P vs NP Problem Solutions Generalized

Abstract

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

This paper covers the principles and procedures for producing the solution of a problem given the procedure for checking the solution of the problem and vice versa. If a problem can be checked in polynomial time, it can be solved in polynomial time, provided a complete checking procedure is available. From a point A, if one uses one's feet to measure a certain distance by counting steps forwards to a point B, and one wants to check the correctness of the measurement, one would count backwards from the point B using one's feet to see if one returns to exactly the point A. If one returns to A, the forward counting is correct, otherwise it is incorrect. If one counted backwards first from the point B to the point A, one could also count forwards from A to B. Before computers were used in filing taxes in the United States, when one prepared a tax return and wanted to check for arithmetic errors, one would reverse the arithmetic steps from the last arithmetic statement backwards all the way to the first entry on the tax form; and if one obtains a zero after reversing the steps, one was sure that there were no arithmetic errors on the tax form (That is, one began with zero entry going forward and one returned with a zero entry). So also, if one is able to check quickly and complletely, the correctness of the solution to a problem, one should also be able to produce the solution of the problem by reversing the steps of the checking process while using opposite operations in each step. If a complete checking process is available, the solution process can be obtained by reversing the steps of the checking while using opposite operations in each step. In checking the correctness of the solution to a problem, one should produce the complete checking process which includes the end of the problem and the beginning of the problem. Checking only the final answer or statement is incomplete checking. Since the solution process and the checking process are inverses of each other, knowing one of them, one can obtain the other by reversing the steps while using opposite operations. To facilitate complete checking, the question should always be posed such that one is compelled to show a checking procedure from which the solution procedure can be deduced. Therefore P is always equal to NP.

P vs NP Problem Solutions Generalized

One assumes that it is possible to design a computer hardware and /or software which can perform a complete checking of a mathematical solution. Then, if it is easy to check, it would be relatively easy to solve, since the complete checking process provides sufficient information to reverse the steps to obtain the solution. Furthermore, if it is difficult to check, it would not be relatively difficult to solve. The question should always be posed such that one is compelled to show a complete checking procedure from which the solution procedure can be deduced.
If the checking is partial, one may not be able to produce the complete solution process. Two cases of checking and solving are covered.

Case 1: Detailed steps for checking the solution are available. In this case, the solution to the original problem is obtained by reversing the steps of the checking process, while using opposite operations in each step. Also, in this case, $P = NP$

Case 2: There are no detailed steps for checking the solution of the original problem. In this case, there may not be obvious steps to reverse.

Example 1 for Case 1

Proof If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

Plan: 1. From conclusion to hypothesis; followed by
2. Reversing the steps while using opposite operations to obtain the conclusion

A: From **conclusion** to **hypothesis**	**B : Proof** (From **hypothesis** to **conclusion**) Reverse the steps in **A** (on left) to obtain the proof
1. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$	1. $\frac{a}{b} = \frac{c}{d}$
2. $(a-b)(c+d) = (a+b)(c-d)$	2. $ad = bc$
3. $ac + ad - bc - bd = ac - ad + bc - bd$	3. $2ad = 2bc$
4. $ad - bc = -ad + bc$	4. $ad - bc = -ad + bc$
5. $2ad = 2bc$	5. $ac + ad - bc - bd = ac - ad + bc - bd$
6. $ad = bc$	6. $(a-b)(c+d) = (a+b)(c-d)$
7. $\frac{a}{b} = \frac{c}{d}$	7. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ Q.E.D.

Observe above that Step 7 of **A** becomes Step 1 of **B;** Step 1 of **A** becomes Step 7 of **B**

B Expanded

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

$2ad = 2bc$

$ad + ad = bc + bc$

$ac + ad + ad = ac + bc + bc$ (add ac to both sides)

$ac + ad - bc = ac - ad + bc$ (subtract ad from both sides and subtract bc from both sides)

$ac + ad - bc - bd = ac - ad + bc - bd$ (subtract bd from both sides)

$a(c+d) - b(c+d) = a(c-d)) + b((c-d)$ (factoring by grouping)

$(a-b)(c+d) = (a+b)((c-d)$

$\frac{a-b}{a+b} = \frac{c-d}{c+d}$

Example 2: Solve for $x: 2x-6=10$

A Solving Step 1: Add 6 $\quad 2x-6=10$ $\quad\quad\quad\quad\quad\quad \underline{+6 \quad +6}$ Step 2: Divide by 2 $\quad 2x=16$ $\quad\quad\quad\quad\quad\quad x=8$	**B Checking solution** Replace x by 8 in $2x-6=10$ Step 1: Multiply 2. $2(8)-6\overset{?}{=}10$ Step 2: Subtract 6 $16-6\overset{?}{=}10$ $10\overset{?}{=}10$ Yes
Given B find A Replace x by 8 in $2x-6=10$ Step 1: Multiply 2. $2(8)-6\overset{?}{=}10$ Step 2: Subtract 6 $16-6\overset{?}{=}10$ $10\overset{?}{=}10$ Yes From **B** to **A** } →	**From checking procedure to solution procedure** From **B** to **A** Step 1: Add 6: $2x-6=10$ Step 1 $\quad\quad\quad\quad \underline{+6 \quad +6}$ ((Step 2 of B becomes Step 1, and the subtraction is changed to addition) Step 2: Divide by 2 $\quad 2x=16$ $\quad\quad\quad\quad x=8$ (Step 1 of B becomes Step 2, and the multiplication is changed to division) Observe that from the checking procedure one can deduce and obtain the solution method.

In reversing the steps, the last step becomes the first step, and the first step becomes the last step.
Analogy: If one reverses the steps for walking to work from home, one obtains the steps for walking home from work.

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If two processes are inverses of each other, then knowing the steps and operations in one of them, one can deduce the steps and operations in the other process.

Example 3: Solve for $x: \frac{x^2}{2} - 8 = 10$

A Solving

$$\frac{x^2}{2} - 8 = 10$$

Step 1: Add 8 $\frac{x^2}{2} = 18$

Step 2: Multiply by 2 :

$$x^2 = 36$$

Step 3: Find the square root

$$x = \sqrt{36}$$

$$x = 6$$

B Checking the solution

Replace x by 6.in $\frac{x^2}{2} - 8 = 10$

Step 1: Find the square

$$\frac{(6)^2}{2} - 8 = 10$$

Step 2: Divide by 2

$$\frac{36}{2} - 8 \stackrel{?}{=} 10$$

Step 3: Subtract 8:

$$18 - 8 \stackrel{?}{=} 10$$

$10 \stackrel{?}{=} 10$ Yes.

B Checking solution

Replace x by 6.in $\frac{x^2}{2} - 8 = 10$

Step 1: Find the square

$$\frac{(6)^2}{2} - 8 = 10$$

Step 2: Divide by 2

$$\frac{36}{2} - 8 \stackrel{?}{=} 10$$

Step 3: Subtract 8:

$$18 - 8 \stackrel{?}{=} 10$$

$10 \stackrel{?}{=} 10$ Yes.

From **B** to **A** } →

From B to A **(From checking to solution)**

$$\frac{x^2}{2} - 8 = 10$$

Step 1: Add 8 $\frac{x^2}{2} = 18$

(Step 3 of B becomes Step 1, and the subtraction changed to addition)

Step 2: Multiply by 2 :

$$x^2 = 36$$

(Step 2 of B becomes Step 2, and the division is changed to multiplication)

Step 3: Find the square root

$$x = \sqrt{36}$$

$$x = 6$$

(Step 1 of B becomes Step 3, and the square is changed to square root)

In **A** and **B**, above, observe the reversal of the order of the steps and the opposite operations involved. The above implies that given A, one can find B; and given B, one can find A.

Example 4: Let us do the following example; and then reverse the steps.

A Integration	**B Differentiation to check the integration**
$\frac{dy}{dx} = 4x^3 + 2x$. find y Step 1: Increase exponent by 1 and Step 2: Divide by new exponent . $y = \frac{4x^{3+1}}{3+1} + \frac{2x^{1+1}}{1+1} + C$ $= \frac{4x^4}{4} + \frac{2x^2}{2} + C$ $y = x^4 + x^2 + C$	If $y = x^4 + x^2 + C$ Step 1: Exponent **multiplies** the base; and Step 2: Exponent **decreases** by 1 $\frac{dy}{dx} = 4x^3 + 2x$ Note above: Step 2 of **A** becomes Step 1 of **B**; with opposite operations; and Step 1 of **A** becomes Step 2 of B with opposite operations;

B	From **B to A**
Using Differentiation to check the integration If $y = x^4 + x^2 + C$ Step 1: Exponent **multiplies** the base; and Step 2: Exponent **decreases** by 1 $\frac{dy}{dx} = 4x^3 + 2x$ From **B** to **A** } →	**Integration** Reverse the order of the steps while using opposite operations of differentiation $\int (4x^3 + 2x)dx$ Step 1: Exponent **increases** by 1. (Step 2 of differentiation becomes Step 1) Step 2: New exponent **divides** (Step 1 of differentiation becomes Step 2) $= \frac{4x^{3+1}}{3+1} + \frac{2x^{1+1}}{1+1} + C$ $= \frac{4x^4}{4} + \frac{2x^2}{2} + C$ $= x^4 + x^2 + C$

Case 2: There are no detailed steps for checking solution of the original problem. In this case, there are no obvious steps to reverse.

Example Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Solution For details, see P vs NP: Solutions of NP Problems, viXra:1408.0204.

A

$Q_A = 100+97+96+93+92+89+88+85+84+81+80+77+76+ 73+72+69+68+65+ 64+61+60+57+56+53+52+49+48+45+44+41+40+37+36+33+32+29+28+ 25+24+21+20+17+16+13+12+9+8+5+4+1=\mathbf{2525}.$

B

$Q_B = 99+98+95+94+91+90+87+86+83+82+79+78+75+74+71+70+67+66+63+ 62+59+58+55+54\ 51+50+47+46+43+42+39+38+35+34+31+30+27+26+ 23+22+19+18+15+14+11+10+7+6+3+2=\mathbf{2525}.$

Checking the solution **1.** $\sum_1^{100} n = 5050$; .2. $Q_A = 2525$, **3.** $Q_B = 2525$; and

4. $Q_A + Q_B = 2525 + 2525 = 5050$

Conclusion: A receives \$2525 and B receives \$2525, Note **t**he zero error for A and B.

Using term numbers

$\mathbf{Q_A} = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} + a_{80} + a_{81} + a_{84} + a_{85} + a_{88} + a_{89} + a_{92} + a_{93} + a_{96} + a_{97} + a_{100}$

$\mathbf{Q_B} = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34} + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} + a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} + a_{79} + a_{82} + a_{83} + a_{86} + a_{87} + a_{90} + a_{91} + + a_{94} + a_{95} + a_{98} + a_{99}$

Discussion

If one checked the solution to the above problem by looking at the numbers 5050 (adding the bills sequentially from 1 to 100) and 2525 (obtained by the proper choices of the dollar bills), one could conclude that the sum of the dollar bills has been divided equally between A and B. Nevertheless, one could not reverse the steps in the above "checking solution" to obtain the list of the bills for A or B. However, if one checked the solution for the problem by providing $\sum_1^{100} n = 5050$ and the reversal of the addends order in boxes **A** and **B** above, one could produce the listings of the dollar bills from which one could deduce the solution process for the problem. For example, from Q_A, for A in box **C** below, one could reverse the order to deduce the solution process for the original problem.

C **Checking:** Reversing solution process Reversal of addends order (Ignoring the minus signs)

$Q_A = 1+4+5+8+9+12+13+16+17+20+21+24+25+28+29+32+33+36+37+40+ 41+44+45+48+49+52+53+56+57+60+61+64+65 +68+69+72+73+76+ 77+80+81+84+85+88+89+92+93+96+97+100=2525$

D

$$Q_A - 2525 = -1-4-5-8-9-12-13-16-17-20-21-24-25-28-29-32-33-36$$
$$-37-40-41-44-45-48-49-52-53-56-57-60-61-64-65 \; -68$$
$$-69-72-73-76-77-80-81-84-85-88-89-92-93-96-97-100=0$$

Note above that the checking in Box D is similar to how one checks for arithmetic errors on tax return forms in the United States.

Alternative posing of the above problem

If the original problem had been posed as " List the dollar bills for A and B so that the total value of the bills is divided equally between A and B"; to check the solution to the problem, one would be compelled to provide for instance the reversals for A in box **C**. From the reversals of the order of the addends as in box C, one could produce the solution as in box **A.**

The next example surely compels one to list the items in checking the solution. From such listings, one could construct the solution for the problem.

Example 2 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B.

The solution to this problem will consist of listings of the masses of concrete blocks. In checking the solution, one will be compelled to produce listings from which the solution process could be deduced.

Checking $\sum_{1}^{1000} n = 500{,}500$**;** Total for Q_A **= 250,250** units; Total for Q_B **= 250,250** units

Q_A $\quad Q_A + Q_B = 250{,}250 + 250{,}250 = 500{,}500$

For the listings of the concrete masses, see P vs NP: Solutions of NP Problems, viXra:1408.0204 (p,14-18)**.**

From the above two examples, the proper posing of the question and the compelled detailed checking process will produce a checking process which can then be reversed to produce the solution process.

Thus, if the question is posed properly, the checking process will be reversible; and then yes, P would be equal to NP.

Conclusion

Completely solving and completely checking mathematical problems are inverse processes of each other. If the checking process is easy, the solution process should also be relatively easy, and vice versa. Computer hardware and software should be designed in such a way that the checking process is complete. Just checking answers is incomplete checking. In Example 2, above, in checking the solution, if one does not list the individual masses of the blocks, but states the total mass for A and the total mass for B, one would not have done any checking, according to how the problem was posed. The checking process must cover from the final answer to the beginning of the problem. For practical purposes, checking answers only is sometimes insufficient. As a student, in high school, on a chemistry class test, there were two parts in the solution of a problem. In one part, one was supposed to multiply by 2, but one did not multiply by 2. In a subsequent part of the solution, one was supposed to divide by 2, but one did not divide by 2, and consequently one obtained the correct answer. The teacher commented that by not multiplying by 2 and subsequently not dividing by 2, one would not receive full credit for the solution of the problem. If the question were of the multiple-choice answer type, one would have received full credit for getting the correct answer. Perhaps it is more difficult or impossible to design a computer hardware and /or software which can completely perform the above checking. Even, if it is difficult to check, it would not be difficult to solve, since the complete checking process provides sufficient information to reverse to obtain the solution. Furthermore, if it is easy to check, it would relatively be easy to solve. The question should always be posed such that one is compelled to show a checking procedure from which the solution procedure can be deduced. Therefore, P = NP.

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