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Generalizations of Desargues Theorem

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GENERALIZATIONS OF DESARGUES THEOREM*

Let's consider the points $A_1,...,A_n$ situated on the same plane, and $B_1,...,B_n$ situated on another plane, such that the lines A_iB_i are concurrent. Let's prove that if the lines A_iA_j and B_iB_j are concurrent, then their intersecting points are collinear.

Solution. Let α be the plane that contains the points $A_1,...,A_n$ (in the case in which the points are non-collinear α is unique), and analogously, let $\beta = P(B_1,...,B_n)$, and consider $\alpha \cap \beta = d$.

Because the lines A_iA_j and B_iB_j are concurrent, $A_iA_j \subset \alpha$, and $B_iB_j \subset \beta$, therefore their intersection belongs to line d.

Remark 1.

For $n = 3$ and A_1, A_2, A_3 non-collinear, B_1, B_2, B_3 non-collinear, and $A_i \neq B_j$ we obtain Desargues theorem.

Remark 2.

An extension of this generalization is: If we consider $A_1,...,A_n$ situated in a plane, and $B_1,...,B_m$ situated on another plane, prove that if A_iA_j and B_kB_r are concurrent, then their intersection points are concurrent.

Remark 3.

For $n = m$, and A_iB_i concurrent lines, we obtain the first generalization.

Remark 4.

If in addition we also have $n = m = 3$ along with the previous conditions, we obtain the Desargues theorem.

* Gamma, Anul X, nr. 1-2, Oct. 1987.