The Theory Of Ultralogics,
the Modified Robinson Approach, and GID

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Abstract: The basic mathematical aspects of the GGU and GID models are discussed. As an illustration, the modified Robinson approach is used to give a more direct prediction as to the composition of the ultra-propertons. The refined developmental paradigm is applied to the General Intelligent Design (GID) model and the basic GID statements are given.

1. The Theory of Ultralogics and Axiomatics.

In the original Theory of Ultralogics (Herrmann, (1978-93, 99)), a formal axiom system **ZFH** was included in the discussion relative to consistency. However, this seems to be the only place within the Nonstandard Analysis literature that such a discussion is included. This system is the usual formal $\mathbf{ZF} + \mathbf{AC} + A = \mathbf{ZFA}$ the Zermelo-Fraenkel system with the Axiom of Choice and with "atoms" and the requirement that, at least, the A be "countably infinite." A model was introduced that made this system consistent relative to **ZFC** and that yields a set **A** with the appropriate properties. This system with the "consistency" so modeled was denoted by **ZFH**. For this system, as pointed out in [6], atoms are not considered as sets. They are also termed as "individuals."

In many texts on Nonstandard Analysis (Hurd and Loeb, (1985)) that use the "superstructure" approach, an actual set-theoretic axiom system is not mentioned. It is after the set-theoretic construction that a standard set-theoretic language is defined using the symbols $\in$ and $=$ as well as expressions for n-ary relations. The language is a standard first-order langrage restricted to these superstructure entities as constructed below. Algebra, in its general form, is a study of symbols and rules for symbol manipulation that require "human" physical notions to apply. One must know what it means to express a string of symbols from left-to-right or right-to-left. The expression, in abbreviated form, $\forall x((x \in A) \to \exists y((y \in B) \wedge (y \in x)))$ needs to be recognized by a logician as "the correct form," while $\forall x((x \in A) \to \exists y((y \in B) \wedge (y \in x))$ is not correct. But why is it not correct? It seems that the second expression has a parenthesis missing on the far "right." For the student, the relation of mathematics to such human physical notions as "left-to-right," "right-to-left" "top-down," and the like for

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symbol formation is mostly not mentioned. One finds in a standard algebra book the "commutative law" relative to an operator $\cdot$. One sees this law stated as $a \cdot b = b \cdot a$ (or simply $ab = ba$) without mentioning that this is symbol manipulation relative to expressing symbolic forms from right-to-left or left-to-right. One might also state this in "operator" language. Is it ever stated as

$$\begin{array}{c} a \\ \cdot \\ b \\ =? \\ b \\ \cdot \\ a \end{array}$$

Throughout Mathematical Logic human abilities are strongly applied. As previously illustrated, one needs to follow an explicit set of rules as to the formation of a finite symbolic expression as written from left-to-right and recognize when such an expression is identical to another such expression in order to obtain the objects of study (Herrmann, (2006a); Mendelson, (1987)). Then a set-theoretic language is applied to "sets" of such expressions. The expressions are considered as "objects," and, usually, not defined as sets themselves. From this viewpoint, I consider the most appropriate set-theory to apply is, at least, **ZFA**.

For The Theory Of Ultralogics, such set-theoretic notions as the power-set and finite power-set operators, restricted to the superstructure, are also used. Via the work of Shoenfield and others, with one exception, the consistency of the **ZF** seems beyond doubt. As usual, the one exception is the Axiom of Infinity. Considering the presently widely accepted finite nature of our universe, this axiom is not open to a concrete matter model. However, there is now available a well-described mental imaging approach to modeling the denumerably infinite, at the least, (Herrmann,(2013)). Further, for the Extended Grundlegend Structure, EGS, and the axioms system **ZFR** with the cardinality of the atoms is that of the real numbers, there is a model for this system using **ZF**. (Of course, trivially, the consistency of the set-theory employed then depends upon the additional collection of defined sets used for any application.)

Throughout Nonstandard Analysis the superstructure approach is widely employed. Let X be the set of atoms. Using the informal (metamathematical) natural numbers, denote $X_0 = X$. Define $X_1 = X_0 \cup \mathcal{P}(X_0)$, where $\mathcal{P}()$ is the power-set operator. Then by induction let $X_{n+1} = X_n \cup \mathcal{P}(X_n)$. Then the superstructure is $S = \bigcup\{X_n \mid n \in \mathbb{N}$. For this article, the $\mathbb{N}$ is not the following N, the "natural number symbols," in what follows. In the Theory of Ultralogics, the Robinson approach uses the symbol N for the natural number symbols n$'$ as a subset of the set of words $\mathcal{W}$ or, for the EGS, $\mathcal{W}'$ where the coding i is not employed (Herrmann, 1978-93, Section 9.3). Further, in The Theory of Ultralogics, the symbol $\mathbb{N}$ may be used in two context, as the informal natural numbers and as the natural numbers as represented by a set of atoms in the superstructure. Further, the **ZFA** constructed natural numbers ω are used in Herrmann (1978-93, 99) to construct the superstructure that is considered a **ZFA** produced entity. The informal natural numbers are usually used for this purpose.

For the superstructure, the given set of atoms X is assumed, at least, to contain a denumerable $\mathbb{N}$. The general claim is that the $\mathbb{N}$ is the set of natural numbers. But, what are the "natural numbers"? There are, of course as indicated above, different entities that carry this name. The informal set used within Mathematical Logic to define, via general induction, many entities cannot be this set since we must use a first-order type model. They are not the "natural numbers," ω, constructed via the **ZFA** axioms as listed by Jech (1971, p. 122). *For superstructures, the set $\mathbb{N}$ is actually but a collection of atoms that is endowed with various "natural number" styled relations that exist within the superstructure constructed from X.* Without a set-theoretic axiom system mentioned, one assumes that the notions of general set theory are employed as they are within the metamathematics used throughout Mathematical Logic (Mendelson, E., 1987, p. 4-9) and a dual langrage approach is applied. *This customary approach, with the atoms concept adjoined, suffices for all of the results in The Theory of Ultralogics.*

One should consider the set $\mathbb{N}$ as an abstract collection of elements and use the customary symbolic "names" for the elements. The superstructure S does have enough structure to accommodate the algebraic natural number relations and, such symbol strings as $0 < 1 < 2 < 3 < 4 < \cdots$ have the customary meaning. (Note: For The Theory of Ultralogics equivalence class construction the form $\cdots > 4 > 3 > 2 > 1 > 0$ is more appropriate.) The same idea holds where the set $\mathbb{N}$ is replaced with "rational" or "real numbers" (under extended cardinality). Mathematical Logic investigates concrete collections of symbol-strings. In this sense, it is applied mathematics. The modified Robinson approach is more relative to this type of investigation than the original Theory of Ultralogics, where the coding leads to a pure number theory approach.

For significant reasons, the set-theory employed needs to have individuals (atoms, urelements) as the foundation for superstructure construction. However, the actual requirement for such a basic set X is that it act like a set of atoms with respect to superstructure elements. In 2000, it was shown that the atomic requirement can be replaced with a general or **ZF** entity, say Y, that can be allowed to act like (is isomorphic to) the superstructure endowed natural numbers $\mathbb{N}$ and satisfies the actually requirement that for any $y \in Y$, there is no superstructure entity x such that $x \in y$. Using this approach a polysaturated model is constructed. However, this approach is not needed to obtain such a model. For The Theory of Ultralogics, the ultralimit construction is sufficient (Stroyan and Luxemburg, 1976, p. 183)).

In The Theory of Ultralogics (Herrmann, 1978-93, 99, 2016), the "sigma" notation used is not the same as it appears elsewhere within Nonstandard Analysis (NSA). It takes a nonempty standard set A and ${}^{\sigma}A = \{{}^*a \in A\}$. Then, in general, when one has an infinite standard B, one can readily consider ${}^*B - {}^{\sigma}B$, which is the set of all internal entities that are not extended standard entities. Today, in the usual rendering of NSA, the ultralimit is used with an appropriate indexing set I. Then. for the usual case where a is only considered as an element of A, the notation ${}^*a$ can be used as an abbreviation for the equivalence class containing the constant a valued I sequence, which is identified as an a in A if A is a set of atoms.

The superstructure approach employs the monomorphism, a function * from the standard superstructure into a second superstructure based upon ${}^*X$. Given an index set I and the set of all maps $a\colon I \to X$. then for an appropriate ultrafilter $\mathcal{U}$, the ${}^*X$ is the set of all ultrapower formed equivalence classes. Among these equivalence classes are the equivalence classes $[\overline{a}]$ that contain the constant X valued I sequence. As mentioned, some authors (Stroyan and Luxemburg, 1976) generally define a map ${}^{\sigma}A$ as the collection of all the $[\overline{a}]$. Then ${}^{\sigma}X \subset {}^*X$. For the set of atoms X, this all comes from the consideration of an isomorphic copy of the standard superstructure within a superstructure entity based upon ${}^*X$. But, due to the atomic properties of X, it is more usual to find the customary approach, where one assumes that $X \subset {}^*X$ and, indeed, all finite subsets of X within any further superstructure based upon ${}^*X$ are also considered as composed of members X. But, when the ultrapower approach is used to model the needed properties of the monomorphism, obviously, this identification is not so modeled.

In later articles, I usually avoid the use of the $^{\sigma}$ notation, but adhere to the identification process for finite sets of atoms. This identification is a type of interpretation, an imposed axiom, and greatly simplifies the modified Robinson approach. In general, it is the monomorphism that relates the standard superstructure to the one based upon a ${}^*X$.

2. The Modified Robinson Approach.

The modified Robinson method is the word forming equivalence classes applied to the following idea, where superstructures had not as yet become an approach to Nonstandard Analysis. (I was not aware of this paper until after 1993).

> We now suppose that certain subsets of U are regarded as the constituents of a language L of the first order predicate calculus. That is to say, there are certain disjoint sets of individuals of U, of adequate cardinal numbers, which serve as brackets, commas, connectives ($\sim$, $\wedge$, and $\vee$), quantifiers ($\forall$ and $\exists$), variables, individuals constants, relations, and functions of L. . . . Going further, we suppose that the terms and well-ordered formulae (wff) of L also constitute subsets of U ("L-terms, L-wff"). . . . Passing to U' (his nonstandard model), we see that the relations which in U define various sets of L-symbols and L-formulae, define corresponding sets in U'. . . . The extended language will be denoted by L'. . . . The set of L'-wff and more particularly, of L'-sentences, is quite varied. Thus, for every non-standard natural number l in U' there exists an L'-sentence whose length exceeds l (Robinson, 1963, p. 90-91).

Hence, the modified Robinson approach is to let $\mathcal{W}$ be a general language as defined in Herrmann (1997) and it includes symbols N for the natural numbers, $\mathbb{N}$, and when necessary rational or real numbers symbols and the like. The set $\mathcal{W}$ is employed in two context. First, it is employed informally. Then with the added set-theoretic notion of each member being an atom, an individual. When an informal word is constructed it can be very meaningful when symbols for numerical quantities are included since they are usually associated with measurements. To avoid confusion, if, for example, "natural numbers" form a part of the word-form, then a different symbol is often used for the numbers as represented by members of $\mathcal{W}$ than the ones used in $\mathbb{N}$. This is done so that $\mathcal{W} \cap \mathbb{N} = \emptyset$ and the analysis of members of $\mathcal{W}$ is not confused with the language being analyzed. Although the formations of the words comes from their informal meanings, when analyzed they carry no such meanings. But, after analysis they can then be "interpreted." Within the superstructure there is the obvious bijection that relates these two symbolic forms. The word building equivalences classes are defined informally as in Herrmann ,(2016, pp. 5-6) without the coding i and trivially embedded into the superstructure based upon $\mathcal{W} \cup \mathbb{N}$. This set of equivalence classes is denoted by $\underline{\mathcal{W}}$ or $\underline{\mathcal{W}}'$ if $\mathbb{N}$ is changed.

Although the GGU-model is a cosmogony, the actual constituents of an instruction-entity can be specified for properton formation. Instruction-entities are characterized via the same primitive sequence and its refinement as are members of a developmental paradigm (Herrmann,(2006, 2013a)). We predict the behavior of entities within our universe via a collection of numerical parameters. If the value were to change for one or more parameters, we would have a much different physical environment, if there was one at all. As Feynman might have said,"Nature has selected a set of parameters from an infinite collection, so She could produce a universe in which humans would evolve and now continue to survive."

Assuming a universe is formed by gatherings of intermediate and ultra-propertons (Herrmann, (2013a), then the rationality of their existence and applications is essential. The modified Robinson approach more directly yields this rationality. It gives a somewhat more directly obtained characterization for the "ultra-properton" notion. These entities can only be known by us in their n-tuple form and how they form physical-systems (i.e. the material physical events so identified) can only be described via a modified linear algebra. The ultra-properton is "predicted" by the non-standard model. Consider the informal collection of symbols, where $\mathrm{m}' \notin \{0', 1'\} \subset \mathrm{N} \subset \mathcal{W}',\ \mathrm{m}' \in \mathrm{N}$. $\mathrm{Each|||of|||the|||m' - corrdinates|||for|||a|||properton|||is|||denoted|||by|||+1'/n'|||or|||-1'/n'.}$, where $\mathrm{n}' \neq 0'$. This is the extended structure, where in place of $\mathbb{N}$ either the rational or real numbers are used as a disjoint set of atoms. This statement besides being just a set of symbols also carries a meaningful interpretation.

Now consider the

$$\{\mathrm{Each|||of|||the|||m' - corrdinates|||for|||a|||properton|||is|||} \\ \mathrm{denoted|||by|||+1'/n'|||or|||-1'/n'.} \mid 0 \neq \mathrm{n}' \in \mathrm{N}\}$$

For the Robinson approach, this set is a subset of $\mathcal{W}'$. Further, one considers the informal word forming equivalence class $[f]$ for this combination of symbols, but without the coding i (Herrmann 1978-93, 99). The spacial member $f \in [f]$ identifies each "alphabet" element of the word. As done in the proof of Theorem 9.3.1 (Herrmann, 1978-93, 99), this yields an internal $[g]$, where g yields the exact same form as f except for the symbol n', which is replaced by an abstract member of ${}^*\mathcal{W}$. A new symbol $\wr$, not a member of the standard alphabet, is assigned to this abstract object. Then the mathematical entities symbolized by $+1'/\wr$ and $-1'/\wr$ are employed. The symbol $\wr$ corresponds to a meta-mathematically denoted infinite Robinson number 10^ω and the symbols $+1'/\wr$ and $-1'/\wr$ correspond to the two infinitesimals $+1/10^\omega$ and $-1/10^\omega$, respectively. Thus, the GGU-model methods predict the mathematical composition of the "ultra-propertons."

3. The Refined GID Developmental Paradigm.

The method described in Herrmann (2013a) to refine the GGU-model processes to include the generation of the physical-systems that comprise a universe-wide frozenframe (UWFF), as mentioned, should be considered as applied to GID and each UWFF. The hyper-algorithm produces the intertwining of the collection of all such physical-systems and the primitive sequence of UWFFs. However, thus far, a GID intelligent actions have not been mentioned as producing each of the physical-systems.

Consider a specific finite collection of material entities that we combine to build a physical-system. It does require a specific aspect of human intelligence to follow directions and perform a specific ordered counting process. This can be modeled by the binary logic-system $\{1', (1', 2'), (2', 3'), (3', 4'), \ldots, (\mathrm{n}' - 1', \mathrm{n}')\}$ This is exactly what needs to be done when the members of an instruction-entity are applied to form members of an info-field except the process is hyperfinite. That is, there is a Robinson infinite number, here denoted by λ, and the *logic-system is of the form $\{1', (1', 2'), (2', 3'), (3', 4'), \ldots, (\lambda - 1', \lambda)\}$. Such specific actions define an hyper-intelligent agent, "Any entity that takes an active role and/or produces specific results." In this case, the actions are specifically stated and when the ultra-properton combinations are realized a physical-system is the result. Thus, the combined GID and GGU-models additionally enhanced the (GID) intelligently designed concept relative the formation of a universe.

Hence, it has been rationally predicted from observable behavior that (1) *intelligent actions yield each described physical-system. (2) An *intelligent action yields the intertwining of the collection all descriptions for the physical-systems that comprise any moment during the development of a universe. (3) An *intelligent action yields the moment-to-moment descriptions for the development of a universes. The *intelligent action of statement (2) and (3) is also modeled via a *logic-system of the same form as that employed for statement (1).

Under the basic scientific approach that descriptions for physical entities are in one-to-one correspondence with the actual material entities they describe, then the three statements can be rephrased relative to indirect evidence. Evidence indirectly verifies that (1) *intelligent actions yield each material physical-event (i.e. physical-system). (2) An *intelligent action yield the intertwining of the collection all material physical-events that comprise any moment during the development of a universe. (3) An *intelligent action yields the moment-to-moment development of a universe. Obviously, these three predicted statements also have various theological interpretations.

A basic interpretation for the *intelligent action notion is rather trivial. One simply assumes that certain ultranatural processes present substratum behavior that is "intelligent-like." That is, the processes simply satisfy such patterns and no further implications need to be made. On the other hand, as pointed out in Herrmann (2013a), the GID logic-system approach can be considered as a more detailed model for the ultraword approach and modus ponens deduction. This holds for both the developmental and instruction paradigms. This is especially so for the theological notion of "changing thoughts into various realities," where such "thoughts" are better described via the ultraword concept.

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