

Note on Uniqueness Solutions of Navier-Stokes Equations

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Remembering the need of impose the boundary condition $u(x, t) = 0$ at infinity to ensure uniqueness solutions to the Navier-Stokes equations.

Recently I wrote a paper named "A Naive Solution for Navier-Stokes Equations"^[1] where I concluded that it is possible does not exist the uniqueness of solutions in these equations for $n = 3$, even with all terms and for any $t > 0$.

This conclusion inhibited me to publish officially my other article "Three Examples of Unbounded Energy for $t > 0$ "^[2], also a very important paper.

This distressful and no way out situation disappears when we impose the boundary condition $\lim_{|x| \rightarrow \infty} u(x, t) = 0$, which guarantees the desired uniqueness of solutions at least in a finite and not null time interval $[0, T]$. Possibly others boundary conditions also arrive at the uniqueness, but null velocity at infinite is according Millenium Problem^[3].

Thus, and fortunately, with some changes in the expressions of external forces, pressures and velocities used in [2] is possible to establish again the breakdown solution in [3], due to occurrence of unbounded energy $\int_{\mathbb{R}^3} |u|^2 dx \rightarrow \infty$ in $t > 0$. In special, a general example (excluding identically null velocity v), for $1 \leq i \leq 3$ and $\nabla \cdot u = \nabla \cdot u^0 = \nabla \cdot v = 0$, is

$$u_i(x, t) = u_i^0(x)e^{-t} + v_i(y)(1 - e^{-t}), \quad u, x \in \mathbb{R}^3, \quad v, y \in \mathbb{R}^m, \quad m = 1, 2,$$

$$u_i^0(x) \in S(\mathbb{R}^3), \quad v_i(y) \in S(\mathbb{R}^m),$$

$$p \in S(\mathbb{R}^3 \times [0, \infty)),$$

$$f_i = \frac{\partial p}{\partial x_i} + \frac{\partial u_i}{\partial t} + \sum_{j=1}^3 u_j \frac{\partial u_i}{\partial x_j} - \nu \nabla^2 u_i.$$

It is appropriate to review in detail the uniqueness proofs (Leray, Ladyzhenskaya, Temam, Kreiss and Lorenz or even more) and in which conditions they exist. Really, I am sorry. It's like having to start all over again. But it is necessary.

June-30-2016

References

- [1] Godoi, Valdir M.S., *A Naive Solution for Navier-Stokes Equations*, in <http://vixra.org/abs/1604.0107> (2016).
- [2] Godoi, Valdir M.S., *Three Examples of Unbounded Energy for $t > 0$* , in <http://vixra.org/abs/1602.0246> (2016).
- [3] Fefferman, Charles L., *Existence and Smoothness of the Navier-Stokes Equation*, in <http://www.claymath.org/sites/default/files/navierstokes.pdf> (2000).