Note on Uniqueness Solutions of Navier-Stokes Equations

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Remembering the need of impose the boundary condition $u(x,t) = 0$ at infinity to ensure uniqueness solutions to the Navier-Stokes equations.

Recently I wrote a paper named "A Naive Solution for Navier-Stokes Equations"[1] where I concluded that it is possible does not exist the uniqueness of solutions in these equations for $n = 3$, even with all terms and for any $t > 0$.

This conclusion inhibited me to publish officially my other article "Three Examples of Unbounded Energy for $t > 0$"[2], also a very important paper.

This distressful and no way out situation disappears when we impose the boundary condition $\lim_{|x|\to\infty} u(x,t) = 0$, which guarantees the desired uniqueness of solutions at least in a finite and not null time interval $[0,T]$. Possibly others boundary conditions also arrive at the uniqueness, but null velocity at infinite is according Millenium Problem[3].

Thus, and fortunately, with some changes in the expressions of external forces, pressures and velocities used in [2] is possible to establish again the breakdown solution in [3], due to occurrence of unbounded energy $\int_{\mathbb{R}^3} |u|^2 dx \to \infty$ in $t > 0$. In special, a general example, for $1 \le i \le 3$ and $\nabla \cdot u = \nabla \cdot u^0 = \nabla \cdot v = 0$, is

$$u_i(x,t) = u_i^0(x)e^{-t} + v_i(x)(1 - e^{-t}),\ u, u^0, v, x \in \mathbb{R}^3,$$

$$u_i^0(x) \in S(\mathbb{R}^3),\ v_i(x) \in C^\infty(\mathbb{R}^3),\ v \notin L^2(\mathbb{R}^3),\ \lim_{|x|\to\infty} v(x) = 0,$$

$$p \in C^\infty\big(\mathbb{R}^3 \times [0,\infty)\big),$$

$$f_i = \left(\frac{\partial p}{\partial x_i} + \frac{\partial u_i}{\partial t} + \sum_{j=1}^{3} u_j \frac{\partial u_i}{\partial x_j} - \nu\, \nabla^2 u_i\right) \in S\big(\mathbb{R}^3 \times [0,\infty)\big).$$

It is appropriate to review in detail the uniqueness proofs (Leray, Ladyzhenskaya, Temam, Kreiss and Lorentz or even more) and in which conditions they exist. Really, I am sorry. It's like having to start all over again. But it is necessary.

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References

[1] Godoi, Valdir M.S., *A Naive Solution for Navier-Stokes Equations*, in http://vixra.org/abs/1604.0107 (2016).

[2] Godoi, Valdir M.S., *Three Examples of Unbounded Energy for* $t > 0$, in http://vixra.org/abs/1602.0246 (2016).

[3] Fefferman, Charles L., *Existence and Smoothness of the Navier-Stokes Equation*, in http://www.claymath.org/sites/default/files/navierstokes.pdf (2000).