Half proof of Collatz problem

T.Nakashima
E-mail address
tainakashima@mbr.nifty.com

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Abstract

We prove Collatz sequence has not general m-cycle.Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

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In this paper,we prove partial result of Collatz problem.This problem is bred by Lothar Collatz at 1937. For 80 years,proved best result is 68-cycle case.We prove general m-cycle case.

Collatz problem

$$\begin{cases} n \ is \ even \ number & \Rightarrow divides \ 2 \\ n \ is \ odd \ number & \Rightarrow times \ 3 \ plus \ 1 \end{cases}$$

repeat this prosess,this sequence reaches to 1

Next result is known already.

Theorem 1.1. *(68-cycle)*
Except trivial case,some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.

example:

$$43 \to 130 \to 65 \to 196 \to 49 \to 148 \to 37 \to 112 \to 7 \to 22 \to 11$$
$$\to 34 \to 17 \to 52 \to 13 \to$$

This sequence has possibility to be the part of 8 or more cycle.

Theorem 1.2. *(Half proof of Collatz problem)*
Except trivial case,In the Collatz sequence,some number does not go to same number

Corollary 1.1. *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

proof. We prove the theorem.We assume next formula.

$$(\frac{1}{2})^n(\frac{3}{2})^m N + \alpha = M$$

$$\alpha = (\frac{1}{2})^{n_1} + (\frac{1}{2})^{n_2}(\frac{3}{2}) + (\frac{1}{2})^{n_3}(\frac{3}{2})^2 + \cdots + (\frac{1}{2})^{n_{m-1}}(\frac{3}{2})^{m-1}$$
$$n_1 \le n_2 \le n_3 \le \cdots \le n_{m-1}$$

M is the number the $(m-1)$th odd number from N.

example.
$N = 43, M = 13$ case.

$$M(=13) = (\frac{1}{2})^6(\frac{3}{2})^7 N(=43) + \alpha$$

$$\alpha = (\frac{1}{2})^2 + (\frac{1}{2})^2(\frac{3}{2}) + (\frac{1}{2})^2(\frac{3}{2})^2 + (\frac{1}{2})^5(\frac{3}{2})^3 + (\frac{1}{2})^6(\frac{3}{2})^4$$
$$+(\frac{1}{2})^7(\frac{3}{2})^5 + (\frac{1}{2})^7(\frac{3}{2})^6$$

In this formula,$\frac{1}{2}$ appear at the time $\frac{3N+1}{2}$ occur.Thereafter,we multiply $\frac{3}{2}, \frac{1}{2}$ according to calculation.This formula easily can be checked by computer.

We assume that M and N is equal and lead contradiction.For easy to understand,we take $M = 13, N = 43$ and assume $M = N$.Ofcourse,$43 \neq 13$.

$$N = (\frac{1}{2})^6(\frac{3}{2})^7 N + \alpha$$

We multiply 2^{13}.

$$2^{13}N = 3^7 N + 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

We calculate α's value.$\alpha = 1.520386\cdots$.

$$N2^{m+n} - N3^m = 2^{m+n}\alpha$$

The case $\alpha > 1$.

$$L > \alpha > L - 1 > 1 \Leftrightarrow 2 > \frac{L}{L-1} > \frac{\alpha}{L-1} > 1$$

$$N2^{m+n} - N3^m = \alpha \Leftrightarrow 2^{m+n} - 3^m = \frac{2^{m+n}\alpha}{N}$$

$$2^{m+n} - 3^m = \frac{2^{m+n} \times (L-1)}{N}\frac{\alpha}{L-1}$$

Left side of last formula is natural number.So we get $N > (L-1)2^{m+n} \geq 2^{m+n}$.
The case $1 > \alpha > 0$.

$$\frac{1}{k-1} > \alpha > \frac{1}{k} \Leftrightarrow 2 > \frac{k}{k-1} > \alpha k > 1$$

$$N2^{m+n} - N3^m = \alpha \Leftrightarrow 2^{m+n} - 3^m = \frac{2^{m+n}\alpha}{N}$$

$$k(2^{m+n} - 3^m) = \frac{2^{m+n}}{N}\alpha k$$

$$N > 2^{m+n}$$

$$N2^{m+n} - N3^m = 2^{m+n}\alpha$$

$$N > 3^m + \alpha > 3^m$$

$k\alpha = 1$ happens $k = 2^{n+m}$,α's numerator is 1.This case is none.
Later,We assume 2 or more large cycle.1-cycle has counter example.We see it last.

$$A \times 2 \times 3 + B = N, 2 \times 3 > B$$

We think first this formula.We assume N is the largest number of m-cycle.After this,the number is divded by 2 as much as we can. The next number of $2 \times 3A + B$ is

$$\frac{3 \times 2 \times 3A + 3B + 1}{2} = 9A + B'(9 > B')$$

This number is the bigger number than $2 \times 3A + B$.By the assumption N is the largest number,$9A + B'$ is even number.This thing is used later.
We go m-cycle backward.$6A + B$ is the largest number.So times 2 and minus 1.

$$2(6A + B) - 1 = 12A + 2B - 1$$

This number is multiple of 3.We take $2B - 1 = 3B_1$.We get

$$\frac{12A + 2B - 1}{3} = 4A + B_1(4 > B_1)$$

Next,We take $A = A_1$,$A_1 = 3A_2$ or $A_1 = 3A_2 + 1$ or $A_1 = 3A_2 + 2$.

$$\frac{2 \times 4 \times 3A_2 + 3B_2}{3} = 8A_2 + B_2$$

Actually,the calculation reaches here at best.But we go backward till we take $m - 1$ times odd number,We see first number N.

$$N = 2 \times 3^{m-1} A_{m-1} + C(2 \times 3^{m-1} > C)$$

We assume important condition.First, less than $m-1$-cycle does not exist.Second,$2 \times 3^{m-1} > C$.

Next result is the new condition got by calculation.Final 2 step

$$\frac{3\frac{3A+B+1}{2} + 1}{2} = \frac{9}{4}A + \frac{3}{4}B + \frac{5}{4}$$

We proved element of m-cycle bigger than 3^m,so we get $N > 2 \times 3^m$.$2 \times 2 \times 3^{m-1} < 2 \times 3^m$

$$N = 2 \times 3^{m-1} A_{m-1} + C > 2 \times 3^m > 2 \times 2 \times 3^{m-1}$$

We get $A_{m-1} \geq 1$.

Next step is the final step of our proof.Are $9A + B' = 9 \times 2 \times 3^{m-1} A_{m-1} + D$ and $2^l(2^{m-1}A_{m-1} + B_{m-1})$ coinside?

An indirect way,we calculate general case.We take $2^n A+B$,but n, A, B is not that used before.$2^n A + B$ may be even number. From this term,go forward Collatz sequence.We get the number's form $2^{n'} 3^{m'} A+B'$.We prove 2 property

First,n', m' is not coinside both at same timing.

Second,These numbers are all different.

This is very delicate argument,we see in the example.A is the most baddest number 1,$B = 15, n = 16$

$$2^{16} \times 1 + 15 \to 2^{15} 3 \times 1 + 23 \to 2^{14} 3^2 \times 1 + 35 \to 2^{13} 3^3 \times 1 + 53$$

$$\to 2^{12} 3^4 \times 1 + 80 \to 2^8 3^4 \times 1 + 5 \to 2^7 3^5 \times 1 + 8 \to 2^4 3^5 \times 1 + 1$$

$$\to 2^3 3^6 \times 1 + 2 \to 2^2 3^6 \times 1 + 1 \to 2^1 3^7 \times 1 + 2 \to 3^7 \times 1 + 1$$

this way can not use more.Our case,$N > 3^m$,so 2 does not exhausted.(Actually,$2 \times 3^{m-1} A_{m-1} + C$ is the final term,2 remain.)

First question is clearlly in the example,m', n' does not coinside another m'', n''.This is clear.

Before second condition,$2^n A + B$ assume odd number.If needed,we take 2^l multiple of $2^n A + B$.
Second condition,$2^n A + B = 2^{n'} 3^{m'} A + B'$ is possible or not.

$$2^n A - 2^{n'} 3^{m'} A = B' - B$$

$$B' = B(\frac{1}{2})^{n-n'} 3^{m'} + \alpha$$

$$2^n A - 2^{n'} 3^{m'} A = B(\frac{1}{2})^{n-n'} 3^{m'} - B + \alpha$$

$$(2^{n'} A + (\frac{1}{2})^{n-n'} B)(2^{n-n'} - 3^{n'}) = \alpha$$

$$(2^n A + B)(1 - (\frac{1}{2})^{n-n'} 3^{m'}) = \alpha$$

We get

$$B(1 - (\frac{1}{2})^{n-n'} 3^{m'}) = \alpha \Leftrightarrow B = B'$$

B makes m-cycle or more less m'-cycle.We assume less than $m-1$-cycle is not exist.And if B makes m-cycle, then B is bigger than 3^m.Please remember C.$2^n A + B$ goes to $2 \times 3^{m-1} A_{m-1} + C (2 \times 3^{m-1} > C)$. We can think $A = A_{m-1}$ and in the Collatz sequence B goes to C.$C > 3^m$,but $2 \times 3^{m-1} > C$.This is the contradiction. There is no m-cycle.m is finite and m is all number. Proof is finished. □

Finally we calculate 1-cycle.

$$N = (\frac{1}{2})^n \frac{3}{2} N + (\frac{1}{2})^{n+1}$$

multiply 2^{n+1}.

$$2^{n+1} N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence $1 \to 4 \to 1 \to 4$.This result is easilly checked.

$$1 = (\frac{1}{2})\frac{3}{2} \times 1 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$