# Half proof of Collatz problem

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**Abstract**

We prove Collatz sequence has not general m-cycle. Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

## 1

In this paper, we prove partial result of Collatz problem. This problem is bred by Lothar Collatz at 1937. For 80 years, proved best result is 68-cycle case. We prove general m-cycle case.

### Collatz problem

$$\begin{cases} n \textit{ is even number} & \Rightarrow \textit{divides } 2 \\ n \textit{ is odd number} & \Rightarrow \textit{times } 3 \textit{ plus } 1 \end{cases}$$

repeat this prosess, this sequence reaches to 1

Next result is known already.

**Theorem 1.1.** *(68-cycle)*
*Except trivial case, some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.*

example:

$$43 \to 130 \to 65 \to 196 \to 49 \to 148 \to 37 \to 112 \to 7 \to 22 \to 11$$
$$\to 34 \to 17 \to 52 \to 13 \to$$

This sequence has possibility to be the part of 8 or more cycle.

**Theorem 1.2.** *(Half proof of Collatz problem)*
*Except trivial case, In the Collatz sequence, some number does not go to same number*

**Corollary 1.1.** *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

**proof.** We prove the theorem. We assume next formula.

$$(\frac{1}{2})^n(\frac{3}{2})^m N + \alpha = M$$

$$\alpha = (\frac{1}{2})^{n_1} + (\frac{1}{2})^{n_2}(\frac{3}{2}) + (\frac{1}{2})^{n_3}(\frac{3}{2})^2 + \cdots + (\frac{1}{2})^{n_{m-1}}(\frac{3}{2})^{m-1}$$
$$n_1 \leq n_2 \leq n_3 \leq \cdots \leq n_{m-1}$$

$M$ is the number the $(m-1)$th odd number from $N$.

example.
$N = 43, M = 13$ case.

$$M(=13) = (\frac{1}{2})^6(\frac{3}{2})^7 N(=43) + \alpha$$

$$\alpha = (\frac{1}{2})^2 + (\frac{1}{2})^2(\frac{3}{2}) + (\frac{1}{2})^2(\frac{3}{2})^2 + (\frac{1}{2})^5(\frac{3}{2})^3 + (\frac{1}{2})^6(\frac{3}{2})^4$$
$$+(\frac{1}{2})^7(\frac{3}{2})^5 + (\frac{1}{2})^7(\frac{3}{2})^6$$

In this formula, $\frac{1}{2}$ appear at the time $\frac{3N+1}{2}$ occur. Thereafter, we multiply $\frac{3}{2}, \frac{1}{2}$ according to calculation. This formula easily can be checked by computer.

We assume that $M$ and $N$ is equal and lead contradiction. For easy to understand, we take $M = 13, N = 43$ and assume $M = N$. Ofcourse,$43 \neq 13$.

$$N = (\frac{1}{2})^6(\frac{3}{2})^7 N + \alpha$$

We multiply $2^{13}$.

$$2^{13}N = 3^7 N + 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

We calculate $\alpha$'s value. $\alpha = 1.520386\cdots$.

Later, We assume 2 or more large cycle. 1-cycle has counter example. We see it last.

$$N(1 - (\frac{1}{2})^n 3^m) = \alpha$$

$$N(2^n - 3^m) = 2^{n-l} + 2^{n-l'}3 + \cdots + 3^{m-1}$$

We use next result.

$$2^m - 3^m = (2-3)(2^{m-1} + 2^{m-2}3 + \cdots + 3^{m-1})$$

$$N(2^n - 2^m) = 2^{n-l} + N(2^{m-1}) + 2^{n-l'}3 + N(2^{m-2}3) + \cdots + 3^{m-1} + N(3^{m-1})$$

In this formula, left hand can be devided by $2^m$. But right hand can not devided by $2^m$ for any $N$. $N = 1, n = 2, m = 1, l = 2$ case exists. So we take $m \geq 2$. We see in the example again.

$$2^{13}N = 3^7 N + 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

$$2^{13}N - 2^7 N + 2^7 N - 3^7 N = 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

$$2^{13}N - 2^7 N = 2^{11} + 2^6 N + 2^{10}3 + 2^5 3N + 2^9 3^2 + 2^4 3^2 N + 2^5 3^3 +$$

$$2^3 3^3 N + 2^3 3^4 + 2^2 3^4 N + 23^5 + +23^5 N + 3^6 + 3^6 N$$

This formula's right hand can not be devided by $2^7$. This is the contradiction. There is no $m$-cycle. $m$ is finite and $m$ is all number. Proof is finished. $\square$

Finally we calculate 1-cycle.

$$N = (\frac{1}{2})^n \frac{3}{2} N + (\frac{1}{2})^{n+1}$$

multiply $2^{n+1}$.

$$2^{n+1}N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence $1 \to 4 \to 1 \to 4$. This result is easilly checked.

$$1 = (\frac{1}{2})\frac{3}{2} \times 1 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$