Why Tau Replaces Pi

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Tau (τ) is a constant equal to about 6.28 - which is twice the value of the constant pi (π). There are many good reasons why we should replace pi with tau: tau is simpler, more intuitive, easier to teach and learn, etc.

However, the pi loyalists have always responded that there was no need to change since there is nothing that τ can do that 2π can not also do (meaning 2π will always work in a formula in place of τ). They were right - until now. This short paper explains how we started off on the wrong foot using pi, and shows us that the reasons why we need to change were right in front of us all along.

Pi has been used for thousands of years. People somehow learned that the length across a circle (diameter) multiplied by pi was equal to the circumference (the length around the edge of a circle).

But today our mathematical system has evolved to favor the use of the radius (r) of a circle - instead of the diameter - in our formulas. For example:

$$\text{diameter of a circle} = 2\mathrm{r} \tag{1}$$
$$\text{area of a circle} = \pi\mathrm{r}^2 \tag{2}$$
$$\text{volume of a sphere} = \frac{4}{3}\pi\mathrm{r}^3 \tag{3}$$

But if the radius is really more important mathematically than the diameter, then what is the constant that relates the radius of a circle to its circumference? The answer is tau - and we should be using tau when we work with "round objects" - not pi.

The proof for why tau is the constant that we should be using lies within the above series of formulas. However, to recognize it we need to briefly review a series of terms created by Leonhard Euler more than two centuries ago.

Euler produced this series of terms to model exponential growth:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \tag{4}$$

What we are interested in, is that each term in the series is a derivative of the next term. For example the derivative of $x^3/3!$ is $x^2/2!$. It is remarkable that the exponential value of each term is always equal to the factorial value, even after each derivative.

This eloquent usage of derivatives and factorials is also found in our series of math formulas. If you look at both "Pi formula" columns, you will find where those math formulas fit into this table.

Step number	Even path	Tau formula	Pi formula	Odd path	Tau formula	Pi formula
0	$\frac{1}{0!}\tau^0 r^0$	1	1			
1				$\frac{2}{1!}\tau^0 r^1$	2r	2r
2	$\frac{1}{2}\tau^1 r^2$	$\frac{1}{2}\tau r^2$	πr^2			
3				$\frac{2}{3}\tau^1 r^3$	$\frac{2}{3}\tau r^3$	$\frac{4}{3}\pi r^3$
4	$\frac{1}{4\times 2}\tau^2 r^4$	$\frac{1}{8}\tau^2 r^4$	$\frac{1}{2}\pi^2 r^4$			
5				$\frac{2}{5\times 3}\tau^2 r^5$	$\frac{2}{15}\tau^2 r^5$	$\frac{8}{15}\pi^2 r^5$
6	$\frac{1}{6\times 4\times 2}\tau^3 r^6$	$\frac{1}{48}\tau^3 r^6$	$\frac{1}{6}\pi^3 r^6$			
7				$\frac{2}{7\times 5\times 3}\tau^3 r^7$	$\frac{2}{105}\tau^3 r^7$	$\frac{16}{105}\pi^3 r^7$
8	$\frac{1}{8\times 6\times 4\times 2}\tau^4 r^8$	$\frac{1}{384}\tau^4 r^8$	$\frac{1}{24}\pi^4 r^8$			
9				$\frac{2}{9\times 7\times 5\times 3}\tau^4 r^9$	$\frac{2}{945}\tau^4 r^9$	$\frac{32}{945}\pi^4 r^9$

This table illustrates a new type of integration called skip integration - odd numbers are skipped on the even path, and even numbers are skipped on the odd path. This affects both the exponents and the factorials. But the same characteristic balancing of the exponential and factorial values is used (look under the Even or Odd path and you will notice the exponential value of r is always equal to the factorial value).

So why do we have an equivalent Tau formula right next to the Pi formula?

To illustrate that this pattern of derivatives is only apparent when tau is used in the formulas - the pi formulas obscure the pattern. Mother nature writes her formulas using tau!

Further, this table of basic formulas - with its relation to derivatives - has gone unnoticed until now because we have been writing our formulas with pi. The writing is on the wall, and it is in mother natures handwriting - it's time to replace pi with tau.