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Counter-Projective Geometry

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COUNTER-PROJECTIVE GEOMETRY

Let P, L be two sets, and r a relation included in $P \times L$. The elements of P are called points, and those of L lines. When (p, l) belongs to r, we say that the line l contains the point p. For these, one imposes the following COUNTER-AXIOMS:

(I) There exist: either at least two lines, or no line, that contains two given distinct points.

(II) Let p_1, p_2, p_3 be three non-collinear points, and q_1, q_2 two distinct points. Supoose that $\{p_1, q_1, p_3\}$ and $\{p_2, q_2, p_3\}$ are collinear triples. Then the line containing p_1, p_2, and the line containing q_1, q_2 do not intersect.

(III) Every line contains at most two distinct points.

Questions 30-31:

Find a model for the Counter-(General Projective) Geometry (the previous I and II counter-axioms hold), and a model for the Counter-Projective Geometry (the previous I, II, and III counter-axioms hold). [They are called COUNTER-MODELS for the general projective, and projective geometry, respectively.]

Questions 32-33:

Find geometric modls for each of the following two cases:

- There are points/lines that verify all the previous counter-axioms, and other points/lines in the same COUNTER-PROJECTIVE SPACE that do not verify any of them;

- Some of the counter-axioms I, II, III are verified, while the others are not (there are particular cases already known).

Question 34:

The study of these counter-models may be extended to Infinite-Dimensional Real (or Complex) Projective Spaces, denying the IV-th axioms, i.e.:

(IV) There exists no set of finite number of points for which any subspace that contains all of them contains P.

Question 35:

Does the Duality Principle hold in a counter-projective space?

What about Desargues's Theorem, Fundamental Theorem of Projective Geometry/Theorem of Pappus, and Staudt Algebra?

Or Pascal's Theorem, Brianchon's Theorem? (I think none of them will hold!)

Question 36:

The theory of Buildings of Tits, which contains the Projective Geometry as a particular case, can be 'distorted' in the same <paradoxist> way by deforming its axiom of a BN-pair (or Tits system) for the triple (G, B, N), where G is a group, and B, N its subgroups; [see J.Tits, "Buildings of spherical type and finite BN-pairs", Lecture notes in math. 386, Springer, 1974].

Notions as: simplex, complex, chamber, codimension, apartment, building will get contorted either...

Develop a Theory of Distorted Buldings of Tits!