Note on Uniqueness Solutions of Navier-Stokes Equations

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Remembering the need of impose the boundary condition $u(x,t)=0$ at infinity to ensure uniqueness solutions to the Navier-Stokes equations.

Recently I wrote a paper named "A Naive Solution for Navier-Stokes Equations"[1] where I concluded that it is possible does not exist the uniqueness of solutions in these equations for $n=3$, even with all terms and for any $t>0$.

This conclusion inhibited me to publish officially my other article "Three Examples of Unbounded Energy for $t>0$"[2], also a very important paper.

This distressful and no way out situation disappears when we impose the boundary condition $\lim_{|x|\to\infty} u(x,t)=0$, which guarantees the desired uniqueness of solutions at least in a finite and not null time interval $[0,T]$. Possibly others boundary conditions also arrive at the uniqueness, but null velocity at infinite may imply a minimum volume of $|u|^2$ and the respective total kinetic energy.

Thus is necessary do some changes in the expressions of external forces, pressures and velocities used in [2] to establish again the breakdown solution in [3], due to occurrence of unbounded energy $\int_{\mathbb{R}^3}|u|^2dx\to\infty$ in $t>0$. In special, a general example, for $1\le i\le 3$ and $\nabla\cdot u=\nabla\cdot u^0=\nabla\cdot v=0$, is

$$u_i(x,t)=u_i^0(x)e^{-t}+v_i(x)e^{-t}(1-e^{-t}),\ u,u^0,v,x\in\mathbb{R}^3,$$

$$u_i^0(x)\in S(\mathbb{R}^3),\ v_i(x)\in C^\infty(\mathbb{R}^3),\ v\notin L^2(\mathbb{R}^3),\ \lim_{|x|\to\infty} v(x)=0,$$

$$p\in C^\infty\big(\mathbb{R}^3\times[0,\infty)\big),$$

$$f_i=\left(\frac{\partial p}{\partial x_i}+\frac{\partial u_i}{\partial t}+\sum_{j=1}^3 u_j\frac{\partial u_i}{\partial x_j}-\nu\,\nabla^2u_i\right)\in S\big(\mathbb{R}^3\times[0,\infty)\big).$$

The conditions (4) for initial velocity and (5) for external force, conforming description given in [3],

(4) $|\partial_x^\alpha u^0(x)|\le C_{\alpha K}(1+|x|)^{-K}:\mathbb{R}^3,\ \forall\alpha,K$

(5) $|\partial_x^\alpha\partial_t^m f(x,t)|\le C_{\alpha mK}(1+|x|+t)^{-K}:\mathbb{R}^3\times[0,\infty)\ ,\forall\alpha,m,K$

is a kind of *straitjacket,* and for me do not seem good conditions to make possible physically reasonable solutions, rather only restricts the solutions to a very limited and very artificial set of possibilities. If it were possible to the external force be in the set

$C^\infty(\mathbb{R}^3 \times [0,\infty))$, such as the velocity and pressure in $t > 0$, even being only limited functions and equals zero as $|x| \to \infty$, instead Schwartz Space, the possible solutions will be much more interesting and realistic.

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References

[1] Godoi, Valdir M.S., *A Naive Solution for Navier-Stokes Equations*, in http://vixra.org/abs/1604.0107 (2016).

[2] Godoi, Valdir M.S., *Three Examples of Unbounded Energy for* $t > 0$, in http://vixra.org/abs/1602.0246 (2016).

[3] Fefferman, Charles L., *Existence and Smoothness of the Navier-Stokes Equation*, in http://www.claymath.org/sites/default/files/navierstokes.pdf (2000).