# On the Cyclic Variations in Newton's Constant

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**Abstract.** Periodic (5.9 year) oscillations are observed in Newton's gravitational constant G which coincide with length of day (LOD) data obtained from the International Earth Rotation and Reference System (IERS). It is shown that the oscillations in G are dualistic with Gauss' gravitational constant $k$ due to variations in the Earth's mean motion during the 5.9 year period. Falsifiable predictions are submitted to test the G/$k$ duality hypothesis.

## Introduction

Measurements [1, 2] of G oscillate between $6.672 \times 10^{-11}$ and $6.675 \times 10^{-11}$ N·(m/kg)$^2$ with a periodicity of 5.9 years (a difference of $10^{-4}$ %). Scientists studying this recently discovered (2015) anomaly have found that the variations can be predicted from length of day (LOD) data obtained from IERS [3]. Although the G/LOD correlation is intriguing, it cannot explain the full $10^{-4}$ % variations observed in G.

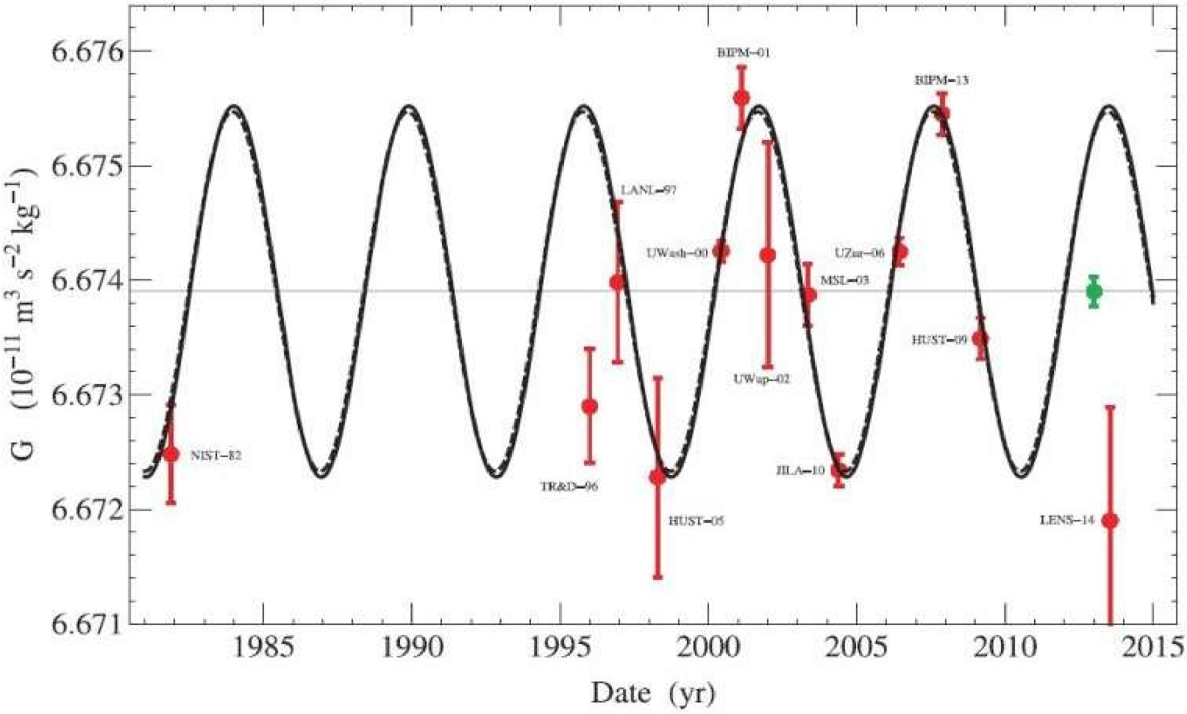


Fig. 1: The solid curve is a CODATA set of G measurements and periodic oscillations in length of day (LOD) measurements are represented by the dashed curve. The green dot, with its one−sigma error bar, is the mean value between the G/LOD measurements. The LENS−13 outlier conducted in 2013 by the MAGIA collaboration was the only measurement which utilized quantum interferometry, while the other 12 measurements were determined macroscopically.

## The Duality Between Newton's Constant and Gauss' Constant

A modern version of Kepler's 3$^{rd}$ law of planetary motion is

$$G(M + m) = \frac{4\pi^2 a^3}{T^2}, \tag{1}$$

where G is the gravitational constant, M is the mass of a primary, m is the mass of a secondary, $a$ is the semi-major axis of the orbit, and T is a secondary's sidereal period. Since the mean motion $n$ of an orbit is

$$n = \frac{2\pi}{T}; \tag{2}$$

Eq. 1 can be rearranged and condensed into

$$G = \frac{n^2 a^3}{M + m} = \frac{a v^2}{M + m}; \tag{3}$$

where $v$ is the secondary's velocity. Note, however, that this is the same formula for a circular orbit, in which case the semi–major axis $a$ is substituted with the radius $r$.

The Gaussian gravitational constant $k$ is

$$k = \frac{2\pi}{T\sqrt{M + m}} = \frac{n}{\sqrt{M + m}}. \tag{4}$$

Rearranging Eq. 4 and squaring it,

$$n^2 = k^2(M + m). \tag{5}$$

Substituting $n^2$ in Eq. 3 with the definition above (where $a$ is in astronomical units),

$$G = k^2 a^3. \tag{6}$$

We can see from Eq. 6 that $k^2$ is inversely proportional to $a^3$ and G is directly proportional to $a^3$.

From Kepler's 2nd law we know that a secondary's areal velocity is constant,

$$\frac{d\vec{A}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t) \times \vec{r}(t + \Delta t)}{2\Delta t} = \frac{\vec{r}(t) \times \vec{v}(t)}{2} = constant. \tag{7}$$

The mean motion $n$ in Eq. 3 assumes a constant speed in a circular orbit, which leads to a contradiction between Kepler's 2nd law and the modern version of his 3rd law in Eq. 1.

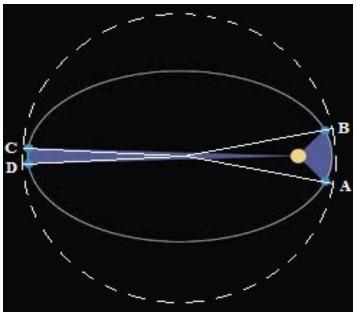


Fig. 2: The mean motion cirumference is represented by the dashed circle. During a secondary's periapsis transit (A, B) it takes less time for the semi–major axis (white) to sweep out a constant sector relative to its areal velocity. During its apoapsis transit (C, D) the opposite is true.

After taking into account the Earth's eccentricity, obliquity, and hysteresis due to its inertia (graphed in the conclusion), it is hypothesized that G will vary according to Kepler's 2nd law by

$$\Delta G = \frac{2\,d\vec{A}\,\vec{v}(t)}{dt\,(M + m)} = \frac{\vec{r}(t) \times \vec{v}(t)^2}{M + m} = \frac{h\vec{v}(t)}{M + m}, \tag{8}$$

where h is the Earth's specific relative angular momentum. Likewise, since $k^2$ and G are spatially dualistic (deduced from Eq. 6), it is hypothesized that $k$ will change inversely with the square root of ΔG when $k^2$ and G are measured with the same dimensions. This G/$k$ duality hypothesis should be relatively simple to test experimentally by graphing G and $k^2$ simultaneously and superimposing the outcome.

## Conclusion

It is proposed that the outlier measurement of G conducted with quantum interferometry by LENS−14 in Fig. 1 is accurate, even though it is inverted relative to the Earth's rotation rate. The macroscopic methods are synchronous with the Earth's rotation rate, indicating they are better for $k^2$ measurements since $k$ is directly proportional to a secondary's angular frequency (2π / T). It is hypothesized that the G/LOD synchronicity is due to spin−orbit coupling:

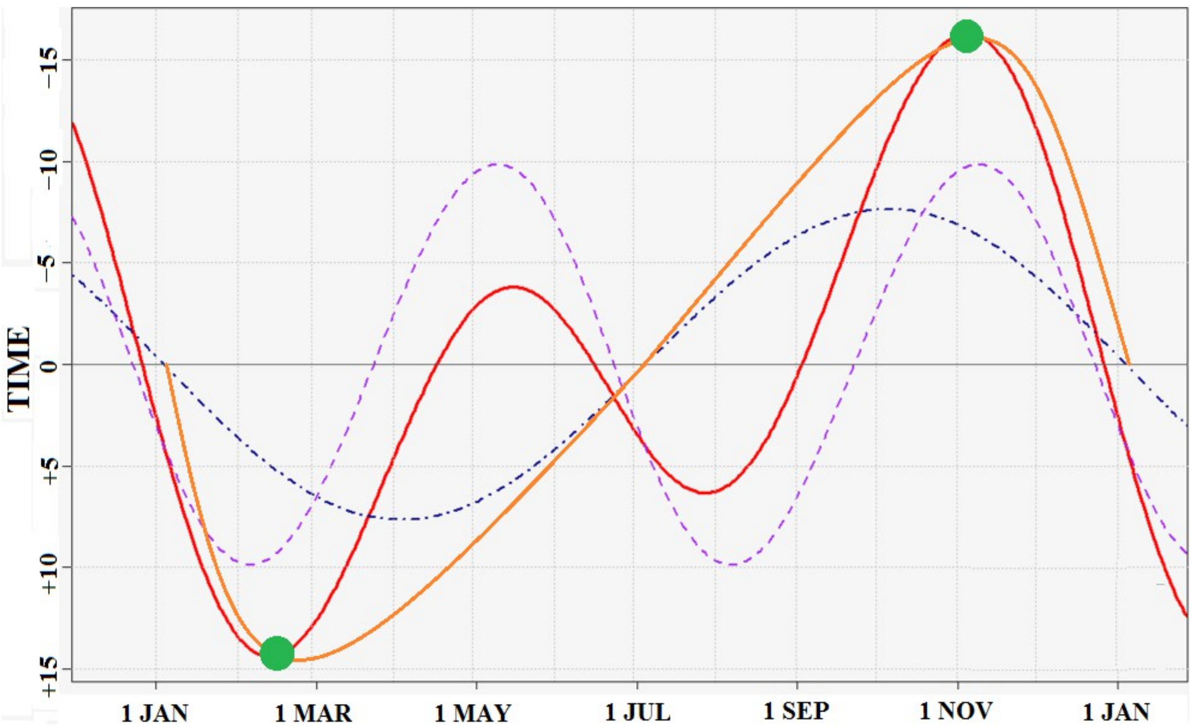


Fig. 3: Positive time values indicate an accurate clock ticking faster than a sundial and negative values indicate the opposite (in minutes throughout a year). Taking into account the Earth's obliquity (mauve dashed curve), eccentricity (blue dash−dot curve), and hysteresis (orange curve), projected variations in $k^2$ (red curve) are hypothesized to be greatest on the dates marked by the green dots. $G > k^2$ is predicted around 12 FEB and $G < k^2$ around 3 NOV (assuming the measurements are made simultaneously and proximal to the equator).

## References


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