FORMULAS AND POLYNOMIALS WHICH GENERATE PRIMES AND FERMAT PSEUDOPRIMES

(COLLECTED PAPERS)

INTRODUCTION

To make an introduction to a book about arithmetic it is always difficult, because even most apparently simple assertions in this area of study may hide unsuspected inaccuracies, so one must always approach arithmetic with attention and care; and seriousness, because, in spite of the many games based on numbers, arithmetic is not a game. For this reason, I will avoid to do a naive and enthusiastic apology of arithmetic and also to get into a scholarly dissertation on the nature or the purpose of arithmetic. Instead of this, I will summarize this book, which brings together several articles regarding primes and Fermat pseudoprimes, submitted by the author to the preprint scientific database Research Gate.

Part One of this book, "Sequences of primes and conjectures on them", brings together twenty-nine papers regarding sequences of primes, sequences of squares of primes, sequences of certain types of semiprimes, also few types of pairs, triplets and quadruplets of primes and conjectures on all of these sequences. There are also few papers regarding possible methods to obtain large primes or very large numbers with very few prime factors, some of them based on concatenation, some of them on other arithmetic operations. It is also introduced a new notion: "Smarandache-Coman sequences of primes", defined as "all sequences of primes obtained from the terms of Smarandache sequences using any arithmetical operation" (for instance, the sequence of primes obtained concatenating to the right with the digit one the terms of Smarandache consecutive numbers sequence).

Part Two of this book, "Sequences of Fermat pseudoprimes and conjectures on them", brings together sixteen papers on sequences of Poulet numbers and Carmichael numbers, i.e. the Fermat pseudoprimes to base 2 and the absolute Fermat pseudoprimes, two classes of numbers that fascinated the author for long time. Among these papers there is a list of thirty-six polynomials and formulas that generate sequences of Fermat pseudoprimes.

Part Three of this book, "Prime producing quadratic polynomials", contains three papers which list some already known such polynomials, that generate more than 20, 30 or even 40 primes in a row, and few such polynomials discovered by the author himself (in a review of records in the field of prime generating polynomials, written by Dress and Landreau, two french mathematicians well known for records in this field, review that can be found on the web adress <http://villemin.gerard.free.fr/Wwwgvmm/Premier/formule.htm>, the author – he says this proudly, of course – is mentioned with 18 prime producing quadratic polynomials). One of the papers proposes seventeen generic formulas that may generate prime-producing quadratic polynomials.

SUMMARY

Part One. Sequences of primes and conjectures on them

Part Two. Sequences of Fermat pseudoprimes and conjectures on them

Part Three. Prime producing quadratic polynomials

Part One.
Sequences of primes and conjectures on them

1. Conjecture on the numbers of the form np^2 – np + p – 2 where p prime

Abstract. In this paper I conjecture that there exist, for any p prime, p greater than or equal to 7, an infinity of positive integers n such that the number n*p^2 – n*p + p – 2 is prime.

Conjecture:

There exist, for any p prime, p greater than or equal to 7, an infinity of positive integers n such that the number n*p^2 – n*p + p – 2 is prime.

The sequence of the numbers n*p^2 – n*p + p – 2 for p = 7:
(in other words the numbers of the form 42*n + 5)
: 47, 89, 131, 173, 215, 257, 299, 341, 383, 425 (...)

The sequence of the primes of the form 42*n + 5:
: 47, 89, 131, 173, 257, 383 (...)

Note that there are also Poulet numbers that can be written as 42*n + 5; two of such numbers are 341 = 11*31 (n = 8) and 8321 = 53*157 (n = 198); these 2-Poulet numbers have also in common the fact that 11*3 – 2 = 31 and 53*3 – 2 = 157.

The sequence of the numbers n*p^2 – n*p + p – 2 for p = 11:
(in other words the numbers of the form 110*n + 9)
: 119, 229, 339, 449, 559, 669, 779, 889, 999, 1109 (...)

The sequence of the primes of the form 110*n + 9:
: 229, 449, 1109 (...)

The sequence of the numbers n*p^2 – n*p + p – 2 for p = 13:
(in other words the numbers of the form 156*n + 11)
: 167, 323, 479, 635, 791, 947, 1103, 1259, 1415 (...)

The sequence of the primes of the form 156*n + 11:
: 47, 89, 131, 173, 257, 383 (...) 2

The sequence of the numbers n*p^2 – n*p + p – 2 for p = 17:
(in other words the numbers of the form 272*n + 15)
: 287, 559, 831, 1103, 1375, 1647, 1919, 2191, 2463 (...)

The sequence of the primes of the form 272*n + 15:
: 1103, 3823, 4639 (...)

Note that there exist numbers that can be written in more than one way as n*p^2 – n*p + p – 2; such a number is 1103 = 7*13^2 – 7*13 + 13 – 2 = 4*17^2 – 4*17 + 17 – 2.

The sequence of the numbers n*p^2 – n*p + p – 2 for p = 19:
(in other words the numbers of the form 342*n + 17)
: 359, 701, 1043, 1385, 1727, 2069, 2411, 2753, 3095 (...)

The sequence of the primes of the form 272*n + 15:
: 359, 701, 2069, 2411, 2753 (...)

The sequence of the primes of the form 110*n + 9, where n is of the form 10^k, k greater than or equal to 0:
: 1109, 1100009, 1100000009 (...)

The sequence of the primes of the form 156*n + 11, where n is of the form 10^k, k greater than or equal to 0:
: 167, 1571, 156011, 1560011, 156000011, 15600000000000000000011, 1560000000000000000000000000011, 1560000000000000000000000000000000011(...)

The sequence of the primes of the form 342*n + 17, where n is of the form 10^k, k greater than or equal to 0:
: 359, 34217, 342000000000000000000000017, 34200000000000000000000000000017 (...)

2. Conjecture on the quadruplets of primes of the form (p, p+4k^2, p+6k^2, p+8k^2)

Abstract. In a strict sence, the term "prime quadruplet" refers strictly to the primes (p, p + 2, p + 6, p + 8) - see Wolfram MathWorld; it is not known if there are infinitely many such prime quadruplets. In this paper I conjecture that for any k non-null positive integer there exist an infinity of quadruplets of primes of the form (p, p+2k^2, p+6k^2, p+8k^2). Finally, I define the generalized Brun's constant for prime quadruplets of the type showed and I estimate its value for the particular case k = 2 (for k = 1 the value it is known being approximately equal to 0.87).

Conjecture:

For any k non-null positive integer there exist an infinity of quadruplets of primes of the form (p, p + 2*k^2, p + 6*k^2, p + 8*k^2).

The first two quadruplets of this form for few values of k:

: **for k = 1** we have (p, p + 2, p + 6, p + 8):
: (5, 7, 11, 13) and (11, 13, 17, 19).

Note that, beside the first quadruplet, the rest of them must have the form (30n+11, 30n+13, 30n+17, 30n+19).

: **for k = 2** we have (p, p + 8, p + 24, p + 32):
: (5, 13, 29, 37) and (29, 37, 53, 61).

Note that, beside the first quadruplet, the rest of them must have the form (30n+29, 30n+37, 30n+53, 30n+61).

: **for k = 3** we have (p, p + 18, p + 54, p + 72):
: (109, 127, 163, 181) and (139, 157, 193, 211).

Note that all of these quadruplets must have the form (30n+19, 30n+37, 30n+73, 30n+91).

: **for k = 4** we have (p, p + 32, p + 96, p + 128):
: (11, 43, 107, 139) and (71, 103, 167, 199).

Note that all of these quadruplets must have the form (30n+11, 30n+43, 30n+107, 30n+139).

The first quadruplet of this form for few other values of k:

: **for k = 5** we have (p, p + 50, p + 150, p + 200): (131, 181, 281, 331).

Note that all of these quadruplets must have the form (30n+11, 30n+61, 30n+161, 30n+211).

: **for k = 6** we have (p, p + 72, p + 216, p + 288): (101, 173, 317, 389).

Note that all of these quadruplets must have the form (30n+11, 30n+83, 30n+227, 30n+299).

: **for k = 7** we have (p, p + 98, p + 294, p + 392): (269, 367, 563, 661).

Note that all of these quadruplets must have the form (30n+29, 30n+127, 30n+323, 30n+421).

: **for k = 8** we have (p, p + 128, p + 384, p + 512): (179, 307, 563, 691).

Note that all of these quadruplets must have the form (30n+29, 30n+157, 30n+413, 30n+541).

: **for k = 9** we have (p, p + 162, p + 486, p + 648): (71, 233, 557, 719).

Note that all of these quadruplets must have the form (30n+11, 30n+173, 30n+497, 30n+659).

: **for k = 10** we have (p, p + 200, p + 600, p + 800): (179, 307, 563, 691).

Note that these quadruplets must have one of the following four forms: (30n+11, 30n+211, 30n+611, 30n+811); (30n+17, 30n+217, 30n+617, 30n+817); (30n+23, 30n+223, 30n+623, 30n+823); (30n+29, 30n+229, 30n+629, 30n+829) .

The generalized Brun's constant for prime quadruplets

It is known that the Brun's constant for prime quadruplets represents the sum of the reciprocals of all prime quadruplets in the restricted sence that a prime quadruplet is (p, p + 2, p + 6, p + 8), that is ((1/5 + 1/7 + 1/11 + 1/13) + (1/11 + 1/13 + 1/17 + 1/19)...) and is approximately equal to 0.87.

Let's see if we can find such constants for the generalized form of this prime quadruplet, i.e. the quadruplet (p, p + 2*k^2, p + 6*k^2, p + 8*k^2).

Let's take the quadruplet (p, p + 8, p + 24, p + 32) obtained from the general quadruplet for k = 2.

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) ≈ 0.435;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) ≈ 0.459;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) ≈ 0.466;

: (1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) + (1/719 + 1/727 + 1/743 + 1/751) ≈ 0.471.

Finally, we conjecture that the value of *generalized Brun's constant* for prime quadruplets of the form (p, p + 2*k^2, p + 6*k^2, p + 8*k^2), for the particular case k = 2, is not greater than 0.49 and not less than 0.48.

3. Conjecture on the primes of the form (q + n)2^n + 1 where q odd prime

Abstract. In this paper I first conjecture that for any non-null positive integer n there exist an infinity of primes p such that the number q = (p – 1)/2^n – n is also prime and than I conjecture that for any odd prime q there exist an infinity of positive integers n such that the number p = (q + n)*2^n + 1 is prime.

Conjecture:

For any non-null positive integer n there exist an infinity of primes p such that the number q = (p – 1)/2^n – n is also prime.

Examples:
(for n = 1)

: for p = 13, (13 – 1)/2^1 – 1 = 5, prime;
: for p = 17, (17 – 1)/2^1 – 1 = 7, prime;
: for p = 29, (29 – 1)/2^1 – 1 = 13, prime;
: for p = 37, (37 – 1)/2^1 – 1 = 17, prime;
: for p = 41, (41 – 1)/2^1 – 1 = 19, prime;
: for p = 61, (61 – 1)/2^1 – 1 = 29, prime;
[...]
: for p = 104537, (104537 – 1)/2^1 – 1 = 52267, prime;
: for p = 104729, (104729 – 1)/2^1 – 1 = 52363, prime.

Examples:
(for n = 2)

: for p = 29, (29 – 1)/2^2 – 2 = 5, prime;
: for p = 37, (37 – 1)/2^2 – 2 = 7, prime;
: for p = 53, (53 – 1)/2^2 – 2 = 11, prime;
: for p = 61, (61 – 1)/2^2 – 2 = 13, prime;
[...]
: for p = 104693, (104693 – 1)/2^2 – 2 = 26171, prime.
: for p = 104717, (104717 – 1)/2^2 – 2 = 26177, prime.

Examples:
(for n = 3)

: for p = 113, (113 – 1)/2^3 – 3 = 11, prime;
: for p = 192, (192 – 1)/2^3 – 3 = 23, prime;
: for p = 257, (256 – 1)/2^3 – 3 = 29, prime;
: for p = 353, (353 – 1)/2^3 – 3 = 41, prime.

Examples:
(for n = 4)

: for p = 113, (113 – 1)/2^4 – 4 = 3, prime;
: for p = 337, (337 – 1)/2^4 – 4 = 17, prime;
: for p = 433, (433 – 1)/2^4 – 4 = 23, prime.

Examples:
(for n = 5)
: for p = 577, (577 – 1)/2^5 – 5 = 13, prime.

Examples:
(for n = 6)
: for p = 577, (577 – 1)/2^6 – 6 = 3, prime;
[...]
: for p = 104513, (104513 – 1)/2^6 – 6 = 1627, prime.

Conjecture:

For any odd prime q there exist an infinity of positive integers n such that the number p = (q + n)*2^n + 1 is prime.

: for q = 3, the least n for which p is prime is n = 4, because (3 + 4)*2^4 + 1 = 113, prime;
: for q = 5, the least n for which p is prime is n = 1, because (5 + 1)*2^1 + 1 = 13, prime;
: for q = 7, the least n for which p is prime is n = 1, because (7 + 1)*2^1 + 1 = 17, prime;
: for q = 11, the least n for which p is prime is n = 2, because (11 + 2)*2^2 + 1 = 53, prime;
: for q = 13, the least n for which p is prime is n = 1, because (13 + 1)*2^1 + 1 = 29, prime;
: for q = 17, the least n for which p is prime is n = 1, because (17 + 1)*2^1 + 1 = 37, prime;
: for q = 19, the least n for which p is prime is n = 1, because (19 + 1)*2^1 + 1 = 41, prime;
: for q = 23, the least n for which p is prime is n = 2, because (23 + 2)*2^2 + 1 = 101, prime;
: for q = 29, the least n for which p is prime is n = 1, because (29 + 1)*2^1 + 1 = 61, prime;
: for q = 31, the least n for which p is prime is n = 5, because (31 + 5)*2^5 + 1 = 1153, prime [note the interesting fact that for n = 4 is obtained (31 + 4)*2^4 + 1 = 561, the first absolute Fermat pseudoprime].

Taking seven larger consecutive primes were obtained:

: for q = 104693, the least n for which p is prime is n = 8, because (104693 + 8)*2^8 + 1 = 26803457, prime;
: for q = 104701, the least n for which p is prime is n = 2, because (104701 + 2)*2^2 + 1 = 418813, prime;
: for q = 104707, the least n for which p is prime is n = 2, because (104707 + 2)*2^2 + 1 = 418837, prime;
: for q = 104711, the least n for which p is prime is n = 4, because (104711 + 4)*2^4 + 1 = 1675441, prime;
: for q = 104717, the least n for which p is prime is n = 7, because (104717 + 7)*2^7 + 1 = 13404673, prime;
: for q = 104723, the least n for which p is prime is n = 1, because (104723 + 1)*2^1 + 1 = 209449, prime;
: for q = 104729, the least n for which p is prime is n = 8, because (104729 + 8)*2^8 + 1 = 26812673, prime;

Note the relative small value of n for which the first prime is found!

4. Two conjectures on the numbers of the form 4p^4-800p^2+5 where p is prime

Abstract. In this paper I state two conjectures on the numbers of the form 4*p^4 – 800*p^2 + 5, where p is prime, i.e. that there exist an infinity of primes of such form respectively that there exist an infinity of sempiprimes q*r of such form, where r = q + 40*n, where n positive integer.

Conjecture 1:

There exist an infinity of primes q of the form q = 4*p^4 – 800*p^2 + 5, where p is prime.

Examples:

: for p = 3, q = 7529, prime;
: for p = 7, q = 48809, prime;
: for p = 13, q = 249449, prime;
: for p = 17, q = 565289, prime;
: for p = 31, q = 4462889, prime;
: for p = 41, q = 12647849, prime;
: for p = 43, q = 15154409, prime;
: for p = 53, q = 33809129, prime;
: for p = 67, q = 84195689, prime;
: for p = 71, q = 105679529, prime;
: for p = 83, q = 195344489, prime;
: for p = 101, q = 424402409, prime;
: for p = 127, q = 1053481769, prime;
: for p = 167, q = 3133496489, prime;
: for p = 239, q = 13096931369, prime;
: for p = 251, q = 15926904809, prime;
: for p = 307, q = 35606895209, prime;
[...]
: for p = 104723, q = 481092181583867300969, prime.

Conjecture 2:

There exist an infinity of semiprimes q*r of the form q*r = 4*p^4 – 800*p^2 + 5, where p is prime, such that r = q + 40*n, where n positive integer.

Examples:

: for p = 19, q*r = 7*115727 and 115727 = 7 + 40*2893;
: for p = 29, q*r = 227*15427 and 15427 = 227 + 40*380;
: for p = 37, q*r = 7*1227407 and 1227407 = 7 + 40*30685;
: for p = 59, q*r = 73*702113 and 702113 = 73 + 40*17551;
: for p = 61, q*r = 7*8337167 and 8337167 = 7 + 40*208429;
: for p = 97, q*r = 797*453757 and 453757 = 797 + 40*11324;
: for p = 109, q*r = 487*1178927 and 1178927 = 487 + 40*29461;

: for p = 113, q*r = 2203*300683 and 300683 = 2203 + 40*7462;
: for p = 137, q*r = 433*3288953 and 3288953 = 433 + 40*82213;
: for p = 151, q*r = 31237*67157 and 67157 = 31237 + 40*898;
: for p = 157, q*r = 233*10515073 and 10515073 = 233 + 40*262871;
: for p = 179, q*r = 10973*376573 and 376573 = 10973 + 40*9140;
: for p = 181, q*r = 14783*292183 and 292183 = 14783 + 40*6935;
: for p = 191, q*r = 7*764662607 and 764662607 = 7 + 40*19116565;
: for p = 197, q*r = 3607*1678847 and 1678847 = 3607 + 40*41881;
: for p = 223, q*r = 967*10270607 and 10270607 = 967 + 40*256741;
: for p = 227, q*r = 66863*159463 and 159463 = 66863 + 40*2315;
: for p = 229, q*r = 7*1577455247 and 1577455247 = 7 + 40*39436381;
[...]

5. Three conjectures on the numbers of the form p(p + 4n) – 60n where p and p + 4n primes

Abstract. In this paper I present three conjectures on the numbers of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, more accurate a general conjecture and two particular ones, on the numbers of the form p*(p + 4) – 60 respectively p*(p + 20) – 300.

Note:

The numbers of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, seem to have special attributes.

Conjecture 1:

There exist an infinity of primes of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, for any n non-null positive integer.

1.

Let's take the positive numbers of the form p*q – 60, where p and q = p + 4 are both primes:

: for (p, q) = (7, 11) is obtained 17, prime;
: for (p, q) = (13, 17) is obtained 161 = 7*23;
: for (p, q) = (19, 23) is obtained 377 = 13*29;
: for (p, q) = (37, 41) is obtained 1457 = 31*47;
[...]
: for (p, q) = (104323, 104327) is obtained 73*101*1033*1429 (we note the prime factors with a, b, c, d, a < b < c < d, and it can be seen that b*c – a*d = 16;
: for (p, q) = (104239, 104243) is obtained 61*1709*104233 (it can be seen that a*b – c = 16);
: for (p, q) = (104707, 104711) is obtained 10963974617 = 104701*104717 (it can be seen that b – a = 16);

Conjecture 2:

For any composite number of the form p*q – 60, where p and q = p + 4 are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 16.

2.

Let's take the positive numbers of the form p*q – 120, where p and q = p + 8 are both primes: the sequence of primes of this form is 83, 953, 3833, 8513, 10889, 18089 (...), obtained for (p, q) = (11, 19), (29, 37), (59, 67), (89, 97), (101, 109), (131, 139)...

3.

Let’s take the positive numbers of the form p*q – 180, where p and q = p + 12 are both primes: the sequence of primes of this form is 73, 313, 409, 1009, 1993, 2593, 4273, 5113 (...), obtained for (p, q) = (11, 23), (17, 29), (19, 31), (29, 41), 41, 53), (47, 59), (61, 73), (67, 79)...

4.

Let’s take the positive numbers of the form p*q – 240, where p and q = p + 16 are both primes: the sequence of primes of this form is 137, 1217, 1721, 6257 (...), obtained for (p, q) = (13, 29), (31, 47), (37, 53), (73, 89)...

5.

Let’s take the positive numbers of the form p*q – 300, where p and q = p + 20 are both primes:

: for (p, q) = (11, 31) is obtained 41, prime;
: for (p, q) = (17, 37) is obtained 329 = 7*47;
: for (p, q) = (23, 43) is obtained 689 = 13*53;
: for (p, q) = (41, 61) is obtained 2201 = 31*71;
[...]
: for (p, q) = (104681, 104701) is obtained 7*19*787*104711 (we note the prime factors with a, b, c, d, a < b < c < d and it can be seen that d – a*b*c = 40;
: for (p, q) = (104639, 104659) is obtained 7*17*47*131*14947 (it can be seen that b*c*d – a*e = 40);
: for (p, q) = (104471, 104491) is obtained 7*31*3371*14923 (it can be seen that b*c – a*d = 40);
: for (p, q) = (104327, 104347) is obtained 11*53*73*179*1429 (it can be seen that a*b*d – c*e = 40);

Conjecture 3:

For any composite number of the form p*q – 300, where p and q = p + 20 are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 40.

6. Conjecture on an infinity of triplets of primes generated by each 3-Poulet number

Abstract. In this paper I present the following conjecture: for any 3-Poulet number (Fermate pseudoprime to base two with three prime factors) P = x*y*z is true that there exist an infinity of triplets of primes [a, b, c] such that x*a + a – x = y*b + b – y = z*c + c – z.

Conjecture:

For any 3-Poulet number (Fermate pseudoprime to base two with three prime factors) P = x*y*z is true that there exist an infinity of triplets of primes [a, b, c] such that x*a + a – x = y*b + b – y = z*c + c – z.

The sequence of 3-Poulet numbers is: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841 (...). See the sequence A215672 that I posted on OEIS.

Examples:

For P = 561 = 3*11*17,
we need to find [a, b, c] such that 4*a – 3 = 12*b – 11 = 18*c – 17; for this, [a, b, c] must be of the form [9*n + 1, 3*n + 1, 2*n + 1], where n can't be odd, can't be of the form 3*k + 1 and also can't have the last digit 2, 6 or 8. The least n for which [a, b, c] are all three primes is n = 20 which gives us [a, b, c] = [181, 61, 41]. The following such triplet is [a, b, c] = [487, 163, 109] corresponding to n = 54.

For P = 645 = 3*5*43,
we need to find [a, b, c] such that 4*a – 3 = 6*b – 5 = 44*c – 43; for this, [a, b, c] must be of the form [33*n + 1, 22*n + 1, 3*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 2 or 8. The least n for which [a, b, c] are all three primes is n = 4 which gives us [a, b, c] = [133, 89, 13]. The following such triplet is [a, b, c] = [199, 133, 19] corresponding to n = 6.

For P = 1105 = 5*13*17,
we need to find [a, b, c] such that 6*a – 5 = 14*b – 13 = 18*c – 17; for this, [a, b, c] must be of the form [21*n + 1, 9*n + 1, 7*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 2, 4 or 6. The least n for which [a, b, c] are all three primes is n = 18 which gives us [a, b, c] = [379, 163, 127]. The following such triplet is [a, b, c] = [631, 271, 211] corresponding to n = 30.

For P = 1729 = 7*13*19, we need to find [a, b, c] such that 8*a – 7 = 14*b – 13 = 20*c – 19; for this, [a, b, c] must be of the form [35*n + 1, 20*n + 1, 14*n + 1], where n can't be odd, can't be of the form 3*k + 1 and also can't have the last digit 6. The least n for which [a, b, c] are all three primes is n = 2 which gives us [a, b, c] = [71, 41, 29]. The following such triplet is [a, b, c] = [491, 281, 197] corresponding to n = 14.

For P = 1905 = 3*5*127, we need to find [a, b, c] such that 4*a – 3 = 6*b – 5 = 128*c – 127; for this, [a, b, c] must be of the form [96*n + 1, 64*n + 1, 3*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 4, 6 or 8. The least n for which [a, b, c] are all three primes is n = 12 which gives us [a, b, c] = [1153, 769, 37]. The following such triplet is [a, b, c] = [2113, 1409, 67] corresponding to n = 22.

For P = 2465 = 5*17*29, we need to find [a, b, c] such that 6*a – 5 = 18*b – 17 = 30*c – 29; for this, [a, b, c] must be of the form [15*n + 1, 5*n + 1, 3*n + 1], where n can't be odd, can't be of the form 3*k + 1 and also can't have the last digit 8. The least n for which [a, b, c] are all three primes is n = 2 which gives us [a, b, c] = [31, 11, 7]. The following such triplet is [a, b, c] = [181, 61, 37] corresponding to n = 12.

For P = 2821 = 7*13*31, we need to find [a, b, c] such that 8*a – 7 = 14*b – 13 = 32*c – 31; for this, [a, b, c] must be of the form [28*n + 1, 16*n + 1, 7*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 2, 4 or 8. The least n for which [a, b, c] are all three primes is n = 16 which gives us [a, b, c] = [449, 257, 113]. The following such triplet is [a, b, c] = [841, 481, 211] corresponding to n = 30.

7. Observation on the numbers 4p^2 – 2p – 1 where p and 2p – 1 are primes

Abstract. In this paper I observe that many numbers of the form 4*p^2 – 2*p – 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d – n + 1; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d + n – 1, and I make few related notes.

Observation:

Many numbers of the form N = 4*p^2 – 2*p – 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d – n + 1; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d + n – 1.

Verifying the observation:
(true for the first 27 odd primes p for which 2*p – 1 is also prime)

Note that if p is prime of the form 10*k + 9 than the least prime factor of N is 5 and obviously then N respects the condition (ii) or (iii).

Also note that if d is equal to 11 and Q is of the form 10k + 1 is obviously respected condition (ii).

: for p = 3, N = 29, prime;
: for p = 7, N = 181, prime;
: for p = 19, N divisible by 5;
: for p = 31, N = 19*199 and 199 = 11*19 – 10;
: for p = 37, N = 11*491, d = 11 and Q = 10k + 1;
: for p = 79, N divisible by 5;
: for p = 97, N = 37441, prime;
: for p = 139, N divisible by 5;
: for p = 157, N = 29*3389 and 3389 = 121*29 – 120;
: for p = 199, N divisible by 5;
: for p = 211, N = 11*16151, d = 11 and Q = 10k + 1;
: for p = 229, N divisible by 5;
: for p = 271, N = 293221, prime;
: for p = 307, N = 89*4229 and 4229 = 47*89 + 46;
: for p = 331, N = 29*15089 and 15089 = 503*29 + 502;
: for p = 337, N = 453601, prime;
: for p = 367, N = 11*48911, d = 11 and Q = 10k + 1;
: for p = 379, N divisible by 5;
: for p = 439, N divisible by 5;
: for p = 499, N divisible by 5;
: for p = 547, N = 1195741, prime;
: for p = 577, N = 241*5521 and 5521 = 23*241 – 22;
: for p = 601, N = 19*75979 and 75979 = 4221*19 – 4220;

: for p = 607, N = 11*133871, d = 11 and Q = 10k + 1;
: for p = 619, N divisible by 5;
: for p = 661, N = 131*13331 and 13331 = 101*131 + 100.

Notes:

: Some numbers of this form meet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m, or respectively Q = (n*d + n – m)/m. An example: for p = 691, N = 149*12809 and 12809 = (427*149 + 427 - 5)/5;

: Some numbers of this form meet yet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and the number Q – d + 1 is prime or respectively the number Q + d - 1 is prime. An example: for p = 727, N = 139*15199 and 15199 – 139 + 1 = 15061, prime.

8. Observation on the numbers 4p^2 + 2p + 1 where p and 2p – 1 are primes

Abstract. In this paper I observe that many numbers of the form 4*p^2 + 2*p + 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d + n – m)/m, and I make few related notes.

Observation:

Many numbers of the form 4*p^2 + 2*p + 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d + n – m)/m.

Verifying the observation:
(true for the first 27 odd primes p for which 2*p – 1 is also prime)

Note that if d is equal to 7 is obviously respected condition (i) or condition (ii).

: for p = 3, N = 43, prime;
: for p = 7, N = 211, prime;
: for p = 19, N = 1483, prime;
: for p = 31, N = 3907, prime;
: for p = 37, N = 7*13*61 so d = 7;
: for p = 79, N = 7*37*97 so d = 7;
: for p = 97, N = 37831, prime;
: for p = 139, N = 77563, prime;
: for p = 157, N = 98911, prime;
: for p = 199, N = 158803, prime;
: for p = 211, N = 7*7*3643 so d = 7;
: for p = 229, N = 13*16171 and 16171 = (2695*13 – 2695 + 2)/2;
: for p = 271, N = 13*22639 and 22639 = (3773*13 – 3773 + 2)/2;
: for p = 307, N = 13*29047 and 29047 = (4841*13 – 4841 + 2)/2;
: for p = 331, N = 7*62701 so d = 7;
: for p = 337, N = 7*64993 so d = 7;
: for p = 367, N = 79*6829 and 6829 = (683*79 + 683 - 8)/8;
: for p = 379, N = 7*82189 so d = 7;
: for p = 439, N = 771763, prime;
: for p = 499, N = 7*7*20347 so d = 7;
: for p = 547, N = 7*171133 so d = 7;
: for p = 577, N = 43*30997 and 30997 = (738*43 – 738 + 1)/1;
: for p = 601, N = 1446007, prime;
: for p = 607, N = 31*47581 and 47581 = (1586*31 – 1586 + 1)/1;
: for p = 619, N = 13*117991 and 117991 = (19665*13 – 19665 + 2)/2;
: for p = 661, N = 13*134539 and 134539 = (22423*13 – 22423 + 2)/2.

Note:

: Some numbers of this form meet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and the number Q – d + 1 is prime or respectively the number Q + d - 1 is prime. An example: for p = 691, N = 43*44449 and 44449 + 43 – 1 = 44491, prime.

9. Conjecture on the numbers 6pq + 1 where p and q primes and q = kp – m k + 1

Abstract. In this paper I make the following conjecture on the numbers of the form n = 6*p*q + 1, where p and q are primes and q = k*p – k + 1: There exist an infinity of n primes for any k positive integer, k > 1. Note that the conjecture implies that there exist an infinity of pairs of primes [p, q] such that q = k*p – k + 1, for any k positive integer, k > 1, which I already conjectured in previous papers, as well as that there exist an infinity of pairs of primes [p, q] such that q = k*p + k – 1, for any k positive integer, k > 1.

Conjecture:

There exist an infinity of primes n of the form n = 6*p*q + 1, where p and q are primes and q = k*p – k + 1, for any k positive integer, k > 1.

Note that the conjecture implies that there exist an infinity of pairs of primes [p, q] such that q = k*p – k + 1, for any k positive integer, k > 1, which I already conjectured in previous papers, as well as that there exist an infinity of pairs of primes [p, q] such that q = k*p + k – 1, for any k positive integer, k > 1.

The sequence of these primes for k = 2 (q = 2*p – 1):

: 547, 4219, 74419, 112327, 627919, 879667, 2310019 (...), obtained for [p, q] = [7, 13], [19, 37], [79, 157], [97, 193], [229, 457], [271, 541], [439, 877]...

See A005382 in OEIS for primes p such that 2*p – 1 also prime.

The sequence of these primes for k = 3 (q = 3*p – 2):

: 2887, 39199, 49927, 79999, 336199, 587527, 3338527 (...), obtained for [p, q] = [13, 37], [47, 139], [53, 157], [67, 199], [137, 409], [181, 541], [431, 1291]...

See A088878 in OEIS for primes p such that 3*p – 2 also prime.

The sequence of these primes for k = 4 (q = 4*p – 3):

: 2707, 82483, 283183, 530143, 872107, 1655323 (...), obtained for [p, q] = [11, 41], [59, 233], [109, 433], [149, 593], [191, 761], [263, 1049]...

See A157978 in OEIS for primes p such that 4*p – 3 also prime.

10. Three conjectures on the numbers 6pq + 1 where p and q primes and q = 2p – 1

Abstract. In this paper I make the following three conjectures on the numbers of the form n = 6*p*q + 1, where p and q are primes and q = 2*p – 1: (I) There exist an infinity of n primes; (II) There exist an infinity of n semiprimes; (III) There exist an infinity of n composites with three or more prime factors, 7 being one of them. Note that for all the first 46 pairs of primes [p, q] with the property mentioned (see the sequence A005382 in OEIS for these primes) the number n obtained belongs to one of the three sequences considered by the three conjectures above.

Conjecture I:

There exist an infinity of primes n of the form n = 6*p*q + 1, where p and q are primes and q = 2*p – 1.

The sequence of these primes is:

: 547 (= 6*7*13 + 1), 4219 (= 6*19*37 + 1), 74419 (= 6*79*157 + 1), 112327 (= 6*97*193 + 1), 627919 (= 6*229*457 + 1), 879667 (= 6*271*541 + 1), 2310019 (= 6*439*877 + 1), 5725627 (= 6*691*1381 + 1), 6337987 (= 6*727*1453 + 1), 16447867 (= 6*1171*2341 + 1), 23478019 (= 6*1399*2797 + 1), 32937847 (= 6*1657*3313 + 1)...

Conjecture II:

There exist an infinity of semiprimes n of the form n = 6*p*q + 1, where p and q are primes and q = 2*p – 1.

The sequence of these semiprimes is:

: 11347 (= 7*1621 = 6*31*61 + 1), 16207 (= 19*853 = 6*37*73 + 1), 1129147 (= 79*14293 = 6*307*613 + 1), 1312747 (= 43*30529 = 6*331*661 + 1), 2985019 (= 163*18313 = 6*499*997 + 1), 4330807 (= 13*333139 = 6*601*1201 + 1), 4417747 (= 19*232513 = 6*607*1213 + 1), 5239087 (= 7*748441 = 6*661*1321), 7887787 (= 151*52237 = 6*811*1621 + 1), 9224287 (= 211*43717 = 6*877*1753 + 1), 10530007 (= 1279*8233 = 6*937*1873 + 1), 13706719 (= 13*1054363 = 6*1069*2137 + 1), 18354607 (= 1153*15919 = 6*1237*2473 + 1), 19622419 (= 61*19622419 = 6*1279*2557 + 1), 20178727 (= 37*545371 = 6*1297*2593 + 1), 24495919 (= 7*3499417 = 6*1429*2857 + 1), 28118347 (= 19*1479913 = 6*1531*3061 + 1), 31056919 (= 1993*15583 = 6*1609*3217 + 1)...

Conjecture III:

There exist an infinity of n composites with three or more prime factors, 7 being one of them, of the form n = 6*p*q + 1, where p and q are primes and q = 2*p – 1.

The sequence of these numbers is:

: 294847 (= 7*73*577 = 6*157*313 + 1), 474019 (= 7*13*5209 = 6*199*397 + 1), 532987 (= 7*13*5857 = 6*211*421 + 1), 1360807 (= 7*31*6271 = 6*337*673 + 1), 1614067 (= 7*13*17737 = 6*367*733 + 1), 1721419 (= 7^2*19*43^2 = 6*379*757 + 1), 3587227 (= 7*31*61*271 = 6*547*1093 + 1), 3991687 (= 7^2*81463 = 6*577*1153 + 1), 4594219 (= 7*19*34543 = 6*619*1237 + 1), 8241919 (= 7*73*127^2 = 6*829*1657 + 1) 11215267 (= 7^2*228883 = 6*967*1933 + 1), 11922127 (= 7*79*21559 = 6*997*1993 + 1), 12210919 (= 7*61*28597 = 6*1009*2017 + 1), 31755787 (= 7*433*10477 = 6*1627*3253 + 1)...

Note:

For all the first 46 pairs of primes [p, q] with the property mentioned (see the sequence A005382 in OEIS for these primes) the number n obtained belongs to one of the three sequences considered by the three conjectures above.

11. Any square of a prime larger than 7 can be written as 30n^2 + 60n + p where p prime or power of prime

Abstract. In this paper I make the following conjecture: Any square of a prime larger than 7 can be written as 30*n^2 + 60*n + p, where p prime or power of prime and n positive integer.

Conjecture:

Any square of a prime larger than 7 can be written as 30*n^2 + 60*n + p, where p prime or power of prime and n positive integer.

Verifying the conjecture:
(for the first fifteen primes larger than 7)

: 11^2 = 121 = 30*1^2 + 60*1 + 31;
: 13^2 = 169 = 30*1^2 + 60*1 + 79;
: 17^2 = 289 = 30*1^2 + 60*1 + 199 = 30*2^2 + 60*2 + 7^2;
: 19^2 = 361 = 30*1^2 + 60*1 + 271 = 30*2^2 + 60*2 + 11^2;
: 23^2 = 529 = 30*1^2 + 60*1 + 439 = 30*2^2 + 60*2 + 17^2 = 30*3^2 + 60*3 + 79;
: 29^2 = 841 = 30*1^2 + 60*1 + 751 = 30*2^2 + 60*2 + 601 = 30*4^2 + 60*4 + 11^2;
: 31^2 = 961 = 30*4^2 + 60*4 + 241;
: 37^2 = 1369 = 30*1^2 + 60*1 + 1279 = 30*2^2 + 60*2 + 1129 = 30*3^2 + 60*3 + 919;
: 41^2 = 1681 = 30*3^2 + 60*3 + 1231 = 30*4^2 + 60*4 + 31^2 = 30*5^2 + 60*5 + 631 = 30*6^2 + 60*6 + 241;
: 43^2 = 1849 = 30*1^2 + 60*1 + 1759 = 30*2^2 + 60*2 + 1609 = 30*3^2 + 60*3 + 1399 = 30*4^2 + 60*4 + 1129 = 30*6^2 + 60*6 + 409;
: 47^2 = 2209 = 30*3^2 + 60*3 + 1759 = 30*4^2 + 60*4 + 1489 = 30*6^2 + 60*6 + 769;
: 53^2 = 2809 = 30*1^2 + 60*1 + 2719 = 30*4^2 + 60*4 + 2089 = 30*5^2 + 60*5 + 1759 = 30*6^2 + 60*6 + 37^2 = 30*7^2 + 60*7 + 919 = 30*8^2 + 60*8 + 409;
: 59^2 = 3481 = 30*1^2 + 60*1 + 3391;
: 61^2 = 3721 = 30*1^2 + 60*1 + 3631 = 30*2^2 + 60*2 + 59^2 = 30*3^2 + 60*3 + 3271 = 30*4^2 + 60*4 + 3001 = 30*5^2 + 60*5 + 2671 = 30*6^2 + 60*6 + 2281 = 30*7^2 + 60*7 + 1831 = 30*8^2 + 60*8 + 1321 = 30*9^2 + 60*9 + 751 = 30*10^2 + 60*10 + 11^2;
: 67^2 = 4489 = 30*4^2 + 60*4 + 3769 = 30*6^2 + 60*6 + 3049 = 30*8^2 + 60*8 + 2089 = 30*11^2 + 60*11 + 199.

12. Any square of a prime larger than 11 can be written as 60n^2 + 90n + p where p prime or power of prime

Abstract. In this paper I make the following conjecture: Any square of a prime larger than 11 can be written as 60*n^2 + 90*n + p, where p prime or power of prime and n positive integer.

Conjecture:

Any square of a prime larger than 11 can be written as 60*n^2 + 90*n + p, where p prime or power of prime and n positive integer.

Verifying the conjecture:
(for the first fifteen primes larger than 11)

: 13^2 = 169 = 60*1^2 + 90*1 + 19;
: 17^2 = 289 = 60*1^2 + 90*1 + 139;
: 19^2 = 361 = 60*1^2 + 90*1 + 211;
: 23^2 = 529 = 60*1^2 + 90*1 + 379 = 60*2^2 + 90*2 + 109;
: 29^2 = 841 = 60*1^2 + 90*1 + 691 = 60*2^2 + 90*2 + 421 = 60*3^2 + 90*3 + 31;
: 31^2 = 961 = 60*1^2 + 90*1 + 811 = 60*2^2 + 90*2 + 541 = 60*3^2 + 90*3 + 151;
: 37^2 = 1369 = 60*4^2 + 90*4 + 7^2;
: 41^2 = 1681 = 60*1^2 + 90*1 + 1531 = 60*4^2 + 90*4 + 19^2;
: 43^2 = 961 = 60*1^2 + 90*1 + 1699 = 60*2^2 + 90*2 + 1429 = 60*3^2 + 90*3 + 1039 = 60*4^2 + 90*4 + 23^2;
: 47^2 = 2209 = 60*2^2 + 90*2 + 1789 = 60*3^2 + 90*3 + 1399;
: 53^2 = 2809 = 60*1^2 + 90*1 + 2659 = 60*2^2 + 90*2 + 2389 = 60*3^2 + 90*3 + 1999 = 60*4^2 + 90*4 + 1489 = 60*5^2 + 90*5 + 859 = 60*6^2 + 90*6 + 109;
: 59^2 = 3481 = 60*1^2 + 90*1 + 3331 = 60*2^2 + 90*2 + 3061 = 60*3^2 + 90*3 + 2671 = 60*4^2 + 90*4 + 2161 = 60*5^2 + 90*5 + 1531;
: 61^2 = 3721 = 60*1^2 + 90*1 + 3571 = 60*2^2 + 90*2 + 3301 = 60*4^2 + 90*4 + 7^4 = 60*6^2 + 90*6 + 1021 = 60*7^2 + 90*7 + 151;
: 67^2 = 4489 = 60*1^2 + 90*1 + 4339 = 60*4^2 + 90*4 + 3169 = 60*5^2 + 90*5 + 2539 = 60*6^2 + 90*6 + 1789 + 60*7^2 + 90*7 + 919;
: 71^2 = 5041 = 60*2^2 + 90*2 + 4621 = 60*3^2 + 90*3 + 4231 = 60*4^2 + 90*4 + 61^2 = 60*6^2 + 90*6 + 2341 + 60*7^2 + 90*7 + 1471.

13. On the numbers of the form pq+10^k where p and q are emirps

Abstract. In this paper I make the following observation: there are many primes among the numbers of the form p*q + 10^k, where p and q are emirps (reversible primes but different one from the other) and k is a positive integer; to highlight the observation I will search the least k for which the number p*q + 10^k is prime, for few pairs of emirps [p, q].

Observation:

There are many primes among the numbers of the form p*q + 10^k, where p and q are emirps (reversible primes but different one from the other) and k is a positive integer.
To highlight the observation I will search the least k for which the number p*q + 10^k is prime, for few pairs of emirps [p, q]. Of course, if there are many low values of k, the observation is verified.

The sequence of emirps:

13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157, 167, 179, 199, 311, 337, 347, 359, 389, 701, 709, 733, 739, 743, 751, 761, 769, 907, 937, 941, 953, 967, 971, 983, 991, 1009, 1021, 1031, 1033, 1061, 1069, 1091, 1097, 1103, 1109, 1151, 1153, 1181, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259, 1279, 1283 (...)

(for more terms see A006567 in OEIS)

: 13*31 + 100 = 503, prime, so the least k is 2;
: 17*71 + 100 = 1307, prime, so the least k is 2;
: 37*73 + 10 = 2711, prime, so the least k is 1;
: 79*97 + 10 = 7673, prime, so the least k is 1;
: 107*701 + 10 = 75017, prime, so the least k is 1;
: 113*311 + 100 = 35153, prime, so the least k is 1;
: 149*941 + 1000 = 141209, prime, so the least k is 3;
: 157*751 + 10 = 117917, prime, so the least k is 1;
: 167*761 + 10000 = 137087, prime, so the least k is 4;
: 179*971 + 10 = 173819, prime, so the least k is 1;
: 337*733 + 10 = 247031, prime, so the least k is 1;
: 347*743 + 100 = 257921, prime, so the least k is 2;
: 359*953 + 1000 = 343127, prime, so the least k is 3;
: 389*983 + 100000 = 482387, prime, so the least k is 5;
: 709*907 + 10 = 643073, prime, so the least k is 1;
: 739*937 + 10 = 692453, prime, so the least k is 1;
: 1009*9001 + 100000000 = 109082009, prime, so the least k is 8;
: 1021*1201 + 100 = 1226321, prime, so the least k is 2;
: 1031*1301 + 100000 = 1441331, prime, so the least k is 5;
: 1033*3301 + 1000 = 3410933, prime, so the least k is 3;
: 1061*1601 + 1000000000 = 1001698661, prime, so the least k is 9;
: 1069*9601 + 10 = 10263479, prime, so the least k is 1;
: 1091*1901 + 100 = 2074091, prime, so the least k is 2;
: 1097*7901 + 100 = 8667497, prime, so the least k is 2;

:	1103*3011 + 100 = 3321233, prime, so the least k is 2;
:	1109*9011 + 10 = 9993209, prime, so the least k is 1;
:	1153*3511 + 100 = 4048283, prime, so the least k is 2;
:	1181*1811 + 10000 = 2148791, prime, so the least k is 4;
:	1193*3911 + 10 = 4665833, prime, so the least k is 1;
:	1213*3121 + 10000 = 3795773, prime, so the least k is 4;
:	1217*7121 + 1000 = 8667257, prime, so the least k is 3;
:	1229*9221 + 100 = 11332709, prime, so the least k is 2;
:	1237*7321 + 10000000 = 19056077, prime, so the least k is 7;
:	1249*9421 + 100 = 11766929, prime, so the least k is 2;
:	1259*9521 + 10000 = 11996939, prime, so the least k is 4;
:	1279*9721 + 1000000 = 13433159, prime, so the least k is 6;
:	1283*3821 + 10 = 4902353, prime, so the least k is 1.

14. Formula that generates a large amount of big primes and semiprimes *i.e.* 529 + 60*10^k

Abstract. In this paper I make the following observation: the formula 529 + 60*10^k, where k positive integer, seems to generate a large amount of big primes and semiprimes. Indeed, up to k = 32, this formula generates 11 primes and 11 semiprimes!

Observation:

The formula 529 + 60*10^k, where k positive integer, seems to generate a large amount of big primes and semiprimes. Indeed, up to k = 35, this formula generates 11 primes and 12 semiprimes!

The following terms are semiprimes:

: 589;
: 60529;
: 60000000529;
: 60000000000529;
: 6000000000000000529;
: 600000000000000000529;
: 6000000000000000000529;
: 60000000000000000000000529;
: 600000000000000000000000529;
: 6000000000000000000000000000000529;
: 600000000000000000000000000000000529;
: 6000000000000000000000000000000000529.

The following terms are primes:

: 1129;
: 6529;
: 600529;
: 6000529;
: 600000000529;
: 6000000000529;
: 6000000000000529;
: 600000000000000529;
: 600000000000000000000529;
: 6000000000000000000000000529;
: 600000000000000000000000000000529.

Note:

This special property of the square of the prime number 23 is not shared by the other squares of primes; for instance, the formula p^2 + 60*10^k generates, up to k = 35, only 3 primes for p = 7 and only 4 primes for p = 11.

15. A sequence of numbers created concatenating the digit 1 twice with a prime of the form 6k – 1

Abstract. In this paper I show an interesting sequence of numbers created concatenating to the right the digit 1, twice, with a prime of the form 6*k – 1 (example of such numbers, terms of this sequence: 12929 and 15353), sequence that has, from the first 50 terms, 21 terms that are primes and 22 that are semiprimes.

Observation:

The sequence created concatenating to the right the digit 1, twice, with a prime of the form 6*k – 1 (example of such numbers, terms of this sequence: 12929 and 15353) seems to be particularly interesting; beside the fact that the sequence contains a lot of terms that are primes, many of the composite terms also share a special property: up to the 50-th term of the sequence, all the composites are semiprimes p*q, in which case many of these have the property that p + q – 1 is a prime, or squarefree composites with three prime factors p*q*r, in which case many of these have the property that p*q + r – 1 is a prime.

The sequence of primes:

: 11717, 12323, 14747, 15959, 1107107, 1131131, 1137137, 1167167, 1173173, 1179179, 1191191, 1197197, 1239239, 1263263, 1281281, 1311311, 1317317, 1401401, 1479479, 1503503, 1509509 (...)

The sequence of semiprimes:

: 11111, 12929, 14141, 15353, 18383, 18989, 1113113, 1227227, 1257257, 1269269, 1293293, 1347347, 1353353, 1383383, 1389389, 1419419, 1431431, 1443443, 1461461, 1467467, 1491491, 1521521 (...)

See that:

: 11111 = 41*271 and 41 + 271 – 1 = 311, prime;
: 14141 = 79*179 and 79 + 179 – 1 = 257, prime;
: 15353 = 13*1181 and 13 + 1181 – 1 = 1193, prime;
: 1227227 = 163*7529 and 163 + 7529 – 1 = 7691, prime;
: 1383383 = 181*7643 and 181 + 7643 – 1 = 7823, prime;
: 1419419 = 461*3079 and 461 + 3079 – 1 = 3539, prime;
: 1431431 = 71*20161 and 71 + 20161 – 1 = 20231, prime.

The sequence of squarefree composites with three prime factors:

: 17171, 1149149, 1233233, 1251251, 1359359, 1449449 (...)

See that:

: 17171 = 7*11*223 and 11*223 + 7 – 1 = 2459, prime; also 7*223 + 11 – 10 = 1571, prime;

: 1149149 = 17*23*2939 and 17*23 + 2939 – 1 = 3329, prime;
: 1233233 = 19*47*11381 and 19*47 + 1381 – 1 = 2273, prime;
: 1251251 = 17*89*827 and 17*89 + 827 – 1 = 2339, prime;
: 1449449 = 29*151*331 and 29*331 + 151 – 1 = 9749, prime.

Note:

Up to the 50-th term of the general sequence of these numbers, 21 terms are primes and 22 are semiprimes! The longest chain of consecutive terms primes met is of 5 terms: 1167167, 1173173, 1179179, 1191191, 1197197.

16. A method based on concatenation to create very large numbers with very few prime factors

Abstract. In this paper I share a very interesting discovery made more or less by accident: taking a number having just even digits, like for instance 224866802226608 (I have chosen this randomly right now when I am writing the Abstract) and concatenating it three times with itself and then to the right with the digit 1 (like in the example taken 2248668022266082248668022266082248668022266081) seems that are great chances to obtain a number with very few prime factors (in the case taken just 4 prime factors).

Observation:

Taking a number having just even digits and concatenating it three times with itself and then to the right with the digit 1 seems that are great chances to obtain a number with very few prime factors.

Examples:

: for 888866824 the number 8888668248888668248888668241 has 3 prime factors;
: for 2244660800 the number 2244660800224466080022446608001 has 2 prime factors;
: for 66624848824 the number 6662484882466624848824666248488241 has 3 prime factors;
: for 668482848284 the number 6684828482846684828482846684828482841 has 3 prime factors;
: for 8000024646480 the number 8000024646480800002464648080000246464801 has 3 prime factors;
: for 22266644488044 the number 2226664448804422266644488044222666444880441 has 4 prime factors;
: for 880008884484828 the number 8800088844848288800088844848288800088844848281 has 3 prime factors;
: for 444666444000804 the number 4446664440008044446664440008044446664440008041 has 4 prime factors;
: for 888866640404202 the number 8888666404042028888666404042028888666404042021 has 2 prime factors; 2
: for 888000444000404 the number 8880004440004048880004440004048880004440004041 has 3 prime factors;
: for 666888000444606 the number 6668880004446066668880004446066668880004446061 has 4 prime factors;
: for 222222222222222 the number 2222222222222222222222222222222222222222222221 has 3 prime factors;
: for 444444444444444 the number 4444444444444444444444444444444444444444444441 has 3 prime factors;
: for 666666666666666 the number 6666666666666666666666666666666666666666666661 has 3 prime factors;
: for 888888888888888 the number 8888888888888888888888888888888888888888888881 has 4 prime factors;
: for 24242424242424 the number 24242424242424 24242424242424 24242424242424 242424242424241 has 4 prime factors;

: for 246802468024680 the number 246802468024680 246802468024680 2468024680246801 has 3 prime factors;

: for 64646464646464 the number 6464646464646464646464646464646464646464641 is prime.

The sequence of primes obtained concatenating the numbers having only even digits three times with themselves and then to the right with the digit 1 (I conjecture that this sequence has an infinity of terms):

: 2221, 4441, 6661, 2424241, 2828281, 4040401, 4242421, 6262621, 6868681, 8282821, 2002002001, 2242242241, 2422422421, 2482482481, 2602602601, 2622622621, 2642642641, 4044044041, 4424424421, 4824824821, 6226226221, 6266266261, 6486486481, 6646646641, 6666666661, 6846846841, 8448448441, 8648648641, 2004200420041, 2024202420241, 2042204220421 (...)

The longest chain, met, with consecutive terms of the general sequence of the numbers obtained like mentioned which are primes has 4 terms: 2482482481, 2602602601, 2622622621, 2642642641.

17. Notable observation on the squares of primes of the form 10k + 9

Abstract. In this paper I conjecture that for any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime and, in case that this conjecture turns out not to be true, I considered three related "weaker" conjectures.

Conjecture:

For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime.

Verifying the conjecture:
(for the first twenty primes p with the property mentioned)

: p = 7 and p^2 = 49; (p^2 – 5)/4 = 11, prime, so [q, r] = [5, 11];

: p = 13 and p^2 = 169; (p^2 – 5)/4 = 41, prime, so [q, r] = [5, 41];

: p = 17 and p^2 = 289; (p^2 – 7)/6 = 47, prime, so [q, r] = [7, 47];

: p = 23 and p^2 = 529; (p^2 – 5)/4 = 131, prime, so [q, r] = [5, 131]; also (p^2 – 13)/12 = 43, prime, so [q, r] = [13, 43];

: p = 37 and p^2 = 1369; (p^2 – 7)/6 = 227, prime, so [q, r] = [7, 227]; also (p^2 – 13)/12 = 113, prime, so [q, r] = [13, 113];

: p = 43 and p^2 = 1849; (p^2 – 5)/4 = 461, prime, so [q, r] = [5, 461]; also (p^2 – 7)/6 = 307, prime, so [q, r] = [7, 307]; also (p^2 – 23)/22 = 83, prime, so [q, r] = [23, 83];

: p = 47 and p^2 = 2209; (p^2 – 7)/6 = 367, prime, so [q, r] = [7, 367]; also (p^2 – 17)/16 = 137, prime, so [q, r] = [17, 137];

: p = 53 and p^2 = 2809; (p^2 – 5)/4 = 701, prime, so [q, r] = [5, 701]; also (p^2 – 7)/6 = 467, prime, so [q, r] = [7, 467]; also (p^2 – 13)/12 = 233, prime, so [q, r] = [13, 233];

: p = 67 and p^2 = 4489; (p^2 – 13)/12 = 373, prime, so [q, r] = [13, 373];

: p = 73 and p^2 = 5329; (p^2 – 7)/6 = 887, prime, so [q, r] = [7, 887]; also (p^2 – 13)/12 = 443, prime, so [q, r] = [13, 443];

: p = 83 and p^2 = 6889; (p^2 – 5)/4 = 1721, prime, so [q, r] = [5, 1721]; also (p^2 – 43)/42 = 163, prime, so [q, r] = [43, 163];

: p = 97 and p^2 = 9409; (p^2 – 5)/4 = 2351, prime, so [q, r] = [5, 2351]; also (p^2 – 7)/6 = 1567, prime, so [q, r] = [7, 1567]; also (p^2 – 17)/16 = 587, prime, so [q, r] = [17, 587]; also (p^2 – 43)/42 = 223, prime, so [q, r] = [43, 223];

: p = 103 and p^2 = 10609; (p^2 – 13)/12 = 883, prime, so [q, r] = [13, 883];

: p = 107 and p^2 = 11449; (p^2 – 5)/4 = 2861, prime, so [q, r] = [5, 2861]; also (p^2 – 7)/6 = 1907, prime, so [q, r] = [7, 1907]; also (p^2 – 13)/12 = 953, prime, so [q, r] = [13, 953]; also (p^2 – 37)/36 = 317, prime, so [q, r] = [37, 317];

: p = 113 and p^2 = 12769; (p^2 – 5)/4 = 3191, prime, so [q, r] = [5, 3191]; also (p^2 – 13)/12 = 1063, prime, so [q, r] = [13, 1063]; also (p^2 – 17)/16 = 797, prime, so [q, r] = [17, 797];

: p = 127 and p^2 = 16129; (p^2 – 7)/6 = 2687, prime, so [q, r] = [7, 2687]; also (p^2 – 43)/42 = 383, prime, so [q, r] = [43, 383]; also (p^2 – 73)/72 = 223, prime, so [q, r] = [73, 223]; also (p^2 – 97)/96 = 167, prime, so [q, r] = [97, 167];

: p = 137 and p^2 = 18769; (p^2 – 5)/4 = 4691, prime, so [q, r] = [5, 4691] ;

: p = 157 and p^2 = 24649; (p^2 – 13)/12 = 2053, prime, so [q, r] = [13, 2053]; : p = 163 and p^2 = 26569; (p^2 – 13)/12 = 2213, prime, so [q, r] = [13, 2213];

: p = 167 and p^2 = 27889; (p^2 – 5)/4 = 6971, prime, so [q, r] = [5, 6971].

Note:

In case that the conjecture above turns out not to be true there are three “weaker” conjectures that may be considered:

(i) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or a power of prime.

Example: p = 73, p^2 = 5329, (p^2 – 5)/4 = 11^3.

(ii) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime.

Examples:

: p = 67, p^2 = 4489, (p^2 – 5)/4 = 19*59 and 59 – 19 + 1 = 41;
: p = 53, p^2 = 2809, (p^2 – 19)/18 = 5*31 and 31 – 5 + 1 = 3^3;
: p = 127, p^2 = 16129, (p^2 – 113)/112 = 11*13 and 13 + 11 - 1 = 23.

(iii) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/((q – 1)*2^n)) is prime.

Examples:

: p = 113, p^2 = 11449, (p^2 – 73)/(72*2) = 79;
: p = 137, p^2 = 18769, (p^2 – 17)/(16*2^2) = 293;
: p = 167, p^2 = 27889, (p^2 – 113)/(112*2^3) = 31.

18. Notable observation on the squares of primes of the form 10k + 1

Abstract. In this paper I conjecture that for any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime and, in case that this conjecture turns out not to be true, I considered three related "weaker" conjectures.

Conjecture:

For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime.

Verifying the conjecture:
(for the first ten primes p with the property mentioned)

: p = 11 and p^2 = 121; (p^2 – 5)/4 = 29, prime;

: p = 19 and p^2 = 361; (p^2 – 5)/4 = 89, prime; also (p^2 – 7)/6 = 59, prime; also (p^2 – 13)/12 = 29, prime;

: p = 29 and p^2 = 841; (p^2 – 7)/6 = 139, prime; also (p^2 – 11)/10 = 83, prime;

: p = 31 and p^2 = 961; (p^2 – 5)/4 = 239, prime; also (p^2 – 13)/12 = 79, prime; also (p^2 – 17)/16 = 59, prime;

: p = 41 and p^2 = 1681; (p^2 – 5)/4 = 419, prime; also (p^2 – 11)/10 = 167, prime; also (p^2 – 13)/12 = 139, prime; also (p^2 – 29)/28 = 59, prime;

: p = 59 and p^2 = 3481; (p^2 – 11)/10 = 347, prime;

: p = 61 and p^2 = 3721; (p^2 – 5)/4 = 929, prime; also (p^2 – 7)/6 = 619, prime;

: p = 71 and p^2 = 3721; (p^2 – 5)/4 = 1259, prime; also (p^2 – 7)/6 = 839, prime; also (p^2 – 11)/10 = 503, prime; also (p^2 – 13)/12 = 419, prime; also (p^2 – 29)/28 = 179, prime; also (p^2 – 31)/30 = 167, prime; also (p^2 – 37)/36 = 139, prime; also (p^2 – 61)/60 = 83, prime;

: p = 79 and p^2 = 6241; (p^2 – 5)/4 = 1559, prime; also (p^2 – 7)/6 = 1039, prime; also (p^2 – 17)/16 = 389, prime; also (p^2 – 61)/60 = 103, prime;

: p = 89 and p^2 = 7921; (p^2 – 5)/4 = 1979, prime; also (p^2 – 7)/6 = 1319, prime; also (p^2 – 13)/12 = 659, prime; also (p^2 – 19)/18 = 439, prime; also (p^2 – 23)/22 = 359, prime; also (p^2 – 31)/30 = 263, prime; also (p^2 – 41)/40 = 197, prime; also (p^2 – 61)/60 = 131, prime; also (p^2 – 73)/72 = 109, prime.

Note:

In case that the conjecture above turns out not to be true there are three “weaker” conjectures that may be considered:

(i) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or a power of prime.

Example:

: p = 59, p^2 = 3481, (p^2 – 13)/12 = 17^2, square of prime.

(ii) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime.

Examples:
: p = 61, p^2 = 3721, (p^2 – 11)/10 = 7*53 and 53 – 7 + 1 = 47, prime; also 53 + 7 – 1 = 59, prime;
: p = 71, p^2 = 5041, (p^2 – 43)/42 = 7*17 and 17 – 7 + 1 = 11, prime; also 17 + 7 – 1 = 23, prime.

(iii) For any square of prime of the form p^2 = 10k + 1, p greater than or equal to 11, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/((q – 1)*2^n)) is prime.

Examples:

: p = 61, p^2 = 3721, (p^2 – 41)/(40*2^2) = 23, prime;
: p = 71, p^2 = 5041, (p^2 – 17)/(16*2) = 157, prime.

19. Conjecture that states that the square of any prime can be written in a certain way

Abstract. In this paper we conjecture that the square of any prime greater than or equal to 5 can be written in one of the following three ways: (i) p*q + q – p; (ii) p*q*r + p*q – r; (iii) p*q*r + p – q*r, where p, q and r are odd primes. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that the square of any prime of the form 11 + 30*k can be written as 3*p*q + p – 3*q, where p and q are odd primes.

Conjecture:

The square of any prime s greater than or equal to 5 can be written in one of the following three ways: (i) p*q + q – p; (ii) p*q*r + p*q – r; (iii) p*q*r + p – q*r, where p, q and r are odd primes.

Verifying the conjecture:
(up to s = 41)

: 5^2 = 25 = 3*7 + 7 - 3;

: 7^2 = 49 = 3*13 + 13 – 3; also 49 = 3*3*5 + 3*3 – 5;

: 11^2 = 121 = 3*31 + 31 - 3; also 121 = 3*3*13 + 13 – 3*3; also 121 = 3*5*7 + 3*7 – 5;

: 13^2 = 169 = 5*29 + 29 - 5; also 169 = 3*43 + 43 – 3; also 169 = 3*5*11 + 3*5 – 11;

: 17^2 = 289 = 7*37 + 37 - 7; also 289 = 3*5*19 + 19 – 3*5; also 289 = 5*5*11 + 5*5 – 11; also 289 = 5*7*7 + 7*7 – 5;

: 19^2 = 361 = 11*31 + 31 - 11; also 361 = 3*7*17 + 3*7 – 17; also 361 = 3*3*37 + 37 – 3*3;

: 23^2 = 529 = 7*67 + 67 - 7; also 529 = 5*89 + 89 – 5;

: 29^2 = 841 = 19*43 + 24; also 841 = 13*61 + 61 – 13; also 841 = 11*71 + 71 – 11;

: 31^2 = 961 = 23*41 + 18; also 961 = 3*11*29 + 3*11 – 29; also 961 = 7*7*19 + 7*7 – 19; also 961 = 3*5*61 + 61 – 3*5;

: 37^2 = 1369 = 7*11*17 + 7*11 – 17;

: 41^2 = 1681 = 23*71 + 71 - 23; also 1681 = 3*13*43 + 43 – 3*13; also 1681 = 3*19*29 + 3*19 – 29; also 1681 = 5*17*19 + 5*17 – 19.

Conjecture:

The square of any prime s of the form 11 + 30*k can be written as 3*p*q + p – 3*q, where p and q are odd primes.

Verifying the conjecture:
(up to s = 131)

: for s = 11 we have [p, q] = [13, 3] (see above);

: for s = 41 we have [p, q] = [43, 13] (see above);

: for s = 71 we have [p, q] = [73, 23];

: for s = 101 we have [p, q] = [1021, 3] and [31, 113];

: for s = 131 we have [p, q] = [331, 17], [79, 73] and [953, 7];

: for s = 191 we have [p, q] = [2281, 5], [229, 53] and [13, 1013].

20. Conjecture on the numbers (p^2 – n)/(n – 1) where p prime

Abstract. In this paper I state the following conjecture: for any p prime there exist at least a value of n, different from p, for which the number (p^2 – n)/(n – 1) is prime.

Conjecture:

For any p prime there exist at least a value of n, different from p, for which the number q = (p^2 – n)/(n – 1) is prime.

Verifying the conjecture:
(for the first 7 primes p)

: for p = 5, q = 23, prime, for n = 2; also q = 11, prime for n = 3; also q = 7, prime, for n = 4;

: for p = 7, q = 47, prime, for n = 2; also q = 23, prime, for n = 3; also q = 11, prime, for n = 5;

: for p = 11, q = 29, prime, for n = 5; also q = 23, prime for n = 6; also q = 19, prime, for n = 7;

: for p = 13, q = 167, prime, for n = 2; also q = 83, prime, for n = 3; also q = 4, prime, for n = 5; also q = 23, prime, for n = 8;

: for p = 17, q = 71, prime, for n = 5; also q = 47, prime, for n = 7; also q = 31, prime, for n = 10; also q = 23, prime, for n = 13;

: for p = 19, q = 359, prime, for n = 1; also q = 179, prime, for n = 3; also q = 89, prime, for n = 5; also q = 71, prime, for n = 6; also q = 59, prime, for n = 7; also q = 29, prime, for n = 13; also q = 23, prime, for n = 16;

: for p = 23, q = 263, prime, for n = 3; also q = 131, prime, for n = 5; also q = 47, prime, for n = 12; also q = 43, prime, for n = 13.

Note that many primes (I conjecture that an infinity of primes) can be written as sqr(24*m – 23):

: 7 = sqr(24*3 – 23);
: 11 = sqr(24*6 – 23);
: 13 = sqr(24*8 – 23);
: 17 = sqr(24*13 – 23);
: 19 = sqr(24*16 – 23).

I also conjecture that there exist an infinity of primes that can be written as sqr(48*m – 47); examples: 7, 17, 23 for n = 2, 7, 12.

21. Conjecture on the numbers 3p(q – 1) – 1 where p and q are primes and p = q + 6

Abstract. In this paper I state the following conjecture: there exist an infinity of primes of the form 3*p*(q – 1) – 1, where p and q are primes and p = q + 6. Note that from the first terms of the sequence of sexy primes we have a chain of consecutive 9 primes: 131, 233, 509, 683, 1103, 1913, 3329, 4643, 5639 (for q = 5, 7, 11, 13, 17, 23, 31, 37, 41).

Conjecture:

There exist an infinity of primes of the form 3*p*(q – 1) – 1, where p and q are primes and p = q + 6. Note that from the first terms of the sequence of sexy primes we have a chain of consecutive 9 primes: 131, 233, 509, 683, 1103, 1913, 3329, 4643, 5639 (for q = 5, 7, 11, 13, 17, 23, 31, 37, 41).

The sequence of primes of this form:

: 3*11*(5 – 1) = 131, prime;
: 3*13*(7 – 1) = 233, prime;
: 3*17*(11 – 1) = 509, prime;
: 3*19*(13 – 1) = 683, prime;
: 3*23*(17 – 1) = 1103, prime;
: 3*29*(23 – 1) = 1913, prime;
: 3*37*(31 – 1) = 3329, prime;
: 3*43*(37 – 1) = 4643, prime;
: 3*47*(41 – 1) = 5639, prime;
: 3*59*(53 – 1) = 9203, prime;
: 3*89*(83 – 1) = 21893, prime;
: 3*103*(97 – 1) = 29663, prime;
: 3*107*(101 – 1) = 32099, prime;
: 3*109*(103 – 1) = 33353, prime;
: 3*113*(107 – 1) = 35933, prime;
: 3*163*(157 – 1) = 76283, prime;
: 3*179*(173 – 1) = 92363, prime;
: 3*197*(191 – 1) = 112289, prime;
: 3*257*(251 – 1) = 192749, prime;
: 3*269*(263 – 1) = 211433, prime;
: 3*283*(277 – 1) = 224369, prime;
: 3*313*(307 – 1) = 287333, prime;
: 3*317*(311 – 1) = 294809, prime;
: 3*359*(353 – 1) = 379103, prime;
: 3*449*(443 – 1) = 595373, prime;
: 3*463*(457 – 1) = 595373, prime;
: 3*509*(503 – 1) = 766553, prime;

(...)

Note:

The sequence of the semiprimes m*n of this form is also interesting because of a property shared by many of these, i.e. that m + n – 1 is prime; examples:

: 3*53*(47 – 1) = 7313 = 71*103 and 71 + 103 – 1 = 173, prime;
: 3*67*(61 – 1) = 12059 = 31*389 and 31 + 389 – 1 = 419, prime;
: 3*79*(73 – 1) = 17063 = 113*151 and 113 + 151 – 1 = 263, prime;
: 3*173*(167 – 1) = 86153 = 101*853 and 101 + 853 – 1 = 953, prime;
: 3*277*(271 – 1) = 224369 = 89*2521 and 89 + 2521 – 1 = 2609, prime.

22. Four conjectures on the numbers obtained concatenating to the right a prime with the digit 9

Abstract. In this paper I state the following four conjectures: (I) There exist an infinity of primes p which, concatenated to the right with the digit 9, form also prime numbers; (II) There exist an infinity of primes obtained concatenating the reversal of p as is defined in Conjecture I to the right with the digit 9; (III) There exist an infinity of semiprimes obtained concatenating primes to the right with the digit 9, semiprimes m*n having the property that n – m + 1 is prime; (IV) There exist an infinity of semiprimes obtained concatenating the reversal of p as is defined in Conjecture I to the right with the digit 9, semiprimes m*n having the property that n – m + 1 is prime.

Conjecture I:

There exist an infinity of primes p which, concatenated to the right with the digit 9, form also prime numbers q.

The sequence of primes q:

: 59, 79, 139, 179, 199, 239, 379, 419, 439, 479, 599, 619, 719, 739, 839, 1019, 1039, 1279, 1319, 1399, 1499, 1579, 1979, 1999, 2239, 2339, 2399, 2579, 2699, 2719, 2819, 2939, 3079, 3119, 3319, 3499, 3539, 3739, 4019, 4099, 4219 (...)

Conjecture II:

There exist an infinity of primes r obtained concatenating the reversal of p as is defined in Conjecture I to the right with the digit 9.

The sequence of primes q:

: 719, 919, 739, 149, 349, 179, 379, 389, 1019, 3019, 7219, 1319, 9319, 9419, 7919, 3229, 3329, 7529, 9629, 3929, 7039, 9439, 3539, 3739, 1049, 9049, 1249 (...)

Conjecture III:

There exist an infinity of semiprimes obtained concatenating primes to the right with the digit 9, semiprimes m*n having the property that n – m + 1 is prime.

The sequence of semiprimes m*n:

: 119 (= 7*17 and 17 – 7 + 1 = 11, prime), 299 (= 13*23 and 23 – 13 + 1 = 11, prime), 319 (= 11*29 and 29 – 11 + 1 = 19, prime), 799 (= 17*47 and 47 – 17 + 1 = 31, prime), 899 (= 29*31 and 31 – 29 + 1 = 3, prime), 979 (= 11*89 and 89 – 11 + 1 = 79, prime), 1079 (= 13*83 and 83 – 13 + 1 = 73, prime), 1099 (= 7*157 and 157 – 7 + 1 = 151, prime), 1379 (= 7*197 and 197 – 7 + 1 = 191, prime), 1639 (= 11*149 and 149 – 11 + 1 = 139, prime), 1739 (= 37*47 and 47 – 37 + 1 = 11, prime), 1799 (= 7*257 and 257 – 7 + 1 = 251, prime), 1919 (= 19*101 and 101 – 19 + 1 = 83, prime), 1939 (= 7*277 and 277 – 7 + 1 = 271, prime), 2119 (= 13*163 and 163 – 13 + 1 = 151, prime), 2279 (= 43*53 and

53 – 43 + 1 = 11, prime), 2419 (= 41*59 and 59 – 41 + 1 = 19, prime), 2839 (= 17*167 and 167 – 17 + 1 = 151, prime), 3139 (= 43*73 and 73 – 43 + 1 = 31, prime), 3379 (= 31*109 and 109 – 31 + 1 = 79, prime), 3599 (= 59*61 and 61 – 59 + 1 = 3, prime), 3679 (= 13*283 and 283 – 13 + 1 = 271, prime), 3799 (= 29*131 and 131 – 29 + 1 = 103, prime), 3979 (= 23*173 and 173 – 23 + 1 = 151, prime)...

Conjecture IV:

There exist an infinity of semiprimes m*n obtained concatenating the reversal of p as is defined in Conjecture I to the right with the digit 9, semiprimes having the property that n – m + 1 is prime.

The sequence of semiprimes m*n:

: 319 (= 11*29 and 29 – 11 + 1 = 19, prime), 329 (= 7*47 and 47 – 7 + 1 = 41, prime), 749 (= 7*107 and 107 – 7 + 1 = 101, prime), 959 (= 7*137 and 137 – 7 + 1 = 131, prime), 7519 (= 73*103 and 103 – 73 + 1 = 31, prime), 1829 (= 31*59 and 59 – 31 + 1 = 29, prime)...

23. Three conjectures on the numbers obtained concatenating to the left the odd numbers with 1234

Abstract. In this paper I state the following three conjectures on the numbers obtained concatenating to the left the odd numbers with 1234: (I) There exist an infinity of primes obtained concatenating to the left odd numbers with 1234; (II) There exist an infinity of primes obtained concatenating to the left prime numbers with 1234; (III) There exist an infinity of primes obtained concatenating to the left Poulet numbers with 1234.

Conjecture 1:

There exist an infinity of primes obtained concatenating to the left odd numbers with 1234.

The sequence of these primes:

: 12343, 12347, 123419, 123427, 123433, 123439, 123449, 123457, 123479, 123491, 123493, 123499, 1234109, 1234117, 1234133, 1234147, 1234187, 1234231, 1234237, 1234241, 1234243, 1234253, 1234271, 1234309, 1234333, 1234349, 1234351, 1234367, 1234379, 1234391, 1234393, 1234439, 1234463, 1234511, 1234517, 1234531, 1234537, 1234543, 1234547, 1234577, 1234603, 1234613 (...)

Conjecture 2:

There exist an infinity of primes obtained concatenating to the left prime numbers with 1234.

The sequence of these primes:

: 12343, 12347, 123419, 123479, 123409, 1234133, 1234241, 1234271, 1234349, 1234379, 1234439, 1234463, 1234547, 1234577, 1234613 (...)

Conjecture 3:

There exist an infinity of primes obtained concatenating to the left Poulet numbers with 1234.

The sequence of these primes:

: 12341729, 12342047, 12342821, 12344681, 12346601, 123412801, 123413747, 123415709, 123415841, 123418721, 123419951, 123433153 (...)

24. Conjecture on the primes obtained deconcatenating to the right the numbers (30k-1)(30k+1) with digit 9

Abstract. In this paper I state the following conjecture: there exist an infinity of primes obtained deconcatenating the numbers of the form (30*k – 1)*(30*k + 1) to the right with digit 9; example: 449*451 = 202499 and 20249 is a prime.

Conjecture :

There exist an infinity of primes p obtained deconcatenating the numbers of the form (30*k – 1)*(30*k + 1) to the right with digit 9.

The sequence of primes p:

: 29*31 = 899 and p = 89 is prime;
: 59*61 = 3599 and p = 359 is prime;
: 89*91 = 8099 and p = 809 is prime;
: 119*121 = 14399 and p = 1439 is prime;
: 209*211 = 44099 and p = 4409 is prime;
: 299*301 = 89999 and p = 8999 is prime;
: 329*331 = 108899 and p = 10889 is prime;
: 359*361 = 129599 and p = 12959 is prime;
: 449*451 = 202499 and p = 20249 is prime;
: 479*481 = 230399 and p = 23039 is prime;
: 599*601 = 359999 and p = 35999 is prime;
: 689*691 = 476099 and p = 47609 is prime;
: 719*721 = 518399 and p = 51839 is prime;
: 749*751 = 562499 and p = 56249 is prime;
: 809*811 = 656099 and p = 65609 is prime;
: 869*871 = 756899 and p = 75689 is prime;
: 989*991 = 980099 and p = 98009 is prime;
: 1019*1021 = 1040399 and p = 104039 is prime;
: 1079*1081 = 1166399 and p = 116639 is prime;
: 2009*2011 = 4040099 and p = 404009 is prime;
: 2039*2041 = 4161599 and p = 416159 is prime;
: 2069*2071 = 4284899 and p = 428489 is prime;
: 2219*2221 = 4928399 and p = 492839 is prime;
: 2339*2341 = 5475599 and p = 547559 is prime;
: 2429*2431 = 5904899 and p = 590489 is prime;
: 2519*2521 = 6350399 and p = 635039 is prime;
: 2669*2671 = 7128899 and p = 712889 is prime;
: 2819*2821 = 7952399 and p = 795239 is prime;
: 2849*2851 = 8122499 and p = 812249 is prime;
: 2939*2941 = 8643599 and p = 864359 is prime;
: 3029*3031 = 9180899 and p = 918089 is prime;
: 3119*3121 = 9734399 and p = 973439 is prime.

(...)

25. Two formulae for obtaining primes based on the prime decomposition of the number 561

Abstract. In this paper I present two formulae which seems to conduct to primes or products of very few prime factors, both of them inspired by the prime decomposition of the first absolute Fermat pseudoprime, the number 561.

Formula I

Observation:

Noting that the number N = 561 = 3*11*17 has the property that conducts to a prime for two values of d from three, where d prime factor, through the formula N – N/d – 1 (i.e. 373 = 561 – 561/3 – 1 and 509 = 561 – 561/11 – 1), I wondered if it is a general property of the numbers of the form N = 3*p*q, where (p, q) is a pair of sexy primes, to conduct often to primes and products of very few prime factors and it seems that, indeed, it is.

Verifying the observation:
(For the first 34 pairs of sexy primes)

: for (p, q) = (5, 11) are obtained the primes 109, 131 and 149;
: for (p, q) = (7, 13) are obtained the primes 181, 233 and 251;
: for (p, q) = (11, 17) are obtained the primes 373 and 509;
: for (p, q) = (13, 19) are obtained the primes 683 and 701;
: for (p, q) = (17, 23) is obtained the prime 1103;
: for (p, q) = (23, 29) are obtained the primes 1913 and 1931;
: for (p, q) = (31, 37) are obtained the primes 2293 and 3329;
: for (p, q) = (37, 43) are obtained the primes 3181 and 4643;
: for (p, q) = (47, 53) is obtained the prime 7331;
: for (p, q) = (53, 59) are obtained the primes 9203 and 9221;
: for (p, q) = (53, 59) are obtained the primes 9203 and 9221;
: for (p, q) = (67, 73) is obtained the prime 9781;
: for (p, q) = (83, 89) are obtained the primes 21893 and 21911;
: for (p, q) = (97, 103) is obtained the prime 29663;
: for (p, q) = (101, 107) are obtained the primes 21613, 32099 and 32117;
: for (p, q) = (103, 109) are obtained the primes 22453 and 33353;
: for (p, q) = (107, 113) are obtained the primes 24181, 35933 and 35951;
: for (p, q) = (151, 157) is obtained the prime 70667;
: for (p, q) = (157, 163) is obtained the prime 76283;
: for (p, q) = (167, 173) are obtained the primes 57781 and 86171;
: for (p, q) = (173, 179) are obtained the primes 61933, 92363 and 92381;
: for (p, q) = (191, 197) are obtained the primes 75253 and 111697;
: for (p, q) = (193, 199) is obtained the prime 114641;
: for (p, q) = (223, 229) is obtained the prime 152531;
: for (p, q) = (227, 233) is obtained the prime 157991;
: for (p, q) = (233, 239) is obtained the prime 111373;
: for (p, q) = (251, 257) are obtained the primes 192749 and 192767;
: for (p, q) = (257, 263) are obtained the primes 135181 and 202001;
: for (p, q) = (263, 269) is obtained the prime 211433;

: for (p, q) = (277, 281) are obtained the primes 156781, 234323 and 234341;
: for (p, q) = (307, 313) is obtained the prime 287333;
: for (p, q) = (311, 317) is obtained the prime 294809;
: for (p, q) = (331, 337) is obtained the prime 333647;

Note: For 30 from the first 34 pairs of sexy primes the formula above conducted to at least one prime from three possible ones.

Formula II

Observation:

Noting that the number N = 32421 = 3*101*107 has the property that conducts to a prime for all three values of d, where d prime factor, through the formula N – N/d – 1 (i.e. 21613 = 32421 – 32421/3 – 1, 32099 = 32421 – 32421/11 – 1 and 32117 = 32421 – 32421/11), I wondered if it is a property of the numbers of the form N = 3*p*q, where p is the form 10^n + 1 and q is the form 10^n + 7, to conduct to big primes and products of very few prime factors and it seems that, indeed, it is.

Verifying the observation:

: for n = 1 are obtained the primes 373 and 509;
: for n = 2 are obtained the primes 21613, 32099 and 32117;
: for n = 3 is obtained the prime 3020999;
: for n = 5 is obtained the prime 30002099999;
: for n = 7 is obtained the prime 300000209999999;
: for n = 10 is obtained the prime 300000000209999999999;
: for n = 22 is obtained the prime 300000000000000000000209999999999999999999999;
: for n = 23 is obtained the prime 20000000000000000000001600000000000000000000013;
: for n = 33 is obtained the prime 2000000000000000000000000000000016000000000000000000000000000000013.

Note: Many values of the number N – N/d – 1 are semiprimes or products of very few prime factors. For instance, the numbers

300000000000000000000002099999999999999999999999999999999999;
300000000000000000000000209999999999999999999999999999999999999999;
300000000000000000000000000000000000210000000000000000000000000000000000017;
30000000000000000000000000000000000000002100000000000000000000000000000000000
000017;
20000000000000000000000000000000000000001600000000000000000000000000000000000
000013;
20000000000000000000000000000000000000000001600000000000000000000000000000000
000000000013;
20000000000000000000000000000000000000000000000000000160000000000000000000000
0000000000000000000000000000013;
20000000000000000000000000000000000000000000000000000000160000000000000000000
000000000000000000000000000000000013
and many others have only two prime factors.

26. Four conjectures on the numbers created concatenating the product of twin primes with 11

Abstract. In this paper I make four conjectures on the numbers n created concatenating to the right the product p*q with number 11, where [p, q] is a pair of twin primes: (I) there exist an infinity of n primes; (II) there exist an infinity of n semiprimes of the form (10k + 1)*(10h + 1); (II) there exist an infinity of n semiprimes of the form (10k + 9)*(10h + 9); (II) there exist an infinity of n semiprimes of the form (10k + 3)*(10h + 7). Note that for 40 from the first 43 pairs of twin primes the number n belongs to one of the four sequences considered by the conjectures above.

Conjecture I:

There exist an infinity of primes created concatenating to the right the product p*q with number 11, where [p, q] is a pair of twin primes.

Example: for the pair of twin primes [p, q] = [59, 61] the product p*q = 3599; concatenating this number to the right with 11 is obtained the number 359911, prime.

The sequence of these primes:

: 1511, 3511, 359911, 518311, 1040311, 1166311, 1904311, 2249911, 3920311, 5759911, 7289911, 12110311, 17639911, 21344311, 27248311, 32489911, 38192311, 43559911, 65768311, 68558311, 77792311, 132710311
(...)
obtained for [p, q] = [3, 5], [5, 7], [59, 61], [71, 73],[101, 103], [107, 109], [137, 139], [149, 151], [197, 199], [239, 241], [269, 271], [347, 349], [419, 421], [461, 463], [521, 523], [569, 571], [617, 619], [659, 661], [821, 823], [827, 829], [881, 883], [1151, 1153].

Note the chain of six primes obtained for six consecutive pairs of twin primes: 359911, 518311, 1040311, 1166311, 1904311, 2249911.

Conjecture II:

There exist an infinity of semiprimes n of the form (10k + 1)*(10h + 1) created concatenating to the right the product p*q with number 11, where [p, q] is a pair of twin primes.

The sequence of these semiprimes:

: n = 14311 = 11*1301 for [p, q] = [11, 13];
: n = 65768311 = 1291*50821 for [p, q] = [809, 811];
: n = 104039911 = 631*164881 for [p, q] = [1019, 1021];
: n = 119246311 = 5741*20771 for [p, q] = [1091, 1093].

Conjecture III:

There exist an infinity of semiprimes n of the form (10k + 9)*(10h + 9) created concatenating to the right the product p*q with number 11, where [p, q] is a pair of twin primes.

The sequence of these semiprimes:

: n = 32311 = 79*409 for [p, q] = [17, 19];
: n = 106502311 = 3989*26699 for [p, q] = [1031, 1033];
: n = 151289911 = 1019*148469 for [p, q] = [1229, 1231];
: n = 1634432311 = 229*7137259 for [p, q] = [1277, 1279].

Conjecture IV:

There exist an infinity of semiprimes n of the form (10k + 3)*(10h + 7) created concatenating to the right the product p*q with number 11, where [p, q] is a pair of twin primes.

The sequence of these semiprimes:

: n = 89911 = 47*1913 for [p, q] = [29, 31];
: n = 176311 = 157*1123 for [p, q] = [41, 43];
: n = 3239911 = 17*190583 for [p, q] = [179, 181];
: n = 3686311 = 607*6073 for [p, q] = [191, 193];
: n = 5198311 = 17*305783 for [p, q] = [227, 229];
: n = 7952311 = 17*467783 for [p, q] = [281, 283];
: n = 9734311 = 47*207113 for [p, q] = [311, 313];
: n = 18662311 = 17*1097783 for [p, q] = [431, 433];
: n = 41216311 = 73*564607 for [p, q] = [641, 643];
: n = 112784311 = 2803*40237 for [p, q] = [1061, 1063].

Note:

For 40 from the first 43 pairs of twin primes the number n belongs to one of the four sequences considered by the conjectures above.

27. Two conjectures on the numbers created concatenating an odd n with 3n-4 and then with 1 or 11

Abstract. In this paper I make two conjectures on the numbers m created concatenating to the right an odd number n, not divisible by 3, with 3*n – 4 and then, if n is of the form 6*k + 1, with 11, respectively, if n is of the form 6*k – 1, with 1: (I) there exist an infinity of m primes; (II) there exist an infinity of m = p*q composites such that p + q – 1 is prime (where p and q may be, or may be not, primes). Note that for 25 from the first 30 odd numbers n not divisible by 3 the number m obtained belongs to one of the two sequences considered by the conjectures above.

Conjecture I:

There exist an infinity of primes m created concatenating to the right an odd number n, not divisible by 3, with 3*n – 4 and then, if n is of the form 6*k + 1, with 11, respectively, if n is of the form 6*k – 1, with 1.

Examples:
: for n = 7 (of the form 6*k + 1), we have 3*n – 4 = 17 and m = 71711, prime;
: for n = 17 (of the form 6*k – 1), we have 3*n – 4 = 47 an m = 17471, prime.

The sequence of the primes m:

: 71711, 17471, 318911, 351011, 531551, 832451, 952811
(...)
obtained for n = 7, 17, 31, 35, 53, 83, 95.

Conjecture II:

There exist an infinity of composites m = p*q, with the property that p + q – 1 is prime (where p and q may be, or may be not, primes), created concatenating to the right an odd number n, not divisible by 3, with 3*n – 4 and then, if n is of the form 6*k + 1, with 11, respectively, if n is of the form 6*k – 1, with 1.

The sequence of the composites m:

: m = 11291 (for n = 11) = 7*1613 and 7 + 1613 – 1 = 1619, prime;
: m = 133511 (for n = 13) = 7*19073 and 7 + 19073 – 1 = 19079, prime;
: m = 23651 (for n = 23) = 67*353 and 67 + 353 – 1 = 419, prime;
: m = 257111 (for n = 25) = 41*6271 and 41 + 6271 – 1 = 6311, prime;
: m = 29831 (for n = 29) = 23*1297 and 23 + 1297 – 1 = 1319, prime;
: m = 411191 (for n = 41) = 29*14179 and 29 + 14179 – 1 = 14207, prime; also 411191 = 319*1289 and 319 + 1289 – 1 = 1607, prime;
: m = 4312511 (for n = 43) = 7*616073 and 7 + 616073 – 1 = 616079, prime;
: m = 5516111 (for n = 55) = 1231*4481 and 1231 + 4481 – 1 = 5711, prime;
: m = 6117911 (for n = 61) = 43*142277 and 43 + 142277 – 1 = 142319, prime; also 6117911 = 1949*3139 and 1949 + 3139 – 1 = 5087, prime;

: m = 65191 (for n = 65) = 7*9313 and 7 + 9313 – 1 = 9319, prime; also 6117911 = 67*973 and 67 + 973 – 1 = 1039, prime; also 6117911 = 139*469and 139 + 469 – 1 = 607, prime;
: m = 6719711 (for n = 67) = 19*353687 and 19 + 353687 – 1 = 353687, prime; also 6719711 = 53*126787 and 53 + 126787 – 1 = 126839, prime;
: m = 712091 (for n = 71) = 509*1399 and 509 + 1399 – 1 = 1907, prime;
: m = 772271 (for n = 77) = 23*33577 and 23 + 33577 – 1 = 33599, prime;
: m = 7923311 (for n = 79) = 11*720301 and 11 + 720301 – 1 = 720311, prime;
: m = 8525111 (for n = 85) = 23*370657 and 23 + 370657 – 1 = 370679, prime;
: m = 9728711 (for n = 97) = 2749*3539 and 2749 + 3539 – 1 = 6287, prime;
: m = 1012991 (for n = 101) = 7*144713 and 7 + 144713 – 1 = 144719, prime; also 1012991 = 47*21553 and 47 + 21553 – 1 = 21599, prime; also 1012991 = 329*3079 and 329 + 3079 – 1 = 21599, prime.

Note:

For 25 from the first 30 odd numbers n not divisible by 3 the number m obtained belongs to one of the two sequences considered by the conjectures above.

28. Seven Smarandache-Coman sequences of primes

Abstract. In a previous paper, "Fourteen Smarandache-Coman sequences of primes", I defined the "Smarandache-Coman sequences" as "all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like a(n) + a(n+2) – a(n+1), or on a term like a(n) + S(a(n)), where S(a(n)) is the sum of the digits of the term a(n) etc." In this paper I extend the notion to the sequences of primes obtained from the Smarandache concatenated sequences using any arithmetical operation and I present seven sequences obtained from the Smarandache concatenated sequences using concatenation between the terms of the sequence and other numbers and also fourteen conjectures on them.

Introduction:

In a previous paper, "Fourteen Smarandache-Coman sequences of primes", I defined the "Smarandache-Coman sequences" as "all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like a(n) + a(n+2) – a(n+1), or on a term like a(n) + S(a(n)), where S(a(n)) is the sum of the digits of the term a(n) etc." In this paper I extend the notion to the sequences of primes obtained from the Smarandache concatenated sequences using any arithmetical operation and I present seven sequences obtained from the Smarandache concatenated sequences using concatenation between the terms of the sequence and other numbers and also fourteen conjectures on them.

Note: The Smarandache concatenated sequences are well known for the very few terms which are primes; on the contrary, many Smarandache-Coman sequences can be constructed that probably have an infinity of terms (primes, by definition).

Note: I shall use the notation a(n) for a term of a Smarandache concatenated sequence and b(n) for a term of a Smarandache-Coman sequence.

SEQUENCE I

Starting from the Smarandache consecutive numbers sequence (defined as the sequence obtained through the concatenation of the first n positive integers, see A007908 in OEIS), we define the following Smarandache-Coman sequence: b(n) = a(n)1, i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(1) = 11, prime;
: b(3) = 1231, prime;
: b(9) = 1234567891, prime;

: b(11) = 12345678910111, prime;
: b(16) = 123456789101112131415161, prime;
: b(26) =12345678910111213141516171819202122232425261, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes (some of them p*q having the interesting property that q – p + 1 is prime; such terms are: b(5) = 123451 = 41*3011 and 3011 – 41 + 1 = 2971; b(6) = 1234561 = 211*5851 and 5851 – 211 + 1 = 5641, prime).

SEQUENCE II

Starting from the Smarandache concatenated odd sequence (defined as the sequence obtained through the concatenation of the odd numbers from 1 to 2*n - 1, see A019519 in OEIS), we define the following Smarandache-Coman sequence: b(n) = a(n)1, i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:
: b(1) = 11, prime;
: b(2) = 131, prime;
: b(9) = 13579111315171, prime;
: b(10) = 1357911131517191, prime;
: b(12) = 13579111315171921231, prime;
: b(15) = 13579111315171921232527291, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes.

SEQUENCE III

Starting from the Smarandache reverse sequence (defined as the sequence obtained through the concatenation of the first n positive integers in reverse order, see A000422 in OEIS), we define the following Smarandache-Coman sequence: b(n) = a(n)1, i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(1) = 11, prime;
: b(2) = 211, prime;
: b(8) = 876543211, prime;
: b(9) = 9876543211, prime;
: b(22) = 222120191817161514131211109876543211, prime;
: b(26) =12345678910111213141516171819202122232425261, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes, some of them having the interesting property that one of the factor is much larger than the other one; such terms are:

: b(15) = 151413121110987654321 1 = 29*522114210727543635 59;

: b(15) = 1514131211109876543211 = 29*52211421072754363559;
: b(17) = 17161514131211109876543211 = 359*47803660532621475979229;
: b(18) = 1817161514131211109876543211 = 31*58618113359071326125049781;
: b(31) = 3130292827262524232221201918171615141312111098765432 11 = 519373*602706114346052688957878426135285265370381421207.

SEQUENCE IV

Starting from the Smarandache n2*n sequence (the n-th term of the sequence is obtained concatenating the numbers n and 2*n, see A019550 in OEIS), we define the following Smarandache-Coman sequence: b(n) = a(n)1, i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(2) = 241, prime;
: b(5) = 5101, prime;
: b(6) = 6121, prime;
: b(8) = 8161, prime;
: b(9) = 9181, prime;
: b(12) = 12241, prime;
: b(14) = 14281, prime;
: b(17) = 17341, prime;
: b(19) = 19381, prime;
: b(22) = 22441, prime;
: b(24) = 24481, prime;
(...)
: b(104) = 1042081, prime;
: b(106) = 1062121, prime;
: b(108) = 1082161, prime;
: b(110) = 1102201, prime;
: b(112) = 1122241, prime;
(...)
: b(1004) = 100420081, prime;
: b(1007) = 100720141, prime;
: b(1011) = 101120221, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes, as well as an infinity of terms b(n) which are squares of primes: such terms are b(1) = 121 = 11^2, b(3) = 361 = 19^2, b(10) = 10201 = 101^2.

SEQUENCE V

Starting again from the Smarandache n2*n sequence (the n-th term of the sequence is obtained concatenating the numbers n and 2*n, see A019550 in OEIS), we define the following Smarandache-Coman sequence: b(n) = 1a(n)1, i.e. the terms of the Smarandache sequence concatenated both to the left and to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(3) = 1361, prime;
: b(4) = 1481, prime;
: b(5) = 15101, prime;
: b(9) = 19181, prime;
: b(12) = 112241, prime;
: b(14) = 114281, prime;
: b(15) = 115301, prime;
: b(18) = 118361, prime;
: b(20) = 120401, prime;
: b(21) = 121421, prime;
(...)
: b(100) = 11002001, prime;
: b(104) = 11042081, prime;
: b(105) = 11052101, prime;
: b(107) = 11072141, prime;
: b(108) = 11082161, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes.

SEQUENCE VI

Starting from the Smarandache nn^2 sequence (the n-th term of the sequence is obtained concatenating the numbers n and n^2, see A053061 in OEIS), we define the following Smarandache-Coman sequence: b(n) = a(n)1, i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(2) = 241, prime;
: b(6) = 6361, prime;
: b(8) = 8641, prime;
: b(9) = 9181, prime;
: b(11) = 111211, prime;
: b(12) = 121441, prime;
: b(29) = 298411, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes.

SEQUENCE VII

Starting again from the Smarandache nn^2 sequence (the n-th term of the sequence is obtained concatenating the numbers n and n^2, see A053061 in OEIS), we define the following Smarandache-Coman sequence: b(n) = 1a(n)1, i.e. the terms of the Smarandache sequence concatenated both to the left and to the right with the number 1. I conjecture that there exist an infinity of terms b(n) which are primes.

We have:

: b(6) = 16361, prime;
: b(7) = 17491, prime;
: b(11) = 111211, prime;
: b(18) = 1183241, prime;
: b(26) = 1266761, prime;
: b(28) = 1287841, prime;
(...)

I also conjecture that there exist an infinity of terms b(n) which are semiprimes.

29. Two conjectures on Smarandache's divisor products sequence

Abstract. In this paper I make the following two conjectures on the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: (1) there exist an infinity of n composites such that the number m = P(n) + n – 1 is prime; (2) there exist an infinity of n composites such that the number m = P(n) – n + 1 is prime.

The *Smarandache's divisor products sequence* (see A007955 in OEIS):
: 1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000, 31, 32768, 1089, 1156, 1225, 10077696, 37, 1444, 1521, 2560000, 41, 3111696, 43, 85184, 91125, 2116, 47, 254803968 (...)

Conjecture 1:

Let P(n) be the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: there exist an infinity of n composites such that the number m = P(n) + n – 1 is prime.

Note that for n primes, because P(n) = n, P(n) + n – 1 = 2*n – 1 and is already conjectured that there exist an infinity of primes of the form 2*q – 1, where q prime.

The sequence of primes m:
: m = 3, prime, for (n, P(n)) = (2, 2);
: m = 11, prime, for (n, P(n)) = (4, 8);
: m = 41, prime, for (n, P(n)) = (6, 36);
: m = 71, prime, for (n, P(n)) = (8, 64);
: m = 109, prime, for (n, P(n)) = (10, 100);
: m = 1739, prime, for (n, P(n)) = (12, 1728);
: m = 239, prime, for (n, P(n)) = (15, 225);
: m = 1039, prime, for (n, P(n)) = (16, 1024);
: m = 5849, prime, for (n, P(n)) = (18, 5832);
: m = 461, prime, for (n, P(n)) = (21, 441);
: m = 149, prime, for (n, P(n)) = (25, 125);
: m = 701, prime, for (n, P(n)) = (26, 676);
: m = 1259, prime, for (n, P(n)) = (35, 1225);
: m = 1481, prime, for (n, P(n)) = (38, 1444);
: m = 2560039, prime, for (n, P(n)) = (40, 2560000);
: m = 2161, prime, for (n, P(n)) = (46, 2116);
(...)

Examples of larger m:
: m = 46656000059, prime, for (n, P(n)) = (60, 46656000000);
: m = 782757789791, prime, for (n, P(n)) = (96, 782757789696);
: m = 1586874323051, prime, for (n, P(n)) = (108, 1586874322944);
: m = 634562281237119143, prime, for (n, P(n)) = (168, 634562281237118976).

Note that m is prime for n = 12, 60, 96, 108, 168. I conjecture that m is prime for an infinity of n of the form 12*k.

Conjecture 2:

Let P(n) be the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: there exist an infinity of n composites such that the number m = P(n) - n + 1 is prime.

Note that for n primes, because P(n) = n, P(n) - n + 1 = 1.

The sequence of primes m:
: m = 5, prime, for (n, P(n)) = (4, 8);
: m = 31, prime, for (n, P(n)) = (6, 36);
: m = 19, prime, for (n, P(n)) = (9, 27);
: m = 211, prime, for (n, P(n)) = (15, 225);
: m = 1009, prime, for (n, P(n)) = (16, 1024);
: m = 421, prime, for (n, P(n)) = (21, 441);
: m = 463, prime, for (n, P(n)) = (22, 484);
: m = 331753, prime, for (n, P(n)) = (24, 331776);
: m = 149, prime, for (n, P(n)) = (25, 125);
: m = 1123, prime, for (n, P(n)) = (34, 1156);
: m = 254803921, prime, for (n, P(n)) = (48, 254803968);
(...)

Examples of larger m:
: m = 531440999911, prime, for (n, P(n)) = (90, 531441000000);
: m = 389328928561, prime, for (n, P(n)) = (208, 389328928768).

Note that m is prime for n = 24, 48. I conjecture that m is prime for an infinity of n of the form 12*k.

Part Two.
Sequences of Fermat pseudoprimes and conjectures on them

1. Generic form for a probably infinite sequence of Poulet numbers *i.e.* 2n^2 + 147n + 2701

Abstract. In this paper I observe that the formula 2*n^2 + 147*n + 2701 produces Poulet numbers, and I conjecture that this formula is generic for an infinite sequence of Poulet numbers.

The sequence of Poulet numbers of the form 2*n^2 + 147*n + 2701:

: 2701, 4371, 8911, 10585, 18721, 33153, 49141, 93961 (...)

These numbers were obtained for the following values of n:

: 0, 10, 30, 36, 60, 92, 120, 180 (...)

Conjecture:

There are infinite many Poulet numbers P of the form 2*n^2 + 147*n + 2701 (see A214016 posted by me on OEIS for a subsequence of the sequence from above, i.e. Poulet numbers of the form 7200*n^2 + 8820*n + 2701).

Observation:

Note the following interesting facts:

: for P = 2701 = 37*73 both 37 (= 2*17 + 3) and 73 (=4*17 + 5) can be written as 17*m + m + 1, where m positive integer;
: for p = 10585 = 5*29*73 both 5*29 = 145 (=8*17 + 9) and 73 (=4*17 + 5) can be written as 17*m + m + 1;
: for p = 93961 = 7*31*433 both 7*31 = 217 (=12*17 + 13) and 433 (=24*17 + 25) can be written as 17*m + m + 1.
: for P = 4371 = 3*31*47 both 31 (= 2*17 - 3) and 47 (=3*17 - 4) can be written as 17*m – m - 1, where m positive integer;
: for P = 18721 = 97*193 both 97 (= 6*17 - 5) and 193 (=12*17 - 11) can be written as 17*m – m – 1;
: for p = 33153 = 3*43*257 both 3*43 = 129 (=8*17 - 7) and 257 (=16*17 - 15) c an be written as 17*m - m – 1. 2

Note the following subsequence of the sequence from above, obtained for n = 10*m:

: 2701, 4371, 8911, 18721, 49141 93961, 226801, 314821, 534061, 665281, 915981 (...)

obtained for m = 0, 1, 3, 6, 12, 18, 30, 36, 48, 54, 64 (...)

2. Generic form for a probably infinite sequence of Poulet numbers *i.e.* 4n^2 + 37n + 85

Abstract. In this paper I observe that the formula 4*n^2 + 37*n + 85 produces Poulet numbers, and I conjecture that this formula is generic for an infinite sequence of Poulet numbers.

The sequence of Poulet numbers of the form 4*n^2 + 37*n + 85:

: 1105, 1387, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 74665, 83665, 88357, 90751 (...)

These numbers were obtained for the following values of n:

: 12, 14, 18, 24, 36, 54, 60, 66, 88, 132, 140, 144, 146 (...)

Conjecture:

There are infinite many Poulet numbers of the form 4*n^2 + 37*n + 85 (see A214017 posted by me on OEIS for a subsequence of the sequence from above, i.e. Poulet numbers of the form 144*n^2 + 122*n + 85).

Observation:

Note that almost all from the first 13 numbers P from the sequence above have a prime factor q of one from the following five forms:

(A) q = 17 (for P = 1105 = 5*13*17);

(B) q is of the form 17*m + m + 1 (q = 73 = 4*17 + 5 for P = 1387, q = 109 = 6*17 + 7 for P = 74665);

(C) q is of the form 17*m + m - 1 (q = 89 = 5*17 + 4 for P = 2047 and P = 31417; q = 233 = 13*17 + 12 for P = 13747, q = 71 = 4*17 + 3 for P = 19951);

(D) q is of the form 17*m - m + 1 (q = 113 = 7*17 - 6 for P = 3277; q = 257 = 16*17 – 15 for P = 13747; q = 353 = 22*17 – 21 for P = 31417, q = 577 = 36*17 – 35 for P = 83665, q = 593 = 37*17 – 36 for P = 88357);

(E) q is of the form 17*m – m – 1.

Exceptions:

: 6601 = 7*23*41; but, even in this case, 7*23 = 161 = 9*17 + 8 (case C), 7*41 = 16*17 + 15 (case C), 23*41 = 59*17 – 60 (case E);

: 90751 = 151*601; but, even in this case, 151*601 = 5672*17 – 5673 (case E).

3. Two conjectures on Poulet numbers of the form mn^2 + 11mn – 23n + 19m – 49

Abstract. In this paper I observe that the formula m*n^2 + 11*m*n – 23*n + 19*m - 49 produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any m non-null positive integer, respectively for any n non-null positive integer.

Conjecture 1:

The formula m*n^2 + 11*m*n – 23*n + 19*m - 49 produces an infinite sequence of Poulet numbers for any n non-null positive integer.

Examples:

Formula becomes 31*m – 72 for n = 1 and we have the following sequence of Poulet numbers P = 31*m – 72 (obtained for m = 259, 367, 5111):
: 7957, 11305, 158369 (...)

Formula becomes 45*m – 95 for n = 2 and we have the following sequence of Poulet numbers P = 45*m – 95 (obtained for m = 888, 928, 2384):
: 39865, 41665, 107185(...)

Formula becomes 61*m – 118 for n = 3 and we have the following sequence of Poulet numbers P = 61*m – 118 (obtained for m = 329, 379):
: 19951, 23001(...)

Formula becomes 99*m – 164 for n = 5 and we have the following sequence of Poulet numbers P = 99*m – 164 (obtained for m = 319, 659, 1387):
: 31417, 65077, 137149(...)

Conjecture 2:

The formula m*n^2 + 11*m*n – 23*n + 19*m - 49 produces an infinite sequence of Poulet numbers for any m non-null positive integer.

Examples:

Formula becomes 3*n^2 + 10*n + 8 for m = 3 and we have the following sequence of Poulet numbers P = 3*n^2 + 10*n + 8 (obtained for n = 9, 13, 27, 29, 35, 41, 51, 71, 91, 101, 149, 165):
: 341, 645, 2465, 2821, 4033, 5461, 8321, 15841, 25761, 31621, 68101, 83333 (...)

Formula becomes 4*n^2 + 21*n + 27 for m = 4 and we have the following sequence of Poulet numbers P = 4*n^2 + 21*n + 27 (obtained for n = 14, 16, 20, 26, 38, 56, 62, 68, 86, 134, 142, 146, 148):
: 1105, 1387, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 83665, 88357, 90751 (...)

4. Three cubic polynomials that generate sequences of Poulet numbers

Abstract. In this paper I present three cubic polynomials that generate (probably infinite) sequences of Poulet numbers.

I.

Poulet numbers of the form 240*n^3 – 2708*n^2 + 10172*n – 12719:

: 340561, 2299081, 4335241, 8041345, 32085041, 153927961, 321524281 (...)

These numbers were obtained for the following values of n:
: 15, 25, 30, 36, 55, 90, 114 (...)

Conjecture:

There are infinite many Poulet numbers of the form 240*n^3 – 2708*n^2 + 10172*n - 12719 (see A182132 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form (30*n – 7)*(90*n – 23)*(300*n – 79).

II.

Poulet numbers of the form 80*n^3 + 788*n^2 + 2584*n + 2821:

: 2821, 63973, 285541, 488881, 7428421(...)

These numbers were obtained for the following values of n:
: 0, 6, 12, 15, 42 (...)

Conjecture:

There are infinite many Poulet numbers of the form 80*n^3 + 788*n^2 + 2584*n + 2821 (see A182085 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form (30*n + 7)*(60*n + 13)*(150*n + 31). 2

III.

Poulet numbers of the form 120*n^3 – 3148*n^2 + 27522*n – 80189:

: 29341, 1152271, 11875821, 16158331, 34901461 (...)

These numbers were obtained for the following values of n:
: 15, 30, 55, 60, 75 (...)

Conjecture:

There are infinite many Poulet numbers of the form 120*n^3 – 3148*n^2 + 27522*n – 80189 (see A182133 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form (30*n – 17)*(90*n – 53)*(150*n – 89).

5. Conjecture on Poulet numbers of the form 8mn^3 + 40n^3 + 38n^2 + 6mn^2 + mn + 11n + 1

Abstract. In this paper I observe that the formula 8*m*n^3 + 40*n^3 + 38*n^2 + 6*m*n^2 + m*n + 11*n + 1 produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any m non-null positive integer.

Conjecture:

The formula 8*m*n^3 + 40*n^3 + 38*n^2 + 6*m*n^2 + m*n + 11*n + 1 produces an infinite sequence of Poulet numbers for any m non-null positive integer.

Examples:

Formula becomes 48*n^3 + 44*n^2 + 12*n + 1 for m = 1 and we have the following sequence of Poulet numbers P = 48*n^3 + 44*n^2 + 12*n + 1 (obtained for n = 3, 7, 15, 18, 33, 45, 66 ...):

: 1729, 18705, 172081, 294409, 1773289, 4463641, 13992265 (...)

Formula becomes 56*n^3 + 50*n^2 + 13*n + 1 for m = 2 and we have the following sequence of Poulet numbers P = 56*n^3 + 50*n^2 + 13*n + 1 (obtained for n = 64, ...):

: 14885697 (...)

Formula becomes 64*n^3 + 56*n^2 + 14*n + 1 for m = 3 and we have the following sequence of Poulet numbers P = 64*n^3 + 56*n^2 + 14*n + 1 (obtained for n = 44, ...):

: 5560809 (...)

Formula becomes 80*n^3 + 68*n^2 + 16*n + 1 for m = 5 and we have the following sequence of Poulet numbers P = 80*n^3 + 68*n^2 + 16*n + 1 (obtained for n = 3, 9, 15, 18, 45 ...):

: 2821, 63973, 285541, 488881, 7428421 (...)

Note that all the solutions obtained for n so far (up to n = 45) are of the form 3k.

Formula becomes 112*n^3 + 92*n^2 + 20*n + 1 for m = 9 and we have the following sequence of Poulet numbers P = 112*n^3 + 92*n^2 + 20*n + 1 (obtained for n = 15, 45, ...):

: 399001, 10393201 (...)

6. Conjecture on Poulet numbers of the form 9mn^3 + 3n^3 – 15mn^2 + 6mn – 2n^2

Abstract. In this paper I observe that the formula 9*m*n^3 + 3*n^3 – 15*m*n^2 + 6*m*n – 2*n^2 produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any m non-null positive integer.

Conjecture:

The formula 9*m*n^3 + 3*n^3 – 15*m*n^2 + 6*m*n – 2*n^2 produces an infinite sequence of Poulet numbers for any m non-null positive integer.

Examples:

Formula becomes 12*n^3 – 17*n^2 + 6*n for m = 1 and we have the following sequence of Poulet numbers P = 12*n^3 – 17*n^2 + 6*n (obtained for n = 5, 11, 23, 29, 35, 65, 71, ...):

: 1105, 13981, 137149, 278545, 493885, 3224065, 4209661 (...)

Note that all the solutions obtained for n so far (up to n = 71) are of the form 6k – 1.

Formula becomes 21*n^3 – 32*n^2 + 12*n for m = 2 and we have the following sequence of Poulet numbers P = 21*n^3 – 32*n^2 + 12*n (obtained for n = 65, ...):
: 5632705 (...)

Formula becomes 30*n^3 – 47*n^2 + 18*n for m = 3 and we have the following sequence of Poulet numbers P = 30*n^3 – 47*n^2 + 18*n (obtained for n = 23, 43, 53, 103, ...):

: 340561, 2299081, 4335241, 32285041 (...)

Note that all the solutions obtained for n so far (up to n = 103) are of the form 10k + 3.
Formula becomes 39*n^3 – 62*n^2 + 24*n for m = 4 and we have the following sequence of Poulet numbers P = 39*n^3 – 62*n^2 + 24*n (obtained for n = 43, ...):

: 2987167 (...)

Formula becomes 48*n^3 – 77*n^2 + 30*n for m = 5 and we have the following sequence of Poulet numbers P = 48*n^3 – 77*n^2 + 30*n (obtained for n = 29, 37, 77...):

: 1106785, 2327041, 21459361 (...)

Note that all the solutions obtained for n so far (up to n = 77) are of the form 8k + 5.

7. A list of thirty-six polynomials and formulas that generate Fermat pseudoprimes

Abstract. In this paper I present a simple list of polynomials (in one or two variables) and formulas having the property that they generate Carmichael numbers or Poulet numbers, polynomials and formulas that I have discovered over time.

Polynomials that generate Carmichael numbers

1.

C = (30*n + 7)*(60*n + 13)*(150*n + 31)
First six such Carmichael numbers: 2821, 488881, 288120421, 492559141, 776176261, 1632785701 (sequence A182085 in OEIS).

2.

C = (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179)
First four such Carmichael numbers: 31146661, 2414829781, 192739365541, 197531244744661 (sequence A182088 in OEIS).

3.

C = (330*n + 7)*(660*n + 13)*(990*n + 19)*(1980*n + 37)
First two such Carmichael numbers: 63973, 461574735553 (sequence A182089 in OEIS).

4.

C = (30*n – 7)*(90*n – 23)*(300*n – 79)
First five such Carmichael numbers: 340561, 4335241, 153927961, 542497201, 1678569121 (sequence A182132 in OEIS).

5.

C = (30*n – 17)*(90*n – 53)*(150*n – 89)
First five such Carmichael numbers: 29341, 1152271, 34901461, 64377991, 775368901 (sequence A182133 in OEIS).

6.

C = (60*n + 13)*(180*n + 37)*(300*n + 61)
First five such Carmichael numbers: 29341, 34901461, 775368901, 1213619761, 4562359201 (sequence A182416 in OEIS).

Polynomials that generate Poulet numbers

1.

P = 7200*n^2 + 8820*n + 2701

First eight such Poulet numbers: 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281 (sequence A214016 in OEIS).

2.

P = 144*n^2 + 222*n + 85
First eight such Poulet numbers: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417 (sequence A214017 in OEIS).

3.

P = 3*(2*n + 1)*(18*n + 11)*(36*n + 17)
First four such Poulet numbers: 561, 62745, 656601, 11921001 (sequence A213071 in OEIS).

4.

P = (6*m – 1)*((6*m – 2)*n + 1)
First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

5.

P = (6*m + 1)*(6*m*n + 1)
First ten such Poulet numbers: 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911 (sequence A214607 in OEIS).

6.

P = m*n^2 + (11*m – 23)*n + 19*m – 49
First ten such Poulet numbers: 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461 (sequence A215326 in OEIS).

Formulas that generate Carmichael numbers

1.

C = (30*n – p)*(60*n – (2*p + 1))*(90*n – (3*p + 2)),
where p, 2*p + 1, 3*p + 2 are all three prime numbers.
First six such Carmichael numbers: 1729, 172081, 294409, 1773289, 4463641, 56052361 (sequence A182087 in OEIS).

Comment: The formula can be reduced to only two possible polynomial forms: C = (30*n – 23)*(60*n – 47)*(90*n – 71) or C = (30*n – 29)*(60*n – 59)*(90*n – 89).

2.

C = (p + 30)*(q + 60)*(p*q + 90),
where p and q are primes.
First two such Carmichael numbers: 488881, 1033669.

3.

C = (30*p + 1)*(60*p + 1)*(90*p + 1),
where p is prime.
First four such Carmichael numbers: 56052361, 216821881, 798770161, 1976295241.

4.

C = p*(2*p – 1)*(3*p – 2)*(6*p – 5),
where p is prime.
First seven such Carmichael numbers: 63973, 31146661, 703995733, 21595159873, 192739365541, 461574735553, 3976486324993 (sequence A182518 in OEIS).

5.

C = p*(2*p – 1)*(n*(2*p – 2) + p),
where p and 2*p – 1 are primes.
First ten such Carmichael numbers: 1729, 2821, 41041, 63973, 101101, 126217, 172081, 188461, 294409, 399001 (sequence 182207 in OEIS).

Comment: I conjecture that any Carmichael number C divisible by p and 2*p – 1 (where p and 2*p – 1 are primes) can be written this way.

6.

C = n*(2*n – 1)*(p*n – p + 1)*(2*p*n – 2*p + 1),
where p is odd and n natural.
Seven such Carmichael numbers: 63973, 172081, 31146661, 167979421, 277241401, 703995733, 1504651681 (sequence 212882 in OEIS).

7.

C = p*n*(3*p*n + 2)*(6*p*n – 1),
where p is prime and n natural.
Ten such Carmichael numbers: 2465, 62745, 11119105, 3249390145 (obtained for p = 5); 6601 (obtained for p = 7); 656601 (obtained for p = 11); 41041, 271794601 (obtained for p = 13); 11119105, 2159003281 (obtained for p = 17) (sequence 212882 in OEIS).

Formulas that generate Poulet numbers

1.

P = (2^(3*k + 1) – 1)/3,
where k natural.
First three such Poulet numbers: 341, 1398101, 5726623061.
Comment: The formula can be generalized as (n^(n*k + k + n – 1) – 1)/(n^2 – 1), formula which generates, I conjecture, an infinity of Fermat pseudoprimes to base n for any integer n, n > 1 (for n = 3 the formula becomes (3^(4*k + 2) – 1)/8 and generates Fermat pseudoprimes to base 3 for 14 values of k from 1 to 20).

2.

P = q*((n + 1)*q – n*q)*((m + 1)*q – m*q),
where q prime and m, n natural.
Five such Poulet numbers: 10585, 13741, 13981, 29341, 137149.

3.

P = q*((n*q – (n + 1)*q)*(m*q – (m + 1)*q),
where q prime and m, n natural.
Such Poulet number: 6601.

4.

P = q*(q + 2*n)*(q + 2^2*n – 2),
where q prime, n natural:
Two such Poulet numbers: 561, 1105.

5.

P = q*(q + 2*n)*(q + 2^k*n),
where q prime and n, k natural.
Four such Poulet numbers: 1729, 2465, 2821, 29341.

6.

P = (1 + 2^k*m)*(1 + 2^k*n)*(1 + 2^k*(m + n)),
where k, m, n natural.
Two such Poulet numbers: 13981, 252601.

7.

P = 3*(3 + 2^k)*(3 + q*2^h),
where q prime and k, h natural.
Three such Poulet numbers: 645, 1905, 8481.

8.

P = q^2 + 81*q + 3*q*r,
where q, r primes.
Four such Poulet numbers: 2821, 6601, 14491, 19951.

Comment: Note that the numbers (2821, 6601) and (14491, 19951) are "pairs" because 2821 = 13^2 + 81*13 + 3*13*41 while 6601 = 41^2 + 81*41 + 3*13*41 and also the values of the [q, r] for 14491 and 19951 are [43, 71] respectively [71, 43].

9.

P = r*q*(n*(q – 1) + r),
where r, q primes and n natural.
Six such Poulet numbers: 137149, 340561, 852841, 950797, 1052503, 1357621.

Comment: I conjecture that any Poulet number having as prime factors both the numbers 23 and 67 can be written this way, also any Poulet number having as prime factors both the numbers 11 and 61.

10.

P = 3*q^3*(3*n + 1) – q^2*(15*n + 2) + 6*q*n,
where q prime and n natural.
Six such Poulet numbers: 4335241, 13421773, 17316001, 17098369, 93869665, 170640961.
Comment: I conjecture that any Poulet number having as prime factors both a number of the form 30*k + 23 and a number of the form 90*k + 67 can be written this way.

11.

P = 6*q^3*(6*n + 1) – q^2*(66*n + 5) + 30*q*n,
where q prime and n natural.
Six such Poulet numbers: 5148001, 7519441, 9890881, 12262321.

Comment: I conjecture that any Poulet number having as prime factors both a number of the form 30*k + 11 and a number of the form 180*k + 61 can be written this way.

12.

P = ((2^n)^k)*((2^n)^(k + 1) + 2^n + 1) + 1,
where k, n natural.
Ten such Poulet numbers: 561, 33153 (obtained for n = 1); 1105, 16705 (obtained for n = 2); 4369, 1052929, 268505089 (obtained for n = 4), 266305 (obtained for n = 6); 2113665 (obtained for n = 7); 16843009 (obtained for n = 8).

13.

P = 2*q^2 – q,
where q is also a Poulet number.
First six such Poulet numbers: 831405, 5977153, 15913261, 21474181, 38171953, 126619741 (sequence A215343 in OEIS).

14.

P = (q^2 + 2*q)/3,
where q is also a Poulet number.
First six such Poulet numbers: 997633, 1398101, 3581761, 26474581, 37354465, 63002501 (sequence A216276 in OEIS).

15.

P = q^2*n – q*n + p,
where q is also a Poulet number and n natural.
First six such Poulet numbers: 348161, 831405, 1246785, 1275681, 2077545, 2513841 (sequence A217835 in OEIS).

16.

P = (n^m + n*m)/(m + 1),
where m, n natural.
Ten such Poulet numbers: 341, 645, 2465, 2821, 4033 (obtained for m = 2); 341, 1729, 188461, 228241, 1082809 (obtained for m = 3) (sequence A216170 in OEIS).

17.

P = (6*k – 1)*((6*k – 2)*n + 1),
where k, n natural.
First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

8. A list of 15 sequences of Poulet numbers based on the multiples of the number 6

Abstract. In previous papers, I presented few applications of the multiples of the number 30 in the study of Carmichael numbers, i.e. in finding possible infinite sequences of such numbers; in this paper I shall list 15 probably infinite sequences of Poulet numbers that I discovered based on the multiples of the number 6.

A list with 15 probably infinite sequences of Poulet numbers based on the multiples of the number 6.

(1) Poulet numbers of the form
P = (6*n + 7)*(12*n + 13).
First 4 terms: 2701 (= 37*73), 8911 (= 7*19*67), 10585 (= 5*29*73), 18721 (= 97*193),
obtained for n = 5, 10, 11.

(2) Poulet numbers of the form
P = (6*n + 7)*(30*n + 31).
First 6 terms: 1729 (= 7*13*19), 4681 (= 31*151), 30889 (= 17*23*157), 41041 (= 7*11*13*41), 46657 (= 13*37*97), 52633 (= 7*73*103),
obtained for n = 2, 4, 12, 16.

(3) Poulet numbers of the form
P = (12*n + 13)*(30*n + 31).
First term: 23377 (= 97*241),
obtained for n = 7.

(4) Poulet numbers of the form
P = (6*n + 7)*(12*n + 13)*(30*n + 31).
First 5 terms: 2821 (= 7*13*31), 63973 (= 7*13*19*37), 285541 (= 31*61*151), 488881 (= 37*73*181), 7428421 (= 7*11*13*41*181),
obtained for n = 0, 2, 4, 5, 14.
Conjecture: The number (6*n + 7)*(12*n + 13)*(30*n + 31) is a Poulet number if (but not only if) 6*n + 7, 12*n + 13 and 30*n + 31 are all three prime numbers.

(5) Poulet numbers of the form
P = (6*n + 1)*(12*n + 1).
First 4 terms: 2701 (= 37*73), 8911 (= 7*19*67), 10585 (= 5*29*73), 18721 (= 97*193),
obtained for n = 6, 11, 12, 16.

(6) Poulet numbers of the form
P = (6*n + 1)*(18*n + 1).
First 4 terms: 2821 (= 7*13*31), 4033 (= 37*109), 5461 (43*127), 15841 (= 7*31*73),
obtained for n = 5, 6, 7, 12.

(7) Poulet numbers of the form
P = (12*n + 1)*(18*n + 1).
First term: 7957 (73*109),
obtained for n = 6.

(8) Poulet numbers of the form
P = (6*n + 1)*(12*n + 1)*(18*n + 1).
First 6 terms: 1729 (= 7*13*19), 172081 (= 7*13*31*61), 294409 (= 37*73*109), 464185 (= 5*17*43*127), 1773289 (= 67*133*199), 4463641 (= 7*13*181*271),
obtained for n = 1, 5, 6, 7, 11, 15.
Note: The numbers (6*n + 1)*(12*n + 1)*(18*n + 1), when 6*n + 1, 12*n + 1 and 18*n + 1 are all three primes, are the well known Chernick numbers, so of course they are consequently Poulet numbers, but note that there exist such numbers which are Poulet numbers though 6*n + 1, 12*n + 1 and 18*n + 1 are not all three primes.

(9) Poulet numbers of the form
P = (6*n + 1)*(12*n + 1)*(18*n + 1)*(36*n + 1).
First 4 terms: 63973 (= 7*13*19*37), 31146661 (= 7*13*31*61*181), 703995733 (= 7*19*67*199*397), 2414829781 (= 7*13*181*271*541),
obtained for n = 1, 5, 11, 15.
Note: The numbers (6*n + 1)*(12*n + 1)*(18*n + 1)*(36*n + 1), when 6*n + 1, 12*n + 1, 18*n + 1 and 36*n + 1 are all four primes, are known that are Carmichael numbers, so of course they are consequently Poulet numbers, but note that there exist such numbers which are Poulet numbers though 6*n + 1, 12*n + 1, 18*n + 1 and 36*n + 1 are not all four primes.

(10) Poulet numbers of the form
P = (6*n + 1)*(24*n + 1).
First 5 terms: 1387 (= 19*73), 83665 (= 5*29*577), 90751 (= 151*601), 390937 (= 313*1249), 748657 (= 7*13*19*433),
obtained for n = 3, 24, 25, 52, 72.

(11) Poulet numbers of the form
P = (6*n – 1)*(12*n – 3).
First 2 terms: 561 (= 3*11*17), 4371 (= 3*31*47),
obtained for n = 3, 8.

(12) Poulet numbers of the form
P = (6*n – 1)*(18*n – 5).
First 3 terms: 341 (= 11*31), 2465 (5*17*29), 8321 (53*157),
obtained for n = 2, 5, 9.

(13) Poulet numbers of the form
P = (6*n – 1)*(24*n – 7).
First 5 terms: 1105 (= 5*13*17), 2047 (= 23*89), 3277 (= 29*113), 6601 (= 7*23*41), 13747 (= 59*233),
obtained for n = 3, 4, 5, 7, 10.

(14) Poulet numbers of the form
P = (6*n – 1)*(18*n – 5)*(60*n – 19).
First 2 terms: 340561 (= 13*17*23*67), 4335241 (= 53*157*521),
obtained for n = 4, 9.

(15) Poulet numbers of the form
P = (6*n + 1)*(18*n + 1)*(30*n + 1).
First 2 terms: 29341 (= 13*37*61), 1152271 (= 43*127*211), obtained for n = 2, 7.

9. Bold conjecture on Fermat pseudoprimes

Abstract. In many of my previous papers I showed various methods, formulas and polynomials designed to generate sequences, possible infinite, of Poulet numbers or Carmichael numbers. In this paper I state that there exist a method to place almost any Fermat pseudoprime to base two (Poulet number) in such a sequence, as a further term or as a starting term.

Conjecture:

If the prime factors of a Poulet number not divisible with 3 can be expressed in the following way, i.e. the least from them P as 6*n + 1, 6*n – 1, 6*n + 5 or 6*n – 5 and the product of the rest of them Q as 6*m*n + 1, 6*m*n – 1, 6*m*n + 5 or 6*m*n – 5, then there exist an infinity of Poulet numbers of the form P*Q.

As example, the first Poulet number, 341, is equal to 11*31 and we have 11 = 6*2 – 1 (so 6*n – 1) and 31 = 18*2 – 5 (so 18*n – 5); the conjecture states that there exist an infinity of Poulet numbers of the form (6*n – 1)*(18*n – 5).

Note that not any Poulet number not divisible by 3 (though the most of them) can be written the way described above; as example, the 2-Poulet number 3277 = 29*113 (29 is equal to 6*4 + 5 and to 6*5 – 1, but 113 isn't equal either to 6*4*m ± 1 or 6*5*m ± 1 neither with 6*4*m ± 5 or 6*5*m ± 5).

Examples:
(for few from the first Poulet numbers not divisible by 3)

: 341 = 11*31 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (6*n – 1)*(18*n – 5) which has the following terms: 2465, 8321, 83333, 162401 (...) obtained for n = 5, 9, 28, 39 (...);

: 1105 = 5*13*17 = 5*221 is the starting term, obtained for n = 1, in the sequence of Poulet numbers (6*n – 1)*(222*n – 1) which has the following terms: 11305 (...) obtained for n = 3 (...);

: 1387 = 19*73 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (6*n + 1)*(24*n + 1) which has the following terms: 83665, 90751 (...) obtained for n = 24, 25 (...);

: 1729 = 7*13*19 = 7*247 is the starting term, obtained for n = 1, in the sequence of Poulet numbers (6*n + 1)*(246*n + 1) which has the following terms: 1082809, 1615681 (...) obtained for n = 27, 33 (...);

: 2047 = 23*89 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (6*n + 5)*(30*n – 1);

: 2465 = 5*17*19 = 5*493 is the starting term, obtained for n = 1, in the sequence of Poulet numbers (6*n – 1)*(492*n + 1);

: 2701 = 37*73 is the starting term, obtained for n = 6, in the sequence of Poulet numbers (6*n + 1)*(12*n + 1) which has the following terms: 8911, 10585, 18721 (...) obtained for n = 11, 12, 16 (...);

: 2821 = 7*13*31 = 7*403 is the starting term, obtained for n = 1, in the sequence of Poulet numbers (6*n + 1)*(402*n + 1);

: 4033 = 37*109 is the second term, obtained for n = 6, in the sequence of Poulet numbers (6*n + 1)*(18*n + 1) which has as the first term, obtained for n = 5, the Poulet number 2821, and as the following terms: 5461, 15841 (...) obtained for n = 7, 12 (...);

Note that the Poulet number 2821 is a term in both of the distinct sequences (6*n + 1)*(402*n + 1) and (6*n + 1)*(18*n + 1).

: 4369 = 17*257 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (6*n + 5)*(126*n + 5);

: 4681 = 31*151 is the second term, obtained for n = 5, in the sequence of Poulet numbers (6*n + 1)*(30*n + 1) which has as the first term, obtained for n = 3, the Poulet number 1729, and as the following terms: 41041, 46657, 52633 (...) obtained for n = 15, 16, 17 (...);

Note that the Poulet number 1729 is a term in both of the distinct sequences (6*n + 1)*(246*n + 1) and (6*n + 1)*(30*n + 1).

: 5461 = 43*127 is the second term, obtained for n = 7, in the sequence of Poulet numbers (6*n + 1)*(18*n + 1) which has as the first term, obtained for n = 5, the Poulet number 2821.

10. Another bold conjecture on Fermat pseudoprimes

Abstract. In my previous paper “Bold conjecture on Fermat pseudoprimes” I stated that there exist a method to place almost any Fermat pseudoprime to base two (Poulet number) in an infinite subsequence of such numbers, defined by a quadratic polynomial, as a further term or as a starting term of such a sequence. In this paper I conjecture that there is yet another way to place a Poulet number in such a sequence defined by a polynomial, this time not necessarily quadratic.

Conjecture:

If we express the prime factors of a Poulet number, not divisible by 3, P = d1*d2*...di, where d1, d2, ..., di are its prime factors, as P = (2n + 1)*(m1*n + 1)*...*(mi*n + 1), then there exist an infinity of Poulet numbers of this form.

As example, the first Poulet number, 341, is equal to 11*31 and we have 11 = 2*5 + 1 (so 2*n + 1) and 31 = 6*5 + 1 (so 6*n + 1); the conjecture states that there exist an infinity of Poulet numbers of the form (2*n + 1)*(6*n + 1).

Note that not any Poulet number not divisible by 3 (though the most of them) can be written the way described above; as example, the 2-Poulet number 6601 = 7*23*41 (7 is equal to 2*3 + 1, but 23 isn’t equal to m*3 + 1 neither 41).

Examples:
(for few from the first Poulet numbers not divisible by 3)

: 341 = 11*31 is the starting term, obtained for n = 5, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1) which has the following terms: 645, 2465, 2821, 4033 (...) obtained for n = 7, 14, 15, 18 (...);

: 1105 = 5*13*17 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1)*(8*n + 1) which has the following terms: 13981, 137149, 278545, 493885 (...) obtained for n = 5, 11, 14, 17 (...);

: 1387 = 19*73 is the second term, obtained for n = 9, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1) which has as the first term, obtained for n = 8, the Poulet number 1105, and as the following terms: 2047, 3277, 6601 (...) obtained for n = 11, 14, 20 (...);

: 1729 = 7*13*19 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1)*(6*n + 1) which has the following terms: 18705, 172081, 294409 (...) obtained for n = 7, 15, 18 (...);

: 2047 = 23*89 is the third term, obtained for n = 11, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1) which has as the first and second terms, obtained for n = 8 and n = 9, the Poulet numbers 1105 and 1387;

: 2465 = 5*17*29 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1)*(14*n + 1) which has the following terms: 176149 (...) obtained for n = 9 (...);

: 2701 = 37*73 is the second term, obtained for n = 18, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1) which has as the first term, obtained for n = 8, the Poulet number 561;

: 2821 = 7*13*31 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1)*(10*n + 1) which has the following terms: 63973, 285541, 488881 (...) obtained for n = 9, 15, 18 (...);

: 4033 = 37*109 is the fifth term, obtained for n = 18, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1) which has as the previous terms, obtained for n = 5, 7, 14, 15 (...) the Poulet numbers 341, 645, 2465, 2821 (...);

: 4369 = 17*257 is the starting term, obtained for n = 8, in the sequence of Poulet numbers (2*n + 1)*(32*n + 1);

: 4681 = 31*151 is the third term, obtained for n = 15, in the sequence of Poulet numbers (2*n + 1)*(10*n + 1) which has as the first and second terms, obtained for n = 5 and n = 9, the Poulet numbers 561 and 1729;

: 5461 = 43*127 is a term, obtained for n = 21, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1);

: 7957 = 73*109 is a term, obtained for n = 36, in the sequence of Poulet numbers (2*n + 1)*(3*n + 1);

: 8321 = 53*157 is a term, obtained for n = 26, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1);

: 8911 = 7*19*67 is the first term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1)*(22*n + 1) which has the following terms: 63973 (...) obtained for n = 6 (...).

11. Generic form of the Poulet numbers having a prime factor of the form 30n + 23

Abstract. In this paper I observe that many Poulet numbers P having a prime factor q of the form 30*n + 23, where n positive integer, can be written as P = m*(q^2 – q) + q^2, where m positive integer, and I conjecture that any Poulet number P having 23 as a prime factor can be written as P = 506*m + 529, where m positive integer.

Observation:

Many Poulet numbers P having a prime factor q of the form 30*n + 23, where n positive integer, can be written as P = m*(q^2 – q) + q^2, where m positive integer.

Examples:

: 8321 = 53*157 and 8321 = 2*(53^2 – 53) + 53^2, so [q, m] = [53, 2];
: 85489 = 53*1613 and 85489 = 30*(53^2 – 53) + 53^2, so [q, m] = [53, 30];
: 88561 = 11*83*97 and 88561 = 12*(83^2 – 83) + 83^2, so [q, m] = [83, 12];
: 91001 = 17*53*101 and 91001 = 32*(53^2 – 53) + 53^2, so [q, m] = [53, 32];
: 208465 = 5*173*241 and 208465 = 6*(173^2 – 173) + 173^2, so [q, m] = [173, 6];
: 215265 = 3*5*113*127 and 215265 = 16*(113^2 – 113) + 113^2, so [q, m] = [113, 16];
: 275887 = 263*1049 and 275887 = 3*(263^2 – 263) + 263^2, so [q, m] = [263, 3];
: 278545 = 5*17*29*113 and 278545 = 21*(113^2 – 113) + 113^2, so [q, m] = [113, 21];
: 422659 = 3*53*2687 and 422659 = 154*(53^2 – 53) + 53^2, so [q, m] = [53, 154].

Conjecture:

Any Poulet number P having 23 as a prime factor can be written as P = 506*m + 529, where m positive integer.

Verifying the conjecture:
(For the first seven such Poulet numbers)

: 2047 = 23*89 and 2047 = 3*506 + 529, so m = 3;
: 6601 = 7*23*41 and 6601 = 12*506 + 529, so m = 12;
: 15709 = 23*683 and 15709 = 30*506 + 529, so m = 30;
: 30889 = 17*23*79 and 30889 = 60*506 + 529, so m = 60;
: 137149 = 23*67*89 and 30889 = 270*506 + 529, so m = 270;
: 272251 = 7*19*23*89 and 272251 = 537*506 + 529, so m = 537;
: 340561 = 13*17*23*67 and 340561 = 672*506 + 529, so m = 672.

Note the following 13 Poulet numbers having a prime factor of the form 23*n + 30 (from the first 29 such Poulet numbers) which can't be written in the way showed above: 13747 = 59*233, 3277 = 29*113, 31417 = 89*353, 60787 = 89*683, 65077 = 59*1103, 72885 = 3*5*43*113, 88357 = 149*593, 130561 = 137*953, 194221 = 167*1163, 196021 = 7*41*683, 253241 = 157*1613, 256999 = 233*1103, 280601 = 277*1013. In all of these cases, the prime factor of the form 23*n + 30 is the biggest prime factor (a particular case is the number 256999 having both of the factors of the form 23*n + 30).

12. Notable observation on a property of Carmichael numbers

Abstract. In this paper I conjecture that for any Carmichael number C is true one of the following two statements: (i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/(q – 1) is prime, power of prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime; (ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/((q – 1)*2^n) is prime or power of prime. In two previous papers I made similar assumptions on the squares of primes of the form 10k + 1 respectively 10k + 9 and I always believed that Fermat pseudoprimes behave in several times like squares of primes.

Conjecture:

For any Carmichael number C is true one of the following two statements:

(i) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/(q – 1) is prime, power of prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime;

(ii) there exist at least one prime q, q lesser than Sqr (C), such that p = (C – q)/((q – 1)*2^n) is prime or power of prime.

Verifying the conjecture:

(for the first ten Carmichael numbers)

: C = 561 and (C – 5)/4 = 139, prime; also (C – 11)/10 = 5*11, semiprime such that 11 – 5 + 1 = 7, prime; also (C – 17)/(16*2) = 17, prime;

: C = 1105 and (C – 7)/6 = 3*61, semiprime such that 61 – 3 + 1 = 59, prime; also (C – 13)/12 = 7*13, semiprime such that 13 – 7 + 1 = 7, prime and 13 + 7 – 1 = 19, prime; also (C – 17)/(16*2^2) = 17, prime;

: C = 1729 and (C – 5)/4 = 431, prime; also (C – 17)/16 = 107, prime; also (C – 37)/36 = 47, prime;

: C = 2465 and (C – 29)/28 = 3*29, semiprime such that 29 – 3 + 1 = 27 = 3^3, power of prime and 29 + 3 – 1 = 31, prime;

: C = 2821 and (C – 7)/6 = 7*67, semiprime such that 67 – 7 + 1 = 61, prime and 67 + 7 – 1 = 73, prime; also (C – 11)/10 = 281, prime; also (C – 31)/30 = 3*31, semiprime such that 31 – 3 + 1 = 29, prime;

: C = 6601 and (C – 5)/4 = 17*97, semiprime such that 97 – 17 + 1 = 81 = 3^4, power of prime and 97 + 17 – 1 = 113, prime; also (C – 7)/6 = 7*157, semiprime such that 157 – 7 + 1 = 151, prime and 157 + 7 – 1 = 163, prime; also (C – 11)/10 = 659, prime; also (C – 23)/22 = 13*23, semiprime such that 23 – 13 + 1 = 11, prime; also (C – 31)/30 = 3*73, semiprime such that 73 – 3 + 1 = 71, prime; also (C – 61)/60 = 109, prime;

: C = 8911 and (C – 23)/(22*2^2) = 101, prime; also (C – 31)/(30*2^3) = 37, prime; also (C – 67)/(66*2) = 67, prime;

: C = 10585 and (C – 7)/6 = 41*43, semiprime such that 43 – 41 + 1 = 3, prime and 43 + 1 – 1 = 83, prime; also (C – 13)/12 = 881, prime; also (C – 19)/18 = 587, prime; also (C – 29)/28 = 13*29, semiprime such that 29 – 13 + 1 = 17, prime and 13 + 29 – 1 = 41, prime; also (C – 37)/36 = 293, prime; also (C – 43)/42 = 251, prime; also (C – 73)/(73*2) = 73, prime;

: C = 5841 and (C – 5)/4 = 37*107, semiprime such that 107 – 37 + 1 = 71, prime; also (C – 11)/10 = 1583, prime; also (C – 13)/12 = 1319, prime; also (C – 61)/60 = 263, prime; also (C – 67)/66 = 239, prime; also (C – 73)/72 = 3*73, semiprime such that 73 – 3 + 1 = 71, prime; also (C – 89)/88 = 179, prime; also (C – 97)/(96*2^2) = 41, prime;

: C = 29341 and (C – 7)/6 = 4889, prime; also (C – 31)/30 = 977, prime; also (C – 61)/(60*2^3) = 61, prime.

13. Conjecture which states that any Carmichael number can be written in a certain way

Abstract. In this paper we conjecture that any Carmichael number C can be written as C = (p + 270)*(n + 1) – n, where n non-null positive integer and p prime. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that there exist an infinity of Poulet numbers P2 that could be written as (P1 + n)/(n + 1) – 270, where n is non-null positive integer and P1 is also a Poulet number.

Conjecture:

In this paper we conjecture that any Carmichael number C can be written as C = (p + 270)*(n + 1) – n, where n non-null positive integer and p prime.

Verifying the conjecture:
(for the first eight Carmichael numbers)

: 561 = (11 + 270)*2 – 1, so [n, p] = [1, 11];

: 1105 = (283 + 270)*2 – 1, so [n, p] = [1, 283]; also 1105 = (7 + 270)*4 – 3, so [n, p] = [3, 7];

: 1729 = (307 + 270)*3 – 2, so [n, p] = [2, 307]; also 1729 = (163 + 270)*4 – 3, so [n, p] = [3, 163]; also 1729 = (19 + 270)*6 – 5, so [n, p] = [5, 19];

: 2465 = (347 + 270)*4 – 3, so [n, p] = [3, 347]; also 2465 = (83 + 270)*7 – 6, so [n, p] = [6, 83];

: 2821 = (941 + 270)*3 – 2, so [n, p] = [2, 941]; also 2821 = (13 + 270)*10 – 9, so [n, p] = [9, 13];

: 6601 = (1931 + 270)*3 – 2, so [n, p] = [2, 1931]; also 6601 = (1381 + 270)*4 – 3, so [n, p] = [3, 1381]; also 6601 = (1051 + 270)*5 – 4, so [n, p] = [4, 1051]; also 6601 = (331 + 270)*11 – 10, so [n, p] = [10, 331]; also 6601 = (281 + 270)*12 – 11, so [n, p] = [11, 281];

: 8911 = (541 + 270)*11 – 10, so [n, p] = [10, 541]; also 8911 = (61 + 270)*27 – 26, so [n, p] = [26, 61];

: 10585 = (5023 + 270)*2 – 1, so [n, p] = [1, 5023]; also 10585 = (3529 + 270)*3 – 2, so [n, p] = [2, 3529]; also 10585 = (2377 + 270)*4 – 3, so [n, p] = [3, 2377]; also 10585 = (907 + 270)*9 – 8, so [n, p] = [8, 907]; also 10585 = (613 + 270)*12 – 11, so [n, p] = [11, 613]; also 10585 = (487 + 270)*14 – 13, so [n, p] = [13, 487]; also 10585 = (109 + 270)*28 – 27, so [n, p] = [27, 109].

Note:

We have not verified, but it would be interesting if the number 1729 would be the first number that could be written as C = (p + 270)*(n + 1) – n, where n non-null positive integer and p prime, in three distinct ways, or if the number 6601 would be the first number that could be written such this in five distinct ways, or if the number 10585 would be the first number that could be written such this in seven distinct ways, or if the first number that could be written such this in k different ways would be a Carmichael number.

Conjecture:

There exist an infinity of Poulet numbers P2 that could be written as (P1 + n)/(n + 1) – 270, where n is non-null positive integer and P1 is also a Poulet number.

Example:
: 701 = (8911 + 2)/3 - 270, so [n, P1, P2] = [2, 8911, 2701].

14. Sequence of Poulet numbers obtained by formula mn – n + 1 where m of the form 270k + 13

Abstract. In this paper we conjecture that there exist an infinity of Poulet numbers of the form m*n – n + 1, where m is of the form 270*k + 13. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that there exist an infinity of numbers s of the form 270*k + 13 which are semiprimes s = p*q having the property that q – p + 1 is prime or power of prime.

Conjecture:

There exist an infinity of Poulet numbers of the form m*n – n + 1, where m is of the form 270*k + 13.

Examples:

: for k = 1, m = 283 and the following numbers are Poulet numbers:
: 2821 = 283*10 – 10 + 1 (...)
: for k = 2, m = 553 and the following numbers are Poulet numbers:
: 1105 = 553*2 – 2 + 1 (...)
: for k = 4, m = 1093 and the following numbers are Poulet numbers:
: 3277 = 1093*3 – 3 + 1;
: 4369 = 1093*4 – 4 + 1;
: 5461 = 1093*5 – 4 + 1 (...)

The sequence of Poulet numbers of the form m*n – n + 1, where m is of the form 270*k + 13:
: 1105, 2821, 3277, 4369, 5461 (...)

Conjecture:

There exist an infinity of numbers s of the form 270*k + 13 which are semiprimes s = p*q having the property that q – p + 1 is prime or power of prime.

Examples:

: for k = 2, s = 553 = 7*79 and 79 – 7 + 1 = 73, prime;
: for k = 5, s = 1363 = 29*47 and 47 – 29 + 1 = 19, prime;
: for k = 6, s = 1633 = 23*71 and 71 – 23 + 1 = 49, power of prime (7^2);
: for k = 7, s = 1903 = 11*173 and 173 – 11 + 1 = 163, prime;
: for k = 8, s = 2173 = 41*53 and 53 – 41 + 1 = 13, prime;
: for k = 9, s = 2443 = 7*349 and 349 – 7 + 1 = 343, power of prime (7^3);
: for k = 11, s = 2983 = 19*157 and 157 – 19 + 1 = 139, prime;
: for k = 15, s = 4063 = 17*239 and 239 – 17 + 1 = 223, prime;
: for k = 16, s = 4333 = 7*619 and 619 – 7 + 1 = 613, prime;
: for k = 18, s = 4873 = 11*443 and 443 – 11 + 1 = 433, prime;
: for k = 19, s = 5143 = 37*139 and 139 – 37 + 1 = 103, prime;
: for k = 24, s = 6493 = 43*151 and 151 – 43 + 1 = 109, prime;

: for k = 26, s = 7033 = 13*541 and 541 – 13 + 1 = 529, power of prime (23^2);
: for k = 27, s = 7303 = 67*109 and 109 – 67 + 1 = 43, prime;
: for k = 33, s = 8383 = 83*101 and 101 – 83 + 1 = 19, prime;
(...)
: for k = 20000, s = 5400013 = 1627*3319 and 3319 – 1627 + 1 = 1693, prime;
(...)
: for k = 190000, s = 51300013 = 1487*34499 and 34499 – 1487 + 1 = 33013, prime.

Note:

Many other numbers s of the form 270*k + 13 are semiprimes s = p1*q1 having the property that q1 – p1 + 1 is a semiprime p2*q2 having the property that q2 – p2 is prime.

Example:

: for k = 2000000, s = 540000013 = 7*77142859 and 77142859 – 7 + 1 = 77142853 = 41*1881533 and 1881533 – 41 + 1 = 1881493, prime.

15. Two conjectures on Super-Poulet numbers with two respectively three prime factors

Abstract. In this paper I make two conjectures on Super-Poulet numbers with two, respectively three prime factors.

Definition:

Super-Poulet numbers are the Poulet numbers whose divisors d all satisfy the relation d divides 2^d – 2 (see the sequence A050217 in OEIS for the list of Super-Poulet numbers).

Note:

Every 2-Poulet number (Poulet number with only two prime factors) is also a Super-Poulet number (see the sequence A214305 for the list of 2-Poulet numbers).

Conjecture 1:

For any 2-Poulet number q*r (obviously q and r primes, distinct (q < r) beside the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes) is true one of the following two statements:

i) there exist n positive integer such that r = n*q – n + 1;

ii) there exist p prime, p greater than 7, also n and m positive integers, such that q = n*p – n + 1 and r = m*p – m + 1.

Verifying the conjecture:
(For the first twenty-two 2-Poulet numbers)

: 341 = 11*31 and 31 = 11*3 – 2;
: 1387 = 19*73 and 73 = 19*4 – 3;
: 2047 = 23*89 and 89 = 23*4 – 3;
: 2701 = 37*73 and 73 = 37*2 – 1;
: 3277 = 29*113 and 113 = 29*4 – 3;
: 4033 = 37*109 and 109 = 37*3 – 2;
: 4369 = 17*257 and 257 = 17*6 – 5;
: 4681 = 31*151 and 151 = 31*5 – 4;
: 5461 = 43*127 and 127 = 43*3 – 2;
: 7957 = 73*109 and 73 = 6*13 – 5 while 109 = 9*13 – 8;
: 8321 = 53*157 and 157 = 53*3 – 2;
: 10261 = 31*331 and 331 = 31*11 – 10;
: 13747 = 59*233 and 233 = 59*4 – 3;
: 14491 = 43*337 and 337 = 43*8 – 7;
: 15709 = 23*683 and 683 = 23*31 – 30;
: 18721 = 97*193 and 193 = 97*2 – 1;
: 19951 = 71*281 and 281 = 71*4 – 3;
: 23377 = 97*241 and 97 = 6*17 – 5 while 241 = 15*17 – 14;
: 31417 = 89*353 and 353 = 89*4 – 3;

: 31609 = 73*433 and 433 = 73*6 – 5;
: 31621 = 103*307 and 307 = 103*3 – 2;
: 35333 = 89*397 and 89 = 4*23 – 3 while 397 = 18*23 – 17.

Note that the conjecture is obviously true for the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes, i.e. 1194649 = 1093^2 and 12327121 = 3511^2. For instance, the prime 1093 can be written in seven distinct ways like n*p – p + 1, where p prime: 1093 = 2*547 – 1 = 7*157 – 6 = 14*79 – 13 = 21*53 – 20 = 26*43 – 25 = 39*29 – 38 = 197*7 – 6 (and, of course, 1093 = 1093*1 – 0).

Conjecture 2:

For any Super-Poulet number with three prime factors p*q*r (obviously p, q and r primes, p < q < r) is true one of the following two statements:

iii) there exist n and m positive integers such that q = n*p – n + 1 and r = m*p – m + 1;

iv) there exist s prime, s greater than 7, also a, b and c positive integers, such that p = a*s – a + 1, q = b*s – b + 1 and r = c*s – c + 1.

Verifying the conjecture:
(For the first 18 such Super-Poulet numbers)

: 294409 = 37*73*109 and 73 = 37*2 – 1 while 109 = 37*3 – 2;
: 1398101 = 23*89*683 and 89 = 23*4 – 3 while 683 = 23*31 – 30;
: 1549411 = 31*151*331 and 151 = 31*5 – 4 while 331 = 31*11 – 10;
: 1840357 = 43*127*337 and 127 = 43*3 – 2 while 337 = 43*8 – 7;
: 12599233 = 97*193*673 and 193 = 97*2 – 1 while 673 = 97*7 – 6;
: 13421773 = 53*157*1613 and 157 = 53*3 – 2 while 1613 = 53*31 – 30;
: 15162941 = 59*233*1103 and 233 = 59*4 – 3 while 1103 = 59*19 – 18;
: 15732721 = 97*241*673 and 97 = 17*6 – 5 while 241 = 17*15 – 14 also 673 = 17*42 – 41;
: 28717483 = 59*233*2089 and 233 = 59*4 – 3 while 2089 = 59*36 – 35;
: 29593159 = 43*127*5419 and 127 = 43*3 – 2 while 5419 = 43*129 – 128;
: 61377109 = 157*313*1249 and 313 = 157*2 – 1 while 1249 = 157*8 – 7;
: 66384121 = 89*353*2113 and 353 = 89*4 – 3 while 2113 = 89*24 – 23;
: 67763803 = 103*307*2143 and 307 = 103*3 – 2 while 2143 = 103*21 – 20;
: 74658629 = 89*397*2113 and 89 = 23*4 - 3 while 397 = 23*18 – 17 while 2113 = 23*96 – 95;
: 78526729 = 43*337*5419 and 337 = 43*8 – 7 while 5419 = 43*129 – 128;
: 90341197 = 103*307*2857 and 307 = 103*3 – 2 while 2857 = 103*28 – 27;
: 96916279 = 167*499*1163 and 499 = 499*3 – 2 while 1163 = 167*7 – 6;
: 109322501 = 101*601*1801 and 601 = 101*6 – 5 while 1801 = 101*18 – 17.

16. Observation on the period of the rational number P/d + d/P where P is a 3-Poulet number and d its least prime factor

Abstract. In this paper I make the following observation: let P be a 3-Poulet number, d its least prime factor and q one of the other two prime factors; then the lenght of the period of the rational number P/d + d/P is for almost any P equal to q – 1 or equal to (q – 1)/n or equal to (q – 1)*n, where n positive integer.

Observation:

Let P be a 3-Poulet number, d its least prime factor and q one of the other two prime factors; then the lenght of the period of the rational number P/d + d/P is for almost any P equal to q – 1 or equal to (q – 1)/n or equal to (q – 1)*n, where n positive integer.

Note:

The sequence of 3-Poulet numbers: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841, 16705, 25761, 29341, 30121, 30889, 33153, 34945, 41665, 52633, 57421, 68101, 74665, 83665, 87249, 88561, 91001, 93961, 113201, 115921, 121465, 137149 (...). See the sequence A215672 that I submitted on OEIS.

Verifying the observation:
(true for 29 from the first 31 such Poulet numbers)

: for P = 561 = 3*11*17, the period of P/d + d/P is equal to 5347593582887700, which has the lenght 16 = 17 – 1;

: for P = 645 = 3*5*43, the period of P/d + d/P is equal to 465116279069767441860, which has the lenght 21 = (43 – 1)/2;

: for P = 1105 = 5*13*17, the period of P/d + d/P is equal to 452488687782805429864253393665158371040723981900, which has the lenght 48 = (17 – 1)*3;

: for P = 1729 = 7*13*19, the period of P/d + d/P is equal to 404858299595141700, which has the lenght 18 = 19 – 1;

: for P = 1905 = 3*5*127, the period of P/d + d/P is equal to 157480314960629921259842519685039370078740, which has the lenght 42 = (127 – 1)/3;

: for P = 2465 = 5*17*29, the period of P/d + d/P has the lenght 112 = (29 – 1)*4;

: for P = 2821 = 7*13*31, the period of P/d + d/P has the lenght 30 = 31 – 1;

: for P = 4371 = 3*31*47, the period of P/d + d/P has the lenght 690 = (47 – 1)*15;

: for P = 6601 = 7*23*41, the period of P/d + d/P has the lenght 110 = (23 – 1)*5;

: for P = 8481 = 3*11*257, the period of P/d + d/P has the lenght 256 = 257 – 1;

: for P = 8911 = 7*19*67, the period of P/d + d/P has the lenght 198 = (67 – 1)*3;

: for P = 10585 = 5*29*73, the period of P/d + d/P has the lenght 56 = (29 – 1)*2;

: for P = 12801 = 3*17*251, the period of P/d + d/P has the lenght 400 = (17 – 1)*25;

: for P = 13741 = 7*13*151, the period of P/d + d/P has the lenght 150 = 151 – 1;

: for P = 13981 = 11*31*41, the period of P/d + d/P has the lenght 15 = (31 – 1)/2;

: for P = 15841 = 7*31*73, the period of P/d + d/P has the lenght 120 = (31 – 1)*4;

: for P = 16705 = 5*13*257, the period of P/d + d/P has the lenght 768 = (257 – 1)*3;

: for P = 29341 = 13*37*61, the period of P/d + d/P has the lenght 60 = 61 – 1;

: for P = 30121 = 7*13*331, the period of P/d + d/P has the lenght 330 = 331 – 1;

: for P = 30889 = 17*23*79, the period of P/d + d/P has the lenght 286 = (23 – 1)*13;

: for P = 33153 = 3*43*257, the period of P/d + d/P has the lenght 5376 = (257 – 1)*21;

: for P = 34945 = 5*29*241, the period of P/d + d/P has the lenght 420 = (29 – 1)*15;

: for P = 41665 = 5*13*641, the period of P/d + d/P has the lenght 96 = (13 – 1)*8;

: for P = 57421 = 7*13*631, the period of P/d + d/P has the lenght 630 = 631 – 1;

: for P = 68101 = 11*41*151, the period of P/d + d/P has the lenght 75 = (151 – 1)/2;

: for P = 74665 = 5*109*137, the period of P/d + d/P has the lenght 216 = (109 – 1)*2;

: for P = 83665 = 5*29*577, the period of P/d + d/P has the lenght 4032 = (577 – 1)*7;

: for P = 87249 = 3*127*229, the period of P/d + d/P has the lenght 1596 = (229 – 1)*7;

: for P = 88561 = 11*83*97, the period of P/d + d/P has the lenght 3936 = (97 – 1)*41.

Exceptions:

: for P = 25761 = 3*31*277, the period of P/d + d/P has the lenght 345;

: for P = 52633 = 7*73*103, the period of P/d + d/P has the lenght 136.

Part Three.
Prime producing quadratic polynomials

1. A list of known root prime-generating quadratic polynomials producing more than 23 distinct primes in a row

Abstract. A simple list of known such polynomials, indexed by the value of discriminants, containing no analysis but the introduction of the "root prime generating polynomial" notion.

I listed below the polynomials (after the value of discriminant). In the brackets we have the polynomials that generate same primes but in reverse order (any prime-producing polynomial has such a reversal). The list contains 42 polynomials (84 with their reversals). I didn't consider redundant primes to not complicate the list furthermore. I discovered myself all the polynomials with the font italic (32(64)). I know the other ones from the articles available on Internet like *Prime-Generating Polynomial* from Wolfram Math World or sites like Rivera's *The Prime Puzzles & Problems Connection*.

Note: because of the special nature of the number 1, I considered the polynomials that generate that number too as prime-generating polynomials, but, for the purists, I indexed with specification "d.p." distinct primes and "d.p.1." distinct primes plus number 1 (in absolute value).

Note: a **"root prime-generating polynomial"** I consider to be the prime-generating polynomial that has two properties:

(1) for n = -1 gives a non-prime term (for instance, 8n^2 + 88n + 43 is not a root prime-generating polynomial because for n = -1 we have the prime term (in absolute value) -37 and for n = n – 39 we have the "complete" root prime-generating polynomial: 8n^2 – 488n + 7243);

(2) there is no other prime-generating polynomial with the same value of discriminant that generates the same amount of primes in a row, having coefficients of smaller values.

Note: I submitted few of these polynomials to OEIS.

Discriminant equal to -222643:
35 d.p.: 43n^2 – 537n + 2971 (43n^2 – 2387n + 34421).

Discriminant equal to -23472:
26 d.p.: 36n^2 – 408n + 1319 (36n^2 – 1392n + 13619).

Discriminant equal to -13203:
28 d.p.: 81n^2 – 1323n + 5443 (81n^2 – 3051n + 28771);
25 d.p.: 9n^2 – 219n + 1699 (9n^2 – 213n + 1627).

Discriminant equal to -10432:
23 d.p.: 64n^2 – 1192n + 5591 (64n^2 – 1624n + 10343).

Discriminant equal to -8523:
23 d.p.: 27n^2 – 489n + 2293 (27n^2 – 699n + 4603).

Discriminant equal to -7987:
23 d.p.: 49n^2 – 469n + 1163 (49n^2 – 1687n + 14561).

Discriminant equal to -4075:
32 d.p.: 25n^2 – 365n + 1373 (25n^2 – 1185n + 14083).

Discriminant equal to -2608:
31 d.p.: 16n^2 – 292n + 1373 (16n^2 – 668n + 7013);
30 d.p.: 16n^2 – 300n + 1447 (16n^2 – 628n + 6203).

Discriminant equal to -1467:
40 d.p.: 9n^2 – 231n + 1523 (9n^2 – 471n + 6203).

Discriminant equal to -708:
29 d.p.: 6n^2 + 6n + 31 (6n^2 – 342n + 4903).

Discriminant equal to -652:
40 d.p.: 4n^2 – 154n + 1523 (4n^2 – 158n + 1601).

Discriminant equal to -232:
29 d.p.: 2n^2 + 29 (2n^2 – 112n + 1597).

Discriminant equal to -163:
40 d.p.: n^2 + n + 41 (n^2 – 79n + 1601).

Discriminant equal to 293:
24 d.p.1.: n^2 + n – 73 (n^2 – 47n + 479).

Discriminant equal to 437:
28 d.p.1.: n^2 + n – 109 (n^2 + 55n + 647).

Discriminant equal to 677:
25 d.p.1.: 13n^2 – 313n + 1871 (13n^2 – 311n + 1847);
23 d.p.: n^2 + 3n – 167 (n^2 – 49n + 431).

Discriminant equal to 1077:
24 d.p.1.: 3n^2 + 3n - 89 (3n^2 – 141n + 1567).

Discriminant equal to 1172:
29 d.p.1.: 4n^2 – 90n + 433 (4n^2 – 142n + 1187).

Discriminant equal to 1253:
27 d.p.1.: 7n^2 + 7n - 43 (7n^2 – 371n + 4871).

Discriminant equal to 1592:
28 d.p.1.: 2n^2 – 199 (2n^2 + 108n + 1259).

Discriminant equal to 6368:
31 d.p.: 8n^2 + 8n – 197 (8n^2 – 488n + 7243).
Discriminant equal to 19808:
23 d.p.: 104n^2 – 2200n + 11587 (104n^2 – 2376n + 13523).

Discriminant equal to 25472:
35 d.p.: 4n^2 + 12n - 1583 (4n^2 – 284n + 3449);
31 d.p.: 32n^2 – 944n + 6763 (32n^2 – 976n + 7243);
29 d.p.: 16n^2 – 408n + 2203 (16n^2 – 488n + 3323).

Discriminant equal to 57312:
35 d.p.: 72n^2 - 1416n + 6763 (72n^2 - 1752n + 10459).

Discriminant equal to 64917:
35 d.p.1.: 27n^2 – 741n + 4483 (27n^2 – 1095n + 10501);
33 d.p.: 81n^2 – 2247n + 15383 (81n^2 – 2937n + 26423);
32 d.p.: 27n^2 – 753n + 4649 (27n^2 – 921n + 7253);
24 d.p.: 9n^2 + 9n – 1801 (9n^2 – 423n + 3167).

Discriminant equal to 78008:
28 d.p.: 98n^2 – 2128n + 11353 (98n^2 – 3164n + 25339).

Discriminant equal to 101888:
31 d.p.: 4n^2 – 428n + 5081 (4n^2 + 188n – 4159);
24 d.p.1.: 128n^2 – 1216n + 2689 (128n^2 – 4672n + 42433);

Discriminant equal to 159200:
27 d.p.: 100n^2 – 2820n + 19483 (4n^2 – 2380n + 13763).

Discriminant equal to 259668:
45 d.p.: 36n^2 – 810n + 2753 (36n^2 – 2358n + 36809);
24 d.p.: 108n^2 – 2130n + 9901 (108n^2 – 2838n + 18043).

Discriminant equal to 979373:
43 d.p.: 47n^2 – 1701n + 10181 (47n^2 – 2247n + 21647).

Discriminant equal to 1038672:
29 d.p.: 144n^2 – 2196n + 6569 (144n^2 – 5868n + 57977).

Discriminant equal to 1398053:
43 d.p.: 103n^2 – 4707n + 50383 (103n^2 – 3945n + 34381).

I also submit the following problem: find a value of discriminant, beside the ones from the following list: -222643, -4075, -2608, -1467, -652, -163, 6368, 25472, 57312, 64917, 101888, 259668, 979373, 1398053, for which a quadratic polynomial having this discriminant generates 30 or more distinct primes in a row.

I list below the polynomials that I know that generates 30 or more distinct primes in a row (in the brackets are the reverse polynomials, that generates same primes in reverse order):

43n^2 – 537n + 2971 (43n^2 – 2387n + 34421);
9n^2 – 231n + 1523 (9n^2 – 471n + 6203);
4n^2 – 154n + 1523 (4n^2 – 158n + 1601);
n^2 + n + 41 (n^2 – 79n + 1601);
8n^2 + 8n – 197 (8n^2 – 488n + 7243);
36n^2 – 810n + 2753 (36n^2 – 2358n + 36809);
47n^2 – 1701n + 10181 (47n^2 – 2247n + 21647);
103n^2 – 4707n + 50383 (103n^2 – 3945n + 34381).

I list below the polynomials that I discovered myself that generates 30 or more distinct primes in a row (few of them are posted on OEIS):

25n^2 – 365n + 1373 (25n^2 – 1185n + 14083);
16n^2 – 292n + 1373 (16n^2 – 668n + 7013);
16n^2 – 300n + 1447 (16n^2 – 628n + 6203);
4n^2 + 12n – 1583 (4n^2 – 284n + 3449);
32n^2 – 944n + 6763 (32n^2 – 976n + 7243);
72n^2 – 1416n + 6763 (72n^2 – 1752n + 10459);
81n^2 – 2247n + 15383 (81n^2 – 2937n + 26423);
27n^2 – 753n + 4649 (27n^2 – 921n + 7253);
4n^2 – 428n + 5081 (4n^2 + 188n – 4159).

2. Ten prime-generating quadratic polynomials

Abstract. In two of my previous papers I treated quadratic polynomials which have the property to produce many primes in a row: in one of them I listed forty-two such polynomials which generate more than twenty-three primes in a row and in another one I listed few generic formulas which may conduct to find such prime-producing quadratic polynomials. In this paper I will present ten such polynomials which I discovered and posted in OEIS, each accompanied by its first fifty terms and some comments about it.

I.

The polynomial 16*n^2 – 300*n + 1447.
Its first fifty terms:

1447, 1163, 911, 691, 503, 347, 223, 131, 71, 43, 47, 83, 151, 251, 383, 547, 743, 971, 1231, 1523, 1847, 2203, 2591, 3011, 3463, 3947, 4463, 5011, 5591, 6203, 6847, 7523, 8231, 8971, 9743, 10547, 11383, 12251, 13151, 14083, 15047, 16043, 17071, 18131, 19223, 20347, 21503, 22691, 23911, 25163, 26447.

Comments:

This polynomial generates 30 primes in a row starting from n = 0.

The polynomial 16*n^2 – 628*n + 6203 generates the same primes in reverse order.

I found in the same family of prime-generating polynomials (with the discriminant equal to -163*2^p, where p is even), the polynomials 4n^2 - 152n + 1607, generating 40 primes in row starting from n = 0 (20 distinct ones) and 4n^2 - 140n + 1877, generating 36 primes in row starting from n = 0 (18 distinct ones).

The following 5 (10 with their "reversal" polynomials) are the only ones I know from the family of Euler’s polynomial n^2 + n + 41 (having their discriminant equal to a multiple of -163) that generate more than 30 distinct primes in a row starting from n = 0 (beside the Escott’s polynomial n^2 – 79n + 1601):
(1) 4n^2 – 154n + 1523 (4n^2 – 158n + 1601);
(2) 9n^2 – 231n + 1523 (9n^2 – 471n + 6203);
(3) 16n^2 – 292n + 1373 (16n^2 – 668n + 7013);
(4) 25n^2 – 365n + 1373 (25n^2 – 1185n + 14083);
(5) 16n^2 – 300n + 1447 (16n^2 – 628n + 6203).

II.

The polynomial 2*n^2 – 108*n + 1259.
Its first fifty terms:

1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181, -191, -197, -199, -197, -191, -181, -167, -149, -127, -101, -71, -37, 1, 43, 89, 139, 193, 251, 313, 379, 449, 523, 601, 683, 769.

Comments:

This polynomial generates 92 primes (66 distinct ones) for n from 0 to 99 (in fact the next two terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for n = m – 35, which is m^2 – 69*m + 1231, but having six distinct primes more than this one.

The non-prime terms in the first 100 are: 1 (taken twice), 1369 = 37^2, 1849 = 43^2, 4033 = 37*109, 5633 = 43*131, 7739 = 71*109 and 8251 = 37*223.

For n = 2*m – 34 we obtain the polynomial 8*m^2 – 488*m + 7243, which generates 31 primes in a row starting from m = 0.

For n = 4*m – 34 we obtain the polynomial 32*m^2 – 976*m + 7243, which generates 31 primes in row starting from m = 0.

III.

The polynomial 2*n^2 – 212*n + 5419.
Its first fifty terms:

5419, 5209, 5003, 4801, 4603, 4409, 4219, 4033, 3851, 3673, 3499, 3329, 3163, 3001, 2843, 2689, 2539, 2393, 2251, 2113, 1979, 1849, 1723, 1601, 1483, 1369, 1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181.

Comments:

This polynomial generates 92 primes (57 distinct ones) for n from 0 to 99 (in fact the next seven terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for n = m – 35, which is m^2 – 69*m + 1231.

The non-prime terms in the first 100 are: 1, 1369 = 37^2, 1849 = 43^2, 4033 = 37*109 (all taken twice).

For n = 2*m + 54 we obtain the polynomial 8*m^2 + 8*m – 197, which generates 31 primes in a row starting from m = 0 (the polynomial 8*m^2 – 488*m + 7243 generates the same 31 primes, but in reverse order).

IV.

The polynomial 25*n^2 – 1185*n + 14083.
Its first fifty terms:

14083, 12923, 11813, 10753, 9743, 8783, 7873, 7013, 6203, 5443, 4733, 4073, 3463, 2903, 2393, 1933, 1523, 1163, 853, 593, 383, 223, 113, 53, 43, 83, 173, 313, 503, 743, 1033, 1373, 1763, 2203, 2693, 3233, 3823, 4463, 5153, 5893, 6683, 7523, 8413, 9353, 10343, 12473, 13613, 14803, 16043, 17333.

Comments:

The polynomial generates 32 primes in row starting from n = 0.

The polynomial 25*n^2 – 365*n + 1373 generates the same primes in reverse order.

This family of prime-generating polynomials (with the discriminant equal to -4075 = -163*5^2) is interesting for generating primes of same form: the polynomial 25*n^2 – 395(n + 1601 generates 16 primes of the form 10*k + 1 (1601, 1231, 911, 641, 421, 251, 131, 61, 41, 71, 151, 281, 461, 691, 971, 1301) and the polynomial 25*n^2 + 25*n + 47 generates 16 primes of the form 10*k + 7 (47, 97, 197, 347, 547, 797, 1097, 1447, 1847, 2297, 2797, 3347, 3947, 4597, 5297, 6047).

Note:
All the polynomials of the form 25*n^2 + 5*n + 41, 25*n^2 + 15*n + 43,..., 25*n^2 + 5*(2k + 1)*n + p,..., 25*n^2 + 5*79*n + 1601, where p is a (prime) term of the Euler's polynomial p = k^2 + k + 41, from k = 0 to k = 39, have their discriminant equal to -4075 = -163*5^2.

V.

The polynomial 16*n^2 – 292*n + 1373.
Its first fifty terms:

1373, 1097, 853, 641, 461, 313, 197, 113, 61, 41, 53, 97, 173, 281, 421, 593, 797, 1033, 1301, 1601, 1933, 2297, 2693, 3121, 3581, 4073, 4597, 5153, 5741, 6361, 7013, 7697, 8413, 9161, 9941, 10753, 11597, 12473, 13381, 14321, 15293, 16297, 17333, 18401, 20633, 21797, 22993, 24221, 25481, 26773.

Comments:

The polynomial generates 31 primes in row starting from n = 0.

The polynomial 16*n^2 – 668*n + 7013 generates the same primes in reverse order.

Note:
All the polynomials of the form p^2*n^2 ± p*n + 41, p^2*n^2 ± 3*p*n + 43, p^2*n^2 ± 5*p*n + 47, ..., p^2*n^2 ± (2k+1)*p*n + q, ..., p^2*n^2 ± 79*p*n + 1601, where q is a (prime) term of the Euler's polynomial q = k^2 + k + 41, from k = 0 to k = 39, have their discriminant equal to -163*p^2; the demonstration is easy: the discriminant is equal to b^2 – 4*a*c = (2*k + 1)^2*p^2 - 4*q*p^2 = - p^2 ((2*k + 1)^2 – 4*q) = - p^2*(4*k^2 + 4*k + 1 – 4*k^2 – 4*k - 164) = - 163*p^2.

Observation:
Many of the polynomials formed this way have the capacity to generate many primes in row.

Examples:
: 9*n^2 + 3*n + 41 generates 27 primes in row starting from n = 0 (and 40 primes for n = n – 13);
: 9*n^2 – 237*n + 1601 generates 27 primes in row starting from n = 0;
: 16*n^2 + 4*n + 41 generates, for n = n - 21 (that is 16*n^2 – 668*n + 7013) 31 primes in row.

VI.

The polynomial 4*n^2 – 284*n + 3449.
Its first fifty terms:

3449, 3169, 2897, 2633, 2377, 2129, 1889, 1657, 1433, 1217, 1009, 809, 617, 433, 257, 89, -71, -223, -367, -503, -631, -751, -863, -967, -1063, -1151, -1231, -1303, -1367, -1423, -1471, -1511, -1543, -1567, -1583, -1591, -1591, -1583, -1567, -1543, -1511, -1471, -1367, -1303, -1231, -1151, -1063, -967, -863, -751.

Comments:

The polynomial successively generates 35 primes or negative values of primes starting at n = 0.

This polynomial generates 95 primes in absolute value (60 distinct ones) for n from 0 to 99, equaling the record held by the Euler's polynomial for n = m - 35, which is m^2 - 69*m + 1231.

The non-prime terms (in absolute value) up to n = 99 are: 1591 = 37*43, 3737 = 37*101, 4033 = 37*109; 5633 = 43*131; 5977 = 43*139; 9017 = 71*127.

The polynomial 4*n^2 + 12*n - 1583 generates the same 35 primes in row starting from n = 0 in reverse order.

Note:
In the same family of prime-generating polynomials (with the discriminant equal to 199*2^p, where p is odd) there are the polynomial 32*n^2 - 944*n + 6763 (with its "reversed polynomial" 32*m^2 - 976*m + 7243, for m = 30 - n), generating 31 primes in row, and the polynomial 4*n^2 - 428*n + 5081 (with 4*m^2 + 188*m - 4159, for m = 30-n), generating 31 primes in row.

VII.

The polynomial n^2 + 3*n – 167.
Its first fifty terms:

-167, -163, -157, -149, -139, -127, -113, -97, -79, -59, -37, -13, 13, 41, 71, 103, 137, 173, 211, 251, 293, 337, 383, 431, 481, 533, 587, 643, 701, 761, 823, 887, 953, 1021, 1091, 1163, 1237, 1313, 1391, 1471, 1553, 1637, 1723, 1811, 1901, 1993, 2087, 2183, 2381, 2483.

Comments:

The polynomial generates 24 primes in absolute value (23 distinct ones) in row starting from n = 0 (and 42 primes in absolute value for n from 0 to 46).

The polynomial n^2 – 49*n + 431 generates the same primes in reverse order.
Note:

We found in the same family of prime-generating polynomials (with the discriminant equal to 677) the polynomial 13*n^2 – 311*n + 1847 (13*n^2 – 469*n + 4217) generating 23 primes and two noncomposite numbers (in absolute value) in row starting from n = 0 (1847, 1549, 1277, 1031, 811, 617, 449, 307, 191, 101, 37, -1, -13, 1, 41, 107, 199, 317, 461, 631, 827, 1049, 1297, 1571, 1871).

Note:
Another interesting algorithm to produce prime-generating polynomials could be N = m*n^2 + (6*m + 1)*n + 8*m + 3, where m, 6*m + 1 and 8*m + 3 are primes. For m = 7 then n = t - 20 we get N = 7*t^2 – 237*t + 1999, which generates the following primes: 239, 163, 101, 53, 19, -1, -7, 1, 23, 59, 109, 173, 251 (we can see the same pattern: …, -1, -m, 1, …).

VIII.

The polynomial 81*n^2 – 2247*n + 15383.
Its first forty terms:

15383, 13217, 11213, 9371, 7691, 6173, 4817, 3623, 2591, 1721, 1013, 467, 83, -139, -199, -97, 167, 593, 1181, 1931, 2843, 3917, 5153, 6551, 8111, 9833, 11717, 13763, 15971, 18341, 20873, 23567, 26423, 29441, 32621, 35963, 39467, 43133, 46961, 50951.

Comments:
The polynomial generates 33 primes/negative values of primes in row starting from n = 0.

The polynomial 81*n^2 – 2937*n + 26423 generates the same primes in reverse order.

Note:
We found in the same family of prime-generating polynomials (with the discriminant equal to 64917 = 3^2*7213) the polynomial 27*n^2 – 753*n + 4649 (with its "reversed polynomial" 27*n^2 – 921*n + 7253), generating 32 primes in row and the polynomial 27*n^2 – 741*n + 4483 (27*n^2 – 1095*n + 10501), generating 35 primes in row, if we consider that 1 is prime (which seems to be constructive in the study of prime-generating polynomials, at least).

Note:
The polynomial 36*n^2 – 810*n + 2753, which is the known quadratic polynom that generates the most distinct primes in row (45), has the discriminant equal to 259668 = 2^2*3^2*7213.

IX.

The polynomial 4*n^2 + 12*n – 1583.
Its first forty terms:

-1583, -1567, -1543, -1511, -1471, -1423, -1367, -1303, -1231, -1151, -1063, -967, -863, -751, -631, -503, -367, -223, -71, 89, 257, 433, 617, 809, 1009, 1217, 1433, 1657, 1889, 2129, 2377, 2633, 2897, 3169, 3449, 3737, 4033, 4337, 4649, 4969. 14561, 14083, 13613, 13151, 12697, 12251, 11813, 11383, 10961, 10547, 10141, 9743, 9353, 8971, 8597, 8231, 7873, 7523, 7181, 6847, 6521, 6203, 5893, 5591, 5297, 5011, 4733, 4463, 4201, 3947, 3701, 3463, 3233, 3011, 2797, 2591, 2393, 2203, 2021, 1847.

Comments:

The polynomial generates 35 primes/negative values of primes in row starting from n = 0.

The polynomial 4*n^2 – 284*n + 3449 generates the same primes in reverse order.

Other related polynomials are:

: for n = 6*n + 6 then n = n – 11 we get 144*n^2 – 2808*n + 12097 which generates 16 primes in a row starting from n = 0 (with the discriminant equal to 2^9*3^2*199);

: for n = 12*n + 12 then n = n – 15 we get 576*n^2 – 15984*n + 109297 which generates 17 primes in a row starting from n = 0 (with the discriminant equal to 2^11*3^2*199).

Note: so this polynomials opens at least two directions of study:
(1) polynomials of type 4*n^2 + 12*n – p, where p is prime (could be of the form 30k + 23);
(2) polynomials with the discriminant equal to 2^n*3^m*199, where n is odd and m is even
(an example of such polynomial, with the discriminant equal to 2^5*3^4*199 is 36*n^2 – 1020*n + 3643 which generates 32 primes for values from 0 to 34).

X.

The polynomial 4*n^2 – 482*n + 14561.
Its first forty terms:

14561, 14083, 13613, 13151, 12697, 12251, 11813, 11383, 10961, 10547, 10141, 9743, 9353, 8971, 8597, 8231, 7873, 7523, 7181, 6847, 6521, 6203, 5893, 5591, 5297, 5011, 4733, 4463, 4201, 3947, 3701, 3463, 3233, 3011, 2797, 2591, 2393, 2203, 2021, 1847.

Comments:

This polynomial generates 88 distinct primes for n from 0 to 99, just two primes less than the record held by the polynomial discovered by N. Boston and M. L. Greenwood, that is 41*n^2 - 4641*n + 88007 (this polynomial is sometimes cited as 41*n^2 + 33*n - 43321, which is the same for the input values [-57, 42].

Note:
The non-prime terms in the first 100 are: 10961 = 97*113; 10547 = 53*199; 9353 = 47*199; 7181 = 43*167; 6847 = 41*167; 5893 = 71*83; 3233 = 53*61; 2021 = 43*47; 1681 = 41^2; 1763 = 41*43; 2491 = 47*53; 4331 = 61*71.

Note:
For n = m + 41 we obtain the polynomial 4*m^2 - 154*m + 1523, which generates 40 primes in a row starting from m = 0.

3. Seventeen generic formulas that may generate prime-producing quadratic polynomials

Abstract. In one of my previous papers I listed forty-two quadratic polynomials which generate more than twenty-three primes in a row, from which ten were already known from the articles available on Internet and thirty-two were discovered by me. In this paper I list few generic formulas which may conduct to find such prime-producing quadratic polynomials.

I.

The formula 8*n^2 + (2*p + 2)*n + p, where p is prime.

Examples:

: for p = 43 we have the polynomial 8*n^2 + 88*n + 43 which generates 26 distinct primes for values of n from 0 to 25; also, for m = n – 39 is obtained the root prime-generating polynomial 8*m^2 – 488*m + 7243 which generates, from values of m from 0 to 30, thirty-one distinct primes in a row;

: for p = 29 we have the polynomial 8*n^2 + 60*n + 29 which generates 20 distinct primes or squares of primes for values of n from 0 to 19;

: for p = 19 we have the polynomial 8*n^2 + 40*n + 19 which generates 20 distinct primes for values of m from 0 to 19, where m = n – 12, in other words from this polynomial is obtained the root prime-generating polynomial 8*m^2 – 152*m + 691.

II.

The formula 2*m^2*n^2 + 40*m*n + 1, where m is positive integer.

Examples:

: for m = 1 we have the polynomial 2*n^2 + 40*n + 1 which generates 36 distinct primes or squares of primes for values of n from 0 to 35; also, for m = 6*n + 1, is obtained the polynomial 72*m^2 + 264*m + 43 which generates 9 distinct primes in a row; for m = 7*n + 5 is obtained the polynomial 98*m^2 + 420*m + 251 which generates 14 distinct primes in a row; for m = 8*n + 6 is obtained the polynomial 128*m^2 + 512*m + 313 which generates 13 distinct primes or squares of primes in a row;

: for m = 8 we have the polynomial 128*n^2 + 320*n + 1 which generates 17 distinct primes in a row for values of n from 0 to 16.

III.

The formula 2*m^2*n^2 – 199, where m is positive integer.

Examples:

: for m = 1 we have the polynomial 2*n^2 – 199 which generates 28 distinct primes in a row for values of n from 0 to 27; also, for m = 2*n + 29, is obtained the polynomial 8*m^2 + 232*m + 1483 which generates 31 distict primes respectively 62 redundant primes in a row; also, for m = 2*n – 1, is obtained the polynomial 8*m^2 – 8*m – 197 which generates 31 distict primes in a row;

: for m = 2 we have the polynomial 8*n^2 – 199 which generates 14 distinct primes in a row; also, for m = n – 13 we have the polynomial 8*m^2 – 208*m + 1153 which generates 31 distinct primes and 44 redundant primes in a row;

: for m = 3 we have the polynomial 18*n^2 – 199 which generates 18 distinct primes in a row;

: for m = 4 we have the polynomial 32*n^2 – 199 which generates 27 distinct primes or squares of primes in a row.

IV.

The formula 2*m^2*n^2 + 29, where m is positive integer.

Examples:

: for m = 1 we have the Sierpinski's polynomial 2*n^2 + 29 which generates 29 distinct primes in a row;

: for m = 2 we have the polynomial 8*n^2 + 29 which generates 15 distinct primes in a row.

V.

The formula m^2*n^2 + m*n + 41, where m is positive integer.

Examples:

: for m = 1 we have the Euler's polynomial n^2 + n + 41 which generates 40 distinct primes in a row;

: for m = 2 we have the polynomial 4*n^2 + 2*n + 41 which generates 20 distinct primes in a row; also, for m = 2*n + 1 is obtained the polynomial 16*m^2 + 20*m + 47 which generates 20 distinct primes in a row; also for t = t – 10 we have the polynomial 16*t^2 – 300*t + 1447 which generates 31 primes in a row;

: for m = 3 we have the polynomial 9*n^2 + 3*n + 41 which generates 27 distinct primes in a row; also, for m = n – 13 is obtained the polynomoal 9*n^2 – 231*n + 1523 which generates 40 distinct primes in a row.

VI.

The formula m^2*n^2 + 2*m*n + 59, where m is positive integer.

Examples:

: for m = 2 we have the polynomial 4*n^2 + 4*n + 59 which generates 14 distinct primes in a row;

: for m = 6 we have the polynomial 36*n^2 + 12*n + 59 which generates 15 distinct primes in a row; also for m = n – 4 is obtained the polynomial 36*m^2 – 276*m + 587 which generates 19 distinct primes in a ro;

: for m = 12 we have the polynomial 144*n^2 + 24*n + 59 which generates 12 distinct primes in a row; also for m = n – 7 is obtained a polynomial which generates 19 distinct primes in a row.

VII.

The formula 8*m^2*n^2 + 60*m*n + 29, where m is positive integer.

Examples:

: for m = 1 we have the polynomial 8*n^2 + 60*n + 29 which generates 20 distinct primes or squares of primes in a row; also for m = n – 17 is obtained the polynomial 8*m^2 – 212*m + 1321 which generates 22 distinct primes restpectively 37 primes or squares of primes in a row.

VIII.

The formula 11*n^2 + (2*p – 13)*n + p, where p is prime.

Examples:

: for p = 11 we have the polynomial 11*n^2 + 9*n + 11 which generates 11 distinct primes in a row; also for m = n – 10 is obtained the polynomial 11*m^2 – 211*m + 1021 which generates 21 distinct primes in a row;

: for p = 13 we have the polynomial 11*n^2 + 13*n + 13 which generates 10 distinct primes in a row; also for m = n – 11 is obtained the polynomial 11*m^2 – 427*m + 4153 which generates 21 distinct primes in a row.

IX.

The formula 8*n^2 – (2*p – 2)*n – p, where p is prime.

Examples:

: for p = 13 we have the polynomial 8*n^2 – 24*n – 13 which generates 10 distinct primes in a row;
: for p = 37 we have the polynomial 8*n^2 – 72*n – 37 which generates also many primes in a row.

X.

The formula m^2*n^2 – 57*m*n + 853, where m is positive integer.

Examples:

: for m = 1 and t = n – 11 is obtained the polynomial t^2 – 79*t + 1601 which generates 40 distinct primes in a row (the same primes generated by Euler's polynomial in reversed order);

: for m = 2 and t = n – 5 is obtained the polynomial 4*t^2 – 154*t + 1523 which generates 40 distinct primes in a row;

: for m = 3 and t = n – 3 is obtained the polynomial 9*t^2 – 225*t + 1447 which also generates many distinct primes in a row;

: for m = 4 and t = n – 2 is obtained the polynomial 16*t^2 – 292*t + 1373 which generates 31 distinct primes in a row;

: for m = 5 and t = n – 18 is obtained the polynomial 25*t^2 – 1185*t + 14083 which generates 32 distinct primes in a row;

: for m = 9 and t = n – 5 is obtained the polynomial 81*t^2 – 1323*t + 5443 which generates 28 distinct primes in a row.

XI.

The formula m^2*n^2 – 69*m*n + 1231, where m is positive integer.

Examples:

: for m = 2 and t = n – 2 is obtained the polynomial 4*t^2 – 154*t + 1523 which generates many primes in a row;

: for m = 3 and t = n – 1 is obtained the polynomial 9*t^2 – 225*t + 1447 which generates many primes in a row;

: for m = 4 and t = n – 12 is obtained the polynomial 16*t^2 – 628*t + 6203 which generates 30 distinct primes in a row;

: for m = 9 and t = n – 15 is obtained the polynomial 81*t^2 – 3051*t + 28771 which generates 28 distinct primes in a row.

XII.

The formula m^2*n^2 – 149*m*n + 5591, where m is positive integer.

XIII.

The formula m^2*n^2 – 157*m*n + 6203, where m is positive integer.

XIV.

The formula m^2*n^2 – 77*m*n + 1523, where m is positive integer.

XV.

The formula 2*m^2*n^2 – 60*m*n + 251, where m is positive integer.

XVI.

The formula 2*m^2*n^2 – 140*m*n + 2251, where m is positive integer.

XVII.

The formula 2*(m*n + m + 1)^2 – 199, where m is positive integer.

Examples:

: for m = 1 is obtained the polynomial 2*n^2 + 8*n – 191 which generates 26 distinct primes in a row;

: for m = 2 is obtained the polynomial 8*n^2 + 24*n – 181 which generates 30 distinct primes in a row;

: for m = 4 and t = n – 6 is obtained the polynomial 32*n^2 – 944*n + 6763 which generates 31 distinct primes in a row.

Note:
In this paper I considered to be primes the number 1 and the negative integers which are primes in absolute value.