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Geometric Conjecture

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GEOMETRIC CONJECTURE

a) Let M be an interior point in an $A_1A_2\dots A_n$ convex polygon and P_i the projection of M on

A_iA_{i+1} i=1, 2, 3, ..., n.

Then,

$$\sum_{i=1}^{n}\overline{MA_i} \geq c\sum_{i=1}^{n}\overline{MP_i}$$

where c is a constant to be found.

For $n = 3$, it was conjectured by Erdõs in 1935 and solved by Mordell in 1937 and Kazarinoff in 1945. In this case $c = 2$ and the result is called the Erdõs-Mordell Theorem.

Question: What happens in 3-space when the polygon is replaced by a polyhedron ?

b) More generally: If the projections P_i are considered under a given oriented angle $\alpha \neq 90$ degrees, what happens with the Erdõs-Mordell Theorem and the various generalizations ?

c) In 3-space, we make the same generalization for a convex polyhedron

$$\sum_{i=1}^{n}\overline{MA_i} \geq c_1\sum_{j=1}^{m}\overline{MP_j}$$

where $P_j, 1 \leq j \leq m$, are projections of M on all the faces of the polyhedron.

Futhermore,

$$\sum_{i=1}^{n}\overline{MA_i} \geq c_2\sum_{k=1}^{r}\overline{MT_k}$$

where $T_k, 1 \leq k \leq r$, are projections of M on all sides of the polyhedron and c_1 and c_2 are constants to be determined.

[Kazarinoff conjectured that for the tetrahedron

$$\sum_{i=1}^{4}\overline{MA_i} \geq 2\sqrt{2}\sum_{i=1}^{4}\overline{MP_i}$$

and this is the best possible.

References

[1] P.Erdõs, Letter to T.Yau, August, 1995.

[2] Alain Bouvier et Michel George, <Dictionnaire des Mathématiques>, Press Universitaires de France, Paris, p. 484.