpha}}{\sqrt{\alpha}}\right) \qquad (6.6)$$

Note that $R_p(r_0) = 0\ \forall r_0\ \forall n$.

The constants r_0 and n are arbitrary. Setting $r_0 = 0$, $n = 3$, $r_0 \le r$ yields Schwarzschild's actual solution [16]. Setting $r_0 = 0$, $n = 1$, $r_0 \le r$ yields Brillouin's solution [36]. Setting $r_0 = \alpha$, $n = 1$, $r_0 \le r$ yields Droste's solution [37]. Hilbert's solution is not an element of the infinite equivalence class. Note that Hilbert's solution is an alleged 'extension' of Droste's solution. Owing to equivalence, if any element of the infinite equivalence class cannot be extended then none can be extended, owing to equivalence. It is immediately apparent that none can be extended because $|r - r_0|^n \ge 0$. This is amplified by the case $r_0 = 0$, $n = 2$, in which case Eq.(6.5) is defined for all real values of r except $r = r_0 = 0$. In this case the black hole requires that,

$$-\alpha^2 \le r^2 = \left(x^2 + y^2 + z^2\right)$$

Thus, the Schwarzschild black hole is invalid because it violates the rules of pure mathematics – it requires the square of a real number to be less than zero. The same violation of the rules of pure mathematics produces all the black hole universes [4,7,23-35]. All purported means of extending Droste's solution to Hilbert's are consequently invalid [4,31,32].

The Kerr-Newman solution adds charge and angular momentum to the Schwarzschild solution. The infinite equivalence class for Kerr-Newman spacetime is given by [4,23,24,35],

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\varphi\right)^2 + \\ + \frac{\sin^2\theta}{\rho^2}\left[\left(R_c^2 + a^2\right)d\varphi - a\,dt\right]^2 + \\ + \frac{\rho^2}{\Delta}dR_c^2 + \rho^2 d\theta^2$$

$$\Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad \rho^2 = R_c^2 + a^2\cos^2\theta$$

$$R_c = \left(\left| r - r_0 \right|^n + \psi^n \right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

$$\psi = \frac{\alpha + \sqrt{\alpha^2 - 4q^2 - 4a^2 \cos^2\theta}}{2}, \quad a^2 + q^2 < \frac{\alpha^2}{4}$$

No black hole is possible without a violation of the rules of pure mathematics, as the case $r_0 = 0$, $n = 2$ amplifies.

7 The paradox of black hole mass

Although one violation of the rules of pure mathematics is sufficient to invalidate it, the black hole violates other rules of logic. Einstein maintains that although Eq.(6.1) contains no terms for material sources (since $T_{\mu\nu} = 0$), a material source is nonetheless present, in order to cause a gravitational field. The material source is rendered present linguistically by the assertion that Eq.(6.1) describes the gravitational field *outside a body such as a star*. Indeed, concerning Hilbert's solution, Einstein writes,

$$\text{“} ds^2 = \left(1 - \frac{A}{r}\right) dl^2 - \left[\frac{dr^2}{1 - \frac{A}{r}} + r^2 \left(\sin^2\theta \, d\varphi^2 + d\theta^2 \right) \right]$$

$$A = \frac{\kappa M}{4\pi} \tag{109a}$$

> *M denotes the sun's mass centrally symmetrically placed about the origin of co-ordinates; the solution (109a) is valid only outside this mass, where all the $T_{\mu\nu}$ vanish.*" Einstein [18]

Note that Einstein has incorrectly asserted, in the standard fashion of cosmologists, that his mass M in his Eq.(109a) is "*centrally symmetrically placed about the origin of co-ordinates*".

Einstein's field equations,

> *"... couple the gravitational field (contained in the curvature of spacetime) with its sources."* Foster & Nightingale [6]

On the one hand, Einstein removes all material sources by setting $T_{\mu\nu} = 0$ and on the other hand immediately reinstates the presence of a massive source with words, by alluding to a mass "*outside*" of which equations $R_{\mu\nu} = 0$ apply; since his gravitational field must be caused by matter. According to Einstein his equation (109a) contains a massive source, at "*the origin*", yet also according to Einstein Eq.(6.1), from which (109a) is obtained, contains no material source. This contradiction is readily amplified. That Eq.(6.1) contains no material sources whatsoever is easily reaffirmed by the field equations,

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{7.1}$$

The constant λ is the so-called 'cosmological constant'. The solution for Eq.(7.1) is de Sitter's empty universe, which is empty precisely because the energy-momentum tensor for material sources is zero, i.e. $T_{\mu\nu} = 0$. De Sitter's universe contains no matter:

> "*This is not a model of relativistic cosmology because it is devoid of matter*." d'Inverno [5]
>
> "*the de Sitter line element corresponds to a model which must strictly be taken as completely empty*." Tolman [21]
>
> "*the solution for an entirely empty world*." Eddington [38]
>
> "*there is no matter at all*!" Weinberg [39]

Note that in Eq.(6.1) and Eq.(7.1), $T_{\mu\nu} = 0$. Thus, according to Einstein and the cosmologists, material sources are both present and absent by the very same mathematical constraint, which is a violation of the rules of logic. Since de Sitter's universe is devoid of material sources by virtue of $T_{\mu\nu} = 0$, the 'Schwarzschild solution' must also be devoid of material sources by the very same constraint. Thus, Eq.(6.1) contains no matter and its solution physically meaningless. But it is upon Eq.(6.1) that black hole theory relies.

8 Localisation of gravitational energy and conservation laws

Without a theoretical framework by which the usual conservation laws for the energy and momentum of a closed system hold, as determined by a vast array of experiments, there is no means to produce gravitational waves by General Relativity. Einstein was aware of this and so devised a means for his theory to satisfy the usual conservation of energy and momentum for a closed system. However, Einstein's method of solving this problem is invalid because he violated the rules of pure mathematics. There is in fact no means by which the usual conservation laws can be satisfied by General Relativity. Consequently the concept of gravitational waves has no valid theoretical basis in Einstein's theory.

It must first be noted that when Einstein talks of the conservation of energy and momentum he means that the sum of the energy and momentum of his gravitational field and its material sources is conserved *in toto*, in the usual way for a closed system, as experiment attests, for otherwise his theory is in conflict with experiments and therefore invalid.

> *"It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum of matter alone."*
>
> Einstein [18]

The meaning of Einstein's 'matter' needs to be clarified.

> *"We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter'. Our use of the word therefore includes not only matter in the ordinary sense, but the electromagnetic field as well."* Einstein [40 §14]

The energy-momentum of Einstein's matter alone is contained in his energy-momentum tensor $T_{\mu\nu}$. To account for the energy-momentum of his gravitational field alone Einstein introduced his pseudotensor $t^{\alpha}{}_{\sigma}$, defined by (Einstein [40 §15]),

$$\kappa t^{\alpha}_{\sigma} = \frac{1}{2}\delta^{\alpha}_{\sigma} g^{\mu\nu}\Gamma^{\lambda}_{\mu\beta}\Gamma^{\beta}_{\nu\lambda} - g^{\mu\nu}\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\sigma} \tag{8.1}$$

where κ is a constant and $\delta^{\alpha}{}_{\sigma}$ is the Kronecker-delta.

> "*The quantities $t^{\alpha}{}_{\sigma}$ we call the 'energy components' of the gravitational field*". Einstein [40 §15]

But $t^{\alpha}{}_{\sigma}$ is not a tensor. As such it is a coordinate dependent quantity, contrary to the basic coordinate independent tenet of General Relativity.

> "*It is to be noted that $t^{\alpha}{}_{\sigma}$ is not a tensor*" Einstein [40 §15]

The justification is that $t^{\alpha}{}_{\sigma}$ acts 'like a tensor' under linear transformations of coordinates when subjected to certain strict conditions. Einstein then takes an ordinary divergence,

$$\frac{\partial t^{\alpha}_{\sigma}}{\partial x_{\alpha}} = 0 \tag{8.2}$$

and claims a conservation law for the energy and momentum of his gravitational field alone.

> "*This equation expresses the law of conservation of momentum and of energy for the gravitational field*."
>
> Einstein [40 §15]

Einstein added his pseudotensor for his gravitational field alone to his energy-momentum tensor for matter alone to obtain the total energy-momentum equation for his gravitational field and its material sources.

$$E = \left(t_{\mu}^{\sigma} + T_{\mu}^{\sigma}\right) \tag{8.3}$$

Not being a tensor equation, Einstein cannot take a tensor divergence. He therefore takes an ordinary divergence, [40 §17],

$$\frac{\partial\left(t_{\mu}^{\sigma} + T_{\mu}^{\sigma}\right)}{\partial x_{\alpha}} = 0 \tag{8.4}$$

and claims the usual conservation laws of energy and momentum for a closed system,

> "*Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied. … here, instead of the energy components t^{σ}_{μ} of the gravitational field, we have to introduce the totality of the energy components of matter and gravitational field*."
>
> Einstein [40 §17]

The mathematical error is profound, but completely unknown to cosmologists. Contract Einstein's pseudotensor by setting $\sigma = \alpha$ to yield the invariant $t = t^{\alpha}_{\alpha}$, thus,

$$\kappa t_{\alpha}^{\alpha} = \kappa t = \frac{1}{2}\delta_{\alpha}^{\alpha} g^{\mu\nu}\Gamma_{\mu\beta}^{\lambda}\Gamma_{\nu\lambda}^{\beta} - g^{\mu\nu}\Gamma_{\mu\beta}^{\alpha}\Gamma_{\nu\alpha}^{\beta} \tag{8.5}$$

Since the $\Gamma^{\alpha}_{\beta\sigma}$ are functions only of the components of the metric tensor and their first derivatives, *t* is seen to be a first-order intrinsic differential invariant [4,23,24,41], i.e. it is an invariant that depends solely upon the components of the metric tensor and their first derivatives. However, the pure mathematicians proved in 1900 that first-order intrinsic differential invariants do not exist [42]. Thus, by *reductio ad absurdum*, Einstein's pseudotensor is a meaningless collection of mathematical symbols. Contrary to Einstein and cosmologists, it cannot therefore be used to represent anything in physics or to make any calculations, including those for the energy of Einstein's gravitational waves. Nevertheless, cosmology calculates:

"*It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy within a definite (three dimensional) region at a certain time is independent of the coordinate system. Thus, in general, gravitational energy cannot be localized. The best we can do is to use the pseudotensor, which satisfies condition (i) but not condition (ii). It gives us approximate information about gravitational energy, which in some special cases can be accurate*." Dirac [43]

"*Let us consider the energy of these waves. Owing to the pseudo-tensor not being a real tensor, we do not get, in general, a clear result independent of the coordinate system. But there is one special case in which we do get a clear result; namely, when the waves are all moving in the same direction.*" Dirac [43]

Consider the following two equivalent forms of Einstein's field equations,

$$R^{\mu}_{\nu} = -\kappa\left(T^{\mu}_{\nu} - \frac{1}{2}Tg^{\mu}_{\nu}\right) \tag{8.6}$$

$$T^{\mu}_{\nu} = -\frac{1}{\kappa}\left(R^{\mu}_{\nu} - \frac{1}{2}Rg^{\mu}_{\nu}\right) \tag{8.7}$$

By Eq.(8.6), according to Einstein, if $T^{\mu}{}_{\nu} = 0$ then $R^{\mu}{}_{\nu} = 0$. But by Eq.(8.7), if $R^{\mu}{}_{\nu} = 0$ then $T^{\mu}{}_{\nu} = 0$. In other words, $R^{\mu}{}_{\nu}$ and $T^{\mu}{}_{\nu}$ must vanish identically – if either is zero then so is the other, and the field equations reduce to the identity 0 = 0 [4,23,24,41]. Hence, if there are no material sources (i.e. $T^{\mu}{}_{\nu} = 0$) then there is no gravitational field, and no universe. Bearing this in mind, with Eq.(8.3) and Eq.(8.4), consideration of the conservation of energy and momentum, and tensor relations, Einstein's field equations must take the following form [4,23,24,41],

$$\frac{G^{\mu}_{\nu}}{\kappa} + T^{\mu}_{\nu} = 0 \tag{8.8}$$

where $G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2}Rg^{\mu}_{\nu}$

Comparing this to Eq.(8.3) it is clear that the $G^{\mu}{}_{\nu}/\kappa$ actually constitute the energy-momentum components of Einstein's gravitational field [4,23,24,41], which is rather natural since the Einstein tensor $G^{\mu}{}_{\nu}$ describes the curvature of Einstein's spacetime (i.e. his gravitational field). Eq.(8.8) also constitutes the total energy-momentum equation for Einstein's gravitational field and its material sources combined.

"*space as opposed to 'what fills space', which is dependent on the coordinates, has no separate existence*" Einstein [44]

"*I wish to show that space-time is not necessarily something to which one can ascribe a separate existence, independently of the actual objects of physical reality*." Einstein [45]

Unlike Eq.(8.3), Eq.(8.8) is a tensor equation. The tensor (covariant derivative) divergence of the left side of Eq.(8.8) is zero and therefore constitutes the conservation law for Einstein's gravitational field and its material sources. However, the total energy-momentum by Eq.(8.8) is always zero, the $G^{u}{}_{v}/\kappa$ and the $T^{u}{}_{v}$ must vanish identically because spacetime and matter have no separate existence in General Relativity, and hence gravitational energy cannot be localised, i.e. there is no possibility of gravitational waves [4,23,24,41]. Moreover, since the total energy-momentum is always zero the usual conservation laws for energy and momentum of a closed system cannot be satisfied [4,23,24,41]. General Relativity is therefore in conflict with a vast array of experiments on a fundamental level.

The so-called 'cosmological constant' can be easily included as follows,

$$\frac{\left(G_{\nu}^{\mu}+\lambda g_{\nu}^{\mu}\right)}{\kappa}+T_{\nu}^{\mu}=0 \tag{8.9}$$

In this case the energy-momentum components of Einstein's gravitational field are the $(G^{\mu}{}_{\nu} + \lambda g^{\mu}{}_{\nu})/\kappa$. When $G^{\mu}{}_{\nu}$ *or* $T^{\mu}{}_{\nu}$ is zero, all must vanish identically, and all the same consequences ensue just as for Eq. (8.8). Thus, once again, if there is no material source, not only is there no gravitational field, there is no universe, and Einstein's field equations violate the usual conservation of energy and momentum for a closed system.

The so-called 'dark energy' is attributed to λ by cosmologists. Dark energy is a mysterious æther *ad arbitrium*, because, according to Einstein [18,46], λ is not a material source for a gravitational field, but is vaguely implicated by him in his gravitational field,

"*... by introducing into Hamilton's principle, instead of the scalar of Riemann's tensor, this scalar increased by a universal constant*" Einstein [46]

The 'cosmological constant' however falls afoul of de Sitter' empty universe, which possesses spacetime curvature but contains no matter, and is therefore physically meaningless. By Eq.(8.9), if λg_{ν}^{μ} is to be permitted, for the sake of argument, it must be part of the energy-momentum of the gravitational field, which necessarily vanishes when $T_{\nu}^{\mu}=0$. Recall that according to Einstein, everything except his gravitational field is matter and that matter is the cause of his gravitational field. The insinuation of λ can be more readily seen by writing Eq.(8.9) as,

$$\frac{\left[R_{\nu}^{\mu}-\frac{1}{2}(R-2\lambda)g_{\nu}^{\mu}\right]}{\kappa}+T_{\nu}^{\mu}=0 \tag{8.9b}$$

Einstein's "*scalar increased by a universal constant*" is clearly evident; it is the term -(R-2λ)/2. Hence Einstein's field equations "*in the absence of matter*" [40 §14], i.e. $R_{\mu\nu}=0$, once again, have no physical meaning, and so the Schwarzschild solution too has no physical meaning, despite putative observational verifications. Consequently, the theories of black holes and gravitational waves are invalid.

9 Numerical relativity and perturbations

Numerical analysis of merging black holes and perturbation of black holes are ill-posed procedures for the simple fact that such mathematical means cannot validate a demonstrable fallacy. Similarly, no amount of observation and experiment can legitimise entities that are the products of violations of the rules of pure mathematics and logic. Since the premises are false and the conclusions drawn from them inconsistent with them, such numerical and perturbation procedures are consequently of no scientific merit. Nonetheless the LIGO-Virgo Collaboration has stated,

> *"The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0 × 10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole."*
>
> *"Using the fits to numerical simulations of binary black hole mergers in [92,93], we provide estimates of the mass and spin of the final black hole, the total energy radiated in gravitational waves, and the peak gravitational-wave luminosity [39]."*
>
> Abbott *et al.* [1]

> *"Several analyses have been performed to determine whether or not GW150914 is consistent with a binary black hole system in general relativity [94]. A first consistency check involves the mass and spin of the final black hole. In general relativity, the end product of a black hole binary coalescence is a Kerr black hole, which is fully described by its mass and spin. For quasicircular inspirals, these are predicted uniquely by Einstein's equations as a function of the masses and spins of the two progenitor black holes. Using fitting formulas calibrated to numerical relativity simulations [92], we verified that the remnant mass and spin deduced from the early stage of the*

> *coalescence and those inferred independently from the late stage are consistent with each other, with no evidence for disagreement from general relativity."*
>
> Abbott *et al.* [1]

Signal GW150914 was extracted from a database containing tens of thousands of numerically determined waveforms generated on the false assumptions of the existence of black holes and gravitational waves. A 'generic' signal cGW was initially reported by LIGO, after which powerful computers extracted GW150914 from the waveform database for a best fit element.

> *"The initial detection was made by low-latency searches for generic gravitational-wave transients [41] and was reported within three minutes of data acquisition [43]. Subsequently, matched-filter analyses that use relativistic models of compact binary waveforms [44] recovered GW150914 as the most significant event from each detector for the observations reported here."* Abbott *et al.* [1]

With such powerful computing resources and so many degrees of freedom it is possible to best fit just about any LIGO instability with an element of its numerically determined waveform database. This is indeed the outcome for the LIGO-Virgo Collaborations, as they have managed to best fit a numerically determined waveform for and to entities that not only do not exist, but are not even consistent with General Relativity itself. This amplifies the futility of applying numerical and perturbation methods to ill-posed problems.

There are no known Einstein field equations for two or more masses and hence no known solutions thereto. There is no existence theorem by which it can even be asserted that Einstein's field equations contain latent capability for describing configurations of two or more masses [4,23,24,47]. General Relativity cannot account for the simple experimental fact that two fixed suspended masses approach one another upon release. It is for precisely these reasons that all the Big Bang models treat the universe as a single mass, an ideal indivisible fluid of uniform macroscopic density and pressure that permeates the entire universe.

10 Conclusions

LIGO did not detect Einstein gravitational waves and black holes. Black holes and Einstein's gravitational waves do not exist; they are not even consistent with General Relativity. The LIGO instability has been interpreted as gravitational waves produced by two merging black holes by a combination of theoretical fallacies, wishful thinking, and conformational bias. Black holes are products entire of violations of the rules of pure mathematics. General Relativity is riddled with logical inconsistencies, invalid mathematics, and impossible physics. The General Theory of Relativity cannot satisfy the usual conservation of energy and momentum for a closed system and is thereby in conflict with a vast array of experiments, rendering it untenable at a fundamental level.

LIGO is reported to have so far cost taxpayers $1.1 billion [48]. Just as with the Large Hadron collider at CERN, such large sums of public money demand justification by eventually finding what they said they would.

Appendix A

Einstein's gravitational waves are extracted from his linearisation of his nonlinear field equations. Accordingly the metric tensor is written as,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{A.1}$$

where the $h_{\mu\nu} << 1$ and $\eta_{\mu\nu}$ represents the Galilean values (1,-1,-1,-1). In the linearisation game the $h_{\mu\nu}$ slightly perturb the flat Minkowski spacetime $g_{\mu\nu} = \eta_{\mu\nu}$ from its flatness, and so suffixes are raised and lowered on the $h_{\mu\nu}$ by the $\eta_{\mu\nu}$. Here the $h_{\mu\nu}$ and their first derivatives $\partial h_{\mu\nu} / \partial x^{\sigma} \equiv h_{\mu\nu,\sigma}$, and higher derivatives, are small, and all products of them are neglected. Since the $\eta_{\mu\nu}$ are constants, the derivatives of Eq. (A.1) are $g_{\mu\nu,\sigma} = h_{\mu\nu,\sigma}$. The validity of the linearisation game is merely taken on trust because it leads to the desired result.

The selection of a specific coordinate system in order to ensure that gravitational waves propagate at the presupposed speed of light *c*=2.998x10^{8}m/s is exposed by the approximation of the Ricci tensor $R_{\mu\nu}$, in accordance with Eq.(A.1). The Ricci tensor can be first written in the following form by a contraction on the Riemann-Christoffel curvature tensor $R_{\rho\mu\nu\sigma}$, as follows,

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right) + \Gamma_{\beta\rho\sigma}\Gamma^{\beta}_{\mu\nu} - \Gamma_{\beta\rho\nu}\Gamma^{\beta}_{\mu\sigma}\right] \tag{A.2}$$

Since the last two terms of Eq.(A.2) are products of the components of the metric tensor $g_{\mu\nu}$ and their first derivatives, by the linearisation game they are neglected, so that the Ricci tensor reduces to,

$$R_{\mu\nu} = g^{\sigma\rho}\left[\frac{1}{2}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho} + g_{\mu\nu,\sigma\rho}\right)\right] \tag{A.3}$$

which can be broken into two parts,

$$R_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} g_{\mu\nu,\sigma\rho} + \frac{1}{2} g^{\sigma\rho}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right) \tag{A.4}$$

If the coordinates x^{α} are chosen so that the second part of Eq.(A.4) vanishes, the Ricci tensor reduces further as follows,

$$R_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} g_{\mu\nu,\sigma\rho} = \frac{1}{2} g^{\sigma\rho} \frac{\partial^2 g_{\mu\nu}}{\partial x^\rho \partial x^\sigma} \tag{A.5}$$

$$g^{\sigma\rho}\left(g_{\sigma\rho,\mu\nu} - g_{\nu\rho,\sigma\mu} - g_{\mu\sigma,\nu\rho}\right) = 0 \tag{A.6}$$

According to Eq.(A.1), $g_{\mu\nu,\beta} = h_{\mu\nu,\beta}$ and so on for higher derivatives. Hence,

$$R_{\mu\nu} = \frac{1}{2}\eta^{\sigma\rho} h_{\mu\nu,\sigma\rho} = \frac{1}{2}\eta^{\sigma\rho} \frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} \tag{A.5b}$$

$$\eta^{\sigma\rho}\left(h_{\sigma\rho,\mu\nu} - h_{\nu\rho,\sigma\mu} - h_{\mu\sigma,\nu\rho}\right) = 0 \tag{A.6b}$$

(remembering that suffixes on the kernel *h* are raised and lowered by $\eta_{\mu\nu}$ according to tensor type). Contracting Eq.(A.5b) yields the Ricci scalar,

$$R = \eta^{\nu\mu} R_{\mu\nu} = \frac{1}{2}\eta^{\nu\mu}\eta^{\sigma\rho} \frac{\partial^2 h_{\mu\nu}}{\partial x^\sigma \partial x^\rho} = \frac{1}{2}\eta^{\sigma\rho} \frac{\partial^2 h}{\partial x^\sigma \partial x^\rho} \tag{A.7}$$

Einstein's field equations (without cosmological constant) are,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} \tag{A.8}$$

In terms of $h_{\mu\nu}$ these become, using Eq.(A.5b) and Eq.(A.7),

$$\eta^{\sigma\rho} \frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x^\sigma} - \frac{1}{2}\eta^{\sigma\rho} \frac{\partial^2 h}{\partial x^\sigma \partial x^\rho} = -2\kappa T_{\mu\nu} \tag{A.8b}$$

The d'Almbertian operator is defined by,

$$\Box \equiv -\eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} \tag{A.9}$$

Recalling that $\eta^{\mu\nu}$ represents the Galilean values and that hence $\eta^{\mu\nu} = 0$ when $\mu \neq \nu$, Eq.(A.9) gives,

$$\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{A.10}$$

where ∇ is the differential operator *del* (or *nabla*), defined as,

$$\nabla \equiv \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle .$$

Taking the dot product of *del* with itself gives the Laplacian operator ∇^2,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Setting $x^0=ct$, $x^1=x$, $x^2=y$, $x^3=z$, Eq.(A.8b) can be written as,

$$\Box\left(h_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu}h\right) = -2\kappa T_{\mu\nu} \qquad \text{(A.11)}$$

These are the linearised field equations. They are subject to the condition (A.6b), which can be condensed to the following condition [38],

$$\frac{\partial}{\partial x^\alpha}\left(h_\mu^\alpha - \frac{1}{2}\delta_\mu^\alpha h\right) = 0 \qquad \text{(A.6c)}$$

Using Eq.(A.9), Eq.(A.5b) can be written as,

$$\Box\, h_{\mu\nu} = 2R_{\mu\nu} \qquad \text{(A.12)}$$

For empty space this becomes,

$$\Box\, h_{\mu\nu} = 0 \qquad \text{(A.13)}$$

which by Eq.(A.10) describes a wave propagating at the speed of light *in vacuo*. However, the crucial point of the foregoing mathematical development is that Einstein's gravitational waves do not have a unique speed of propagation. The speed of the waves is coordinate dependent, as the condition at Eq.(A.6) attests. It is the constraint at Eq.(A.6) that selects a set of coordinates to produce the propagation speed *c*. A different set of coordinates yields a different speed of propagation, as Eq.(A.3) does not have to be constrained by Eq.(A.6). Einstein deliberately chose a set of coordinates that yields the desired speed of propagation at that of light in vacuum (i.e. c = 2.998x10^8 m/s) in order to satisfy the presupposition that propagation is at speed *c*. There is no *a priori* reason why this particular set of coordinates is better than any other. The sole purpose for the choice is to obtain the desired and presupposed result.

> *"All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials* $g_{\mu\nu}$ *pertain not only to the gravitational influence which has objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with* ***the speed of thought****, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of* $g_{\mu\nu}$.
>
> *"The statement that in the relativity theory gravitational waves are propagated with the speed of light has, I believe,*

been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle." Eddington [38 §57]

References

[1] Abbott, B.P. *et al*., Observation of Gravitational Waves from a Binary Black Hole Merger, *PRL* 116, 061102 (2016)

[2] Conover, E., Gravitational Waves Caught in the Act, *APS News*, February 11 2016, accessed online 3rd March 2016

[3] Abbott, B.P. *et al*., LOCALIZATION AND BROADBAND FOLLOW-UP OF THE GRAVITATIONAL-WAVE TRANSIENT GW150914, arXiv:1602.08492v1 [astro-ph.HE] 26 Feb 2016

[4] Crothers, S.J., General Relativity: In Acknowledgement Of Professor Gerardus 't Hooft, Nobel Laureate, 4 August, 2014, http://viXra.org/abs/1409.0072

[5] Tolman, R. C., Relativity Thermodynamics and Cosmology, Dover Publications Inc., New York, (1987)

[6] Foster,J., nightingale, J.D., A Short Course in General Relativity, 2nd Ed., Springer-Verlag, New York, (1995)

[7] Crothers, S.J., On Corda's 'Clarification' of Schwarzschild's Solution, http://vixra.org/abs/1602.0221

[8] Abbott, B.P. *et al.*,Tests of general relativity with GW150914, arXiv:1602.03841v1 [gr-qc] 11 Feb 2016

[9] Dictionary of Geophysics, Astrophysics, and Astronomy, Matzner, R. A., Ed., CRC Press LLC, Boca Raton, LA, (2001)

[10] Penrose, R., Gravitational Collapse: The role of General Relativity, *General Relativity and Gravitation*, Vol. 34, No. 7, July 2002

[11] Wald, R. M., General Relativity, The University of Chicago Press, (1984)

[12] Hawking, S. W., The Theory of Everything, The Origin and Fate of the Universe, New Millennium Press, Beverly Hills, CA, (2002)

[13] Dodson, C. T. J. & Poston, T., Tensor Geometry - The Geometric Viewpoint and its Uses, 2nd Ed., Springer-Verlag, (1991)

[14] Carroll, B. W. & Ostlie, D. A., An Introduction to Modern Astrophysics, Addison-Wesley Publishing Company Inc., (1996)

[15] C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation, W.H. Freeman and Company, New York, (1973)

[16] Schwarzschild, K., On the Gravitational Field of a Point Mass According to Einstein's Theory, *Sitzungsber. Preuss. Akad. Wiss., Phys. Math.* **Kl**: 189 (1916),

[17] Abrams, L. S., Black holes: the legacy of Hilbert's error. *Can. J. Phys*., v. 67, 919, (1989)

[18] Einstein, A., The Meaning of Relativity, expanded Princeton Science Library Edition, (2005)

[19] McMahon, D., Relativity Demystified, A Self teaching Guide, McGraw-Hill, New York,(2006)

[10] Schutz, B., A First Course in General Relativity, 2nd Ed., Cambridge University Press, (2009)

[21] d'Inverno, R., Introducing Einstein's Relativity, Oxford University Press, (1992)

[22] 't Hooft, G., Introduction to General Relativity, online lecture notes, 8/4/2002, http://www.phys.uu.nl/thooft/lectures/genrel.pdf http://www.staff.science.uu.nl/~hooft101/lectures/genrel_2013.pdf

[23] Crothers, S. J., 'Flaws in Black Hole Theory and General Relativity', for the Proceedings of the XXIXth International Workshop on High Energy Physics, Protvino, Russia, 26-28 June 2013, http://www.worldscientific.com/worldscibooks/10.1142/9041 http://viXra.org/abs/1308.0073

[24] Crothers, S.J., The Schwarzschild solution and its implications for gravitational waves, for 'Mathematics, Physics and Philosophy in the Interpretations of Relativity Theory', Budapest, (4-6 September, 2009), www.sjcrothers.plasmaresources.com/Budapest09-b.pdf

[25] Crothers, S. J., Black Hole and Big Bang: A Simplified Refutation, 2013, http://viXra.org/abs/1306.0024

[26] Crothers, S. J., On the Geometry of the General Solution for the Vacuum Field of the Point-Mass, *Progress in Physics,* v.2, pp. 3-14, 2005, http://vixra.org/abs/1101.0003

[27] Crothers, S.J., A Short Discussion or Relativistic Geometry, *Bulletin of Pure and Applied Sciences*, Vol. 24E (No.2) 2005:P.267-273, http://viXra.org/abs/1208.0229

[28] Crothers, S. J., On the General Solution to Einstein's Vacuum Field and Its Implications for Relativistic Degeneracy, *Progress in Physics,* v.1, pp. 68-73, 2005, http://vixra.org/abs/1012.0018

[29] Crothers, S. J., Gravitation on a Spherically Symmetric Metric Manifold, *Progress in Physics,* v.1, pp. 68-74, 2007, http://vixra.org/abs/1101.0005

[30] Crothers, S. J., On The 'Stupid' Paper by Fromholz, Poisson and Will, http://viXra.org/abs/1310.0202

[31] Crothers, S. J. On the Invalidity of the Hawking-Penrose Singularity 'Theorems' and Acceleration of the Universe with Negative Cosmological Constant, *Global Journal of Science Frontier Research Physics and Space Science*, Volume 13 Issue 4, Version 1.0, June 2013, http://viXra.org/abs/1304.0037

[32] Crothers, S. J. On the Generation of Equivalent 'Black Hole' Metrics: A Review, *American Journal of Space Science*, July 6, 2015, http://vixra.org/abs/1507.0098

[33] Crothers, S.J., The Painlevé-Gullstrand 'Extension' - A Black Hole Fallacy, *American Journal of Modern Physics,* 5, Issue 1-1 , February 2016, Pages:33-39, http://vixra.org/abs/1512.0089

[34] Crothers, S. J., On Isotropic Coordinates and Einstein's Gravitational Field, *Progress in Physics,* v.3, pp. 7-12, 2006, http://vixra.org/abs/1101.0008

[35] Crothers, S. J., On the Ramifications of the Schwarzschild Space-Time Metric, *Progress in Physics,* v.1, pp. 75-80, 2005, http://vixra.org/abs/1101.0002

[36] Brillouin, M., The singular points of Einstein's Universe. *Journ Phys. Radium*, v. 23, 43, (1923)

[37] Droste, J., The field of a single centre in Einstein's theory of gravitation, and the motion of a particle in that field, *Ned. Acad. Wet*., S.A., v.19, 197, (1917)

[38] Eddington, A.S., The mathematical theory of relativity, Cambridge University Press, Cambridge, (1963)

[39] Weinberg, S., Gravitation and Cosmology: Principles and Applications of the General theory of Relativity, John Wiley & Sons, Inc., (1972)

[40] Einstein, A., The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, (1916)

[41] Levi-Civita, T., Mechanics. – On the Analytical Expression that Must be Given to the Gravitational Tensor in Einstein's Theory, *Rendiconti della Reale* Accadmeia dei Lincei 26: 381, (1917)

[42] Ricci-Curbastro, G., Levi-Civita, T., Méthodes de calcul différentiel absolu ET leurs applications, *Matematische Annalen*, B. 54, p.162, (1900)

[43] Dirac, P.A.M., General Theory of Relativity, Princeton Landmarks in Physics Series, Princeton University Press, Princeton, New Jersey, (1996)

[44] Einstein, A., 'Relativity and the problem of space', Appendix 5 in the 15th edition of Relativity: The Special and the General Theory, Methuen, London, 1954, pp. 135-157

[45] Einstein, A., Preface to the 15th edition of Relativity: The Special and the General

Theory, Methuen, London, (1954)

[46] Einstein, A., Cosmological Considerations on the General Theory of Relativity, Sitzungsberichte der Preussischen Akad. d. Wissenschaften, (1917)

[47] McVittie, G. C.,Laplace's Alleged 'Black Hole', *Observatory* **98**: 272-274 (Dec 1978)

[48] Horgan, J., Is the Gravitational-Wave Claim True? And Was It Worth the Cost?, *Scientific American*, http://blogs.scientificamerican.com/cross-check/is-the-wave-claim-true-and-was-it-worth-the-cost/
Accessed online 7 March 2016