FLORENTIN SMARANDACHE

P-Q Relationships and Sequences

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P-Q Relationships and Sequences

Let $A = \{a_n\}, n \geq 1$ be a sequence of numbers and q, p integers ≥ 1.

We say that the terms $a_{k+1}, a_{k+2}, \ldots, a_{k+p}, a_{k+p+1}, a_{k+p+2}, \ldots, a_{k+p+q}$ satisfy a $p-q$ relationship if

$$a_{k+1} \diamond a_{k+2} \diamond \ldots \diamond a_{k+p} = a_{k+p+1} \diamond a_{k+p+2} \diamond \ldots \diamond a_{k+p+q}$$

where $\diamond$ may be any arithmetic operation, although it is generally a binary relation on A. If this relationship is satisfied for any $k \geq 1$, then $\{a_n\}, n \geq 1$ is said to be a $p - q - \diamond$ sequence. For operations such as addition, where $\diamond = +$, the sequence is called a $p-q$-additive sequence.

As a specific case, we can easily see that the Fibonacci/Lucas sequence ($a_n + a_{n+1} = a_{n+2}$, for $n \geq 1$), is a $3-1$-additive sequence.

Definition. *Given any integer $n \geq 1$, the value of the Smarandache function $S(n)$ is the smallest integer m such that n divides $m!$.*

If we consider the sequence of numbers that are the values of the Smarandache function for the integers $n \geq 1$,

$1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, \ldots$

they can be incorporated into questions involving the $p - q - \diamond$ relationships.

a) How many ordered quadruples are there of the form $(S(n), S(n+1), S(n+2), S(n+3))$ such that $S(n+1)+S(n+2)=S(n+3)+S(n+4)$ which is a $2-2$-additive relationship?

The three quadruples

$S(6)+S(7)=S(8)+S(9),\ \ 3+7=4+6;$

$S(7)+S(8)=S(9)+S(10),\ \ 7+4=6+5;$

$S(28)+S(29)=S(30)+S(31),\ \ 7+29=5+31.$

are known. Are there any others? At this time, these are the only known solutions.

b) How many quadruples satisfy the $2-2$-subtrac relationship $S(n+1)-S(n+2)=S(n+3)-S(n+4)$?

The three quadruples

$S(1)-S(2)=S(3)-S(4),\ \ 1-2=3-4;$

$S(2)-S(3)=S(4)-S(5),\ \ 2-3=4-5;$

$S(49)-S(50)=S(51)-S(52),\ \ 14-10=17-13$

are known. Are there any others?

c) How many 6-tuples satisfy the $2-3$-additive relationship $S(n+1)+S(n+2)+S(n+3)=S(n+4)+S(n+5)+S(n+6)$?

The only known solution is

$$S(5)+S(6)+S(7)=S(8)+S(9)+S(10),\ 5+3+7=4+6+5.$$

Charles Ashbacher has a computer program that calculates the values of the Smarandache function. Therefore, he may be able to find additional solutions to theese problems.

More general, if f_p is a p-ary raltion and g_q a q-ary relation, both defined on the set $\{a_1, a_2, a_3, \ldots\}$, then $a_{i_1}, a_{i_2}, \ldots, a_{i_p}, a_{j_1}, a_{j_2}, \ldots, a_{j_q}$ satisfies a $f_p - g_q$ relationship if

$$f(a_{i_1}, a_{i_2}, \ldots, a_{i_p}) = g(a_{j_1}, a_{j_2}, \ldots, a_{j_q}).$$

If this relationship holds for all terms of the sequence, then $\{a_n\}, n \geq 1$ is called a $f_p - g_q$ sequence.

Study some $f_p - g_p$ relationship for welll-known sequences, such as the perfect numbers, Ulam numbers, abundant numbers, Catalan numbers and Cullen numbers. For example, a $2-2$-additive, subtractive or multiplicative relationship.