ROOT OF A SUM AS THE SUM OF ROOTS

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Abstract

In the following document shows a particular form of simplify the root of a sum as the sum of roots, through an algebraic expression entitled: "***Camacho Identity".***

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<u>Camacho Identity</u>

"Root of a sum as the sum of roots"

When we work the laws of the exponents and algebraic properties we noticed that there is a way to simplify a root of a product as shown below:

$$\sqrt{a * b * c} = \sqrt{a} * \sqrt{b} * \sqrt{c}$$

But it is impossible to say the same for the root of a sum:

$$\sqrt{a + b + c} \neq \sqrt{a} + \sqrt{b} + \sqrt{c}$$

In the following document would share a particular form of simplify the root of a sum as the sum of roots, through an algebraic expression entitled : ***"Identity of Camacho".***

The identity of Camacho makes use of algebraic resources to synthesize and reach a simple expression that applies to the trinomios perfectly square.

"If we exploit resources and the large arsenal that offers us the algebra, we can find mathematical poems"

- *CAMACHO MEDINA J.J.* -

Be a perfect square triad

$$ax^2 + bx + c = 0$$

$$Where\ a, b, c\ \ are\ real\ numbers.$$

Identity of Camacho

$$\sqrt{ax^2 + bx + c} = \sqrt{ax^2} + \sqrt{c}$$

That can be simplified as:

$$\sqrt{ax^2 + bx + c} = \sqrt{a}x + \sqrt{c}$$

Must meet that the discriminatory:

$$b^2 - 4ac = 0$$

For all 'x' positive.

Example 1:

$$\sqrt{x^2 + 2x + 1}$$

Clearly identify that

$$a = 1, \qquad b = 2, \qquad c = 1$$

$$b^2 - 4ac = 0$$

$$2^2 - 4(1)(1) = 0$$

Then we have:

$$\sqrt{x^2 + 2x + 1} = \sqrt{x^2} + \sqrt{1} = x + 1$$

Example 2:

$$\sqrt{64x^2 + 32x + 4}$$

Clearly identify that

$$a = 64, \qquad b = 32, \qquad c = 4$$

$$b^2 - 4ac = 0$$

$$32^2 - 4(64)(4) = 0$$

Then we have:

$$\sqrt{64x^2 + 32x + 4} = \sqrt{64x^2} + \sqrt{4} = 8x + 2$$

Applications

Suppose that we want to solve the derivative and integral of a function that contains a perfect square triad: $\sqrt{49x^2+84x+36}$. es a common practice to ignore the algebraic resources on the part of the students and even the same Teacher, this leads them to have access to the techniques of derivation and integration Looking for a formula of salvation according to the expression.

If we apply to our identity, we will observe that the process of derivation and integration will be more simple:

$$\sqrt{49x^2+84x+36}$$

$$a = 49\,, \qquad b = 84\,, \qquad c = 36$$

$$b^2 - 4ac = 0$$

$$84^2 - 4(49)(36) = 0$$

Then we have:

$$\sqrt{49x^2+84x+36} = \sqrt{49x^2} + \sqrt{36} = 7x + 6$$

$$\boldsymbol{Dx}\sqrt{49x^2+84x+36} = \boldsymbol{Dx(7x+6)} = \mathbf{7}$$

$$\int \sqrt{49x^2+84x+36}\,\boldsymbol{dx} = \int \boldsymbol{(7x+6)dx} = \boldsymbol{\frac{7x^2}{2}} + \boldsymbol{6x} + \boldsymbol{C}$$

For another type of expressions in relation to the triad perfect square:

Be a perfect square trinomial

$$ax^n + bx^{\frac{n}{2}} + c = 0$$

$$where\ a, b, c\ \ are\ real\ numbers\ and\ \ n\ \ is\ even > 2.$$

We make a change of variable and replace:

$$u = x^n, v = x^{\frac{n}{2}}$$

$$au + bv + c = 0$$

The identity we can generalize as:

$$\sqrt{au + bv + c} = \sqrt{au} + \sqrt{c}$$

If is met $b^2 - 4ac = 0$, for all 'x' positive.

Be a perfect square trinomial of the form:

$$ax^n + bx^{\frac{n}{2}}y^{\frac{n}{2}} + cy^n = 0$$

$$where\ a, b, c\ are\ real\ numbers\ and\ \ n\ is\ even > 2.$$

We make a change of variable and replace:

$$u = x^n, v = x^{\frac{n}{2}}y^{\frac{n}{2}}, w = y^n$$

$$au + bv + cw = 0$$

The identity we can generalize as:

$$\sqrt{au + bv + cw} = \sqrt{au} + \sqrt{cw}$$

If is met $b^2 - 4ac = 0$, for all 'x' positive.

Example 3:

$$\sqrt{x^6 + 2x^3 + 1}$$

Clearly identify that

$a = 1,\ b = 2,\ c = 1$,

$$u = x^6, \qquad v = x^3$$

Checked the discriminatory:

$$b^2 - 4ac = 0$$

$$2^2 - 4(1)(1) = 0$$

$$\sqrt{x^6 + 2x^3 + 1} = \sqrt{x^6} + \sqrt{1} = x^3 + 1$$

Example 4:

$$\sqrt{9x^2 + 18xy + 9y^2}$$

Clearly identify that

$a = 9,\ b = 18,\ c = 9$,

$$u = x^2, \qquad v = xy, \qquad w = y^2$$

Checked the discriminatory:

$$b^2 - 4ac = 0$$

$18^2 - 4(9)(9) = 0$; $\sqrt{9x^2 + 18xy + 9y^2} = \sqrt{9x^2} + \sqrt{9y^2} = 3x + 3y$

Demonstration

Why does the identity?

Be a perfect square trinomial

$$ax^2 + bx + c = 0$$

$$where\ a, b, c, x\ \ are\ areal\ numbers$$

This triad can develop as a binomial squared

$$Si\ \ se\ satisface\ \ que\ el\ discriminante\ \ b^2 - 4ac = 0$$

$$\left(\sqrt{ax^2} + \sqrt{c}\right)^2 = \sqrt{ax^2}^2 + \left(2\sqrt{acx^2}\right) + \sqrt{c}^2$$

$$= ax^2 + 2\sqrt{acx^2} + c$$

$$= ax^2 + 2\sqrt{ac}\,x + c$$

$$Como\ \ b^2 - 4ac = 0 \rightarrow b = \sqrt{4ac} \rightarrow b = 2\sqrt{ac}$$

$$= ax^2 + bx + c$$

Then we apply square root to the triad perfect square expressed as quadratic binomial and by law of exponents canceled root and square power:

$$\sqrt{(\sqrt{ax^2} + \sqrt{c})^2} = \sqrt{ax^2} + \sqrt{c}$$

Therefore :

$$\boxed{\sqrt{ax^2 + bx + c} = \sqrt{ax^2} + \sqrt{c}}$$

If you met the discriminatory $b^2 - 4ac = 0$

For all 'x' positive.

Other Identities Related

We can get other variants of ***the identity of Camacho*** that can serve as a great help in the resolution of mathematical exercises.

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$Donde\ a, b, c\ \ son\ positivos\ \ Reales.$$

$$\boldsymbol{If\ discriminant\ \ \Delta = 0}$$

$$\sqrt{a^2 + b^2 + c^2 + 2ab + 2ac + 2bc} = \sqrt{a^2} + \sqrt{b^2} + \sqrt{c^2}$$

$$\boldsymbol{we\ simplify:\ \ a + b + c}$$

Demonstration:

$$Como\ \ (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$Where\ a, b, c\ \ are\ positive\ real\ numbers\ and\ \ \Delta = \mathbf{0}.$$

Apply the laws of exponents , we have :

$$\sqrt{a^2 + b^2 + c^2 + 2ab + 2ac + 2bc} = \sqrt{(a+b+c)^2} = a + b + c$$

Cubic perfect pair polinomyal

$$ax^3 + bx^2 + cx + d = 0$$

$$Where\ \ \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{x}\ \ are\ positive\ real\ numbers.$$

$$\sqrt[3]{ax^3 + bx^2 + cx + d} = \sqrt[3]{\left(\sqrt[3]{ax^3} + \sqrt[3]{d}\right)^3} = \sqrt[3]{ax^3} + \sqrt[3]{d} = \sqrt[3]{a}x + \sqrt[3]{d}$$

$Sí\ \Delta = \mathbf{0}$

CAMACHO - OUROBOROS IDENTITY

Be **a, b, c,** and **x** positive real numbers

$ax^2 + bx + c$: Triad Perfect Square

$$\sqrt{ax^2 - c + \frac{b}{\sqrt{a}}(\sqrt{ax^2 + bx + c})} = \sqrt{ax^2} + \sqrt{c}$$

If $\quad b^2 - 4ac = 0$

Example:

$$\sqrt{9x^2 - 16 + \frac{24}{\sqrt{9}}(\sqrt{9x^2 + 24x + 16})} = \sqrt{9x^2} + \sqrt{16} = 3x + 4$$

Verification:

$$b^2 - 4ac = 0, \qquad 24^2 - 4(9)(16) = 0$$

$$= \sqrt{9x^2 - 16 + \frac{24}{3}(3x + 4)}$$

$$= \sqrt{9x^2 - 16 + 8(3x + 4)}$$

$$= \sqrt{9x^2 - 16 + 24x + 32}$$

$$= \sqrt{9x^2 + 24x + 16} = \sqrt{9x^2} + \sqrt{16} = 3x + 4$$

APPLICATIONS OF THE IDENTITY CAMACHO - OUROBOROS

Imagine that is intended to solve the derivative and integral of the function :

$$f(x) = \sqrt{(2x+1)(2x-1) + 2(\sqrt{4x^2+4x+1})}$$

We note two important points:

- The function can be expressed in the format of the identity Ouroboros: $\sqrt{4x^2 - 1 + \frac{4}{\sqrt{4}}(\sqrt{4x^2+4x+1})} \rightarrow$ $\sqrt{ax^2 - c + \frac{b}{\sqrt{a}}(\sqrt{ax^2+bx+c})}$
- The expression $4x^2 + 4x + 1$ is a perfect square triad.

Therefore :

$$\sqrt{4x^2 - 1 + \frac{4}{\sqrt{4}}(\sqrt{4x^2+4x+1})} = \sqrt{4x^2} + \sqrt{1} = 2x + 1$$

DERIVATIVE: $f'(x) = 2$

INTEGRAL: $\int f(x)\,dx = \int 2x + 1\,dx = x^2 + x + K$

For all 'x' positive.

DEMONSTRATION OF THE IDENTITY CAMACHO - OUROBOROS

$$\sqrt{ax^2 - c + \frac{b}{\sqrt{a}}(\sqrt{ax^2 + bx + c})} = \sqrt{ax^2} + \sqrt{c}$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{ax^2 + bx + c}\right) = (\sqrt{ax^2} + \sqrt{c})^2$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{ax^2 + bx + c}\right) = ax^2 + 2\sqrt{ax^2c} + c$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{ax^2 + bx + c}\right) = ax^2 + 2\sqrt{ac}x + c$$

$$Como \quad b^2 - 4ac = 0 \to b = \sqrt{4ac} \to b = 2\sqrt{ac}\,,\ a = \frac{b^2}{4c}\ ,\ c = \frac{b^2}{4a}$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{ax^2 + bx + c}\right) = ax^2 + bx + c$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{(\sqrt{ax^2} + \sqrt{c})^2}\right) = ax^2 + bx + c$$

$$ax^2 - c + \frac{b}{\sqrt{a}}\left(\sqrt{ax^2} + \sqrt{c}\right) = ax^2 + bx + c$$

$$\frac{b}{\sqrt{a}}\left(\sqrt{ax^2} + \sqrt{c}\right) = bx + 2c$$

$$\frac{b}{\sqrt{a}}\left(\sqrt{a}x + \sqrt{c}\right) = bx + 2c$$

$$\frac{b\sqrt{a}x}{\sqrt{a}} + \frac{b\sqrt{c}}{\sqrt{a}} = bx + 2c$$

$$bx + \frac{b\sqrt{c}}{\sqrt{a}} = bx + 2c$$

$$\frac{b\sqrt{c}}{\sqrt{a}} = 2c$$

$$Como\ a = \frac{b^2}{4c} \to \sqrt{a} = \frac{b}{2\sqrt{c}}$$

$$\frac{\frac{b\sqrt{c}}{1}}{\frac{b}{2\sqrt{c}}} = 2c$$

$$\frac{2b\sqrt{c^2}}{b} = 2c\ ;\ \frac{2bc}{b} = 2c$$

$\boldsymbol{2c = 2c} \quad$ ***Q.E.D.***

This allows us to even deduct a formula for the derivative and the integral of functions whose structure resembles the *Identity Camacho Ouroboros:*

FORMULA OF DERIVATION CAMACHO - OUROBOROS

Be ***a, b, c*** *and* ***x positive.***

$$ax^2 + bx + c\ ,\ \ b^2 - 4ac = 0 : Square\ perfect\ Trinomial$$

$$\boldsymbol{Dx\sqrt{ax^2 - c + \frac{b}{\sqrt{a}}(\sqrt{ax^2 + bx + c})} = \sqrt{a}}$$

FORMULA OF INTEGRATION CAMACHO - OUROBOROS

For the whole of ***a, b, c*** *and* ***x positive.***

$$ax^2 + bx + c\ ,\ \ b^2 - 4ac = 0 : Square\ perfect\ Trinomial$$

$$\boldsymbol{\int \sqrt{ax^2 - c + \frac{b}{\sqrt{a}}(\sqrt{ax^2 + bx + c})}\, dx = \frac{\sqrt{a}}{2}x^2 + \sqrt{c}x + K}$$