Why Tau Replaces Pi

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The constant pi (π) has been used for thousands of years to relate the diameter of a circle to its circumference. But today our mathematical system has evolved to favor the use of the radius (r) of a circle - instead of the diameter - in our formulas. For example:

$$\begin{aligned} \text{diameter of a circle} &= 2\mathrm{r} && (1)\\ \text{area of a circle} &= \pi\mathrm{r}^2 && (2)\\ \text{volume of a sphere} &= \frac{4}{3}\pi\mathrm{r}^3 && (3) \end{aligned}$$

But if the radius is really more important mathematically than the diameter, then what is the constant that relates the radius of a circle to its circumference?

The answer is tau (τ), and its value is equal to about 6.28 (twice π). Tau is the constant that we should be using when we work with "round objects" - not pi.

The proof for why tau is the constant that we should be using lies within the above series of formulas. However, to recognize it we need to briefly review a series of terms created by Leonhard Euler more than two centuries ago.

Euler produced this series of terms to model exponential growth:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \tag{4}$$

What we are interested in, is that each term in the series is a derivative of the next term. For example the derivative of $x^3/3!$ is $x^2/2!$. It is remarkable that the exponential value of each term is always equal to the factorial value, even after each derivative.

This eloquent usage of derivatives and factorials is also found in our series of math formulas. If you look at the "Pi formulas" for each "Step number", you will find where those math formulas fit into this table.

Step number	Even path	Tau formula	Pi formula	Odd path	Tau formula	Pi formula
0	$\frac{1}{0!}\tau^0 r^0$	1	1			
1				$\frac{2}{1!}\tau^0 r^1$	2r	2r
2	$\frac{1}{2}\tau^1 r^2$	$\frac{1}{2}\tau r^2$	πr^2			
3				$\frac{2}{3}\tau^1 r^3$	$\frac{2}{3}\tau r^3$	$\frac{4}{3}\pi r^3$
4	$\frac{1}{4\times2}\tau^2 r^4$	$\frac{1}{8}\tau^2 r^4$	$\frac{1}{2}\pi^2 r^4$			
5				$\frac{2}{5\times3}\tau^2 r^5$	$\frac{2}{15}\tau^2 r^5$	$\frac{8}{15}\pi^2 r^5$
6	$\frac{1}{6\times4\times2}\tau^3 r^6$	$\frac{1}{48}\tau^3 r^6$	$\frac{1}{6}\pi^3 r^6$			
7				$\frac{2}{7\times5\times3}\tau^3 r^7$	$\frac{2}{105}\tau^3 r^7$	$\frac{16}{105}\pi^3 r^7$
8	$\frac{1}{8\times6\times4\times2}\tau^4 r^8$	$\frac{1}{384}\tau^4 r^8$	$\frac{1}{24}\pi^4 r^8$			
9				$\frac{2}{9\times7\times5\times3}\tau^4 r^9$	$\frac{2}{945}\tau^4 r^9$	$\frac{32}{945}\pi^4 r^9$

There are differences between these two types of integration, but the similarity is that the exponential value of each term always equals the "factorial" value, even after each derivative.

So why do we have an equivalent Tau formula right next to the Pi formula?

The derivatives are only apparent when tau is used in the formulas - the pi formulas obscure the pattern. In this type of a series tau acts like a key - no other constant will unlock and reveal these patterns. Mother nature writes her formulas using tau!

This table of formulas - with its relation to derivatives - has gone unnoticed because we have been writing our formulas with pi. If we quibble with mother nature about which constant to use, we will not be able to see all of her beauty.