On the cyclic variations in Newton's constant

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Abstract

Oscillations observed in Newton's gravitational constant G are shown to be associated with inverse oscillations in the Gaussian gravitational constant *k*. A falsifiable prediction is submitted to test the G/*k* duality hypothesis.

Introduction

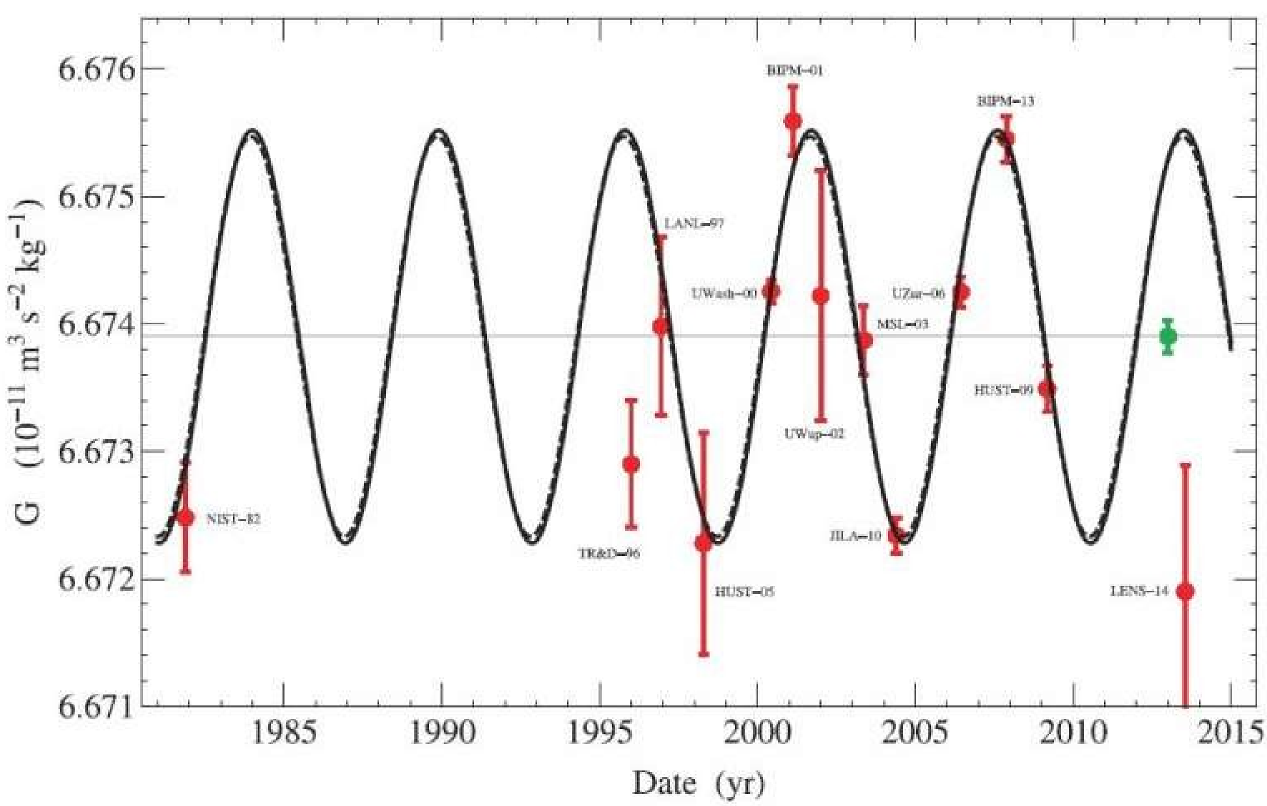

FIG. 1: A correlation between the 5.9 year period of the Earth's rotation rate and variations in Newton's constant G are observed from a set of 13 consecutive measurements. An estimate of the mean value of G is indicated by the green dot, which is almost perfect with the mean length of day (LOD) variations.

Measurements[1], [2] of G oscillate within the range of 6.672×10^{-11} and 6.675×10^{-11} N·(m/kg)^2 on a periodic basis (a difference of 10^{-4} %). Scientists currently studying this anomaly have discovered that the variations can be predicted from length of day (LOD) data[3] obtained from the International Earth Rotation and Reference System (IERS). Although the G/LOD correlation is intriguing, it cannot fully explain the full 10^{-4} % variations observed in Newton's constant.

The duality between G and *k*

A modern version of Kepler's 3rd law of planetary motion is

$$[1]\ G(M + m) = \frac{4\pi^2 a^3}{T^2},$$

where G is the gravitational constant, M is the mass of a primary body, m is the mass of a secondary, *a* is the semi-major axis of the orbit, and T is a secondary's sidereal period. Angular frequency ω is

$$[2]\ \omega = 2\pi f = \frac{2\pi}{T} = n,$$

where f is frequency and *n* is mean motion, so [1] can be rearranged and condensed into

$$[3]\ G = \frac{\omega^2 a^3}{(M + m)}.$$

Note, however, that this is the same formula for a circular orbit, in which case the semi-major axis *a* is substituted with the radius *r*.

The Gaussian gravitational constant *k* is

$$[4]\ k = \frac{2\pi}{T\sqrt{M + m}} = \frac{\omega}{\sqrt{M + m}}.$$

Rearranging [4] and squaring it,

$$[5]\ \omega^2 = k^2(M + m).$$

Substituting ω^2 in [3] with $k^2(M + m)$,

$$[6]\ G = k^2 a^3,$$

(*a* in astronomical units). We can see from [4] and [6] that *k* is directly proportional to ω while G is directly proportional to a^3. This inverse relationship between G and *k* will be referred to as G/*k* duality.

From Kepler's 2nd law we know that a secondary's areal velocity is constant,

$$[7]\ \frac{d\vec{A}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t) \times \vec{r}(t + \Delta t)}{2\Delta t} = constant.$$

In effect, the angular frequency of a secondary's orbit is inversely proportional to the secondary's distance from the center of mass. Since ω and *a* in [3] are static, the precision of [3] is dependent upon the eccentricity of an orbit. It is hypothesized that the cyclic variations in G will coincide with a slightly modified version of [3],

$$[8]\ \Delta G = \frac{\Delta\omega^2 \Delta r^3}{(M + m)}.$$

Likewise, variations in *k* are hypothesized to change inversely with the square root of ΔG when k^2 and G are measured with the same dimensions. This G/*k* duality hypothesis should be relatively simple to test experimentally by measuring G and *k* simultaneously and superimposing the outcome. Spin−orbit coupling could explain why the cyclic variations in G are synchronous with LOD data. The angular frequency ω should therefore be the Earth's total angular frequency (the sum of its spin and orbit).

Conclusion

If G/k duality is observed, it is proposed that the radius *r* in [8] be defined by the distance when ΔG is at its mean value G_{MEAN} (indicated by the green dot in FIG. 1). The total angular frequency ω would then be relative to the period of G_{MEAN} as determined from a sidereal frame of reference. If G_{MEAN} occurs periodically at a constant distance which differs from *a*, the sum of the masses in a 2−body system may need to be adjusted accordingly. To include Einstein's special theory of relativity, the relativistic masses of the primary and secondary should also be considered.

G is spatially dependent while *k* is temporally dependent. In some ways, each of these “constants” are analogous to electric and magnetic fields. It is well known that Newton's gravitational force law is analogous to Coulomb's electric force law. From the relationship given in [6], an alternative version of Newton/Coulomb's force law can be given as

$$[9]\ F = \gamma\mu_0 \Delta r \Delta\omega^2$$

where $\gamma\mu_0$ is the reduced (relativistic) mass of the system (the “reduced charge” in an atomic system).

Dedication

This paper is dedicated to Cynthia Cashman Lett, without whom it would not have been possible.

References

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