

# Half proof of Collatz problem

T.Nakashima

E-mail address

tainakashima@mbr.nifty.com

May 11, 2016

## Abstract

We prove Collatz sequence has not general m-cycle. Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

## 1

In this paper, we prove partial result of Collatz problem. This problem is bred by Lothar Collatz at 1937. For 80 years, proved best result is 68-cycle case. We prove general m-cycle case.

## Collatz problem

$$\begin{cases} n \text{ is even number} & \Rightarrow \text{divides } 2 \\ n \text{ is odd number} & \Rightarrow \text{times } 3 \text{ plus } 1 \end{cases}$$

repeat this process, this sequence reaches to 1

Next result is known already.

### **Theorem 1.1.** *(68-cycle)*

*Except trivial case, some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.*

example:

$$43 \rightarrow 130 \rightarrow 65 \rightarrow 196 \rightarrow 49 \rightarrow 148 \rightarrow 37 \rightarrow 112 \rightarrow 7 \rightarrow 22 \rightarrow 11 \\ \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow$$

This sequence has possibility to be the part of 8 or more cycle.

**Theorem 1.2.** *(Half proof of Collatz problem)*

*Except trivial case, In the Collatz sequence, some number does not go to same number*

**Corollary 1.1.** *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

**proof.** We prove the theorem. We assume next formula.

$$\left(\frac{1}{2}\right)^n \left(\frac{3}{2}\right)^m N + \alpha = M \\ \alpha = \left(\frac{1}{2}\right)^{n_1} + \left(\frac{1}{2}\right)^{n_2} \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^{n_3} \left(\frac{3}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^{n_{m-1}} \left(\frac{3}{2}\right)^{m-1} \\ n_1 \leq n_2 \leq n_3 \leq \dots \leq n_{m-1}$$

$M$  is the number the  $(m - 1)$ th odd number from  $N$ .

example.

$N = 43, M = 13$  case.

$$M(= 13) = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N(= 43) + \alpha \\ \alpha = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^5 \left(\frac{3}{2}\right)^3 + \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^4 \\ + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^5 + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^6$$

In this formula,  $\frac{1}{2}$  appear at the time  $\frac{3N+1}{2}$  occur. Thereafter, we multiply  $\frac{3}{2}, \frac{1}{2}$  according to calculation. This formula easily can be checked by computer.

We assume that  $M$  and  $N$  is equal and lead contradiction. For easy to understand, we take  $M = 13, N = 43$  and assume  $M = N$ . Of course,  $43 \neq 13$ .

$$N = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N + \alpha$$

We multiply  $2^{13}$ .

$$2^{13}N = 3^7N + 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

We calculate  $\alpha$ 's value.  $\alpha = 1.520386 \dots$

Later, We assume 2 or more large cycle. 1-cycle has counter example. We see it last.

$$A \times 2 \times 3 + B = N, 2 \times 3 > B$$

We think first this formula. We assume  $N$  is the largest number of  $m$ -cycle. After this, the number is divided by 2 as much as we can. The next number of  $2 \times 3A + B$  is

$$\frac{3 \times 2 \times 3A + 3B + 1}{2} = 9A + B' (9 > B')$$

This number is the bigger number than  $2 \times 3A + B$ . By the assumption  $N$  is the largest number,  $9A + B'$  is even number. This thing is used later.

We go  $m$ -cycle backward.  $6A + B$  is the largest number. So times 2 and minus 1.

$$2(6A + B) - 1 = 12A + 2B - 1$$

This number is multiple of 3. We take  $2B - 1 = 3B_1$ . We get

$$\frac{12A + 2B - 1}{3} = 4A + B_1 (4 > B_1)$$

Next, We take  $A = A_1, A_1 = 3A_2$  or  $A_1 = 3A_2 + 1$  or  $A_1 = 3A_2 + 2$ .

$$\frac{2 \times 4 \times 3A_2 + 3B_2}{3} = 8A_2 + B_2$$

Actually, the calculation reaches here at best. But we go backward till we take  $m - 1$  times odd number, We see first number  $N$ .

$$N = 2 \times 3^{m-1} A_{m-1} + C (2 \times 3^{m-1} > C)$$

We get  $N > 4 \times 3^{m-1}$  by the calculation. We get  $A_{m-1} \geq 1$ . Clearly,  $N = 1$  case can not be written this form.

Next step is the final step of our proof. Are  $9A + B' = 9 \times 2 \times 3^{m-1}A_{m-1} + D$  and  $2^l(2^{m-1}A_{m-1} + B_{m-1})$  coincide?

An indirect way, we calculate general case. We take  $2^n A + B$ , but  $n, A, B$  is not that used before.  $2^n A + B$  may be even number. From this term, go forward Collatz sequence. We get the number's form  $2^{n'} 3^{m'} A + B'$ . We prove 2 property

First,  $n', m'$  is not coincide both at same timing.

Second, These numbers are all different.

This is very delicate argument, we see in the example.  $A$  is the most baddest number 1,  $B = 15, n = 16$

$$\begin{aligned} 2^{16} \times 1 + 15 &\rightarrow 2^{15} 3 \times 1 + 23 \rightarrow 2^{14} 3^2 \times 1 + 35 \rightarrow 2^{13} 3^3 \times 1 + 53 \\ &\rightarrow 2^{12} 3^4 \times 1 + 80 \rightarrow 2^8 3^4 \times 1 + 5 \rightarrow 2^7 3^5 \times 1 + 8 \rightarrow 2^4 3^5 \times 1 + 1 \\ &\rightarrow 2^3 3^6 \times 1 + 2 \rightarrow 2^2 3^6 \times 1 + 1 \rightarrow 2^1 3^7 \times 1 + 2 \rightarrow 3^7 \times 1 + 1 \end{aligned}$$

this way can not use more. Our case,  $N > 3^m$ , so 2 does not exhausted. (Actually,  $2 \times 3^{m-1}A_{m-1} + C$  is the final term, 2 remain.)

First question is clearly in the example,  $m', n'$  does not coincide another  $m'', n''$ . This is clear.

Before second condition,  $2^n A + B$  assume odd number. If needed, we take  $2^l$  multiple of  $2^n A + B$ .

Second condition, we think only  $2^n A + B = 3^m A + B'$ .

$$2^n A - 3^m A = B' - B$$

$$B' = B \left(\frac{1}{2}\right)^n 3^m + \alpha$$

$$2^n A - 3^m A = B \left(\frac{1}{2}\right)^n 3^m - B + \alpha$$

$$(2^n A + B) \left(1 - \left(\frac{1}{2}\right)^n 3^m\right) = \alpha$$

This formula is identical equation, but we lead contradiction.

$$2^n A - 3^m A + B - B' = 0$$

$2^n - 3^m > 0$  is the precondition.

$$2^n - 3^m > 0, B - B' < 0$$

$$B - B' + \alpha > 0$$

This thing is  $B - B' + \alpha = B(1 - (\frac{1}{2})^n 3^m)$ . By  $2^n - 3^m > 0$ , we get this formula.

$$(2^n - 3^m)A - \alpha < 0$$

We take natural number  $L$  well,

$$L - 1 < (2^n - 3^m)A < L$$

$(2^n - 3^m)A$  is natural number. this formula takes 2 different natural numbers in 1 natural number. This is the contradiction. There is no  $m$ -cycle.  $m$  is finite and  $m$  is all number. Proof is finished.  $\square$

Finally we calculate 1-cycle.

$$N = (\frac{1}{2})^n \frac{3}{2} N + (\frac{1}{2})^{n+1}$$

multiply  $2^{n+1}$ .

$$2^{n+1} N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence  $1 \rightarrow 4 \rightarrow 1 \rightarrow 4$ . This result is easily checked.

$$1 = (\frac{1}{2}) \frac{3}{2} \times 1 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$