On the Cyclic Variations in Newton's Constant

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Abstract. Periodic (5.9 year) oscillations are observed in Newton's gravitational constant G which coincide with length of day (LOD) data obtained from the International Earth Rotation and Reference System (IERS). It is shown that the oscillations in G are dualistic with Gauss' gravitational constant *k* due to variations in the Earth's mean motion during the 5.9 year period. Falsifiable predictions are submitted to test the G/*k* duality hypothesis.

Introduction

Measurements [1, 2] of G oscillate between 6.672×10^{-11} and 6.675×10^{-11} N·(m/kg)^2 with a periodicity of 5.9 years (a difference of 10^{-4} %). Scientists studying this recently discovered (2015) anomaly have found that the variations can be predicted from length of day (LOD) data obtained from IERS [3]. Although the G/LOD correlation is intriguing, it cannot explain the full 10^{-4} % variations observed in G.

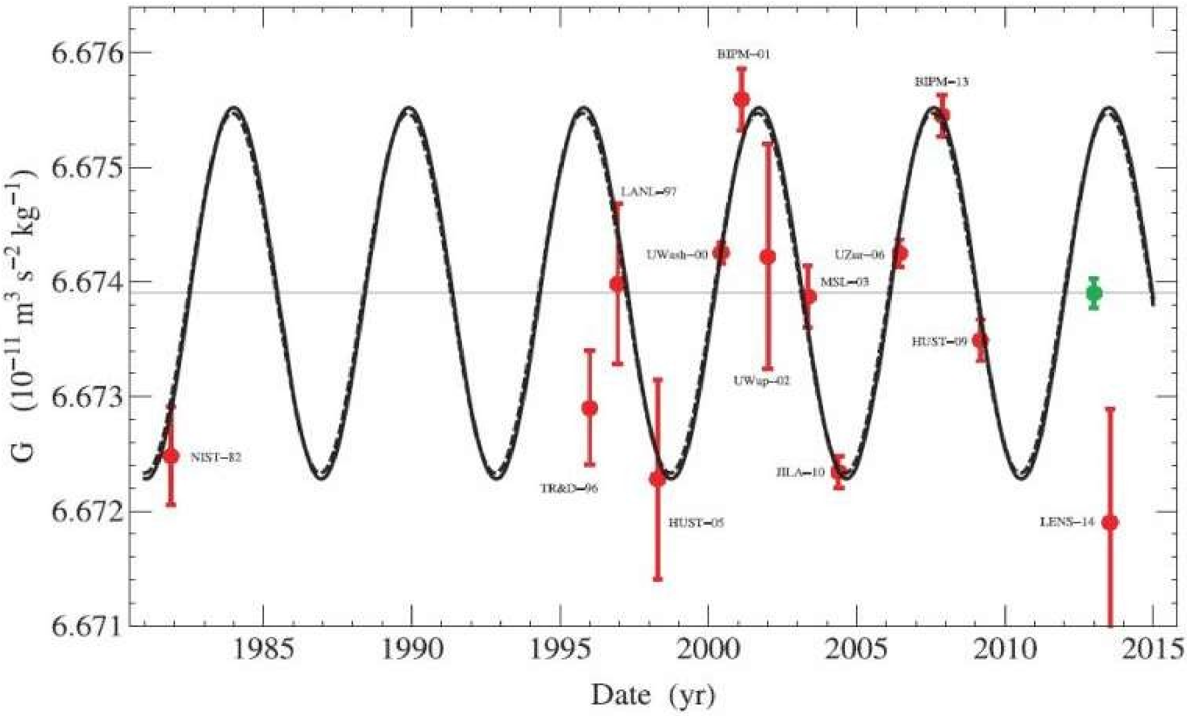

Fig. 1: The solid curve is a CODATA set of G measurements and periodic oscillations in length of day (LOD) measurements are represented by the dashed curve. The green dot, with its one−sigma error bar, is the mean value between the G/LOD measurements. The LENS−13 outlier conducted in 2013 by the MAGIA collaboration was the only measurement which utilized quantum interferometry, while the other 12 measurements were determined macroscopically.

A modern version of Kepler's 3rd law of planetary motion is

$$G(M + m) = \frac{4\pi^2 a^3}{T^2}, \tag{1}$$

where G is the gravitational constant, M is the mass of a primary, m is the mass of a secondary, *a* is the semi-major axis of the orbit, and T is a secondary's sidereal period. Since the mean motion *n* of an orbit is

$$n = \frac{2\pi}{T}, \tag{2}$$

Eq. 1 can be rearranged and condensed into

$$G = \frac{n^2 a^3}{M + m} = \frac{a v^2}{M + m}, \tag{3}$$

where v is the secondary's velocity. Note, however, that this is the same formula for a circular orbit, in which case the semi–major axis a is substituted with the radius r.

The Gaussian gravitational constant k is

$$k = \frac{2\pi}{T\sqrt{M + m}} = \frac{n}{\sqrt{M + m}}. \tag{4}$$

Rearranging Eq. 4 and squaring it,

$$n^2 = k^2(M + m). \tag{5}$$

Substituting n^2 in Eq. 3 with the definition above (where a is in astronomical units),

$$G = k^2 a^3. \tag{6}$$

The Duality Between Newton's Constant and Gauss' Constant

From Kepler's 2nd law we know that a secondary's areal velocity is constant,

$$\frac{d\vec{A}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t) \times \vec{r}(t + \Delta t)}{2\Delta t} = \frac{\vec{r}(t) \times \vec{v}(t)}{2} = constant. \tag{7}$$

The mean motion n in Eq. 3 assumes a constant speed in a circular orbit, which leads to a contradiction between Kepler's 2nd law and the modern version of his 3rd law in Eq. 1.

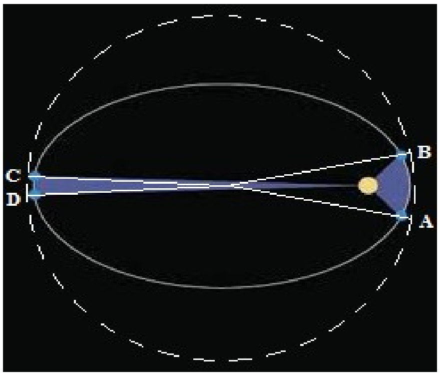

Fig. 2: The mean motion cirumference is represented by the dashed circle. During a secondary's periapsis transit (A, B) it takes less time for the semi–major axis (white) to sweep out a constant sector relative to its areal velocity. During its apoapsis transit (C, D) the opposite is true.

After taking into account the Earth's eccentricity, obliquity, and hysteresis due to its inertia (graphed in the conclusion), it is hypothesized that G will vary according to Kepler's 2nd law by

$$\Delta G = \frac{2\, d\vec{A}\vec{v}(t)}{dt(M+m)} = \frac{\vec{r}(t)\times\vec{v}(t)^2}{M+m} = \frac{h\vec{v}(t)}{M+m}, \tag{8}$$

where h is the Earth's specific relative angular momentum.

From Einstein's general theory of relativity, ΔG in Eq. 8 can be interpreted as a frame−dragging (gravitomagnetic) effect. The gravitomagnetic field B_g of a rotating body is,

$$B_g = \frac{G L_S}{2c^2 r_E^3}, \tag{9}$$

where c is the velocity of light, L_S is the body's spin angular momentum, and r_E is its equitorial radius. Each of these quantities would remain constant in a torque free orbit, but since we know G varies periodically [2], B_g, L, and r_E must also vary. The constant of proportionality between these quantities is $2c^2$. ΔG can therefore be determined relative to a secondary's spin by

$$\Delta G = \frac{\Delta B_g 2c^2 \Delta r_E^3}{\tau_S}, \tag{10}$$

where τ_S is the periodic torque on the Earth's spin ($\tau = \Delta L / t$) due to gravitomagnetic induction. We know from the law of conservation of angular momentum that any change in a secondary's spin must be accompanied by a change in its distance from the center of mass (spin−orbit coupling), hence the definition of ΔG in Eq. 8. Combining Eq. 8 with Eq. 10, the total change in G can be determined by

$$\Delta G = \frac{h v(t) \Delta B_g 2c^2 \Delta r_E^3}{\tau_S (M+m)}. \tag{11}$$

At this point, it is important to note the difference between the Newtonian constant G and the Gaussian constant k since G and k^2 are currently assumed to be equivalent. From Eq. 9 we can see that B_g is directly proportional to L_S. The magnitude of L_S for a ball−shaped body is

$$L_S = I\omega_S, \tag{12}$$

where I is the moment of inertia of the body and ω_S is the angular velocity of its spin,

$$\omega_S = \frac{2\pi}{T_S}, \tag{13}$$

where T_S is the spin period. Notice that this definition of angular velocity is equivalent to the mean motion n given in Eq. 2 when T is substituted with TS. From Eq. 4 and Eq. 6 we can see that the Gaussian costant k is directly proportional to n while being inversely proportional to a^3. On the other hand, the Newtonian constant G is directly proportional to a^3 while being inversely proportional to the square of n. The Newtonian constant is spatially dependent and the Gaussian constant is temporally dependent.

Conclusion

It is hypothesized that k will change inversely with the square root of G when k^2 and G are measured with the same dimensions. This G/k duality hypothesis should be relatively simple to test experimentally by graphing ΔG and Δk^2 simultaneously and superimposing the outcome. It is proposed that the outlier measurement of ΔG conducted with quantum interferometry by LENS−14 in Fig. 1 is accurate, even though it is inverted relative to the Earth's rotation rate. The macroscopic methods are synchronous with the Earth's rotation rate, indicating they are better for k^2 measurements. Spin−orbit coupling could explain the G/LOD synchronicity:

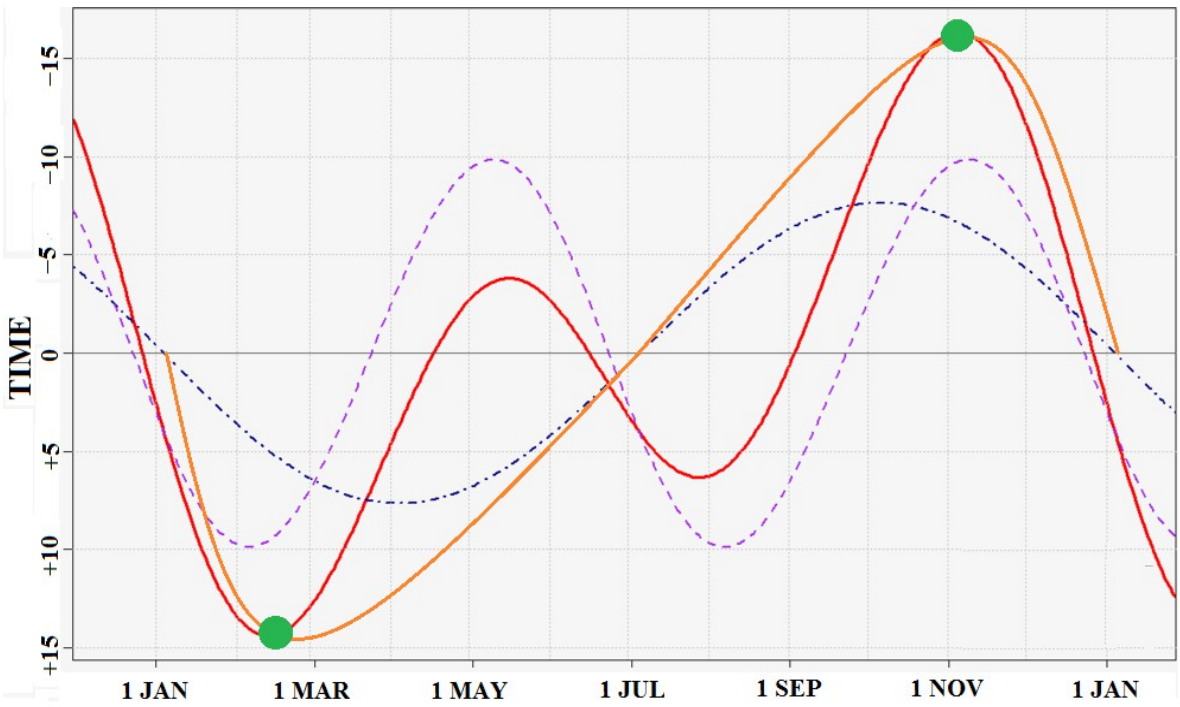

Fig. 3: Positive time values indicate an accurate clock ticking faster than a sundial and negative values indicate the opposite (in minutes throughout a year). Taking into account the Earth's obliquity (mauve dashed curve), eccentricity (blue dash−dot curve), and hysteresis (orange curve), projected variations in k^2 (red curve) are hypothesized to be greatest on the dates marked by the green dots. $G > k^2$ is predicted around 12 FEB and $G < k^2$ around 3 NOV (assuming the measurements are made simultaneously and proximal to the equator).

References

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[3] Holme R. and de Viron 2013 "*Characterization and implications of intradecadal variations in length of day*" *Nature* 499 202