Meaning and Resolution of the Ladder Paradox for Special Relativity Theory

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Abstract

Regarding to the Ladder Paradox, there are some misunderstanding and insufficient understanding of the special relativity theory. Accurate recognition of the theory would make clear the core mechanism of both the theory and the paradox.

1. Introduction

While there are many explanations for the Ladder paradox, we may be able to summarize the essence of these as following.

> About mutually and relatively moving garage and ladder, garage sees contracted ladder based on the contraction effect of special relativity theory, also ladder sees contracted garage on same reason. Then there are the cases that one side of the two sees the ladder can fit in the garage and another side of the two sees the ladder cannot fit in the garage. This looks like a contradiction.

2. View from s system

We consider about following two systems

s system(2dimensions(ct, x)) and s' system(2dimensions(ct', x')). [1]

Here both are moving relatively with velocity v.

This situation can be shown as Minkowsky graph. (Fig.1)

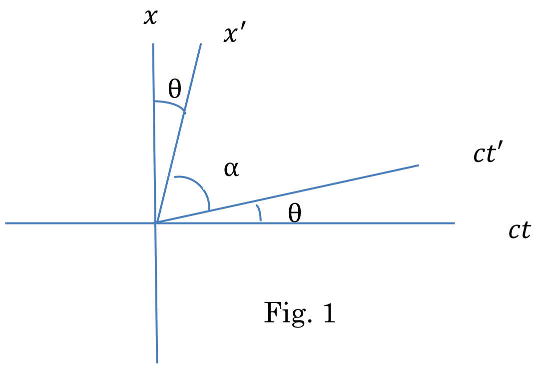


Fig. 1

On Fig.2, point A stays in s' system. Its track is $\overline{PQ}$. This is elapse of time in s' system. Also point B stays in s' system. Its track is $\overline{RS}$. This is elapse of time in s' system. These points A, B are simultaneous for s' system but not for s system. Then regarding to these two tracks, simultaneous points for s system are T, U, for example. On this situation, space distance $\overline{AB}$ for s' system is recognized as space distance $\overline{TU}$ for s system.

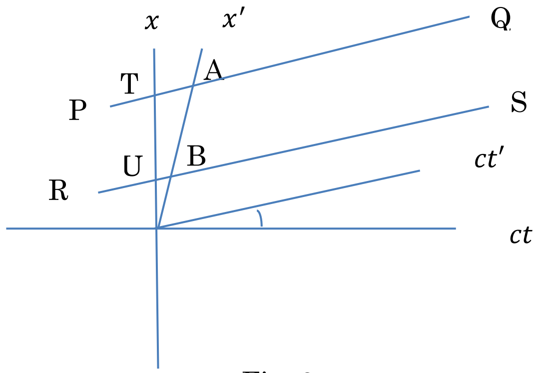


Fig. 2

Lorentz equation is

$$ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1)$$

$$x' = \frac{-vt + x}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2)$$

Here we set

x' value of point A: x'_A,

x' value of point B: x'_B,

x value of point T: x_T,

x value of point U: x_U,

space distance $\overline{AB} = l'$

space distance $\overline{TU} = l$

Here x' value at T point is equal to x'_A.

x' value at U point is equal to x'_B.

Then from (2),

$$x'_A = \frac{-v\times 0 + x_T}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (3)$$

$$x'_B = \frac{-v\times 0 + x_U}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (4)$$

$$x'_A - x'_B = \frac{x_T - x_U}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (5)$$

$$l' = \frac{l}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (6)$$

(6) has been called contraction of moving object.

3. View from s' system

Same as above, we look points of s system from s' system.

On Minkowsky graph. (Fig.3), point A stays in s system. Its track is $\overline{PQ}$. This is elapse of time in s system.

Also point B stays in s system. Its track is $\overline{RS}$. This is elapse of time in s system.

These points A, B are simultaneous for s system but not for s' system. Then regarding to these two tracks, simultaneous points for s' system are T, U, for example. On this situation, space distance $\overline{AB}$ for s system is recognized as space distance $\overline{TU}$ for s' system.

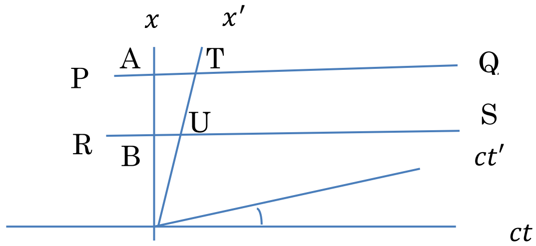


Fig. 3

Lorentz inverse transformation equation is

$$ct = \frac{ct' + \frac{v}{c}x'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

$$x = \frac{vt' + x'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

Here we set

x value of point A: x_A,

x value of point B: x_B,

x' value of point T: x'_T,

x' value of point U: x'_U,

space distance $\overline{AB} = l$

space distance $\overline{TU} = l'$

Here x value at T point is equal to x_A.

x' value at U point is equal to x_B.

Then from (8),

$$x_A = \frac{v \times 0 + x'_T}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

$$x_B = \frac{v\times 0 + x'_U}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

$$x_A - x_B = \frac{x'_T - x'_U}{\sqrt{1-\frac{v^2}{c^2}}} \tag{11}$$

$$l = \frac{l'}{\sqrt{1-\frac{v^2}{c^2}}} \tag{12}$$

(12) has been called contraction of moving object.

4. Time-space distance

On Fig.2 for example, $\overline{AB}$ is (time-)space distance and real existence for s' system. Also $\overline{AB}$ is time-space distance and real existence for s system. For s system, its time element is

$$\frac{\frac{v}{c}(x'_B - x'_A)}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{13}$$

space element is

$$\frac{x'_B - x'_A}{\sqrt{1-\frac{v^2}{c^2}}} \tag{14}$$

based on Lorentz inverse equation.

But actually s system feel space length as own space length. It is its space length at same time of s system.

One example of it is $\overline{TU}$. It is a projection of $\overline{AB}$ as space distance for s system. Here real existence of time-space distance is only one or common for every inertia system. On the other hand, every inertia systems have own projection as space distance.

5. s system and s' system

There is no inertia system which has priority. Every inertia systems are equivalent. But when multiple systems are described at once, there are following two categories of system as shown on Minkowsky graph in this report.

Staying system: I myself and total staying universe

Moving system: moving parts with same velocity and assembly of these

In this report, staying system is s system and a moving system is s′ system.

In s system, I myself am with staying space, and time elapse simultaneously with whole spaces. s′ system is moving in s system, and it has own frame of reference.

Of course, because both systems are equivalent, each system category is exchangeable. But while one scenario is described, each category should be kept.

6. Re-description of the Ladder Paradox

Based on above consideration, ladder paradox could be re-described as following.

Garage length l is in s system. Ladder length l' is in s' system.

From s system, the ladder can be seen as length $\sqrt{1-\frac{v^2}{c^2}}\,l'$ based on the contraction of moving object.

From s' system, the garage can be seen as length $\sqrt{1-\frac{v^2}{c^2}}\,l$ based on the contraction of moving object.

On such situation, we could find the case that one sees the ladder is fit in the garage from a system, and another sees the ladder is not fit in the garage from another system.

In such case, when the ladder is tried to fit in the garage, it faces contradiction.

Here paradox is regarding to 'one sees' or projection. But because there is only one unique real time-space distance even how one sees it, there is no paradox about the real existence. Also it says 'when the ladder is tried to fit', but two systems have to become same system to fit the ladder in the garage. Then at that timing both see same space distance projection of a real time-space distance. Then contradiction regarding to projection could disappear.

7. Another approach

Whole above story is:

-Lorentz transformation is derived on the following definition. [1]

Definition: Time moves toward time direction also toward space direction with speed c. (a)

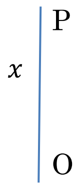
-Length contraction of moving object is derived on the Lorentz transformation and Minkowsky graph which draws the relation of the Lorentz transformation.

-Length contraction depends on each view to see an object. But real time-space existence is unique. Then ladder paradox is resolved on such recognition.

From here, length contraction is explained directly on the definition (a) for intuitive understanding.

8. Explanation of length contraction

There are two point O, P. O is at position 0 and P is at position x.



In the case that point P and O are staying, P receives time t_p from point O at time t after $\frac{x}{c}$ from t_p.

Time distance (distance which time moved in space) between O and P is x. This is equivalent for time to pass $\frac{x}{c}$.

So at point O, time is $t_p + \frac{x}{c}$ after time passed $\frac{x}{c}$.

For point O at point P, time is also $t_p + \frac{x}{c}$ after time traveled distance x.

Two points O, P are synchronized. These two points are in a system.

In the case that point O is moving with velocity v toward to lower, also P receives time t_p from point O at time t after $\frac{x}{c}$ from t_p. For point O, this situation is same as point P is moving toward to upper.

When time from point O reaches to the point P, space distance between O and P becomes $x + v\frac{x}{c}$.

So at point O, time is $t_p + \frac{x}{c}$ after time passed $\frac{x}{c}$,

At point P, also time passed $\frac{x}{c}$.

For point O at point P, time is $t_p + \frac{x}{c} + v\frac{x}{c^2}$ after time traveled distance $x + v\frac{x}{c}$.

Two points O, P are not synchronized. These two points are in different system.

This means for point O when point O is at ct_p, point P is at $ct_p + v\frac{x}{c}$, After time passed x, time reaches to point P at time $ct_p + x + v\frac{x}{c}$. (Fig.4)

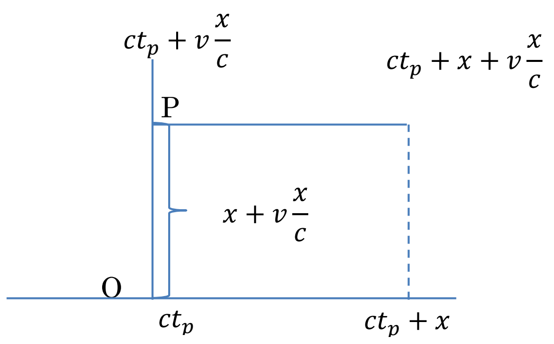


Fig. 4

Because Point P and O is not synchronized in the case that point P is moving, when O at time t_p, point P at time $t_p + v\frac{x}{c^2}$. When point P at time t_p, point P was $-v^2\frac{x}{c^2}$ from the space position at time $t_p + v\frac{x}{c^2}$. Then distance between point O and P at time t_p was

$$x - v^2\frac{x}{c^2} = x(1 - \frac{v^2}{c^2}) \tag{15}$$

Here oblique frame of reference indication and scaling should be applied because moving points are on oblique system to staying system. [1] Then (15) is

$$x\left(1-\frac{v^2}{c^2}\right)\frac{\cos\theta\sqrt{\sin\alpha}}{\sin\alpha} = x\left(1-\frac{v^2}{c^2}\right)\frac{\cos\theta}{\sqrt{\sin\alpha}} = x\left(1-\frac{v^2}{c^2}\right)\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = x\sqrt{1-\frac{v^2}{c^2}}$$

This is contraction of moving object.

9. Conclusion

Lorentz transformation is derived on the definition (a). [1]

Length contraction is derived on the Lorentz transformation.

Also length contraction is derived based on the definition directly.

On both ways, ladder paradox is resolved.

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Reference

[1] Tsuneaki Takahashi, viXra:1611.0077,(http://vixra.org/abs/1611.0077)