

Half proof of Collatz problem

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May 12, 2016

Abstract

We prove Collatz sequence has not general m-cycle. Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

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In this paper, we prove partial result of Collatz problem. This problem is bred by Lothar Collatz at 1937. For 80 years, proved best result is 68-cycle case. We prove general m-cycle case.

Collatz problem

$$\begin{cases} n \text{ is even number} & \Rightarrow \text{divides } 2 \\ n \text{ is odd number} & \Rightarrow \text{times } 3 \text{ plus } 1 \end{cases}$$

repeat this process, this sequence reaches to 1

Next result is known already.

Theorem 1.1. *(68-cycle)*

Except trivial case, some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.

example:

$$43 \rightarrow 130 \rightarrow 65 \rightarrow 196 \rightarrow 49 \rightarrow 148 \rightarrow 37 \rightarrow 112 \rightarrow 7 \rightarrow 22 \rightarrow 11 \\ \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow$$

This sequence has possibility to be the part of 8 or more cycle.

Theorem 1.2. *(Half proof of Collatz problem)*

Except trivial case, In the Collatz sequence, some number does not go to same number

Corollary 1.1. *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

proof. We prove the theorem. We assume next formula.

$$\left(\frac{1}{2}\right)^n \left(\frac{3}{2}\right)^m N + \alpha = M \\ \alpha = \left(\frac{1}{2}\right)^{n_1} + \left(\frac{1}{2}\right)^{n_2} \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^{n_3} \left(\frac{3}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^{n_{m-1}} \left(\frac{3}{2}\right)^{m-1} \\ n_1 \leq n_2 \leq n_3 \leq \dots \leq n_{m-1}$$

M is the number the $(m - 1)$ th odd number from N .

example.

$N = 43, M = 13$ case.

$$M(= 13) = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N(= 43) + \alpha \\ \alpha = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right) + \left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^5 \left(\frac{3}{2}\right)^3 + \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^4 \\ + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^5 + \left(\frac{1}{2}\right)^7 \left(\frac{3}{2}\right)^6$$

In this formula, $\frac{1}{2}$ appear at the time $\frac{3N+1}{2}$ occur. Thereafter, we multiply $\frac{3}{2}, \frac{1}{2}$ according to calculation. This formula easily can be checked by computer.

We assume that M and N is equal and lead contradiction. For easy to understand, we take $M = 13, N = 43$ and assume $M = N$. Of course, $43 \neq 13$.

$$N = \left(\frac{1}{2}\right)^6 \left(\frac{3}{2}\right)^7 N + \alpha$$

We multiply 2^{13} .

$$2^{13}N = 3^7N + 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

We calculate α 's value. $\alpha = 1.520386 \dots$

Later, We assume 2 or more large cycle. 1-cycle has counter example. We see it last.

$$A \times 2 \times 3 + B = N, 2 \times 3 > B$$

We think first this formula. We assume N is the largest number of m -cycle. After this, the number is divided by 2 as much as we can. The next number of $2 \times 3A + B$ is

$$\frac{3 \times 2 \times 3A + 3B + 1}{2} = 9A + B' (9 > B')$$

This number is the bigger number than $2 \times 3A + B$. By the assumption N is the largest number, $9A + B'$ is even number. This thing is used later.

We go m -cycle backward. $6A + B$ is the largest number. So times 2 and minus 1.

$$2(6A + B) - 1 = 12A + 2B - 1$$

This number is multiple of 3. We take $2B - 1 = 3B_1$. We get

$$\frac{12A + 2B - 1}{3} = 4A + B_1 (4 > B_1)$$

Next, We take $A = A_1, A_1 = 3A_2$ or $A_1 = 3A_2 + 1$ or $A_1 = 3A_2 + 2$.

$$\frac{2 \times 4 \times 3A_2 + 3B_2}{3} = 8A_2 + B_2$$

Actually, the calculation reaches here at best. But we go backward till we take $m - 1$ times odd number, We see first number N .

$$N = 2 \times 3^{m-1} A_{m-1} + C (2 \times 3^{m-1} > C)$$

We get $N > 4 \times 3^{m-1}$ by the calculation. We get $A_{m-1} \geq 1$. Clearly, $N = 1$ case can not be written this form.

Next step is the final step of our proof. Are $9A + B' = 9 \times 2 \times 3^{m-1}A_{m-1} + D$ and $2^l(2^{m-1}A_{m-1} + B_{m-1})$ coincide?

An indirect way, we calculate general case. We take $2^n A + B$, but n, A, B is not that used before. $2^n A + B$ may be even number. From this term, go forward Collatz sequence. We get the number's form $2^{n'} 3^{m'} A + B'$. We prove 2 property

First, n', m' is not coincide both at same timing.

Second, These numbers are all different.

This is very delicate argument, we see in the example. A is the most baddest number 1, $B = 15, n = 16$

$$\begin{aligned} 2^{16} \times 1 + 15 &\rightarrow 2^{15}3 \times 1 + 23 \rightarrow 2^{14}3^2 \times 1 + 35 \rightarrow 2^{13}3^3 \times 1 + 53 \\ &\rightarrow 2^{12}3^4 \times 1 + 80 \rightarrow 2^8 3^4 \times 1 + 5 \rightarrow 2^7 3^5 \times 1 + 8 \rightarrow 2^4 3^5 \times 1 + 1 \\ &\rightarrow 2^3 3^6 \times 1 + 2 \rightarrow 2^2 3^6 \times 1 + 1 \rightarrow 2^1 3^7 \times 1 + 2 \rightarrow 3^7 \times 1 + 1 \end{aligned}$$

this way can not use more. Our case, $N > 3^m$, so 2 does not exhausted. (Actually, $2 \times 3^{m-1}A_{m-1} + C$ is the final term, 2 remain.)

First question is clearly in the example, m', n' does not coincide another m'', n'' . This is clear.

Before second condition, $2^n A + B$ assume odd number. If needed, we take 2^l multiple of $2^n A + B$.

Second condition, we think only $N = 2^n A + B = 3^m A + B'$.

$$N(1 - (\frac{1}{2})^n 3^m) = \alpha$$

$$N(2^n - 3^m) = 2^{n-l} + 2^{n-l'} 3 + \dots + 3^{m-1}$$

We use next result.

$$2^m - 3^m = (2 - 3)(2^{m-1} + 2^{m-2}3 + \dots + 3^{m-1})$$

$$N(2^n - 2^m) = 2^{n-l} + N(2^{m-1}) + 2^{n-l'}3 + N(2^{m-2}3) + \dots + 3^{m-1} + N(3^{m-1})$$

In this formula, left hand can be divided by 2^{n-m} . But right hand can not be divided by 2^{n-m} for any N . $N = 1, n = 2, m = 1, l = 2$ case exists. So we take $m \geq 2$. We see in the example again.

$$2^{13}N = 3^7N + 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

$$2^{13}N - 2^7N + 2^7N - 3^7N = 2^{11} + 2^{10}3 + 2^93^2 + 2^53^3 + 2^33^4 + 23^5 + 3^6$$

$$2^{13}N - 2^7N = 2^{11} + 2^6N + 2^{10}3 + 2^53N + 2^93^2 + 2^43^2N + 2^53^3 +$$

$$2^33^3N + 2^33^4 + 2^23^4N + 23^5 + 23^5N + 3^6 + 3^6N$$

This formula's right hand can not be divided by 2^7 . This is the contradiction. There is no m -cycle. m is finite and m is all number. Proof is finished. \square

Finally we calculate 1-cycle.

$$N = \left(\frac{1}{2}\right)^n \frac{3}{2}N + \left(\frac{1}{2}\right)^{n+1}$$

multiply 2^{n+1} .

$$2^{n+1}N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence $1 \rightarrow 4 \rightarrow 1 \rightarrow 4$. This result is easily checked.

$$1 = \left(\frac{1}{2}\right) \frac{3}{2} \times 1 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$$