# Thou shalt construct in a modular way 

By J.A.J. van Leunen.
Retired physicist
16 August 2016


#### Abstract

Look around and you become easily convinced from the fact that all discrete objects are either modules or modular systems. With other words, the creator of this universe must be a modular designer. His motto is "Construct in a modular way". However, also non-discrete items exist. Universe contains continuums and these continuums appear to relate to the discrete objects. Further, we as observers of these facts, want to place everything into an appropriate model, such that we can comprehend our environment.


If you think, then think twice.
In any case, think frankly.

## Observation

The foundation of physical reality must necessarily be very simple and as a consequence its structure must be easily comprehensible by skilled scientists. So, quite probable the structure was long ago discovered and added as a part to mathematics. As a consequence the best way to investigate the foundation of reality is to use mathematical test models. The rediscovery of this structure as a foundation of reality is a complicated task because extending this foundation must automatically lead to a higher level of the structure of physical reality that shows more features that can be recognized as features of physical reality. The lower levels of the structure of physical reality must leave some signatures that are visible in many facets of the universe.

Several of such signatures exist. For example the fact that all discrete items in universe are either modules or they are modular systems is probably a signature of the foundation or of the lower levels of the structure of physical reality. Considering this observation as a signature requires the investigation of the peculiarities of modular design and modular construction. That analysis learns that relations between modules and relations that are relevant inside modules or modular systems play a major role. Especially the relations that determine that an object is a module or is part of a module are important. It is quite probable that the foundation of physical reality is a relational structure. This is a set in which the relations that can exist between the elements are restricted in a well-defined way.

About eighty years ago a relational structure was discovered, which was thought to play a significant role in the description of physical reality by physical theories. The discoverers of the relational structure called it "quantum logic". The mathematicians used a more technical name and called it "orthomodular lattice". In their introductory paper the discoverers showed that the set of closed subspaces of the somewhat earlier discovered Hilbert space has exactly the relational structure of an orthomodular lattice. With other words this Hilbert space is a realization of the orthomodular lattice. The question that arises now is whether this Hilbert space is also a realization of modules and modular systems. This question has a positive answer but the argumentation requires a deep dive into advanced mathematics.

More signatures exist but in this paper we first focus on this one.

## Modular construction

Diving deep into the fundamental structure of physical reality requires a deep dive into advanced mathematics. Usually this goes together with formulas or other descriptions that are incomprehensible to most people. The nice thing about this situation is that the deepest foundation of reality must be rather simple and as a consequence it can be described in a simple way and without any formulas. For example if the observed signature characterizes physical reality, then the most fundamental and most influential law of physical reality can be formulated in the form of a commandment:
"THOU SHALT CONSTRUCT IN A MODULAR WAY"
This law is the direct consequence of the structure of the deepest foundation. That foundation restricts the types of relations that may play a role in physical reality. That structure does not yet contain numbers. Therefore it does not yet contain notions such as location and time.
Modular construction acts very economic with its resources and the law thus includes an important lesson. "DO NOT SPOIL RESOURCES!"

## Modular design

Understanding that the above statements indeed concern the deepest foundation of physics requires deep mathematical insight. Alternatively it requests belief from those that cannot (yet) understand this methodology. On the other hand intuition quickly leads to trust and acceptance that the above major law must rule our existence! Modular design has the intention to keep the relational structure of the target system as simple as is possible.

Modular design is a complicated concept. Successful modular construction involves the standardization of module types and it involves the encapsulation of modules such that internal relations are hidden from the outside. Systems become complicated when many relations exist inside that system, which must be reckoned when the system is configured, operated or changed. The reduction in relational complexity plays a significant role during system configuration. The ability to configure modular systems relies heavily on the ability to couple modules and on the capability to let these modules operate in concordance.

The modular design method becomes very powerful when modules can be constructed from lower level modules. The standardization of modules enables reuse and may generate type communities. The success of a type community may depend on other type communities.

An important category of modules are the elementary modules. This are modules, which are themselves not constructed from other modules. These modules must be generated by a mechanism that constructs these elementary modules. Each elementary module type owns a private generation mechanism.

Another category are modular systems. Modular systems and modular subsystems are conglomerates of connected modules. The constituting modules are bonded. Often the modules are coupled via interfaces that channel the information paths that are used by the relations. Modular subsystems can act as modules and often they can also act as independent modular systems. The hiding of internal relations inside a module eases the configuration of modular (sub)systems. In complicated systems, modular system generation can be several orders of magnitude more efficient than the generation of equivalent monoliths.

The generation of modules and the configuration of modular (sub)systems can be performed in a stochastic or in an intelligent way. Stochastic (sub)system generation takes more resources and requires more trials than intelligent (sub)system generation.

If all discrete objects are either modules or modular systems, then intelligent (sub)system generation must wait for the arrival of intelligent modular systems.
Intelligent species can take care of the success of their own type. This includes the care about the welfare of the types on which its type depends. Thus for intelligent modular systems, modularization also includes the lesson "TAKE CARE OF THE TYPES ON WHICH YOU DEPEND".

In reality the elementary modules appear to be generated by mechanisms that apply stochastic processes. In most cases system configuration occurs in a trial and error fashion. Only when intelligent species are present that can control system configuration will intelligent design occasionally manage the system configuration and binding process. Thus in the first phase, stochastic evolution will represent the modular system configuration drive. Due to restricted speed of information transfer, intelligent design will only occur at isolated locations. On those locations intelligent species must be present.

## Mathematical model

Now we treat some aspects that involve advanced mathematics. We do that in a descriptive way.

In a modular system relations play a major role. The success of the described modular construction methodology depends on a particular relational structure that characterizes modular systems.

In 1936 the discoverers of the orthomodular lattice published their discovery in a paper in which they called that lattice "quantum logic". Garret Birkhoff was an expert in lattice theory and John von Neumann was a broadly oriented scientist that was especially interested in quantum physics. "quantum logic" is a strange name because in the same paper the duo showed that the set of closed subspaces of a separable Hilbert spaces has exactly the relational structure of this orthomodular lattice. The name "quantum logic" is only comprehensible, because the relational structure of this lattice is quite similar to the relational structure of classical logic and the elements of classical logic are logical propositions. It is not likely that the elements of the orthomodular lattice can be represented by logical propositions, but this immediately creates the question what kind of other objects the elements of the orthomodular lattice represent. The answer is that they represent storage locations of dynamic geometric data. This will be elucidated later.

## Hilbert space

The orthomodular lattice extends naturally into a separable Hilbert space. Separable Hilbert spaces are mathematical constructs that act as storage media for dynamic geometric data. Quantum physicists use Hilbert spaces as a base model in which they perform their quantum physical modelling.

Hilbert spaces are linear vector spaces and each pair of Hilbert vectors owns an inner product that represents a number, which is a member of a division ring. Hilbert spaces can only cope with number systems that are division rings. This means that every non-zero element of that number system owns a unique inverse. Only three suitable division rings exist. These are the real numbers, the complex numbers and the quaternions. Also their rational subsets form division rings. The quaternions form the most elaborate division ring and comprise the other division rings. The inner product of two perpendicular vectors equals zero.

Operators describe how Hilbert spaces map into other Hilbert spaces and can describe how Hilbert spaces map onto themselves. In the latter case, the inner product describes the relation between a Hilbert vector and its map. If the vector is mapped onto itself then the inner product adds an eigenvalue to that vector and the vector is called an eigenvector. Thus eigenvalues must be members of a division ring. If two eigenvalues differ, then their eigenvectors are perpendicular.

## Reference operators

Number systems that are division rings can be used to define a category of operators that we will call reference operators. The rational values of the number system are used to enumerate the members of an orthonormal base of the Hilbert space. The reference operator connects the enumerator with the base vector and in this way they become eigenvalue and eigenvector. Each reference operator implements a parameter space that is defined by its eigenspace.

Number systems exist in several versions that differ in the way that they are ordered. For example multidimensional number systems can be ordered by a selected Cartesian coordinate system and subsequently the result can be ordered by a polar coordinate system. The imaginary part of the quaternionic number system represents a three dimensional space that can be ordered in eight independent ways by a Cartesian coordinate system. The ordering affects the arithmetic properties of the quaternions. Left handed multiplying quaternions exist and right handed multiplying quaternions exist.

The Hilbert space can harbor multiple reference operators and in that way it can harbor multiple parameter spaces.

A subset of the reference operators can be considered to be part of the household of the Hilbert space. For example a special reference operator plays the role of the generator of the background parameter space. Other parameter spaces can float with respect to the background parameter space and they can float with respect to each other.

## Defined operators

By starting from a selected reference operator, it is possible to define a category of defined normal operators that use a continuous function in order to replace the parameter value by the function value and connect this value with the corresponding eigenvector of the reference operator. In fact the reference operators are special versions of the defined operators for which the defining functions use the parameter value as the function value. This procedure is very powerful and merges Hilbert space operator technology with function theory.

## Modules as subspaces

According to the discoverers of the orthomodular lattice, the elements of the lattice can be represented as closed subspaces of a separable Hilbert space. It also has sense to consider a subset of these elements as representatives of modules or modular systems. Thus, not every closed subspace of a separable Hilbert space represents a module or modular system, but every module and every modular system is represented by a closed subspace of the separable Hilbert space. Compared to general closed subspaces of the Hilbert space, will modules and modular systems have extra characteristics.

## Dynamic geometric data

Quaternions can be interpreted as a combination of a scalar progression value and a three dimensional spatial location. The scalar part is the real part of the quaternion and the vector part is the imaginary part. Quaternions can represent other subjects, but in this paper the representation of dynamic geometric data plays a major role.
The scanning vane
If a certain reference operator, such as the background reference operator, is singled out, then a special subspace of the Hilbert space can be specified that represents the current static status quo of the Hilbert space. In the subspace all eigenvalues of the selected reference operator share the same real part. This specification divides the Hilbert space into three subspaces. The first subspace represents the past history. The third part represents the future. If the selected real value, which
represents progression is allowed to increase, then the second subspace represents a vane that scans over the Hilbert space.

## Elementary modules

An elementary module cannot be represented by a single Hilbert vector, because that single vector can on the utmost correspond to a static geometric location and from reality we know that modules possess a dynamic geometric location and that fact also holds for the elementary modules. However, elementary modules cannot be split into other modules. Thus the subspaces that represent elementary modules must have multiple dimensions. Still it is possible that at each progression instant each module represents exactly one spatial location. This is a very special condition, but we postulate that this special condition is valid for all elementary modules. As a consequence of this postulate the vane contains representatives of elementary modules that are one-dimensional subspaces. They cannot be split into lower level modules.
The postulate forbids that two elementary modules at the same progression instant take the same geometric location.
The vane contains many subspaces that do not contain Hilbert vectors that represent an elementary module. These subspaces are representing empty space.
Germ operators
The elementary modules are represented by a new category of operators that differ from reference operators and that differ from defined operators but that describe the dynamics of elementary modules. The dynamic location of elementary modules hops as a function of progression. After a while the hops form a swarm and both the hopping path and the swarm represent the elementary module. The operators will be called germ operators. The germ operators are controlled by mechanisms that apply stochastic processes for the generation of the dynamic locations. The germ operator uses its own private reference operator. This means that the elementary modules reside on their own platform, which apply its own private parameter space that may float with respect to the selected background parameter space.

## Hopping paths and swarms

After generation, the dynamic locations will be ordered with respect to the real value of the quaternion that represents the dynamic location. After ordering of the progression values the elementary module appears to walk along a hopping path and the landing positions form a location swarm. An uncontrolled generation would produce an arbitrary hopping path and a chaotic hop landing location swarm. The mechanism ensures that a coherent swarm is generated.

This means that elementary modules are represented by closed subspaces of a Hilbert space that may have a huge dimension. However, at a single progression instant, the elementary modules are represented by a subspace that is spanned by a single Hilbert vector. Thus modules and modular systems are represented by a subset of the orthomodular lattice.
If the swarm contains a large number of hop landing locations, then its geometric center will move in a much less chaotic way. A large number of elements also means an equivalently long regeneration cycle of the swarm. Increasing the number of hop landings will increase the inertness of the swarm.

## Mechanisms

From reality we know that the hopping path is not an arbitrary path and the location swarm is not a chaotic collection. Instead the swarm forms a coherent set of locations that can be characterized by a rather continuous location density distribution.

From physics we know that elementary particles own a wave function and the squared modulus of that wave function forms a continuous probability density distribution, which can be interpreted as a location density distribution of a point-like object. The location density distribution owns a Fourier transform and as a consequence the swarm owns a displacement generator. This means that at first
approximation the swarm can be considered to move as one unit. Thus the swarm is a coherent, rather smoothly moving object, which represents the violent stochastic hopping of a point-like object. For a large part this is due to the fact that the swarm contains a huge number of locations that is refreshed in a cyclic fashion.

The fact that at every progression instant the swarm owns a Fourier transform means that at every progression instant the swarm can be interpreted as a wave package. Wave packages can represent interference patterns, thus they can simulate wave behavior. The problem is that moving wave packages tend to disperse. The swarm does not suffer that problem because at every progression instant the wave package is regenerated. The result is that the elementary module shows wave behavior and at the same time it shows particle behavior. When it is detected it is caught at the precise location where it was at this progression instant.

The Hilbert space is nothing more and nothing less than a structured storage medium for dynamic geometric data. It does not contain functionality that ensures the coherent dynamic behavior of the location swarms. Dedicated mechanisms, which do not belong to the household of the Hilbert space fill the eigenspaces of the stochastic germ operators that control the elementary modules. The hopping path only stops when the elementary module is "detected" and the controlling mechanism changes to a different mode of operation.

## Dynamic model

We did construct a vane that splits the Hilbert space such that all elementary module eigenvalues that have a selected real value have the corresponding eigenvector inside the vane. The vane splits the Hilbert space in an historic part, the vane itself and a future part. The vane then represents a static status quo that corresponds to the current state of the universe.

This represents an interesting possibility. The Hilbert space can be seen as a storage medium that contains a repository of all historic, present and future data. Or it can be interpreted as a scene that is observed by modular systems that travel with the vane. These observers might know part of the stored history, but have no notion of the future. Depending on their capabilities, the observers reflect only a part of their history. Information that inside the vane is generated at a distance has still to travel through an information carrying field that acts as the living space for the elementary modules in order to reach the observer. The encounter will take place in the future. Information that reaches the observer arrives from the past. Those kinds of information travel via information carriers.

The vane forms a subspace of the Hilbert space and for each elementary module that subspace contains a single Hilbert vector that plays as eigenvector for the corresponding geometric location. This location is the landing point of a hop rather than the geometric center of the location swarm.

## Fields

Each infinite dimensional separable Hilbert space owns a unique companion non-separable Hilbert space that features operators, which have continuum eigenspaces. Such eigenspaces can form flat parameter spaces or dynamic fields. This can easily be comprehended when in the non-separable Hilbert space a similar procedure is used as we applied in the separable Hilbert space. In that procedure we specified reference operators and we defined normal operators by using continuous functions. This time we not only use the rational members of the number system, but we also apply the irrational members and we use the same continuous functions. The consequence is that the notion of dimension of the subspaces may lose its sense. The procedure that creates defined operators now links operator technology with function technology, differentiation technology and integration technology.

## Living space

The germ operators have no equivalent inside the non-separable Hilbert space. However, their eigenvalues may be sensed by a field that reside as eigenspace of a defined operators that resides in the non-separable Hilbert space. This can occur when the separable Hilbert space is embedded in its non-separable companion Hilbert space. We will call the mentioned field the living space of the modules and modular systems.

The hopping path that represents an elementary module, corresponds to a coherent location swarm, which is characterized by a location density distribution. Via the convolution of the Green's function of a field and this location density distribution, the swarm corresponds to a deformed part of the field that in this way the field describes all elementary modules. The convolution means that the Green's function blurs the location density distribution. This can be interpreted as if the hopping landing locations influence the field, but the alternative interpretation is that the field is a kind of blurred descriptor of the hopping landing locations. Anyway the landing locations and the discussed field are intimately coupled. The deformed field can be interpreted as the living space of the modules and modular systems.

## Green's function

At every progression instant, the hop landing locations cause the emission of a spherical shape keeping front. That front keeps its shape, but the amplitude of that shape diminishes as $1 / r$ with distance $r$ from the emission location. The shape keeping front is a solution of the homogeneous second order partial differential equation that describes the dynamic behavior of the affected field. If this effect is integrated over the regeneration cycle of the swarm, then the Green's function results. The integration turns the homogeneous second order partial differential equation into an inhomogeneous second order partial differential equation. The extra term that makes the equation inhomogeneous concerns the Green's function. The amplitude of the Green's diminishes as $1 / r$ with distance $r$ from the emission location.

## Stochastic processes

The mechanisms that generate the hopping landing location control the dynamics of the model. These mechanisms use stochastic processes. These processes appear to belong to a category which is mathematically known as inhomogeneous spatial Poisson point processes. In more detail these processes probably are modified Thomas processes.

This fact is supported by a signature that is visible in the visual trajectory of a category of living species that are called vertebrates. It appears that the visual trajectory of all vertebrates is optimized for low dose rate imaging. This visual system contains noise filters that block information for which the signal to noise ratio is too low. This signal to noise ratio is typical for information generated by Poisson processes that are attenuated by subsequent binomial processes that are implemented by spatial point spread functions. The mechanisms appear to apply inhomogeneous spatial Poisson point processes. See: "Low Dose Rate Imaging"; http://vixra.org/abs/1606.0329.

Physical theories stop at the wave function of particles. This exposure of the mechanisms dives deeper and reaches the characteristic function of the stochastic process that controls the generation of the landing locations that form the hopping path.

## Self-coherence

It is difficult to belief in a creator that installs separate mechanisms, which ensure the dynamic coherence of the generated modules. It is easier to accept that the relation between the generated location swarms and the field that describes these swarms is based on a mathematically explainable kind of self-coherence. In case of self-coherence, the interaction between the field and the swarm
restricts the possible location density distribution. This restriction may be influenced by the number of elements that are contained in the swarm. This fact may explain the existence of generations of elementary modules.

In the relation between the swarm and the field, the Green's function of the field plays an important role. It plays the role of a potential that implements an attracting force. Another factor is the kind of stochastic process that generates the individual locations. This process belongs to the category of the inhomogeneous spatial Poisson point processes. Each hop tries to displace the geometric center of the swarm. This displacement represents an acceleration of the geometric center of the swarm. Let the Green's function represent a scalar potential. When the platform on which the elementary object resides moves with respect to the background parameter space with a uniform speed, then the scalar potential will in that coordinate system turn into a vector potential. Differential calculus learns that the dynamic change of the vector field goes together with a new field that counteracts the acceleration. This effect is similar to the phenomenon that is known as inertia. It looks as if the center of geometry of the swarm is attracting the accelerating hopping elementary object. This is an effective kind of self-coherence.

In order to elucidate this obscure description, we will explain this by applying formulas.
The Green's function $G(\boldsymbol{q})$ represents a scalar potential.

$$
\begin{equation*}
G(\boldsymbol{q})=m_{1} /|\boldsymbol{q}-\boldsymbol{c}| \tag{1}
\end{equation*}
$$

If the platform travels with uniform speed $\boldsymbol{v}$, then this corresponds with a vector potential $\boldsymbol{A}(\boldsymbol{q})$ :

$$
\begin{equation*}
\boldsymbol{A}(\boldsymbol{q})=G(\boldsymbol{q}) \boldsymbol{v} \tag{2}
\end{equation*}
$$

Acceleration goes together with a new field $\boldsymbol{E}(\boldsymbol{q})$ :

$$
\begin{equation*}
\boldsymbol{E}(\boldsymbol{q}) \stackrel{\text { def }}{=} \dot{\boldsymbol{A}}(\boldsymbol{q})=G(\boldsymbol{q}) \dot{\boldsymbol{v}} \tag{3}
\end{equation*}
$$

This goes together with an attracting force $\boldsymbol{F}(\boldsymbol{q})$

$$
\begin{equation*}
\boldsymbol{F}(\boldsymbol{q})=m_{2} \boldsymbol{E}(\boldsymbol{q})=\frac{m_{1} m_{2} \dot{\boldsymbol{v}}}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{4}
\end{equation*}
$$

This attractive force acts between the landing location of the hopping object and the geometric center of the swarm. $\dot{\boldsymbol{v}}$ is the acceleration of the geometric center of the swarm that is due to the addition of the individual hop. The swarm covers a huge number of landing locations. This is represented by the factor $m_{2}$. The added hop landing concerns just a single location. This indicates that with respect to inertia the mass of the swarm is proportional to the number of contained hopping locations.

The location density distribution of the swarm is blurred by the Green's function. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green's function. The shape of this example is given by:

$$
\begin{equation*}
\chi_{n}(r)=-\frac{C_{n}}{4 \pi} \frac{E R F(r / \sigma \sqrt{2})}{r} \tag{5}
\end{equation*}
$$

In this function every trace of singularity has disappeared. It is due to the huge number of participating hop locations. This is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The extra contribution is due to the local elementary module.

## The symmetry related field

The convolution involves an integration and the local contribution to the integral involves two parameter spaces. These parameter spaces may differ in their ordering. In order to cope for this difference the platform on which the elementary object resides must be encapsulated. The boundary must only cross regions of the parameter spaces where the field and the extra potential are both continuous and the amplitude of the extra potential becomes negligible. In fact the charges characterize the parameter spaces rather than the deformed fields. For the parameter spaces the condition is automatically fulfilled and therefore the shape of the boundary does not matter. For that reason we select a boundary that has the form of a cube, whose axes are aligned along the axes of the Cartesian coordinate systems that are used to order the concerned parameter spaces. This procedure enables the correct accounting for the differences in the ordering. This accounting process reveals the charges that go together with the difference in ordering. This reveals the short list of electric charges and the color charges that appear in the Standard Model. The charges will be located on the geometric centers of the floating platforms. These symmetry related charges are the source of a new separate basic field that we will call the symmetry related field. This field differs fundamentally of the field that represents the living space of the elementary modules.

## Partial differentiation

We use the quaternionic nabla $\nabla$ in order to provide a compact description of quaternionic partial differentiation.

$$
\begin{equation*}
\nabla=\left\{\frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\}=\frac{\partial}{\partial \tau}+\boldsymbol{i} \frac{\partial}{\partial x}+\boldsymbol{j} \frac{\partial}{\partial y}+\boldsymbol{k} \frac{\partial}{\partial z}=\nabla_{0}+\nabla \tag{1}
\end{equation*}
$$

Bold type face is used for the imaginary parts. The real parts get a suffix ${ }_{0}$.

$$
\begin{equation*}
\nabla f=\sum_{\mu=0}^{3} \frac{\partial f}{\partial q_{\mu}} e_{\mu} \tag{2}
\end{equation*}
$$

This form of the partial differential equation highlights the fact that in first order and second order partial differential equations the nabla operator can be applied as a multiplier. This means that we can apply the quaternionic multiplication rule.

$$
\begin{align*}
& \Phi_{0}=\nabla_{0} \psi_{0}-\langle\boldsymbol{\nabla}, \boldsymbol{\psi}\rangle  \tag{3}\\
& \boldsymbol{\Phi}=\nabla_{0} \boldsymbol{\psi}+\nabla \psi_{0} \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{4}
\end{align*}
$$

The $\pm$ sign indicates that the nabla operator is also afflicted by symmetry properties of the applied quaternionic number system. The above equations represent only low order partial differential equations. In this form the equations can still describe point-like disruptions of the continuity of the field. We can take the conjugate:

$$
\begin{align*}
& \Phi^{*}=(\nabla \psi)^{*}=\nabla^{*} \psi^{*} \mp 2 \boldsymbol{\nabla} \times \boldsymbol{\psi}  \tag{5}\\
& \nabla^{*}\left(\nabla^{*} \psi^{*}\right)^{*}=\nabla^{*} \Phi=\nabla^{*} \nabla \psi \tag{6}
\end{align*}
$$

Two different non-homogeneous second order partial differential equations exist that offer different views on the embedding process. The equation that is based upon the double quaternionic nabla $\nabla \nabla^{*}$ cannot show wave behavior. However, the equation that is based on d'Alembert's operator $\mathfrak{D}$ acts as a wave equation, which offers waves as part of its set of solutions.

$$
\begin{align*}
& \nabla \nabla^{*}=\nabla_{0} \nabla_{0}+\langle\nabla, \nabla\rangle  \tag{7}\\
& \mathfrak{D} \stackrel{\text { def }}{=}-\nabla_{0} \nabla_{0}+\langle\nabla, \nabla\rangle \tag{8}
\end{align*}
$$

In isotropic conditions the homogeneous equations look like:

$$
\begin{equation*}
\frac{\partial^{2} \psi}{\partial x^{2}}+\frac{\partial^{2} \psi}{\partial y^{2}}+\frac{\partial^{2} \psi}{\partial z^{2}} \pm \frac{\partial^{2} \psi}{\partial \tau^{2}}=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial \psi}{\partial r}\right) \pm \frac{\partial^{2} \psi}{\partial \tau^{2}}=0 \tag{9}
\end{equation*}
$$

These equations have special solutions in odd numbers of participating dimensions in the form of shape keeping fronts.

The d'Alembert's equation offers solutions in the form of waves. That does not hold for the operator that is defined in (7).

## Elementary behavior

Fields act on point-like artifacts in an elementary way. The d'Alembert's operator offers spherical waves and both second order equations offer shape keeping fronts as elementary solutions.

## One dimensional fronts

First we focus on the solutions that vary in one dimension. Thus :

$$
\begin{equation*}
\frac{\partial^{2} \psi}{\partial z^{2}} \pm \frac{\partial^{2} \psi}{\partial \tau^{2}}=0 \tag{1}
\end{equation*}
$$

We try a solution in the form $\varphi=f(\alpha z+\beta \tau)$ :

$$
\begin{align*}
& \frac{\partial f}{\partial z}=\alpha f^{\prime} ; \frac{\partial^{2} f}{\partial z}=\alpha \frac{\partial f^{\prime}}{\partial z}=\alpha^{2} f^{\prime \prime}  \tag{2}\\
& \frac{\partial f}{\partial \tau}=\beta f^{\prime} ; \frac{\partial^{2} f}{\partial \tau^{2}}=\beta \frac{\partial f^{\prime}}{\partial \tau}=\beta^{2} f^{\prime \prime}  \tag{3}\\
& \alpha^{2} f^{\prime \prime} \pm \beta^{2} f^{\prime \prime}=0 \tag{4}
\end{align*}
$$

This is solved when $\alpha^{2}=\mp \beta^{2}$.
For the first kind of the second order partial differential equation this means: $\beta= \pm \alpha \boldsymbol{i}$, where $\boldsymbol{i}$ is a normalized imaginary quaternion. With $g(z)=f(\beta z)$ follows:

$$
\begin{equation*}
\varphi=g(z i \pm \tau) \tag{5}
\end{equation*}
$$

The function $g$ represents a shape keeping front. It is not a wave.
The imaginary $\boldsymbol{i}$ represents the base vector in the $x, y$ plane. Its orientation $\theta$ may be a function of $z$.

For the second kind of the second order partial differential equation this means: $\beta= \pm \alpha$. With $g(z)=f(\beta z)$ follows:

$$
\begin{equation*}
\varphi=g(z \pm \tau) \tag{6}
\end{equation*}
$$

The corresponding hop landing that represents the continuity disturbing artifact will be called warp. A warp corresponds to the hop of the geometric center of the platform on which the corresponding elementary particle resides. Thus the location of the hop is defined relative to a geometric location on the carrier field that is defined with respect to the background parameter space. The platform carries its own private parameter space. Subsequent warps occur in "linear" strings that follow the deformation of the carrier field. Such strings will be called messengers. Warps can also occur as single hops.

## Spherical fronts

Next we focus on the three dimensional spherical symmetric condition. In that case the equations can be separated by writing $\psi=r \varphi(r, \tau)$

$$
\frac{\partial^{2} \varphi}{\partial r^{2}}+\frac{2}{r} \frac{\partial \varphi}{\partial r} \pm \frac{\partial^{2} \varphi}{\partial \tau^{2}}=0 \Longrightarrow \frac{\partial^{2} \psi}{\partial r^{2}} \pm \frac{\partial^{2} \psi}{\partial \tau^{2}}=0
$$

With other words $\psi$ fulfills the conditions of the one-dimensional case. Thus solutions in the form $\varphi=f(\alpha r+\beta \tau) / r$ will fit.

For the first kind of the second order partial differential equation this means: $\beta= \pm \alpha \boldsymbol{i}$, where $\boldsymbol{i}$ is a normalized imaginary quaternion. With $g(x)=f(\beta x)$ follows:

$$
\begin{equation*}
\varphi=g(r \boldsymbol{i} \pm \tau) / r \tag{2}
\end{equation*}
$$

$\boldsymbol{i}$ represents a base vector in radial direction.
The corresponding hop landing that represents the continuity disturbing artifact will be called a clamp. The clamp corresponds to a hop relative to the geometric center of the platform on which the elementary particle resides. Thus the location of the hop is defined relative to this geometric location. The description uses the parameter space that is private to the platform and the elementary particle. Clamps occur in coherent swarms. All swarm elements share the same platform. Thus the swarm moves as a single unit.

For the second kind of the second order partial differential equation this means: $\beta= \pm \alpha$. With $g(x)=f(\beta x)$ follows:

$$
\begin{equation*}
\varphi=g(x \pm \tau) / r \tag{3}
\end{equation*}
$$

These solutions feature a fixed speed and a fixed shape. However, their amplitude diminishes as $1 / r$ with distance $r$ from the sources. When integrated over a long enough period of progression the result takes the form of the fields Green's function.

## Clamp hops and warp hops

The homogeneous second order partial differential equation offers two kinds of solutions that represent shape keeping fronts. One kind concerns the spherical shape keeping fronts. The second kind acts in one-dimension and not only keeps its shape. It also keeps its amplitude.

Hops that correspond to solutions, which represent spherical shape keeping fronts will be called clamp hops or clamps. Hops that correspond to solutions, which represent one-dimensional shape keeping fronts will be called warp hops or warps. Clamps occur in coherent swarms and warps occur equidistant in strings.

## Information messengers

Both types of shape keeping solutions act as information messengers that inform about the corresponding hop landing. The second kind appears to operate in linear strings of which the elements represent equidistant hop landings. Since the solutions move with a fixed speed, the one dimensional messengers feature a spatial and a temporal frequency. This frequency share these shape keeping fronts with waves, but these fronts are not waves or parts of waves. The linear messengers also feature a polarization.

## Moving elementary modules

Clamp swarms will not move with respect to the geometric center of its platform. The mechanism that ensures coherence of the swarm will ensure that the geometric center of the swarm stays on the geometric center of the platform. Warps may cause the hopping of the platform with respect to the background parameter space. Thus a mixture of clamps and warps may cause the movement of the swarm relative to the geometry of the carrying field.

## The symmetric pair production and annihilation process

At symmetric pair annihilation a pair of elementary modules that are each other's antiparticle convert into a pair of linear messengers that leave in opposite directions. It is the simplest pair annihilation process. In this process each clamp element of the hopping path of the elementary (anti)particle converts in a warp element of a linear hop string of a leaving messenger. The messengers leave in a direction that is perpendicular to the direction into which the elementary modules were approaching each other. The chance that the geometric centers of the elementary modules will meet head-on is very low. The direction of the difference will further determine the direction in which the messengers leave. The process unravels the two clamp swarms into two corresponding linear warp strings. The size of the displacement of the geometric centers is supposed to determine the polarization of the messenger string.

The leaving messengers are strings of warps that transport empty and thus massless platforms. The lengths of the leaving strings reflect the number of clamps in the annihilated elementary modules.

If each warp in the string carries a fixed bit of energy, then this process explains the mass-energy equivalence.

## Pair creation

At pair creation the reverse process takes place. Two strings of warps that have sufficient length that enter from opposite direction combine in order to generate two swarms of clamps that constitute a particle-antiparticle pair.

## The particle zoo

This paper does not touch all members of the elementary particle zoo that is described by the standard model. It does not treat spin and as a consequence the paper does not differentiate between fermions and bosons. The symmetry flavors distinguish between electrons, quarks and neutrinos. The paper does not treat how electric charges and color charges can be explained and it does not treat the difference between particles and anti-particles and we did not really discuss generations. We did not even touch the very important Pauli principle and we did not touch gluons and color confinement. The targeted Hilbert Book Model does not need the existence of a separate Higgs field.

## More detail

Those that possess sufficient knowledge of mathematics might be interested in the paper "The Hilbert Book Test Model". This pure mathematical model exploits the above view and extends it a bit. It explains how electric charges and color charges can be derived from differences in the ordering of parameter spaces. Further, it introduces new mathematical methods. See:
http://vixra.org/abs/1603.0021

