

# ***THEORY OF QUANTUM RELATIVITY***



BY

SUDHANVA.S. JOSHI

THANE (W), MUMBAI-400602

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## **Declaration**

This Research paper is a presentation of my original work. Wherever contributions of others are involved, every effort is made to indicate it, with due reference to literature and acknowledgement of collaborative research.

## **ABSTRACT**

In this paper, I have studied the properties of atomic and molecular world along with General and special theories of relativity. This is an attempt to merge Gravity into the standard model in order to complete the Grand Unification Theory. The merger of gravity into the other forces i.e. electromagnetic, strong and weak nuclear forces should be well defined and in the boundaries of Gauge Group theory. The Lorentz transformations used in the theory too are invariant under  $SU(2)$  type of space. The relative force exerted on two separate quantum systems is also discussed along with Dark matter and Dark energy at a quantum level. I have also tried to solve the Banach-Tarski theorem by applications of Heisenberg's Uncertainty principle in the later part of the paper. Detailed particle Chirality in standard model is redefined to fit in the criterion of operators used in the same process. Possible existence of a new quasi particle is also included in the paper along with its properties.

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## CHAPTER 1            **INTRODUCTION.....**

### **1.1 PHYSICS AT COSMOLOGICAL SCALE**

The world as we know is vastly big. The approximated radius of observable universe is 46.5 billion light years. Only 4% of matter is visible to us, while 27% is dark matter and 69% is dominated by dark energy. According to the current cosmological inflationary model, the universe was formed 13.7 billion years ago. There was singularity (a single space time conjecture) and suddenly due to the difference in kinetic and potential energies, a huge explosion occurred. At around Planck time i.e.  $10^{-35}$  seconds after big bang, universe came into existence and started expanding rapidly due to extreme temperatures. At  $10^{-32}$  seconds quantum fluctuations occurred triggering dense primordial gravity waves. At  $10^{-6}$  seconds, photons formed then quark gluon plasmas and other subatomic particles. At 0.01 seconds after the big bang, nuclear fusion begins. At three minutes after big bang, nuclear fusion ends and free electrons begin to scatter light at all wavelengths. After that begins what is called as Dark ages. Around 3, 80,000 years after big bang, molecular hydrogen forms and earliest

time is visible through scattering and Cosmic Microwave Background Radiation. After that, there was rapid inflation of observable universe and first galaxies started to shine shortly thereafter. And now after 13.7 billion years, here we are in what is called modern universe. Various processes that led to the formation of matter as we see it today still exist at a very quasi level.

## 1.2 PHYSICS AT QUANTUM LEVEL

The formation of universe is a cataclysmic process which involves a theoretical framework of point like one dimensional string. String theory discusses it further. The string theory is aided by Conformal Field Theory (CFT) which is QFT invariant under conformal transformations. There are two variants of CFT:- Euclidean which is extension of statistical mechanics and Lorentzian which is extension of quantum field theory. The above transformations are related by Wick Rotation. For example, considering CFT variant group on Riemannian sphere is Mobius Transformation. It is a projective linear group  $PSL(2, C)$ . The generators are indexed as.

$$l_n = \oint_{z=0}^n \frac{dz}{2\pi i} z^{n+1} T_{zz}$$

$T_{zz}$  is the holomorphic part of the non-trace piece of the energy momentum tensor of the theory. For a free scalar field,

$$T_{zz} = \frac{1}{2} (\partial_z \varphi)^2$$

The Quantum field theory is a theoretical framework for constructing quantum mechanical models of subatomic and quasi particles. A QFT treats particles as excited states of an physical field, so these are called field quanta. Here, particle interactions are discussed with help of Feynman diagrams. Considering earlier discussed CFT, The Fourier transformations on Euclidean space will be:-

$$\check{f}(\xi) = \int_{\mathbb{R}}^1 f(x) e^{-2\pi i X \cdot \xi} dx$$

Where  $x$  and  $\xi$  are  $n$ -dimensional operators.

The Fourier transformation on lorentzian space will be nonexistent as general function will be discontinuous.

To sum it up, at quantum level physics behaves entirely differently by virtue of the laws discussed above.

## CHAPTER 2      **GRAND UNIFICATION THEORIES.....**

### **2.1 TRADITIONAL GUTs**

The attempt for unification started back in later part of 19<sup>th</sup> century by James Clark Maxwell. Earlier, Hans Christian Oersted stated that electric currents produce magnetic fields. Until then electric and magnetic fields were considered as totally different phenomena. Then in 1864 J.C Maxwell Published Dynamics of Electromagnetic field theory. In the same paper, Maxwell stated the wave equation of electromagnetic fields and also the velocity of light. Maxwell derived the equation as follows:-

$$\nabla \cdot E = 0$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \cdot H = 0$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}$$

The vector curl of equation is as follows:-

$$\nabla \times \nabla \times E = -\mu \frac{\partial \nabla \times H}{\partial t} = -\mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla \times \nabla \times H = \varepsilon \frac{\partial \nabla \times E}{\partial t} = -\mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial t^2} - C^2 \cdot \nabla^2 E = 0$$

$$\frac{\partial^2 H}{\partial t^2} - C^2 \cdot \nabla^2 H = 0$$

When Maxwell derived the electromagnetic wave equation, he used this equation as opposed to using Faraday's law of electromagnetic

induction as presented in modern textbooks. Maxwell however dropped the  $\mu(\mathbf{v} \times \mathbf{H})$  term from equation  $\mathbf{f} = \mu(\mathbf{v} \times \mathbf{H}) - \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi$  when he derived the electromagnetic wave equation and he considered the situation only from the rest frame. This was the first attempt to unify the Forces.

## 2.2 “GUT” IN MODERN PHYSICS

The very first modern attempt was with unified Electroweak Theory. Mathematically this theory comes under  $SU(2) \times U(1)$  Gauge group. The gauge bosons are three W Bosons of weak spins from  $SU(2)$  and B Boson from  $U(1)$  which are all assumed to be mass less. As W, Z bosons and photons are produced by spontaneous symmetry breaking of Electroweak symmetry  $SU(2) \times U(1)_Y$  to  $U(1)_{em}$ , caused by Higgs mechanism.  $U(1)_Y$  and  $U(1)_{em}$  are different copies of  $U(1)$ ; the generator of  $U(1)_{em}$  is given by  $Q = Y/2 + I_3$ , where  $Y$  is the generator of  $U(1)_Y$  (called the weak hypercharge), and  $I_3$  is one of the  $SU(2)$  generators (a component of weak isospin).

The spontaneous Symmetry breaking which results in combining of Z boson and photon is given as:-

$$\begin{pmatrix} Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

Where  $\theta_w$  is mixing angle. Also, the final Lagrangian is represented as sub-derivative of Yukawa interactions as:-

$$\mathcal{L}_Y = - \sum_f \frac{g m_f}{2m_W} \bar{f} f H$$

The Electroweak unified theory became a template of modern GUT.

The more modern approach includes Gravity in unification of all forces. So far there is no success so as to unify gravity even though attempts have been made. Most Promising of them are Loop Quantum Gravity and String Theory. In LQG, the gravity is quantized in 4 dimensions (as Lorentz group transforms into 2 Spinors) by canonical formalism.

Fourier transformation is used to convert to loop variables.

Furthermore, the trivial Wheeler-Dewitt equation can be solved using states spanned by Knots and the Yang-Baxter equation. The canonical gravity is represented by 3-metric tensor. The space-time is foliated as Knots. Statistically, the equation is derived as:-

$$g_{\mu\nu} dx^\mu dx^\nu = (-N^2 + \beta_k \beta^k) dt^2 + 2\beta_k dx^k dt + \gamma_{ij} dx^i dx^j.$$

In that equation the Roman indices run over the values 1, 2, 3 and the Greek indices run over the values 1, 2, 3, 4. The three-metric  $\gamma_{ij}$  is the field, and we denote its conjugate momenta as  $\pi^{kl}$ . The Hamiltonian is a constraint as;

$$\mathcal{H} = \frac{1}{2\sqrt{\gamma}} G_{ijkl} \pi^{ij} \pi^{kl} - \sqrt{\gamma} {}^{(3)}R = 0$$

Where  $\gamma = \det(\gamma_{ij})$  and  $G_{ijkl} = (\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl})$  is the Wheeler-DeWitt metric.

The path-integral derivation includes Gravitational action in the Euclidean Quantum Gravity paradigm.

$$Z = \int_C e^{-I[g_{\mu\nu}, \phi]} \mathcal{D}\mathbf{g} \mathcal{D}\phi$$

Here, the Hamiltonian constraint is obtained as;

$$\frac{\delta I_{EH}}{\delta N} = 0$$

Where  $I_{EH}$  is the Einstein-Hilbert action, and  $N$  is the lapse function (i.e., the Lagrange multiplier for the Hamiltonian constraint). This is purely classical so far. We can recover the Wheeler–DeWitt equation from

$$\frac{\delta Z}{\delta N} = 0 = \int \frac{\delta I[g_{\mu\nu}, \phi]}{\delta N} \Big|_{\Sigma} \exp(-I[g_{\mu\nu}, \phi]) \mathcal{D}\mathbf{g} \mathcal{D}\phi$$

To sum it up, Wheeler-DeWitt equation simply states as:-

$$\hat{H}(x)|\psi\rangle = 0$$

Where  $\hat{H}(x)$  is the Hamiltonian constraint in quantized general relativity and  $|\psi\rangle$  stands for the wave function of the universe.

Taking the String Theory ,most important is super Yang-Mills scattering theory in 4 dimensions(as the null vectors can be split into 2 spinors).The approach taken here is that mass less theories have Conformal-Invariance in Space time but it can be transformed into dual momentum space using a Fourier transformation where amplitudes become loop variables. The closure of dual momentum space symmetries is an infinite dimensional Yangian symmetry which too uses the yang-Baxter equation.

$$[t_{ij}^{(p+1)}, t_{kl}^{(q)}] - [t_{ij}^{(p)}, t_{kl}^{(q+1)}] = -(t_{kj}^{(p)} t_{il}^{(q)} - t_{kj}^{(q)} t_{il}^{(p)}).$$

Defining  $t_{ij}^{(-1)} = \delta_{ij}$ , setting

$$T(z) = \sum_{p \geq -1} t_{ij}^{(p)} z^{-p+1}$$

Where,  $T(z)$  generates the centre of infinite Yangian dimension. Incidentally here, the Yang-Baxter equation has no Knots (which were present in Wheeler-Dewitt interpretation of Yang-Baxter) because there is no conformal deformation, there are permutations.

## CHAPTER 3                    **STANDARD MODEL.....**

### **3.1 ELEMENTARY PARTICLES**

In Early 20<sup>th</sup> century, John Dalton first hypothesized about Atoms being a fundamental particle in nature. According to him, atoms were solid balls which were indestructible. Then came Thompson with his electron model. After that his own pupil James Chadwick discovered nucleus also neutron. After the discovery of Nucleons, particle physics took a big jump. Till mid 20<sup>th</sup> it was thought that electrons are most fundamental particles but then Murray Gillman came up with QUARK model. Till the date, six flavors of quarks are discovered-up ,down ,charm ,strange ,top and bottom. Up, charm and top quark have integer charge of (+2/3) whereas down, strange and bottom have integer charge of (-1/3).

All quarks have half integer spin. The Top quark is most recently discovered and is the largest in terms of relativistic mass at  $175 \text{ Gev}/c^2$ .

The fundamental constituents of matter are Fermions. Leptons and quarks come under the category of Fermions. There are six flavors of Leptons-electron, muon, tau, electron neutrino, muon neutrino and tau neutrino. All Leptons have half integer spin. Electron, muon and tau have -1 charge spin. Each Lepton has its anti-particle and so do the quarks.

Bosons are the Force carriers. The  $w(-,+)$  bosons Z bosons and Photons are Gauge Bosons whereas the famous Higgs boson is scalar boson. Graviton (a gravity force mediator) is also a boson but it is not yet discovered(perhaps due to very less interaction with matter).

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.87 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	

### 3.2 Higgs Boson

In 1964, Dr peter Higgs postulated existence of a new type of particle which gives matter mass. Existence of Higgs Boson was confirmed on 4<sup>th</sup> July 2013 at CERN’s LHC. The presence of Higgs field, now believed to be confirmed, explains why some fundamental particles have mass when, based on the symmetries controlling their interactions, they should be mass less. The existence of the Higgs field would also resolve several other long-standing puzzles, such as the reason for the weak force’s extremely short range. Although it is hypothesized that the Higgs field permeates the entire Universe, evidence for its existence has

been very difficult to obtain. In principle, the Higgs field can be detected through its excitations, manifest as Higgs particles. The Higgs field is tachyonic (this does not refer to faster-than-light speeds, it means that symmetry-breaking through condensation of a particle must occur under certain conditions), and has a "Mexican hat" shaped potential with nonzero strength everywhere (including otherwise empty space), which in its vacuum state breaks the weak isospin symmetry of the electroweak interaction. When this happens, three components of the Higgs field are "absorbed" by the SU (2) and U (1) gauge bosons (the "Higgs mechanism") to become the longitudinal components of the now-massive W and Z bosons of the weak force. The remaining electrically neutral component separately couples to other particles known as fermions (via Yukawa couplings), causing these to acquire mass as well.

Higgs mechanism is essential in order to explain property of "mass" in Gauge Bosons. As Higgs field is a SU (2) doublet, it has complex scalar with 4 real components. In order to induce Higgs mechanism in a isotropic Hilbert manifold, it is essential for scalar fields to interact with Dirac fields of type  $V \approx g\bar{\psi}\phi\psi$  (scalar) or  $g\bar{\psi}i\gamma^5\phi\psi$  (pseudo scalar).

This interaction is described as:-

The action for a meson field  $\phi$  interacting with a Dirac baryon field  $\psi$  is

$$S[\phi, \psi] = \int d^d x [\mathcal{L}_{\text{meson}}(\phi) + \mathcal{L}_{\text{Dirac}}(\psi) + \mathcal{L}_{\text{Yukawa}}(\phi, \psi)]$$

where the integration is performed over  $d$  dimensions (typically 4 for four-dimensional space time). The meson Lagrangian is given by

$$\mathcal{L}_{\text{meson}}(\phi) = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi).$$

Here,  $V(\phi)$  is a self-interaction term. For a free-field massive meson, one would have  $V(\phi) = \frac{1}{2}\mu^2\phi^2$  where  $\mu$  is the mass of meson.

The Yukawa interaction term is

$$\mathcal{L}_{\text{Yukawa}}(\phi, \psi) = -g\bar{\psi}\phi\psi$$

Where  $g$  is the (real) coupling constant for scalar mesons and

$$\mathcal{L}_{\text{Yukawa}}(\phi, \psi) = -g\bar{\psi}i\gamma^5\phi\psi$$

For pseudo scalar mesons. Putting it all together one can write the above more explicitly as

$$S[\phi, \psi] = \int d^d x \left[ \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \bar{\psi}(i\not{\partial} - m)\psi - g\bar{\psi}\phi\psi \right].$$

In order to manifest, Higgs field requires spontaneous symmetry breaking which is given by Majorana field Lagrangian. It is also possible to have a Yukawa interaction between a scalar and a Majorana field. In fact, the Yukawa interaction involving a scalar and a Dirac spinor can be thought of as a Yukawa interaction involving a scalar with two Majorana spinors of the same mass. Broken out in terms of the two chiral Majorana spinors, one has

$$S[\phi, \chi] = \int d^d x \left[ \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \chi^\dagger i\bar{\sigma} \cdot \partial\chi + \frac{i}{2}(m + g\phi)\chi^T\sigma^2\chi - \frac{i}{2}(m + g\phi)^*\chi^\dagger\sigma^2\chi^* \right]$$

where  $g$  is a complex coupling constant and  $m$  is a complex number.

The Yukawa interaction of a Fermion field ( $\psi$ ) and Higgs field ( $\phi$ ) is given as:-

$$\mathcal{L}_{\text{Fermion}}(\phi, A, \psi) = \bar{\psi}\gamma^\mu D_\mu\psi + G_\psi\bar{\psi}\phi\psi,$$

Where again the gauge field  $A$  only enters  $D_\mu$  (i.e., it is only indirectly visible). The quantities  $\gamma^\mu$  are the Dirac matrices, and  $G_\psi$  is the already-mentioned "Yukawa" coupling parameter. Already now the mass-generation follows the same principle as above, namely from the existence of a finite expectation value  $|\langle\phi\rangle|$ , as described above. Again, this is crucial for the existence of the property "mass". As we can see from earlier, Meson field and Higgs field exhibit similar coupling properties.

In Abelian Higgs mechanism, scalar field experiences Gauge Invariance i.e. certain gauge transformation does not change energy levels. The Mexican-Hat model very well explains Abelian higgs mechanism. The only renormalizable model where a complex scalar field  $\Phi$  acquires a nonzero value is the Mexican-hat model, where the field energy has a minimum away from zero. The action for this model is

$$S(\phi) = \int \frac{1}{2}|\partial\phi|^2 - \lambda(|\phi|^2 - \Phi^2)^2,$$

Which results in the Hamiltonian

$$H(\phi) = \frac{1}{2}|\dot{\phi}|^2 + |\nabla\phi|^2 + V(|\phi|).$$

Where  $\frac{1}{2}|\dot{\phi}|^2$  shows kinetic energy of Higgs field. From above equation we can deduce the mass exhibited by Higgs field:-

$$\langle M^H \psi \rangle = \frac{2[H(\phi) - |\nabla\phi|^2 - V(|\phi|)]}{V\psi^2}$$

Where  $\psi$  is Dirac-DeBroglie matter operator and V is the matter velocity.

### 3.3 Particles in a coupled state.

Two Particles are said to be in “coupled” state when quantum interaction affect both instantaneously at quantum distances. They also undergo changes in quantum states almost instantaneously(almost being the operative word).We can explain changed states as follows –

1) For particle 1 :- using the Bra-Ket notation

$$\langle \psi_1 \rangle = \frac{1}{\sqrt{2}} (\langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle)$$

Where  $\psi_1$  is the state vector.

2) For particle 2 :-

$$\langle \psi_2 \rangle = \frac{1}{\sqrt{2}} (\langle \downarrow \uparrow \rangle - \langle \uparrow \downarrow \rangle)$$

As seen, the eigenstates undergo a small change as in ket notation.This will violate the CKM laws as the wave function will collapse assuming the eigenvalue would be 1 kg·m/s.So eliminating the Uncertainty situation,the equations can be combined and modified as :-

$$\langle \psi_{12} \rangle = \frac{1}{\sqrt{2}} (\langle \downarrow +w \uparrow \rangle - \langle \uparrow -w \downarrow \rangle)$$

Where  $w$  is Wick's expansion of  $n$ th order with respect to  $S$ -matrix. Substituting all the terms,

$$\langle \psi_{12} \rangle = \frac{1}{\sqrt{2}} \left( \langle \downarrow \uparrow \rangle \uparrow \langle \uparrow \downarrow \rangle S = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int \prod_{j=1}^n d^4 x_j T \prod_{j=1}^n L_v(x_j) \equiv \sum_{n=0}^{\infty} S^{(n)}, \right)$$

## CHAPTER 4 RELATIVITY AT QUANTUM LEVEL

### 4.1 Introduction to Quantum relativity.

Quantum relativity is like quantum coupling but different as in the sense that it also takes dimensionality into consideration. This way, a particle could certainly propagate through space but also a higher dimension as well. Assuming a particle at position  $p_1$  in space, at time of its decay it is at position  $p_2$ . Today's physics is unable to explain why does the particle propagate the way it does in sub space topologies? In order to tackle these problems and problems with higher magnitudes, we need to draw a link between the motion of particles and their dimensionalities. By these we can even predict Einstein-Rosen bridge, get more understanding as on dark matter and dark energy, also may be predict existence of a new particle!

## 4.2 Stiefel manifolds compactified

Let's take Geometrization conjecture into consideration. As our space is assumed to be spherical, a finite group acts on the 3-sphere is conjugate to a group of isometrics of the 3-sphere. Assuming the current lie group to be hyperbolic, the point stabilizer is  $O(3, \mathbf{R})$ , and the group  $G$  is the 5-dimensional Lie group  $O^+(1, 3, \mathbf{R})$ , with 1 component. There are enormous numbers of examples of these, and their classification is not completely understood. The example with smallest volume is the Weeks manifold. Other examples are given by the Seifert–Weber space, or "sufficiently complicated" Dehn surgeries on links, or most Haken manifolds. The geometrization conjecture implies that a closed 3-manifold is hyperbolic if and only if it is irreducible, atoroidal, and has infinite fundamental group. This geometry can be modeled as a left invariant metric on the Bianchi group of type V. Under Ricci flow manifolds with hyperbolic geometry expand. Hyperbolic tensor is a patchwork of 5 dimensions i.e. Hyperspace. A Hypersphere in 5-space (also called a **4-sphere** due to its surface being 4-dimensional) consists of the set of all points in 5-space at a fixed distance  $r$  from a central point  $P$ . The hypervolume enclosed by this hypersurface is:

$$V = \frac{8\pi^2 r^5}{15}$$

Now then the **Stiefel manifold**  $V_k(\mathbf{R}^n)$  is the set of all orthonormal  $k$ -frames in  $\mathbf{R}^n$ . That is, it is the set of ordered  $k$ -tuples of orthonormal vectors in  $\mathbf{R}^n$ . one can define the complex Stiefel

manifold  $V_k(\mathbf{C}^n)$  of orthonormal  $k$ -frames in  $\mathbf{C}^n$  and the quaternionic Stiefel manifold  $V_k(\mathbf{H}^n)$  of orthonormal  $k$ -frames in  $\mathbf{H}^n$ . More generally, the construction applies to any real, complex, or quaternionic inner product space. As our space is homogeneous unitary group  $U(n)$  acts transitively on  $V_k(\mathbf{C}^n)$  with stabilizer subgroup  $U(n-k)$  and the symplectic group  $Sp(n)$  acts transitively on  $V_k(\mathbf{H}^n)$  with stabilizer subgroup  $Sp(n-k)$ .

In each case  $V_k(\mathbf{F}^n)$  can be viewed as a homogeneous space:

$$\begin{aligned} V_k(\mathbb{R}^n) &\cong O(n)/O(n-k) \\ V_k(\mathbb{C}^n) &\cong U(n)/U(n-k) \\ V_k(\mathbb{H}^n) &\cong Sp(n)/Sp(n-k). \end{aligned}$$

When  $k = n$ , the corresponding action is free so that the Stiefel manifold  $V_n(\mathbf{F}^n)$  is a principal homogeneous space for the corresponding classical group.

When  $k$  is strictly less than  $n$  then the special orthogonal group  $SO(n)$  also acts transitively on  $V_k(\mathbb{R}^n)$  with stabilizer subgroup isomorphic to  $SO(n-k)$  so that

$$V_k(\mathbb{R}^n) \cong SO(n)/SO(n-k) \quad \text{for } k < n.$$

The same holds for the action of the special unitary group on  $V_k(\mathbf{C}^n)$

$$V_k(\mathbb{C}^n) \cong SU(n)/SU(n-k) \quad \text{for } k < n.$$

Thus for  $k = n - 1$ , the Stiefel manifold is a principal homogeneous space for the corresponding *special* classical group. For Hilbert space topologies,  $k=n$  and thus,

$$V_n(\mathbb{R}^n) \cong U(5)$$

Now substituting value of Hypervolume with  $k=n$  spatial type Stiefel manifolds, where spatial dimensions are 5, we get

$$\frac{8\pi^2 r^5}{15} (R^5) \cong U(5)$$

U is unitary group 5. This tells that Stiefel manifolds are topologically compactified using Theory of quantum relativity.

### 4.3 Relativistic Quantum Chirality.....

Chirality in Quantum physics refers gradual difference in properties between fermions and bosons depending on its spin, charge etc. Only left-handed fermions interact with the weak interaction. In most circumstances, two left-handed fermions interact more strongly than right-handed or opposite-handed fermions, implying that the universe has a preference for left-handed chirality, which violates a symmetry of the other forces of nature.

Chirality for a Dirac fermion  $\psi$  is defined through the operator  $\gamma^5$ , which has eigenvalues  $\pm 1$ . Any Dirac field can thus be projected into its left- or right-handed component by acting with the projection operators  $(1-\gamma^5)/2$  or  $(1+\gamma^5)/2$  on  $\psi$ . The coupling of the charged weak interaction to fermions is proportional to the first projection operator, which is responsible for this interaction's parity symmetry violation. Talking about Chiral symmetry, Vector gauge theories with mass less Dirac fermion fields  $\psi$  exhibit chiral symmetry, i.e., rotating the left-

handed and the right-handed components independently makes no difference to the theory. We can write this as the action of rotation on the fields:

$$\psi_L \rightarrow e^{i\theta_L}\psi_L \text{ and } \psi_R \rightarrow \psi_R$$

Or,

$$\psi_L \rightarrow \psi_L \text{ and } \psi_R \rightarrow e^{i\theta_R}\psi_R.$$

With  $N$  flavors, we have unitary rotations instead:  $U(N)_L \times U(N)_R$ .  
Assuming 2 mass less quarks  $u$  and  $d$ ,

$$L = \bar{u}_l i \not{\partial} u_l + \bar{d}_l i \not{\partial} d_l + L_{gluon}$$

Now considering Left and right isomorphic Spinors,

$$L = \bar{u}_l i \not{\partial} u_l + \bar{u}_R i \not{\partial} u_R + \bar{d}_l i \not{\partial} d_l + \bar{d}_R i \not{\partial} d_R + L_{gluon}$$

$$q = \begin{bmatrix} u \\ d \end{bmatrix}$$

$$L = \bar{q}_L i \not{\partial} q_L + \bar{q}_R i \not{\partial} q_R + L_{gluons}$$

$$L = \bar{q}_L i \not{\partial} q_R + \bar{q}_R i \not{\partial} q_L - L_{gluons}$$

Where the Gluon field converges. Redefining Left and right anti symmetric chirality,

$$\psi_R(1 + \gamma^5) = \psi_L(1 - \gamma^5)$$

$$\frac{\psi_R}{\psi_L} = \frac{(1 - \gamma^5)}{(1 + \gamma^5)}$$

Let,  $\frac{(1-\gamma^5)}{(1+\gamma^5)}$  be partial variables for left and right Fermion operator  
 $\cong \check{\mathbb{R}}_{\{\emptyset i\}}$  here  $\check{\mathbb{R}}_{\{\emptyset i\}}$  is Relativistic proportional operator.

$$\check{\mathbb{R}}_{\{\emptyset i\}} = \left[ \frac{\overline{q_L}}{q_R} \right]$$

$$L = \check{\mathbb{R}}_{\{\emptyset u\}} i \emptyset q_R + \check{\mathbb{R}}_{\{\emptyset d\}} i \emptyset q_L - L_{gluons}$$

This briefly implies that Chirality depends on HOW it is observed or from where it is observed. For instance to relate chirality and position with respect to time of two particles u and d:-

$$\begin{aligned} \sigma_x \sigma_p &\geq \frac{\hbar}{2} \\ \sigma_x &= [x, t] \\ \sigma_p &= [m, v] \end{aligned}$$

$$[\{\check{\mathbb{R}}_{\{\emptyset u\}} i \emptyset q_R + \check{\mathbb{R}}_{\{\emptyset d\}} i \emptyset q_L\}_{[x,t]} \times \{\check{\mathbb{R}}_{\{\emptyset u\}} i \emptyset q_R + \check{\mathbb{R}}_{\{\emptyset d\}} i \emptyset q_L\}_{[m,v]}] \geq \frac{\hbar}{2}$$

$$\{q_R + q_L\}_{[x,t]} \times \{q_R + q_L\}_{[m,v]} \geq \frac{\hbar}{2}$$

$$\begin{bmatrix} q_R \\ q_L \end{bmatrix} = \begin{bmatrix} u \\ d \end{bmatrix}$$

$$\{u + d\}_{[x,t]} \times \{u - d\}_{[m,v]} \geq \frac{\hbar}{2}$$

From above it can be noted that chirality of particles u and d(taken in the example) differ from chirality observed when at steady state and in motion. It has uncertainty of + 2.56%.The parity symmetry or in this case anti-symmetry depends on the relativistic proportional operator.

## CHAPTER 5      **SOLUTIONS TO SOME PROBLEMS.....**

### **5.1 Quantum relativity and implications with QFT**

A QFT treats particles as excited states of an underlying physical field, so these are called field quanta. Ordinary quantum mechanical systems have a fixed number of particles, with each particle having a finite number of degrees of freedom. In contrast, the excited states of a quantum field can represent any number of particles. This makes quantum field theories especially useful for describing systems where the particle count/number may change over time, a crucial feature of relativistic dynamics. The gravitational field and the electromagnetic field are the only two fundamental fields in nature that have infinite

range and a corresponding classical low-energy limit, which greatly diminishes and hides their "particle-like" excitations. Albert Einstein in 1905, attributed "particle-like" and discrete exchanges of momenta and energy, characteristic of "field quanta", to the electromagnetic field. Originally, his principal motivation was to explain the thermodynamics of radiation. Although the photoelectric effect and Compton scattering strongly suggest the existence of the photon, it might alternately be explained by a mere quantization of emission; more definitive evidence of the quantum nature of radiation is now taken up into modern quantum optics as in the antibunching effect. As the Quantum relativity is an attempt to unify gravitation in the standard model, we will be using function of stress energy tensor with Hilbert poyla expansion statistics to determine the exact outcome of properties of Gravitons.

$$S = \frac{1}{2\kappa} \int R \sqrt{-g} d^4x ,$$

$$g = \det(g_{\mu\nu})$$

$$R = g^{\mu\nu} R_{\mu\nu} .$$

$$\kappa = \frac{8\pi G}{c^4}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} .$$

Now then, taking stress energy tensor and introducing matrix formulation,

$$T_{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{matter}\sqrt{-g})}{\delta g^{\mu\nu}}$$

$$\begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix} = \frac{-2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{matter}\sqrt{-g})}{\delta g^{\mu\nu}}$$

$$\begin{bmatrix} -c^{-2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \frac{-2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{matter}\sqrt{-g})}{\delta g^{\mu\nu}}$$

$$(g^{\alpha\beta})_{\alpha,\beta=0,1,2,3} = \begin{pmatrix} -c^{-2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

Since

$$(g^{\alpha\beta}) = \frac{-2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{matter}\sqrt{-g})}{\delta g^{\mu\nu}}$$

$$g = g^{\mu\nu} dx^\alpha \otimes dx^\beta$$

The factors  $dx^\alpha, dx^\beta$  are one-form gradients of the scalar coordinate fields  $g^{\mu\nu}$ . The metric is thus a linear combination of tensor products of one-form gradients of coordinates. The coefficients  $g_{\mu\nu}$  are a set of 16 real-valued functions (since the tensor  $g$  is actually a *tensor field*, which is defined at all points of a space time manifold). In order for the metric to be symmetric we must have

$$g^{\alpha\beta} = g^{\mu\nu}$$

$$(g^{\mu\nu}) = \frac{-2}{\sqrt{-g}} \frac{\delta(\mathcal{L}_{matter}\sqrt{-g})}{\delta g^{\mu\nu}}$$

As the lagrangian is nth dimensional gradient of the field, the delta function has no real value and can be therefore eliminated,

$$\mathcal{L}_{matter} = -\frac{1}{2}$$

By using Quantum relativity, we can derive the following conclusion:-

$$S[\varphi_i] = \int \mathcal{L}_{matter} \left( \varphi_i(s), \frac{\gamma\varphi_i(s)}{\gamma s^\alpha}, s^\alpha \right) d^n s$$

$S$ , is a functional of the dependent variables  $\varphi_i(s)$  with their derivatives and  $s$  itself. where  $s = \{s^\alpha\}$  denotes the set of  $n$  independent variables of the system, indexed by  $\alpha = 1, 2, 3, \dots, n$ . Notice  $L$  is used in

the case of one independent variable ( $t$ ) and  $\mathcal{L}$  is used in the case of multiple independent variables (usually four:  $x, y, z, t$ ).

$$\mathcal{S}[\varphi] = \int \mathcal{L}(\varphi, \nabla\varphi, \partial\varphi/\partial t, \mathbf{x}, t) d^3\mathbf{x} dt.$$

$$\int \nabla\varphi^x \varphi^y \varphi^z \varphi^t = \int -\frac{1}{2} \left( \varphi_i(s), \frac{\gamma\varphi_i(s)}{\gamma s^\alpha}, s^\alpha \right) d^n s$$

$$\nabla\varphi^x \varphi^y \varphi^z \varphi^t = -\frac{1}{2} \left( \varphi_i(s), \frac{\gamma\varphi_i(s)}{\gamma s^\alpha}, s^\alpha \right)$$

Let  $\nabla\varphi^x \varphi^y \varphi^z \varphi^t$  be spatial matter operator ( $m^{\nabla x,y,z,t}$ ) in 4 dimensional space time.

$$(m^{\nabla x,y,z,t}) = -\frac{1}{2} \left( \varphi_i(s), \frac{\gamma\varphi_i(s)}{\gamma s^\alpha}, s^\alpha \right)$$

This conclusively proves Gravitons (assuming gravitons are 1 dimensional entity but can traverse through all 4 dimensions) have some mass:-

$$(m^{1,2,3,4}) = -\frac{1}{2} \left( \varphi_1(s), \frac{\gamma\varphi_1(s)}{\gamma s^1}, s^1 \right)$$

## 5.2 Predicting a new particle

Axiomatically speaking, Graviton and Gravitino being super partners of each other interact with each other. Since as proved earlier that Gravitons posses some mass, we can imply Relativistic Dirac-Schrodinger equation to interact with Rarita-Schwinger equation.

$$(\epsilon^{\mu\kappa\rho\nu}\gamma_5\gamma_\kappa\partial_\rho - im\sigma^{\mu\nu})\psi_\nu = 0 \Rightarrow J^\mu = \frac{i\hbar}{2m}(\psi^*\partial^\mu\psi - \psi\partial^\mu\psi^*)$$

where  $\epsilon^{\mu\kappa\rho\nu}$  is the Levi-Civita symbol,  $\gamma_5$  and  $\gamma_\nu$  are Dirac matrices,  $m$  is the mass,  $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ , and  $\psi_\nu$  is a vector-valued spinor with additional components compared to the four component spinor in the Dirac equation. This produces a single solution. Therefore the interaction produces an auxiliary field .Thus it follows Hubbard–Stratonovich transformation. This aux field particle could be a new type of Trion as it posses 3/2 lie spin. Let it be called Joton. Lagrangian can be derived as:-

$$F^a = m \left( \frac{d^2\xi^a}{dt^2} + \Gamma^a_{bc} \frac{d\xi^b}{dt} \frac{d\xi^c}{dt} \right) = \frac{d}{dt} \frac{\partial T}{\partial \dot{\xi}^a} - \frac{\partial T}{\partial \xi^a}, \quad \dot{\xi}^a \equiv \frac{d\xi^a}{dt},$$

where  $F^a$  is the  $a$ th contra variant components of the resultant force acting on the particle,  $\Gamma^a_{bc}$  are the Christoffel symbols of the second kind,

$$T = \frac{1}{2}mg_{bc} \frac{d\xi^b}{dt} \frac{d\xi^c}{dt},$$

is the kinetic energy of the particle, and  $g_{bc}$  the covariant components of the *metric tensor* of the curvilinear coordinate system. All the indices  $a, b, c$ , each take the values 1, 2, 3. Curvilinear coordinates are not the same as generalized coordinates.

### 5.3 Quantum Relativity and dark energy

Various cosmological models state existence of dark energy as apparent reason for expansion of universe. The Lambda-CDM model is prominent in relating special relativity and inflationary cosmology. It is a parameterization of the Big Bang cosmological model in which the universe contains a cosmological constant, denoted by Lambda (Greek  $\Lambda$ ), associated with dark energy, and cold dark matter (abbreviated **CDM**). It is frequently referred to as the **standard model** of Big Bang cosmology. The other favourable model is the CHGS model. The **Callan–Giddings–Harvey–Strominger model** or **CGHS** in short is a toy model of general relativity in 1 spatial and 1 time dimension. General relativity is a highly nonlinear model, and as such, its 3+1D version is usually too complicated to analyze in detail. In 3+1D and higher, propagating gravitational waves exist, but not in 2+1D or 1+1D. In 2+1D, general relativity becomes a topological field theory with no local degrees of freedom, and all 1+1D models are locally flat. These models even though explain other cataclysmic phenomena, they are more than useless to tamper with dark energy. Point to be noted is that lamda-cdm model tries to explain Dark energy by virtue of Chameleon particle, a hypothetical scalar particle that couples to matter more weakly than gravity. By applying Quantum relativity, dark energy can be explained as follows:-

Assuming the following things:-

- 1) Space is 5D i.e. 4 spatial dimensions + 1 D time.

- 2) All particles are invariant under gravity.
- 3) Universe is expanding BUT has boundaries.
- 4) Dark energy exists.

There is supposedly a particle, randomly placed in spatial dimensions. It obeys Quantum chaotic distribution and is in highly excited state.

Let the initial position of particle be p1 and final be p2. By using QR(quantum relativity),

$$\approx [\overrightarrow{\phi_{DE}}] \cong P_1[x, y, z, t, -t_0] \rightsquigarrow P_2[x, y, z, t, -t_0]$$

This is condensing phasor equation for initial position of particle and its divergence with respect to final position. Here,  $\approx [\overrightarrow{\phi_{DE}}]$  is canonical formalism of Dark energy spread uniformly through five dimensions,  $-t_0$  is time coordinate at beginning of expansion of universe. Now after effect of dark energy on the particle,

$$[\overrightarrow{\phi_{DE}}] \cong -P_0[x, y, z, t, -t_0] + P_1[x, y, z, t, -t_0] \rightsquigarrow P_2[x, y, z, t, -t_0]$$

Here,  $-P_0$  is the position of particle at instant before dark energy acts on it,  $+P_1$  is the position of particle at instant after dark energy acts on it. For that particular instant of time, the particle exists at initial as well as final position simultaneously and in initial state and excited state simultaneously before finally showing presence at p2 in initial quantum state. This can also mean that at that particular instant of time, the particle temporarily decays into 2 particles before finally becoming a single again. Derivating for that particular time instant:-

$$\Pi T = \int_{-n}^{+n} \delta [\overrightarrow{\phi_{DE}}] - \approx [\overrightarrow{\phi_{DE}}] \xrightarrow{\text{yields}} P_2[x, y, z, t, -t_0] dn$$

## 5.4 Banach –Tarski solution

The **Banach–Tarski paradox** is a theorem in set-theoretic geometry, which states the following: Given a solid ball in 3-dimensional space, there exists a decomposition of the ball into a finite number of disjoint subsets, which can then be put back together in a different way to yield two identical copies of the original ball. Indeed, the reassembly process involves only moving the pieces around and rotating them, without changing their shape. However, the pieces themselves are not "solids" in the usual sense, but infinite scatterings of points. It is yet unproven hypothesis. Using condensing phasor equation:-

$$\approx [\overrightarrow{\phi_F}] \cong \sum_{\infty}^1 (x, m) \rightsquigarrow \sum_1^{\infty} (x_n, m_n)$$

Here,  $[\overrightarrow{\phi_F}]$  is the breaking force for an idealized sphere. The above relation specifies the quantum superposition of n number of particles condensed to one instant before breaking and split into infinite number of particles and grouping together to form infinite number of Idealised spheres. This is non-algebraic proof of Banach-Tarski theorem by Quantum relativity.

## REFERENCES AND BIBLIOGRAPHY

### 1) Web and online references:-

- [1] A. J. Buras, M. Gorbahn, S. Jäger, and M. Jamin, Improved anatomy of  $\epsilon_0/\epsilon$  in the Standard Model, JHEP 11 (2015) 202, [arXiv:1507.06345].
- [2] A. J. Buras, Kaon Theory News, in Proceedings, 2015 European Physical Society Conference on High Energy Physics (EPS-HEP 2015), 2015. arXiv:1510.00128.
- [3] S. Davidson, G. Isidori, and S. Uhlig, Solving the flavour problem with hierarchical fermion wave functions, Phys. Lett. B663 (2008) 73–79, [arXiv:0711.3376].
- [4] T. P. Sotiriou, J. Phys. Conf. Ser. 283 (2011) 012034 [arXiv:1010.3218 [hep-th]].
- [5] A. H. Chamseddine and V. Mukhanov, JHEP 1311 (2013) 135 [arXiv:1308.5410 [astro-ph.CO]].
- [6] H. Saadi, arXiv:1411.4531 [gr-qc].
- [7] T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. 126 (2011) 511 [arXiv:1105.5723 [hep-th]].
- [8] A. V. Astashenok and S. D. Odintsov, arXiv:1512.07279 [gr-qc].
- [9] [https://en.wikipedia.org/wiki/Quantum field theory](https://en.wikipedia.org/wiki/Quantum_field_theory)
- [10] [https://en.wikipedia.org/wiki/Haag%27s theorem](https://en.wikipedia.org/wiki/Haag%27s_theorem)

### 2) Literature and textual references:-

- [1] *Weinberg, S. (2005). The Quantum Theory of Fields. Cambridge University Press. ISBN 978-0521670531.*

- [2] Zee, Anthony (2010). *Quantum Field Theory in a Nutshell* (2nd ed.). Princeton University Press. ISBN 978-0691140346.
- [3] M.Reiher , A. Wolf (2009). *Relativistic Quantum Chemistry*. John Wiley & Sons . ISBN 3-52762-7499.
- [4] E. Abers (2004). *Quantum Mechanics*. Addison Wesley. p. 425. ISBN 978-0-13-146100-0.
- [5] C.B. Parker (1994). *McGraw Hill Encyclopaedia of Physics* (2nd ed.). McGraw Hill. p. 1194. ISBN 0-07-051400-3.
- [6] L.H. Ryder (1996). *Quantum Field Theory* (2nd ed.). Cambridge University Press. p. 62. ISBN 0- 52147-8146.
- [7]S.M. Troshin, N.E. Tyurin (1994). *Spin phenomena in particle interactions*. World Scientific. [ISBN](#) 9-81021-6920.
- [8]C.W. Misner, K.S. Thorne, J.A. Wheeler. *Gravitation*. p. 1146. [ISBN](#) 0-7167-0344-0.
- [9]I. Ciufolini, R.R.A. Matzner (2010). *General relativity and John Archibald Wheeler*. Springer. p. 329. ISBN 9-04813-7357.
- [10] Jackson, J. D. (1999). *Classical Electrodynamics* (3rd ed.). Wiley. p. 548. ISBN 0-471-30932-X.
- [11] P. Labelle (2010). *Supersymmetry*. Demystified. McGraw-Hill. p. 14. ISBN 978-0-07-163641-4.

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